954/2 **STPM**

MATHEMATICS T

PAPER 2

Three Hours (Tiga

JABATAN PELAJARAN NEGERI SABAH

EXCEL II TINGKATAN 6 ATAS
AUGUST 2010

SIJIL TINGGI PERSEKOLAHAN MALAYSIA

This question paper consists of 6 printed pages.
(Kertas soalan ini terdiri daripada 6 halaman bercetak.)

Instructions to candidates:

Answer **all** *questions. Answers may be written in either English or Malay.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae and graph paper are provided.

Arahan kepada calon:

Jawab **semua** soalan. Jawapan boleh ditulis dalam bahasa Inggeris atau bahasa Melayu.

Semua kerja yang perlu hendaklah ditunjukkan dengan jelas.

Jawapan berangka tak tepat boleh diberikan betul hingga tiga angka bererti, atau satu tempat perpuluhan dalam kes sudut dalam darjah, kecuali aras kejituan yang lain ditentukan dalam soalan.

Sifir matematik, senarai rumus matematik, dan kertas graf dibekalkan.

1 A climber of Mount Kinabalu is on top of the mountain at a certain height , H , sees two small town, Kundasang town and Nabalu town, directly to the left of the mountain. The angles of depression to the Kundasang town and Nabalu town , are α° and β° respectively where α° < β° . If the two towns are *d* meter apart, show

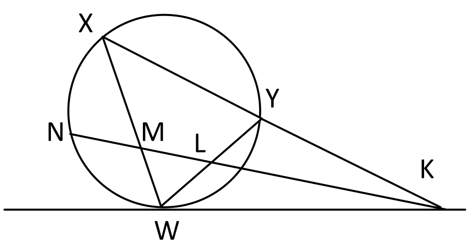
$H = d \sin\alpha \sin\beta \csc(\beta - \alpha)$ [5]

2 Show that the substitution $u = x^2 + y$ transforms the differential equation

$$(1 - x)\frac{dy}{dx} + 2y + 2x = 0$$

into the differential equation $(1 - x)\frac{du}{dx} = -2u$ [4]

3 The diagram belows shows the circumscribed circle of the triangle *ABC*.



The tangent to the circle at *W* meets the line *XY* extended to *K*. The angle bisector of the angle *WKX* cuts *WY* at *L*, *WX* at *M* and the circle at *N*. Show that

(a) triangles *WLK* and *XMK* are similar, [2]

(b) $LK \cdot XK = MK \cdot WK$, [2]

(c) $WL = WM$. [3]

4 A ship *P* moves due east towards a target with a speed of 30 km/h. A ship *Q* moves with a speed of 60 km/h along the course 210^{O}.

(a) Find the magnitude and direction of the velocity of *Q* relative to *P*. [5]

(b) If initially ship *P* is at 20 km to the west of ship *Q*, find the shortest distance between ship *P* and ship *Q*. [3]

5 A swimming pool with a rectangular base and having vertical sides of height, *h* is initially full of water. The cleaner drain out the water through the drain plug outlet. The water drains out from the outlet hole in the horizontal base of the pool at a rate which at any instant, is proportional to the square root of the depth

of the water at that instant.

If *x* is the depth of the water, at time t after the drain started is represented by the equation

$$\frac{dy}{dx} = -k\sqrt{x}$$

If the swimming pool is exactly half empty after an hour, find the further time that elapse before the pool becomes completely empty. [9]

6 A shop sells old lap tops, of which 1 in 5 on average are known to be damaged.

(i) A random sample o 15 lap tops is taken. Find the probability that at most 2 are damaged. [3]

(ii) Find the smallest value of *n* if there is a probability of at least 0.85 that a random sample of *n* lap tops contain at least one damaged. [4]

7 A factory has 360 male workers and 640 female workers, with 100 male workers earning less than RM2000 per month and 170 female workers earning at least RM2000 per month. At the end of the year, workers earning less than RM2000 are given bonus of RM2000 whereas the other receive a bonus of a month's salary.

(a) If 2 workers are randomly chosen, find the probability that only one worker receives a bonus of RM2000. [3]

(b) If a male worker and a female worker are randomly chosen, find the probability that only one worker receives a bonus of RM2000. [6]

8 The discrete random variable X takes the value *k* with the probability

$$P(X = k) = c\left|\frac{3}{2} - k\right|, k = 0, 1, 2$$

Where c is a constant.

(a) Determine the value of c. [2]

(b) Tabulate the distribution. [1]

(c) Calculate the mean and variance. [5]

(d) Write down cumulative distribution of X and sketch the graph of this function. [4]

9 It is known from experience that the probability that an individual will suffer a side effect from a given drug is 0.003 . By using suitable approximation, find the probability that, out of 2000 individual taking the drug,

(a) exactly 2 will suffer a side effect, [2]

(b) more than 3 will suffer a side effect. [3]

Find the probability that in 5 groups of 2000 individuals, 3 groups will have exactly 2 individual suffer a side effect. [3]

10 The weight of fish caught in Kudat has a normal distribution with mean 1.8 tonnes and standard deviation of 350 kilograms. The fish caught in Semporna has a normal distribution with mean 3.8 tonnes and standard deviation of 800 kilograms.

(a) Two fish caught in Kudat are chosen at random. Find the probability that the sum of the weight will be at least 3.4 tonnes. [4]

(b) One fish caught in Kudat and one fish caught in Semporna are chosen at random. Find the probability that the weight of the fish caught in Semporna is more than two times the weight of fish caught in Semporna. [4]

11 The continuous random variable X has probability density function f(x) given by:

$$f(x) = \begin{cases} 1 - \frac{1}{4}x, & 1 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) Calculate the mean and variance of X. [4]

(b) Find the cumulative distribution function F(x) of X. Hence show that the median m of X satisfies the equation $m^2 - 8m + 11 = 0$. [4]

(c) Find the mode of X. [1]

12 The frequency table below shows the examination marks of Math – T test for a group of student.

Marks (%)	**Number of students**
$0 \le x < 20$	1
$20 \le x < 30$	2
$30 \le x < 40$	9
$40 \le x < 50$	15
$50 \le x < 70$	18
$70 \le x < 80$	4
$80 \le x < 100$	1

(a) Plot a histogram for the data above and use your histogram to estimate the mode. [4]

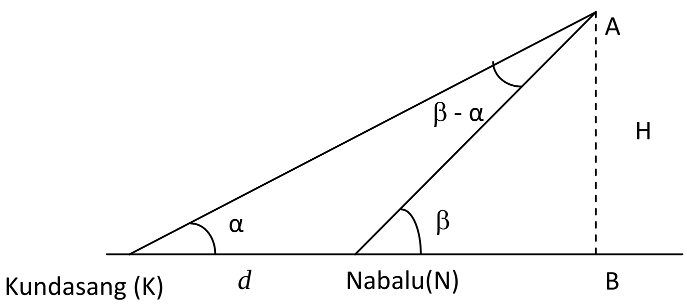
(b) Calculate the variance, median and inter-quartile range for the above data. [9]

(c) Explain why the median is more suitable representation for the marks of the students. [1]

PEPERIKSAAN EXCEL STPM 2010
MATHEMATICS T (PAPER 2)

Answer schemes:

1



D1

M1

ΔABN , $\sin\beta = \frac{H}{AN}$

$\therefore AN = \frac{H}{\sin\beta}$

Use Sine Rule,

M1

$\frac{AN}{\sin\alpha} = \frac{d}{\sin(\beta - \alpha)}$

$\frac{\left(\frac{H}{\sin\beta}\right)}{\sin\alpha} = \frac{d}{\sin(\beta - \alpha)}$

A1

$\therefore H = d\sin\alpha\sin\beta\csc(\beta - \alpha)$ B1

Therefore, $H = d\sin\alpha\sin\beta\csc(\beta - \alpha)$ is shown.

2

$u = x^2 + y$

$\frac{du}{dx} = 2x + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{du}{dx} - 2x$ M1

$(1 - x)\frac{dy}{dx} + 2y + 2x = 0$

$(1 - x)\left(\frac{du}{dx} - 2x\right) + 2(u - x^2) + 2x = 0$ M1

$(1 - x)\frac{du}{dx} - 2x(1 - x) + 2u - 2x^2 + 2x = 0$

$(1 - x)\frac{du}{dx} - 2x + 2x^2 + 2u - 2x^2 + 2x = 0$ M1

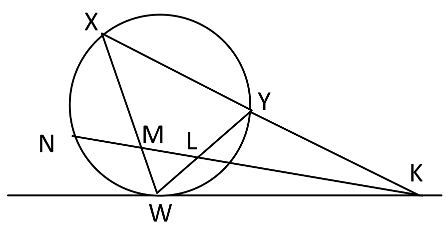
$$(1-x)\frac{du}{dx}+2u=0$$

$$(1-x)\frac{du}{dx}=-2u$$

A1

Shown!!

3



(a)

$\angle XKM=\angle WKM$ {KLMN is the angle bisector of $\angle WKX$} B1

$\angle LWK=\angle MXK$ { Alternate segment}

Hence, $\Delta\frac{WLK}{XMK}$ are similar {shown} A1

(b)

Since, $\Delta\frac{WLK}{XMK}$ are similar, then

$\frac{WK}{XK}=\frac{LK}{MK}$ B1

$LK\cdot XK=MK\cdot WK$

(c) A1

Since, $\Delta\frac{WLK}{XMK}$ are similar, then

$\angle WLK=\angle XMK$

Let $\angle WLK=\angle XMK=\theta$ B1

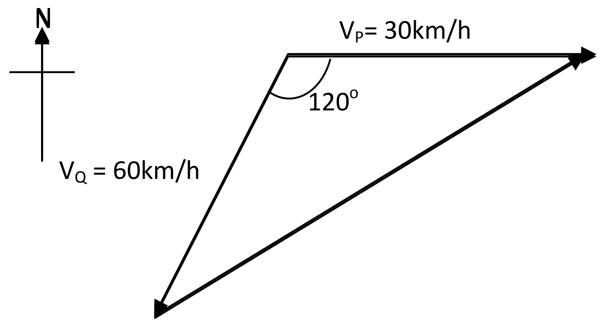
$\angle WLM=180°-\theta$ {Angle on the straight line}

$\angle WML=180°-\theta$ {Angle on the straight line} M1

Since $\angle WLM=\angle WML$, then

ΔWLM is an isosceles triangle where $WL=WM$ {shown} A1

4



(a)

$_QV_P = V_Q - V_P$

$= -60\cos 60^\circ\,\hat{i} - 60\sin 60^\circ\,\hat{j} - 30\hat{i}$

$= -60\hat{i} - 51.96\hat{j}$

$|_QV_P| = \sqrt{(-60)^2 + 51.96^2}$ M1

$= \sqrt{6300} = 30\sqrt{7}\ km/h$ A1

$\tan\theta = \dfrac{-51.96}{-60}$ M1

$\theta = \tan^{-1}\left(\dfrac{-51.96}{-60}\right)$

$= 40.89^\circ$ @ $40^\circ 53'$ A1

Therefore, the magnitude of the velocity of Q relative to P is $30\sqrt{7}$ km/h in the direction of S49.11°W. B1

Altenative method:

$QP = \sqrt{30^2 + 60^2 - (2)(30)(60)\cos 120^\circ}$

$= \sqrt{6300} = 30\sqrt{7}\ km/h$

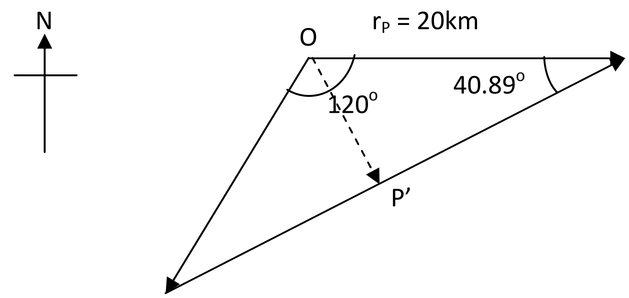
$\dfrac{30\sqrt{7}}{\sin 120^\circ} = \dfrac{60}{\sin\theta}$

$\sin\theta = \dfrac{60}{30\sqrt{7}}(\sin 120^\circ)$

$\theta = 40.89^\circ$

Therefore, the magnitude of the velocity of Q relative to P is $30\sqrt{7}$ km/h in the direction of S49.11°W.

(b)



D1

$shortest\ distance,\ OP' = 20\sin(40.89°)$ M1
$= 13.09\ km$ A1

5 $\frac{dx}{dt} = -k\sqrt{x}$

$\int \frac{dx}{dt} = \int -k\sqrt{x}$

$\int \frac{1}{\sqrt{x}}dx = \int -k dt$ M1

$2x^{\frac{1}{2}} = -kt + c$ A1

When t = 0, x = h

$2\sqrt{h} = c$ M1

$2\sqrt{x} = -kt + 2\sqrt{h}$ A1

When t = 1, x = ½ h

$2\sqrt{\frac{1}{2}h} = -k + 2\sqrt{h}$

$k = (2 - \sqrt{2})\sqrt{h}$ M1

$2\sqrt{x} = -((2 - \sqrt{2})\sqrt{h})t + 2\sqrt{h}$ A1

When x = 0,

$$t = \frac{2\sqrt{h}}{(2 - \sqrt{2})\sqrt{h}}$$ M1

$t = 3.414\ @\ 3\ hour\ 25min$ A1

Hence, the further time required before the pool become empty is 2 hour 25 minutes. B1

6 (i)Let X be the number of damaged lap tops, then $X \sim B(15, 0.2)$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$ B1

$P(X = 0) = {}^{15}C_0(0.2)^0(0.8)^{15} ------ (1)$

$P(X = 1) = {}^{15}C_1(0.2)^1(0.8)^{14} ------ (2)$ M1

$P(X = 2) = {}^{15}C_2(0.2)^2(0.8)^{13} ------ (3)$

$P(X \le 2) = (1) + (2) + (3) = 0.398$ A1

(ii) $X \sim B(n, 0.2)$

$P(X \ge 1) \ge 0.85$ B1

$1 - P(X = 0) \ge 0.85$

$1 - {}^{n}C_0(0.2)^0(0.8)^n \geq 0.85$ M1

$1 - 0.85 \geq (0.8)^n$

$0.15 \geq 0.8^n$

$\log(0.15) \geq n\log(0.8)$ M1

$\frac{\log(0.15)}{\log(0.8)} \geq n$

$8.5 \geq n$

$\therefore n = 9$

The smallest value of *n* is 9. A1

7 (a)

P(a worker recieves a bonus of RM2000 $= \frac{570}{1000}$ A1

P(only one of two workers receives bonus of RM2000) $= {}^{2}C_{1} \times \frac{570}{1000} \times \frac{430}{1000}$ M1

$= 0.4902$ A1

(b)

P(a male workers recieves bonus of RM2000) $= 0.10$ A1

P(a male workers recieves bonus a month's salary) $= 0.26$ A1

P(a female workers recieves bonus of RM2000) $= 0.47$ A1

P(a female workers recieves bonus a month's salary) $= 0.17$ A1

$\therefore$ P(only one worker recieves a bonus of RM2000 when a male and female worker are chosen) M1

$= 0.10 \times 0.17 + 0.47 \times 0.26$

$= 0.1392$ A1

8 (a)

$P(X = k) = c\left|\frac{3}{2} - k\right|$

Known that

$\sum P(X = k) = 1$

$c\left|\frac{3}{2}\right| + c\left|\frac{3}{2} - 1\right| + c\left|\frac{3}{2} - 2\right| = 1$ M1

$c\left[\frac{3}{2} + \frac{1}{2} + \frac{1}{2}\right] = 1$

$c = \frac{2}{5}$ A1

(b)

X = k	0	1	2
P(X = k)	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

D1

(c)

$E(X) = \sum xP(X = k)$

$= 0\left(\frac{3}{5}\right) + 1\left(\frac{1}{5}\right) + 2\left(\frac{1}{5}\right)$ M1

$= \frac{3}{5}$ A1

$E(X^2) = \sum x^2 P(X = k)$

$= 0\left(\frac{3}{5}\right) + 1\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right)$

$= \frac{16}{25}$ A1

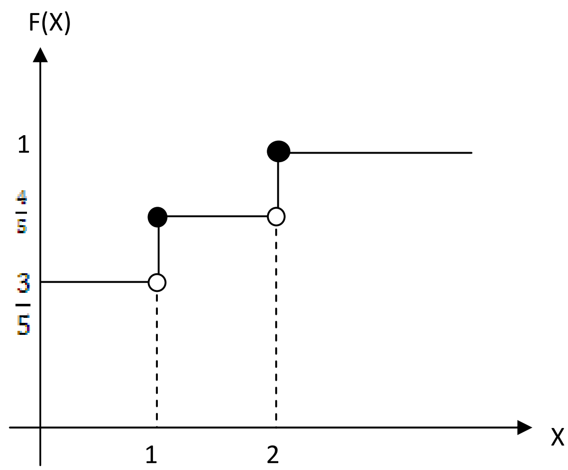
$Var(x) = E(x^2) - [E(x)]^2$ M1

$= \frac{16}{25} - \left[\frac{3}{5}\right]^2 = \frac{7}{25}$ A1

(d) Cumulative distribution function,

$$F(X) = \begin{cases} 0 & , \quad x < 0 \\ \frac{3}{5} & , \quad 0 \le x < 1 \\ \frac{4}{5} & , \quad 1 \le x < 2 \\ 1 & , \quad 2 \le x \end{cases}$$

M1
M1
A1



D1

9 Let X be the number of individual that will suffer a side effect.

$X \sim B(2000, 0.003)$

By using a Poisson Approximation,

$\lambda = np = 2000(0.003) = 6$ B1

Therefore,

$X \sim Po(6)$

(a)

$P(x = 2) = \frac{e^{-6}6^2}{2!} = 0.0446$ A1

(b)

$P(x > 3) = 1 - P(x \leq 3)$ B1

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)]$ M1

$= 1 - e^{-6}[\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!}]$

$= 0.8488$ A1

Let Y be the number of group that will have exactly 2 individual suffer a side effect.

$Y \sim B(5, 0.0446)$ B1

$P(Y = 2) = {}^5C_2(0.0446^2)(1 - 0.0446)^{5-2}$ M1

$= 0.00081$ A1

10 (a)

Let X be the fish that are caught in Kudat

$X \sim N(1.8, 0.35^2)$

$X + X \sim N(3.6, 0.245)$ B1

$P(2X \geq 3.4)$ B1

$P(2X \geq \frac{3.4 - 3.6}{\sqrt{0.245}})$ M1

$= P(Z \geq -0.4041)$

$= 0.6569$ A1

(b)

Let X be the fish that are caught in Kudat

Let Y be the fish that are caught in Semporna

$X \sim N(1.8, 0.35^2)$

$Y \sim N(3.8, 0.8^2)$

Given $Y > 2X$, *therefore*

$Y - 2X \sim N(0.2, 1.13)$ B1

$P(Y > 2X) = P(Y - 2X > 0)$ B1

$= P\left(Z > \frac{0 - 0.2}{\sqrt{1.13}}\right)$ M1

$= P(Z > -0.1881)$

$= 0.5746$ A1

11 (a) Mean, $\mu = \int_1^3 x\left(1 - \frac{1}{4}x\right) dx$

$= \int_1^3 \left(x - \frac{x^2}{4}\right) dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_1^3$ M1

$= \frac{11}{6}$

$E(X^2) = \int_1^3 x^2\left(1 - \frac{1}{4}x\right) dx$

$= \int_1^3 (x^2 - \frac{x^3}{4}) dx$

$= \left[\frac{x^3}{3} - \frac{x^4}{16}\right]_1^3$ M1

$= \frac{11}{3}$

$\therefore$ Var (x) $= E(X^2) - \mu^2$ M1

$= \frac{11}{3} - \frac{121}{36}$

$= \frac{11}{36}$ A1

(b) $X \leq 1 \leq, F(X) = P(X \leq x) = 0$

$1 \leq X \leq 3, F(X) = \int_1^x \left(1 - \frac{t}{4}\right) dt$ M1

$= \left[t - \frac{t^2}{8}\right]_1^x$

$= \frac{1}{8}(8x - x^2 - 7)$

Hence,

$$F(t) = \begin{cases} 0 & , x \le 1 \\ \frac{1}{8}(8x - x^2 - 7) & , 1 \le x \le 3 \\ 1 & , 3 \le x \end{cases}$$ A1

As the median m satisfies $F(X \le m) = \frac{1}{2}$ thus

$F(m) = \frac{1}{2}$

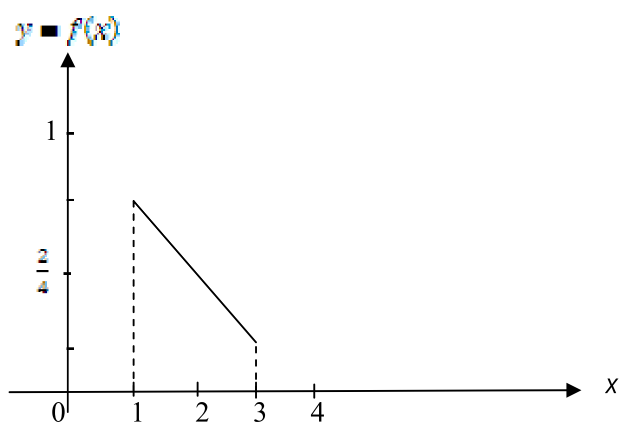
$\therefore \frac{1}{8}(8m^2 - m^2 - 7) = \frac{1}{2}$ M1

$\therefore m^2 - 8m + 11 = 0$

Hence the median *m* satisfies the equation: B1

$\therefore m^2 - 8m + 11 = 0$

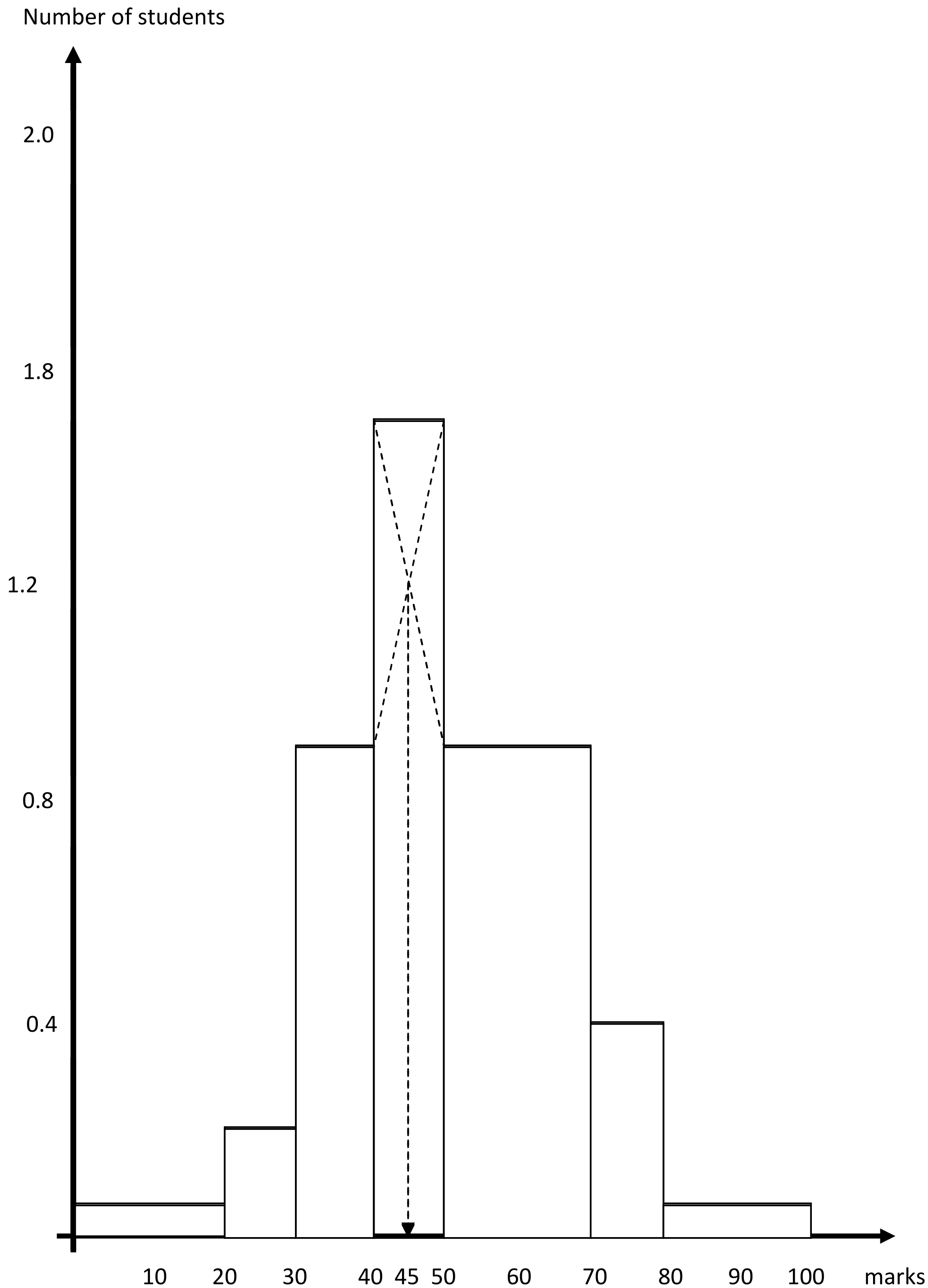
(c) The mode of this probability density function $f(x)$ does not occur at a stationary point, so differentiating the probability density function $f(x)$ is of no help. From the sketch we can see that the maximum value of $f(x)$ occurs at the point when $X = 1$, thus the mode of this distribution is 1. A1



Graph of y = f(x)

12

(a)


Number of students
2.0
1.8
1.2
0.8
0.4
10
20
30
40
45
50
60
70
80
90
100
marks

D1

M1

A1

Mode = 45 A1

Marks (%)	F	Class width	Freq density	x
0 -20	1	20	0.05	10
20-30	2	10	0.2	25
30-40	9	10	0.9	35
40-50	15	10	1.5	45
50-70	18	20	0.9	60
70-80	4	10	0.4	75
80-100	1	20	0.05	90

D1
M1

(b) $var(x) = \frac{\Sigma fx^2}{\Sigma f} - (\frac{\Sigma fx}{\Sigma f})^2$

$= \frac{138150}{50} - (\frac{2520}{50})^2$ A1

$= 222.84$

Median, $m = 40 + \frac{13}{15}(10)$ M1

$= 40.67$ A1

1st quartile, , $Q_1 = 40 + \frac{10.5}{15}(10)$ M1

$= 40.33$ A1

3rd quartile, , $Q_3 = 50 + \frac{10.5}{18}(20)$ M1

$= 61.66$ A1

Interquartile range, $Q_3 - Q_1 = 61.66 - 40.33$ M1

$= 21.33$ A1

(c) Data skewed to the right. B1