1 Given that the shortest distance from point $(\sqrt{3}, 1)$ to the straight line $\sqrt{3}x - y + k = 0$ is 2. Find the values of k.

[3 *marks*]

2 Given that $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

[4 *marks*]

3 The equation of a curve is given by $x^2 + y^2 - 4xy + 3 = 0$.

(a) Show that $\frac{dy}{dx} = \frac{2y - x}{y - 2x}$ [3 *marks*]

(b) Find the coordinates of each of points on the curve where the tangent is parallel to the y-axis.

[4 *marks*]

4 If x is so small that x^3 and the higher powers of x may be neglected, show that $\sqrt{\frac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2$.

By taking $x = \frac{1}{24}$, show that $\sqrt{23} \approx \frac{5525}{1152}$. The approximate value for $\sqrt{23}$ can also be obtained by putting $x = \frac{7}{16}$ in the binomial expansion. State, with reason, which value of x will give a better approximation to $\sqrt{23}$.

[7 *marks*]

5 Show that $\frac{1}{\sqrt{2r+1} + \sqrt{2r+3}} = \frac{1}{2}\left(\sqrt{2r+3} - \sqrt{2r+1}\right)$.

Hence, find the sum of the series

$$\frac{1}{\sqrt{1} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \ldots + \frac{1}{\sqrt{2n+1} + \sqrt{2n+3}}.$$

[7 *marks*]

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6 Find the non-zero values of x that satisfy the equation

$$8^x - 11(4^x) + 31(2^x) - 21 = 0.$$

Giving your answers correct to three decimal places.

[8 *marks*]

7 Let $I = \int_1^4 \frac{1}{x(4-\sqrt{x})}\,dx$.

Use the substitution $u = \sqrt{x}$ to show that $I = \int_1^2 \frac{2}{u(4-u)}\,du$.

Hence, show that $I = \frac{1}{2}\ln 3$.

[8 *marks*]

8 Find the domain of the function $f : x \to 2 + \sqrt{x+4}$.

Find f^{-1} and state its domain and range.

Sketch the graphs of f and f^{-1} on the same diagram.

[8 *marks*]

9 A point P moves on a curve such that the difference of its distance from the point $(4, 0)$ and $(-4, 0)$ is a constant and equals to $2\sqrt{10}$. Find the equation of the locus of P.

Show that the line $y = x - 2$ is a tangent to the curve.

[8 *marks*]

10 The gradient function of a curve is given by $\frac{6}{(2x-1)^2}$. $P(2, 9)$ is a point on the curve. The normal to the curve at P meets the y-axis at Q and the x-axis at R.

(a) Find the coordinates of the mid point of QR. [4 *marks*]

(b) Find the equation of the curve and sketch the curve. [6 *marks*]

(c) A point (x, y) moves along the curve in such a way that the x-coordinate increases at a constant rate of 0.03 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2 *marks*]

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11 The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}, \text{ where } a \neq 2.$$

Find in terms of a,

(a) $|\mathbf{M}|$ [2 *marks*]

(b) cofactor of element -1. [1 *mark*]

(c) adj **M** [3 *marks*]

(d) $\mathbf{M}^{-1}$ [2 *marks*]

Hence, solve the simultaneous equations

$$3x + 2y = 1$$
$$3x + y + 2z = 9$$
$$-y + z = 5$$

[5 *marks*]

12 α and β are the roots of the equation $ax^2 + bx + c = 0$. Show that $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$. [4 *marks*]

(a) If $\alpha = 3\beta$ and $b = a + c$, express a in terms of c. [5 *marks*]

(b) Show that $c^2x^2 + (2ac - b)x + a^2 = 0$ has roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [6 *marks*]

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