Tests of "Fanning Out" of Indifference Curves: Results from Animal and Human Experiments

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In an earlier paper (Raymond C. Battalio, John H. Kagel, and Don N. Mac Donald, 1985), we reported Allais-type violations of the independence axiom of expected utility theory with rats choosing over positively valued payoffs (food rewards). This note extends this research, examining animals' choices over losses, testing for (1) standard Allais-type common ratio effect violations of expected utility theory and (2) fanning out of indifference curves for random prospects, tests of Mark J. Machina's (1982, 1987) hypothesis II (hereafter H2), over previously unexplored areas of the unit probability triangle. Results from a parallel series of experiments using human subjects choosing over real losses are also reported. For both rats and people, we find standard Allais-type violations of expected utility theory and a systematic failure of the fanning out hypothesis in the southeast corner of the unit probability triangle, in the case of losses. Thus, the fanning out hypothesis (Machina 1982, 1987) cannot provide a satisfactory explanation for behavioral deviations from expected utility theory.

I. Allais-Type Common Ratio Effects and New Tests of Fanning Out

A key element of all descriptive alternatives to expected utility theory is to explain Allais-type common ratio effects and a number of other systematic violations of expected utility theory. Common ratio effect violations of expected utility theory include the "certainty effect" of Daniel Kahneman and Amos Tversky (1979) and the "Bergen Paradox" of Ole Hagen (1979), as special cases. Allais-type common ratio violations involve rankings over pairs of prospects of the form:

Example 1

A: x_2 with probability p
x_3 with probability $1-p$
B: x_1 with probability q
x_3 with probability $1-q$

Example 2

C: x_2 with probability rp
x_3 with probability $1-rp$
D: x_1 with probability rq
x_3 with probability $1-rq$

where $p>q$, $0 \geq x_3 > x_2 > x_1$, and $0<r<1$ (the term "common ratio" derives from the equality of $\text{prob}(x_2)/\text{prob}(x_1)$ in A vs. B and C vs. D). An expected utility maximizer would prefer either A and C (if $p[u(x_2)-u(x_3)] > q[u(x_1)-u(x_3)]$), or else B and D (if the opposite were true). However, using primarily hypothetical choice alternatives, researchers find a systematic tendency for subjects to prefer B and C when the outcomes involve losses (Kahneman and Tversky, 1979; Kenneth R. MacCrimmon and Stig Larson, 1979; and the references cited in Machina, 1983).

Machina (1982, 1987) identifies three systematic violations of the independence axiom in addition to the common ratio effect and shows that all four follow from a single

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assumption, H2.[1] More recently, Zvi Safra, Uzi Segal, and Avia Spivak (1988) have used H2 to explain the preference reversal phenomena (Sarah Lichtenstein and Paul Slovic, 1971; David W. Grether and Charles R. Plott, 1979; Machina, 1987). H2 states that in moving from one probability distribution to another, which (first order) stochastically dominates it, the local (linear in the probabilities) utility function retains the same degree of concavity, or becomes more concave at each point. In other words, preferences tend to vary systematically so that a first-order stochastic dominating shift in wealth (i.e., a nonnegative random addition to wealth) results in the same or more risk averse choices.

In terms of the unit probability triangle, this yields indifference curves that are parallel (as expected utility theory predicts) or that fan out relative to the origin. Figure 1 illustrates how fanning out can explain a typical Allais-type common ratio effect over losses. The set of all prospects over the fixed outcome levels $0 \geq x_3 > x_2 > x_1$ can be represented by the set of all probability triples of the form $P = (p_1, p_2, p_3)$ where $p_i = \text{prob}(x_i)$ and $\Sigma p_i = 1$. Since $p_2 = 1 - p_1 - p_3$, we can represent these lotteries by points in the unit triangle in the (p_1, p_3) plane. Figure 1 illustrates this for examples 1 and 2 above when $p = 1.0$, $q = 0.75$, and $r = 1/3$, (so that prospect A is represented by $(p_1, p_3) = (0,0)$, prospect B is represented by $(p_1, p_3) = (0.75, 0.25)$, etc.). Prospects C and D stochastically dominate A and B, as they are derived from A and B by reducing the probability of x_1 and x_2 by a common proportion, $0 < r < 1$, and adding the displaced probability mass to x_3 in both cases.

In Figure 1, the dashed lines represent linear combinations of the prospects chosen over. Solid lines are indifference curves over the random prospects. Since, in expected utility theory, the individual's indifference curves in the unit probability triangle are given by solutions to the linear equation

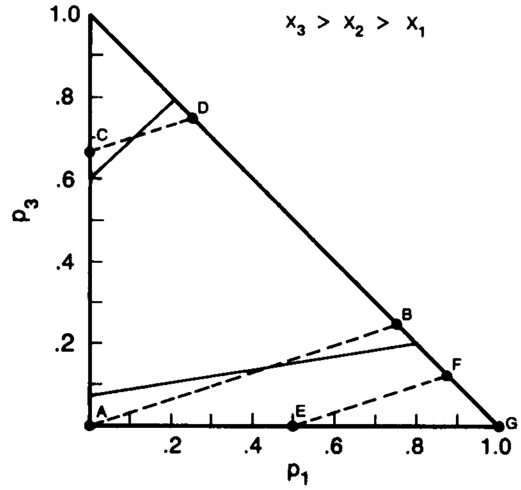

FIGURE 1. PROSPECTS EMPLOYED IN TESTS OF FANNING OUT. DASHED LINES CONNECT PROSPECTS' SUBJECTS CHOSE BETWEEN. STRAIGHT LINES ARE INDIFFERENCE CURVES THAT DISPLAY "FANNING OUT." PROSPECTS CORRESPOND TO THOSE SPECIFIED IN THE TEXT AND TABLE 1

$$\begin{aligned}\bar{U} &= \Sigma u(x_i) p_i \\ &= u(x_1) p_1 + u(x_2)(1 - p_1 - p_3) \\ &\quad + u(x_3) p_3 = \text{constant},\end{aligned}$$

they will consist of parallel straight lines with slope $[u(x_2) - u(x_1)]/[u(x_3) - u(x_2)]$, with more preferred curves lying to the northwest. Since, by construction, the dashed lines are parallel, the parallel linear indifference curves of expected utility theory require preference for prospects A and C or B and D. However, indifference curves that "fan out" relative to the origin can, as shown in Figure 1, explain the preference for prospects B and C.

Fanning out of indifference curves in the unit probability triangle corresponds to the "light hypothesis" of weighted utility theory (Soo Hong Chew and Kenneth R. MacCrimmon, 1979) and is a characteristic of

[1]These involve (1) the common consequence effect, the most famous specific example of which is the so called "Allais Paradox," (2) over sensitivity to changes in small probability-outlying events, and (3) the utility evaluation effect (see Machina, 1982, 1987).

TABLE 1—TREATMENT CONDITIONS FOR LOSSES: ANIMALS

Condition	Prospects		
1	A: 5 sec delay, prob. 1.0 (5.0)	or	B: 13 sec delay, prob. 3/4 1 sec delay, prob. 1/4 (10.0)
2	C: 5 sec delay, prob. 1/3 1 sec delay, prob. 2/3 (2.34)	or	D: 13 sec delay, prob. 1/4 1 sec delay, prob. 3/4 (4.0)
3	E: 13 sec delay, prob. 1/2 5 sec delay, prob. 1/2 (9.0)	or	F: 13 sec delay, prob. 7/8 1 sec delay, prob. 1/8 (11.5)
4	G: 13 sec delay, prob. 1.0 (13.0)	or	F: 13 sec delay, prob. 7/8 1 sec delay, prob. 1/8 (11.5)
	Sequencing of Conditions: 1, 2, 1, 3, 1, 3, 1, 4		

several other generalizations of expected utility theory as well (see Machina, 1983). In the case of losses, the obvious question is whether choices over pairs of prospects that are dominated by A and B, prospects, which lie in the southeast corner of the unit probability triangle and are parallel to A and B, such as E and F in Figure 1, satisfy the requirements of fanning out (preference for F on the part of those choosing B and C).

II. Experiment 1: Rats' Choices Over Losses

We define losses generically as any outcome for which *less* is better. In our experiments, the random variable was the time delay to reinforcement. Given the relatively steep temporal discount functions that rats have, time delay is a potent reinforcer, where it is well established that less is better (see Leonard Green, 1982, and Kagel and Green, 1987, for reviews of experiments most relevant to economists).

Historically, psychologists have studied choices between constant and variable delays to reinforcement, finding strong preference for the variable outcome alternative (risk loving), given equal average delays to reinforcement (Peter Killeen, 1968; see James E. Mazur, 1986, for a review of this literature).[2] In these experiments, there are no auxiliary decisions to be made between the choice point and the time that all uncertainty is resolved. This is important, since it is well established that in choosing over risky prospects whose resolution is not immediate, and where auxiliary decisions are required prior to the resolution of the uncertainty, the independence axiom of expected utility theory will not generally hold (Harry M. Markowitz, 1959, ch. 11; Jan Mossin, 1969).

Table 1 shows the variable outcome alternatives the rats chose between. Each outcome provided 0.1cc of water at the end of the delay period. Mean time delay to reinforcement is shown under each pair of prospects. Conditions 1 and 2 involve standard Allais-type common ratio manipulations as prospects C and D stochastically dominate A and B. Condition 3 checks for fanning out in the southeast corner of the unit probability triangle as prospects A and B stochastically dominate E and F and the dashed lines connecting the two sets of prospects are parallel. Prospects E and F are derived from A and B by multiplying

[2]Risk loving over losses here appears to be characteristic of a general preference for variable as compared to certain losses, consistent with predictions from prospect theory (Kahneman and Tversky, 1979). See Kagel et al. (1988, experiment 1), where we test this prediction by examining rats' preferences over variable versus certain shock with the same mean. The rats prefer the variable shock. Note that unlike humans choosing over monetary losses, there is no opportunity for "asset integration" over these choices, with its implication for inducing risk aversion.

the 5 and 1 sec delay probabilities by 0.5 and adding the displaced probability to the 13 sec delay probability in both cases. Condition 4 is a control condition that tests for the rats' sensitivity to low probability outcome events, since it is generally assumed that the decision maker knows the relevant probabilities of the different payoffs. Payoff frequencies of 1/8 may be sufficiently low that the rats are not cognizant of them. Condition 4 tests for this. Figure 1 shows these prospects in terms of the unit probability triangle.

A discrete trials choice procedure was employed where the rats chose between a single pair of prospects at a time. Choices were recorded and payoffs delivered in response to a single press on one of two choice levers. Each choice trial lasted 39 sec, irrespective of the alternative chosen or the length of the delay to reinforcement. This established a minimum of a 20 sec blackout period between choice trials (a maximum delay to reinforcement of 13 sec, with 6 sec to consume the water). Given the steep time discount rates the rats have, these 20 sec blackout periods create relatively strong independence between trials.

Each experimental session began with 32 forced choice trials where only one lever was operative, followed by 48 free choice trials when both levers were operative. The forced choice trials served to familiarize subjects with the alternatives. During these trials, the empirical distribution function was forced to match the programmed distribution function over each prospect's trial set.[3] Free choice trials served to measure preferences. The random number generator was given free reign during these trials, so that the probability of obtaining a given outcome on any trial was independent of outcomes on other trials.

Experimental sessions were conducted once a day, 7 days a week, at approximately the same time each day. Each experimental condition lasted 38 days, divided into two 19-day, periods. The levers delivering the certain and variable delay alternatives remained constant within each 19-day period, but they were switched between the two 19-day periods. Switching alternatives across levers and averaging choices controls for lever bias, which may be severe at times for rats. Preferences were measured in terms of choices of the certain alternative during the free choice trials for the last 5 days of each 19-day period and averaged across side switches.

Table 2 shows average percentage of free choice trials allocated to the more certain alternative under each condition. A choice frequency of 50 percent indicates that the rats chose the more certain alternative half the time, averaged across side switches, and is interpreted as indifference.

Under baseline pair 1 choices, the rats generally preferred the more variable (B) alternative. On average, the rats chose the A alternative on 38.9 percent of the trials ($t = -2.68$, $p < 0.05$, 2-tailed t-test). Using choice frequencies to measure intensity of preference, six of the eight subjects had relatively strong preference for the B alternative, the two exceptions being subjects 612 and 622. Given the strong preference for variable over fixed delay probabilities reported in the psychology literature, the preference for the B alternative here is not surprising, even though the average delay to reinforcement was twice that of the certain delay probability. Further, a necessary condition for generating Allais-type common ratio effect violations of expected utility theory, similar to those reported with human subjects, is to establish preference for the variable alternative under the high probability loss condition. The parameter values employed were designed to achieve this.

Under pair 2 choices, mean percentage choice for the more certain alternative, C, was 54.5 percent ($t = 1.63$). As indicated by the individual subject t-statistics, five rats were indifferent between C and D, and three had a clear preference for the C alternative. Upon return to pair 1 choices, mean percentage choice for the A alternative fell to 44.8 percent ($t = -2.01$, $p < 0.10$, 2-tailed

[3]Sequences of choices across levers and outcomes were fixed; however, the start point was determined randomly on a daily basis. Details of experimental procedures are reported in Kagel et al., 1988, experiment 2.

TABLE 2—CHOICES OVER LOSSES: ANIMAL SUBJECTS

	Percentage Choice of More Certain Alternative (Absolute Value of t-Statistics in Parentheses)							
Subj.	Baseline (1)	Standard Allais (2)	Baseline (1)	Test of H2 (3)	Baseline (1)	Test of H2 (3)	Baseline (1)	Sensitivity Test (4)
611	21.7	72.3	38.9	64.6	40.0	64.4	48.3	31.5
	(7.26)[b]	(6.18)[b]	(3.58)[b]	(3.40)[b]	(5.18)[b]	(9.30)[b]	(0.60)	(5.56)[b]
612	50.0	50.2	55.6	49.8	49.2	49.8	49.4	27.7
	(0.0)	(0.14)	(3.30)[a]	(0.54)	(1.00)	(0.28)	(1.34)	(10.2)[b]
613	44.6	49.6	50.4	67.7	49.0	59.0	46.0	31.5
	(1.94)	(0.18)	(0.16)	(14.5)[b]	(0.34)	(4.92)[b]	(2.12)	(5.04)[b]
614	20.6	48.1	32.5	44.6	47.5	60.0	44.6	26.9
	(15.1)[b]	(0.74)	(5.00)[b]	(1.68)	(1.22)	(3.20)[a]	(3.36)[b]	(6.02)[b]
621	45.0	51.7	44.2	63.8	50.2	61.7	49.4	41.3
	(3.20)[a]	(0.52)	(3.36)[b]	(8.90)[b]	(0.08)	(3.82)[b]	(0.20)	(4.56)[b]
622	49.8	52.7	50.4	68.1	49.6	61.0	52.9	16.7
	(0.44)	(4.34)[b]	(0.22)	(16.2)[b]	(0.58)	(4.04)[b]	(2.26)	(16.6)[b]
623	36.9	58.3	41.9	51.3	47.5	57.9	46.3	40.6
	(11.6)[b]	(11.3)[b]	(10.6)[b]	(2.14)	(2.14)	(10.2)[b]	(3.50)[b]	(11.4)[b]
624	42.3	53.1	44.6	54.6	45.6	75.8	–	–
	(2.70)[a]	(1.42)	(3.74)[b]	(2.92)[a]	(1.66)	(7.56)[b]	–	–
AV.	38.9	54.5	44.8	58.1	47.3	61.2	48.1	30.9
	(2.68)[a]	(1.63)	(2.01)	(2.52)[a]	(2.31)[a]	(4.35)[b]	(1.81)	(5.97)[b]

[a]Significantly different from 50 percent at the 5 percent significance level, 2-Tailed *t*-Test.

[b]Significantly different from 50 percent at the 1 percent significance level, 2-Tailed *t*-Test.

t-test). Six of the eight rats were under clear control of the contingencies as their choice frequencies decreased back toward their original levels, the two exceptions being rats 612 and 613. Thus, six of the eight subjects and the aggregate choice data show standard Allais-type violations of expected utility theory.

Tests of H2 in the southeast corner of the unit probability triangle show that it fails to hold. Rather, fanning in characterizes the data here. Under the initial implementation of condition 3, average choice of the more certain (E) alternative jumped to 58.1 percent ($t = 2.52$, $p < 0.05$, 2-tailed *t*-test). Equally as important, with the return to pair 1 choices, mean percentage choice of the certain alternative, A, fell to 47.3 percent ($t = -2.31$, $p < 0.05$, 2-tailed *t*-test). Choice was under reversible control, with a pattern of fanning in for six of the eight rats, 611, 613, and 621–624. The two remaining rats, 612 and 614, were unresponsive to the changes in experimental conditions, rather than displaying any hint of fanning out. A second implementation of condition 3 confirmed and strengthened these results, as subject 614 now showed clear fanning in, as well.[4]

The results of the sensitivity test are shown in the last column of Table 2. On average, the rats chose the more certain, dominated alternative (G) 30.9 percent of the time, well below the 50 percent mark. All the rats were responsive to the 1/8 chance of a 1 sec delay probability, as the mean choice of the dominated alternative was below 50 percent in all cases, and one can reject indifference between prospects for all subjects at the 1 percent significance level or better, including rat 612, who was largely unresponsive to the other experimental manipulations. These results show the rats to be sensitive to delay probabilities

[4]MacDonald, Kagel, and Battalio (1989) report comparable failures of fanning out for rats over gains (food rewards) in previously unexplored areas of the unit probability triangle (the northwest corner, in this case).

as small as 1/8. Hence, we cannot attribute the failure of H2 here to any inherent inability to recognize such low probability outcome events.

III. Experiment 2: Humans' Choices Over Losses

Studies of human choices over losses involved responses to a series of questions requiring subjects to indicate which of two gambles they preferred. All questions were of the following form:

Example 3

A: Losing $14 if 1–100
B: Losing $20 if 1–70
 Losing $0 if 71–100
Answer: (1) I prefer *A*. (2) I prefer *B*. (3) Indifference

The numbers following each dollar amount referred to 100 numbered poker chips in a bowl sitting in front of the room, one of which would be drawn to determine payoffs. Thus, if a subject chose A, he or she would lose $14 with certainty as any numbered poker chip drawn gave this result. If, however, the subject chose B, and a chip numbered 1–70 was drawn, he or she would lose $20, but if a chip numbered 71–100 was drawn, the subject would lose $0.

Subjects were paid off on a *single* question, determined at random, through the draw of a poker chip, after all the questions were answered. They then drew a poker chip from the bowl of 100 chips to determine the actual payoff for that question. Immediately prior to answering these questions, subjects answered three hypothetical questions and went through the two-stage chip drawing procedure, to ensure their understanding of the payoff process. Subjects were given $30 cash balances against which to cover the possibility of losses. Payoffs were set so that all subjects would walk away with positive net earnings.[5]

Paying off subjects on a single question eliminates wealth effects from directly distorting preferences. However, papers by Charles Holt (1986) and Adi Karni and Zvi Safra (1987) show conditions under which answers to questions in a series of questions, only one of which will result in a payoff, will not be independent of the set of questions posed. This criticism applies to both the Gordon M. Becker, Morris H. DeGroot, and Jacob Marschak (1964) procedure for determining certainty equivalent values of gambles, and to simple elicitation of preferences over a series of paired alternatives, such as employed here. Responses to each question are independent of the set of questions posed if the independence axiom of expected utility theory holds, or subjects choose as if each gamble will actually be selected (as in the "isolation effect" in prospect theory). To test for independence, Colin Camerer (1989) has permitted subjects to change their choices once the question to be played out had been selected. Few subjects changed their choices. Answers reported here come from two separate series of questions, with several identical questions across the two series producing strong repeat reliability in the answers (Raymond C. Battalio, John H. Kagel, and Komain Jiranyakul, in press). This, too, is consistent with the independence assumption, as there were substantial differences in the other questions included in the two series. Since the data below show frequent violations of the independence axiom, we conclude that subjects treated each gamble separately, either in response to our urging them to do so or out of a natural isolation effect of the sort promoting the absence of asset integration reported in a number of choice experiments (Kahneman and Tversky, 1979; Camerer, 1989; Battalio, Kagel, and Jiranyakul, in press).[6]

[5]See Battalio, Kagel, and Jiranyakul (in press) for details of the experimental procedures employed.

[6]In the experiment using rats, subjects were paid off following each response. However, the payoffs, in terms of water reinforcement, were independent of the alternative chosen. In this case, there were no wealth effects in the sense that the outcomes of the prospects had no effect on the physical state of the rats.

TABLE 3—CHOICES OVER LOSSES: HUMAN SUBJECTS

Set 1: Allais-Type Common Ratio Changes
$A_1(-\$14, 1.0)$ or $B_1(-\$20, 0.70)$
$A_2(-\$14, 0.20)$ or $B_2(-\$20, 0.14)$

Possible Choice Patterns	Patterns Consistent with	Choice Frequencies Set 1
A A	EU	10
B B	EU	5
B A	Fanning Out	10
A B	Fanning In	4

Set 2: New Tests of Fanning Out
2.1 $A_3(-\$20, 0.74; -\$14, 0.20)$ or $B_3(-\$20, 0.88)$
$A_4(-\$14, 0.90)$ or $B_4(-\$20, 0.63)$
2.2 $A_5(.-\$20, 0.60; -\$14, .40)$ or $B_5(-\$20, 0.88)$
$A_4(-\$14, 0.90)$ or $B_4(-\$20, 0.63)$

Possible Choice Patterns[a]	Patterns Consistent with	Choice Frequencies Set 2.1	Set 2.2
A A	EU	7	3
B B	EU	11	17
B A	Fanning out	1	5
A B	Fanning in	15	7

[a]Stochastically dominated alternatives listed first.

Table 3, set 1, shows one set of Allais-type common ratio alternatives employed (these are illustrated in Figure 2). The gambles employed are shown first, with the first number in brackets showing the dollar amount of the loss, followed by its probability (as a convention, we omit displaying zero loss probabilities). Below this are shown individual subject responses to the questions. The four possible choice patterns in moving from higher to lower probability losses are specified, along with their relationship to expected utility theory and the fanning out hypothesis. H2, as specified in Machina (1982, 1987), is a weak inequality that includes expected utility theory as a special case. Consequently, the fanning out hypothesis has power here to the extent that it organizes *deviations* from expected utility theory. It does this nicely, as a little under half the responses violate expected utility theory, and of these, 71.4 percent of the deviations correspond to fanning out rather than fanning in, far more than one would expect on the basis of chance factors alone.[7]

Choice sets 2.1 and 2.2. report tests of fanning out over losses in the southeast corner of the unit probability triangle (see Figure 2). In set 2.1, expected utility theory organizes 52.9 percent of the data compared to 50 percent on the basis of chance factors alone. Fanning out breaks down completely here, as fanning in organizes 44.1 percent of the data, well beyond expectations based on chance factors alone ($p < 0.01$).

Choice set 2.2 replaces prospect A_3 in set 2.1, with its 6 percent chance to escape without losses, with prospect A_5, which guarantees losses, should it be chosen. The certainty of losses reduces the attractiveness of the A alternative, thereby producing greater conformity with expected utility theory and increasing the frequency of clear fanning out, although fanning out still fails to explain a majority of the deviations from expected utility theory.

[7]Fanning out constitutes one of two possible deviations from expected utility theory. A binomial test statistic comparing observed frequencies with expected frequencies and a 10 percent significance level is employed in making these statistical evaluations.

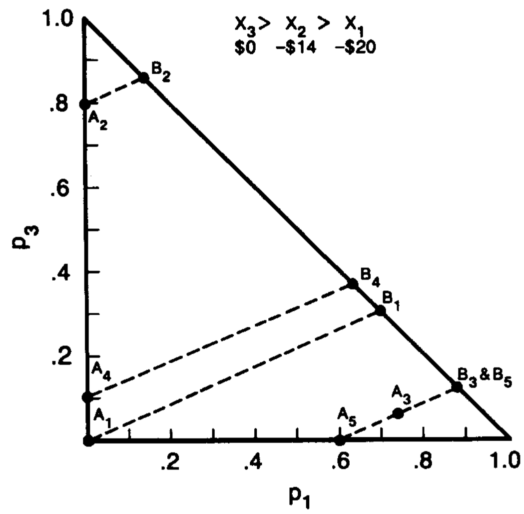

FIGURE 2. PROSPECTS EMPLOYED IN TESTS OF FANNING OUT WITH HUMAN SUBJECTS. PROSPECTS CORRESPOND TO THOSE SPECIFIED IN TABLE 3

The relatively superior performance of fanning out in the southeast portion of the unit probability triangle when the prospects are located on the edges of the triangle, compared to the interior, is characteristic of responses to other loss questions as well (Battalio, Kagel, and Jiranyakul, in press). It indicates that certain losses are much less attractive than uncertain losses with equal expected value when they provide some chance of escaping losses completely. Introducing a chance to escape losses on both alternatives promotes choice on the basis of the lowest loss alternative. This produces fanning out and classic Allais-type common ratio effects as in choice set 1, but it can also produce fanning in, as in choice set 2.1.[8]

[8]Battalio, Kagel, and Jiranyakul (in press) report mirror image responses for tests of fanning out over gains in the northwest corner of the unit probability triangle as well: Prospects located on the edges of the unit probability triangle are much more likely to satisfy weak fanning out compared to cases where both alternatives involve some possibility of zero gains. John Conlisk (1989) reports similar results as well.

IV. Summary and Conclusions

Results from two sets of experiments are reported, testing for fanning out of indifference curves over random prospects involving losses. For both rats and humans, classic Allais-type violations of expected utility theory are reported. However, new tests of fanning out in the southeast corner of the unit probability triangle show systematic fanning in over losses, contrary to Machina's (1982, 1987) H2. Lotteries in this corner of the loss triangle consist of highly positively skewed distributions over losses (i.e., most of the probability mass is on the left of the distribution, toward the maximum negative value), so that there is a "long left tail."

Other tests, over gains, for both human and animal subjects (Battalio, Kagel, and Jiranyakul, in press; MacDonald, Kagel, and Battalio, 1989) show comparable results: Allais-type violations of expected utility theory for standard manipulations, fanning in in previously unexplored areas of the unit probability triangle (in the northwest corner of the triangle in this case). These are largely negative results, as we do not offer an alternative model to explain the data. There are models that can do the job, however; in particular, prospect theory (Kahneman and Tversky, 1979) can organize these patterns of behavior (although it does not necessarily imply them). But more broad-based tests of these alternative theories show important deficiencies as well. (See Camerer, 1989; Battalio, Kagel, and Jiranyakul, in press; David W. Harless, 1987; and Chris Starmer and Robert Sugden, 1987, for comprehensive tests of a number of nonexpected utility models.) Hence, we hesitate to declare a winner in terms of which theory best organizes the data. Nevertheless, the results here are important, given the surprising power of H2 in organizing a number of established violations of expected utility theory (Machina, 1982, 1987) and the preference reversal phenomena (Safra, Segal, and Spivak, 1988). The results also establish some new stylized facts for theorists to organize in their efforts to develop alternatives to expected utility theory as a descriptive model of choice under uncertainty.

REFERENCES

Battalio, Raymond C., Kagel, John H. and MacDonald, Don N., "Animals' Choices Over Uncertain Outcomes: Some Initial Experimental Results," *American Economic Review*, September 1985, *75*, 597–613.

______, ______ **and Jiranyakul, Komain,** "Testing Between Alternative Models of Choice Under Uncertainty: Some Initial Results," *Journal of Risk and Uncertainty* (in press).

Becker, Gordon M., DeGroot, Morris H. and Marschak, Jacob, "Measuring Utility by a Single-Response Sequential Method," *Behavioral Science*, July 1964, *9*, 226–32.

Camerer, Colin, "An Experimental Test of Several Generalized Utility Theories," *Journal of Risk and Uncertainty*, April 1989, *2*, 61–104.

Chew, Soo Hong and MacCrimmon, Kenneth R., "Alpha-Nu Choice Theory: A Generalization of Expected Utility Theory," University of British Columbia Faculty of Commerce and Business Administration Working Paper No. 669, mimeo., 1979.

Conlisk, John, "Three Variants on the Allais Example," *American Economic Review*, June 1989, *79*, 392–407.

Green, Leonard, "Self-Control Behavior in Animals," in V. L. Smith, ed., *Research in Experimental Economics*, Vol. 2, Greenwich, CT: JAI Press, 1982.

Grether, David M., and Plott, Charles R., "Economic Theory of Choice and the Preference Reversal Phenomena," *American Economic Review*, September 1979, *69*, 623–38.

Hagen, Ole, "Towards a Positive Theory of Preferences Under Risk,"in M. Allais and O. Hagen, eds., *Expected Utility Theory and the Allais Paradox*, Dordrecht, Holland: D. Reidel, 1979.

Harless, David W., "Predictions About Indifference Curves in the Unit Probability Triangle: A Test of Some Competing Decision Theories," working paper, Indiana University, 1987.

Holt, Charles, "Preference Reversals and the Independence Axiom," *American Economic Review*, June 1986, *76*, 508–15.

Kagel, John H. and Green, Leonard, "Intertemporal Choice Behavior: Evaluation of Economic and Psychological Models," in L. Green and J. H. Kagel, eds., *Advances in Behavioral Economics*, Vol. 1, Norwood, NJ: Ablex Publishing, 1987.

______, **MacDonald, Don. N., Green, Leonard and Battalio, Raymond C.,** "Risk Preferences Over Losses in Rats: Responses to Variable Shock Levels and Delays to Reinforcement," 1988, mimeo., University of Pittsburgh.

Kahneman, Daniel and Tversky, Amos, "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica*, March 1979, *47*, 263–91.

Karni, Edi and Safra, Zvi, "'Preference Reversal' and the Observability of Preferences by Experimental Methods," *Econometrica*, May 1987, *55*, 675–85.

Killeen, Peter, "On the Measurement of Reinforcement Frequency in the Study of Preference," *Journal of the Experimental Analysis of Behavior*, May 1968, *11*, 263–69.

Lichenstein, Sarah and Slovic, Paul, "Reversals of Preference Between Bids and Choices in Gambling Decisions," *Journal of Experimental Psychology*, July 1971, *89*, 46–55.

MacDonald, Don N., Kagel, John H. and Battalio, Raymond C., "Animals' Choices Over Uncertain Outcomes: Further Experimental Results," mimeo., Northeast Louisiana University, 1989.

Machina, Mark J., "'Expected Utility' Analysis Without the Independence Axiom, " *Econometrica*, March 1982, *50*, 277–324.

______, "The Economic Theory of Individual Behavior Toward Risk: Theory, Evidence and New Directions," IMSSS Technical Report no. 433, Stanford University, 1983.

______, "Choice Under Uncertainty: Problems Solved and Unsolved," *Journal of Economic Perspectives*, Summer 1987, *1*, 121–54.

MacCrimmon, Kenneth R. and Larsson, Stig, "Utility Theory: Axioms Versus 'Paradoxes', " in M. Allais and O. Hagen, eds., *Expected Utility Hypotheses and the Allais Paradox*. Dordrecht, Holland: D. Reidel, 1979.

Markowitz, Harry M., *Portfolio Selection: Efficient Diversification of Investments*, New Haven, CT: Yale University Press, 1959.

Mazur, James E., "Fixed and Variable Ratios and Delays: Further Tests of an Equivalence Rule," *Journal of Experimental Psychology: Animal Behavior Processes*, 1986, *12*, 116–24.

Mossin, Jan, "A Note on Uncertainty and Preferences in a Temporal Context," *American Economic Review*, March 1969, *59*, 172–74.

Safra, Zvi, Segal, Uzi and Spivak, Avia, "Preference Reversals and Nonexpected Utility Behavior," *CARESS* working paper no. 88-19, University of Pennsylvania, 1988.

Starmer, Chris, and Sugden, Robert, "Violations of the Independence Axiom: An Experimental Test of Some Competing Hypotheses," Economics Research Center discussion paper no. 24, University of East Anglia, 1987.