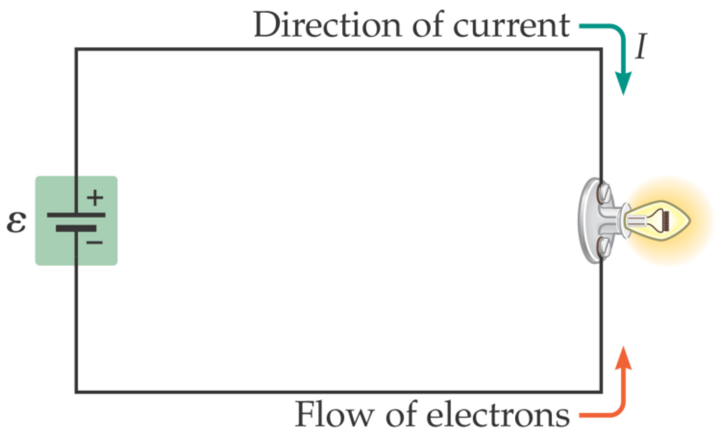
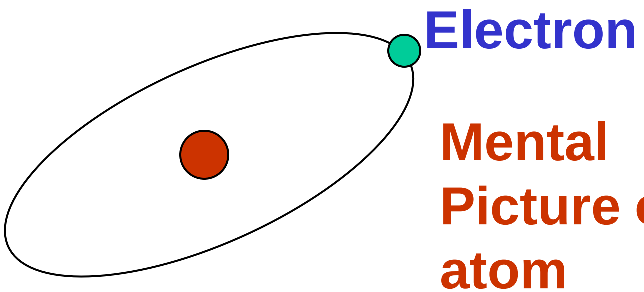
Chapters 21 and 22: Giancoli, 4^{th} Edition
Electrostatics

- Electric Charges
- Coulomb’s Law and Electric force
- The Electric Field
- Electric Field Lines
- Electric flux
- Gauss Law and applications of Gauss Law

Electric Charge

In electrical measurements, Current was defined in terms of movement of electrons. (Electrons have charge)





Mental Picture of an atom

The effects of electric charge were first observed as static electricity:

Static electricity is the build up of electric charges on the surface of some object or materials.

Static electricity is usually created when materials are pulled apart or rubbed together, causing positive (+) charges to collect on one material and negative (−) charges on the other surface.

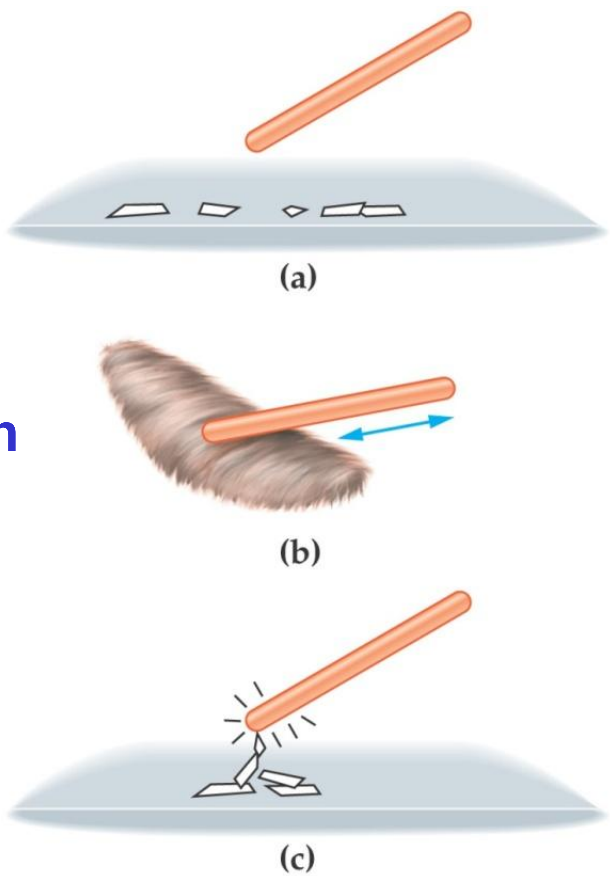
Results from static electricity may be sparks, shocks or materials clinging together or crackle of electricity as synthetic shirts or dresses are removed

Electric Charge

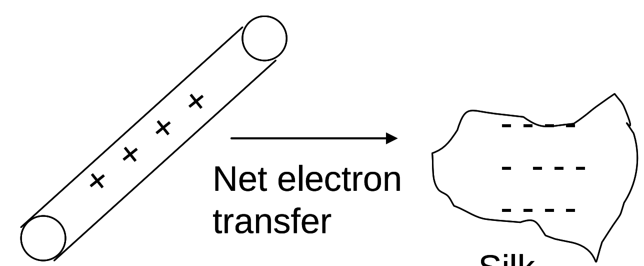
Static charges can be demonstrated through a number of experiments: including rubbing a pen on cloth and picking up paper, rubbing a balloon on cloth and placing it next to your hair etc.

What actually happens is that objects get charged as a result of electron transfer.

After being rubbed on a piece of fur, an amber rod acquires a charge and can attract small objects such as pieces of paper.



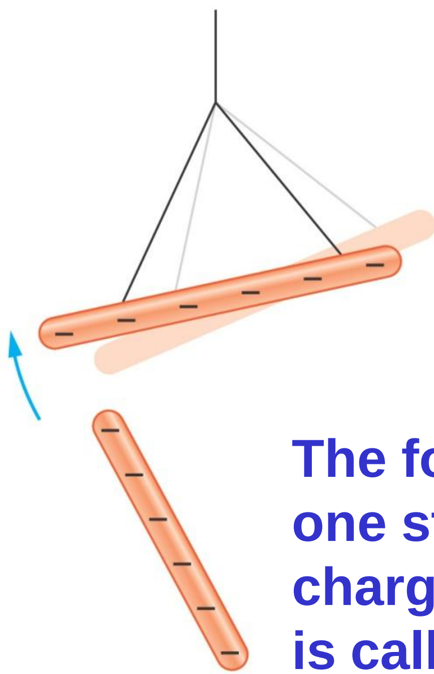
e.g. If a glass rod is rubbed with a piece of silk, electrons are transferred from the glass rod to the silk, thus the glass rod acquires a positive charge



Electric Charge

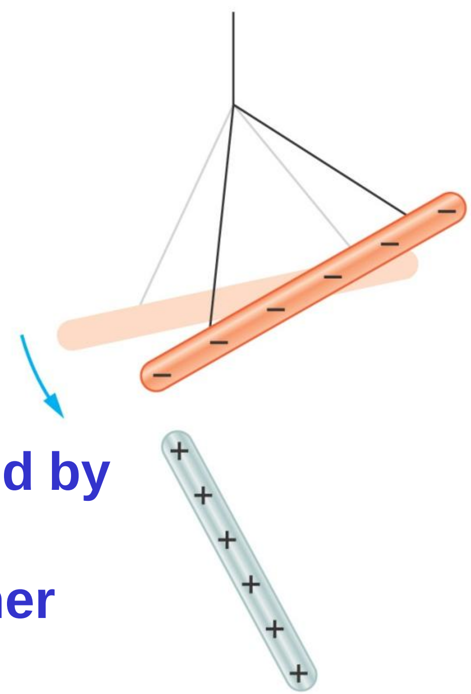
PARTICLE	CHARGE	MASS
Electron	-1.6×10^{-19} C	9.11×10^{-31} kg
Proton	$+1.6 \times 10^{-19}$ C	1.67×10^{-27} kg
Neutron	No charge	1.67×10^{-27} kg

Charging both amber and glass rods shows that there are two types of electric charge; **like charges repel and unlike charges attract.**



(a)

The force exerted by one stationary charge on another is called the electrostatic force.



(b)

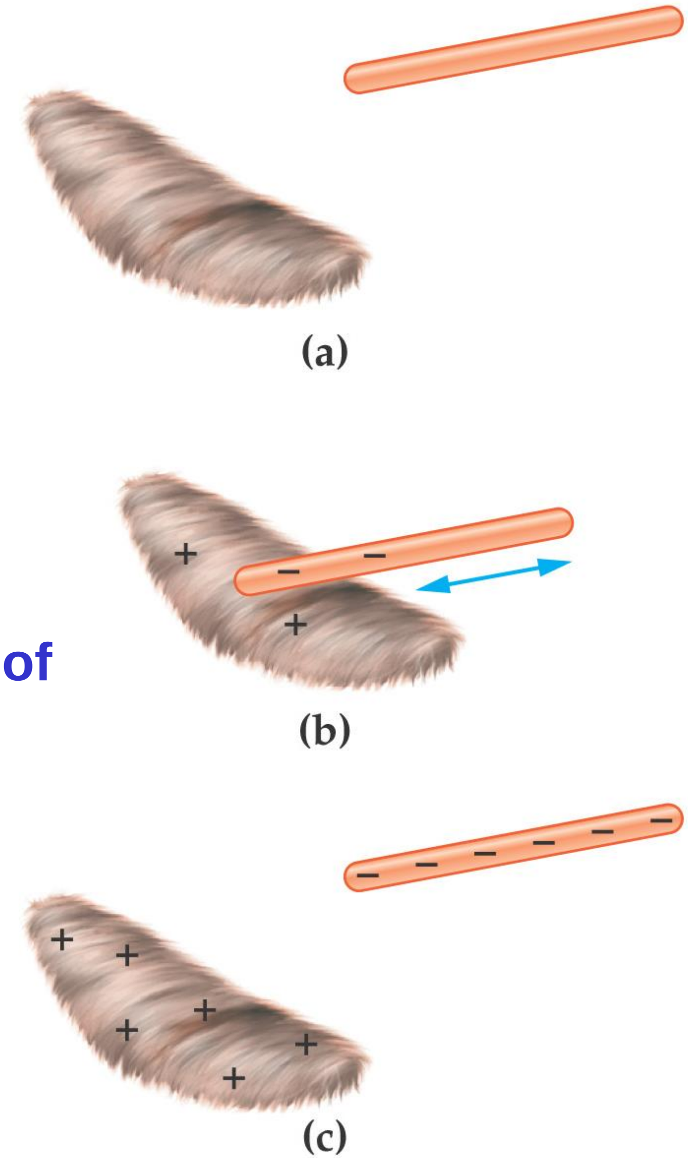
Electric Charge

All electrons have exactly the same charge; the charge on the proton (in the atomic nucleus) has the same magnitude but the opposite sign:

Magnitude of an Electron's Charge, *e*

$$e = 1.60 \times 10^{-19}\ \mathrm{C}$$

SI unit: coulomb, C



When an amber rod is rubbed with fur, some of the electrons on the atoms in the fur are transferred to the amber.

The atom that has lost an electron is now positively charged – it is a positive ion

The atom that has gained an electron is now negatively charged – it is a negative ion

Electric Charge

We find that the total electric charge of the universe is a constant:

Electric charge is conserved.

If the charge on an electron is *e* (the smallest unit of charge) the electric charge on a material will be some multiple of *e*. i.e

Electric charge is quantized in units of *e*. It comes in integer multiples of $e = 1.6 \times 10^{-19}$ C

i.e. $q = ne$ where $n = 0, \pm 1, \pm 2, ...$

Like Charges: Repel one another

Unlike Charges: Attract one another

Example:

Your jumper is neutral. You rub a pen on your jumper and the pen loses 10 electrons, which move to your jumper. What is the charge on your jumper?

$q = Ne$ = number of electrons x the charge of an electron

$= 10 \times -1.602 \times 10^{-19} = -16.02 \times 10^{-19}$ C

Coulomb's Law

Coulomb's law gives the force between two point charges:
The electrostatic force between two charges is directly proportional to the product of the two charges and inversely proportional to the square the distance between them.

$$F \propto q_1 q_2 \qquad F \propto \frac{1}{r^2}$$

$$F = k\frac{|q_1||q_2|}{r^2}$$

SI unit: newton, N

$$k = 8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

$$k = \frac{1}{4\pi\,\varepsilon_0}$$

$$\varepsilon_0 = 8.85\times 10^{-12}\ \mathrm{C^2/N\,m^2}$$

ε_0 = Permittivity of free space

The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if the charges are like.

Coulomb’s Law

This is similar to Newton’s law of gravitation:

Newton's Law of Universal Gravitation

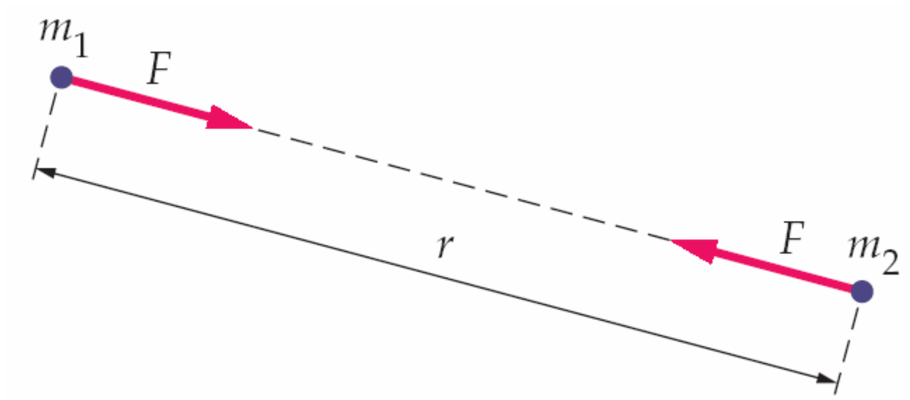
The force of gravity between any two point objects of mass m_1 and m_2 is attractive and of magnitude

$$F = G\frac{m_1 m_2}{r^2} \qquad \textbf{12–1}$$

In this expression, r is the distance between the masses and G is a constant referred to as the **universal gravitation constant**. Its value is

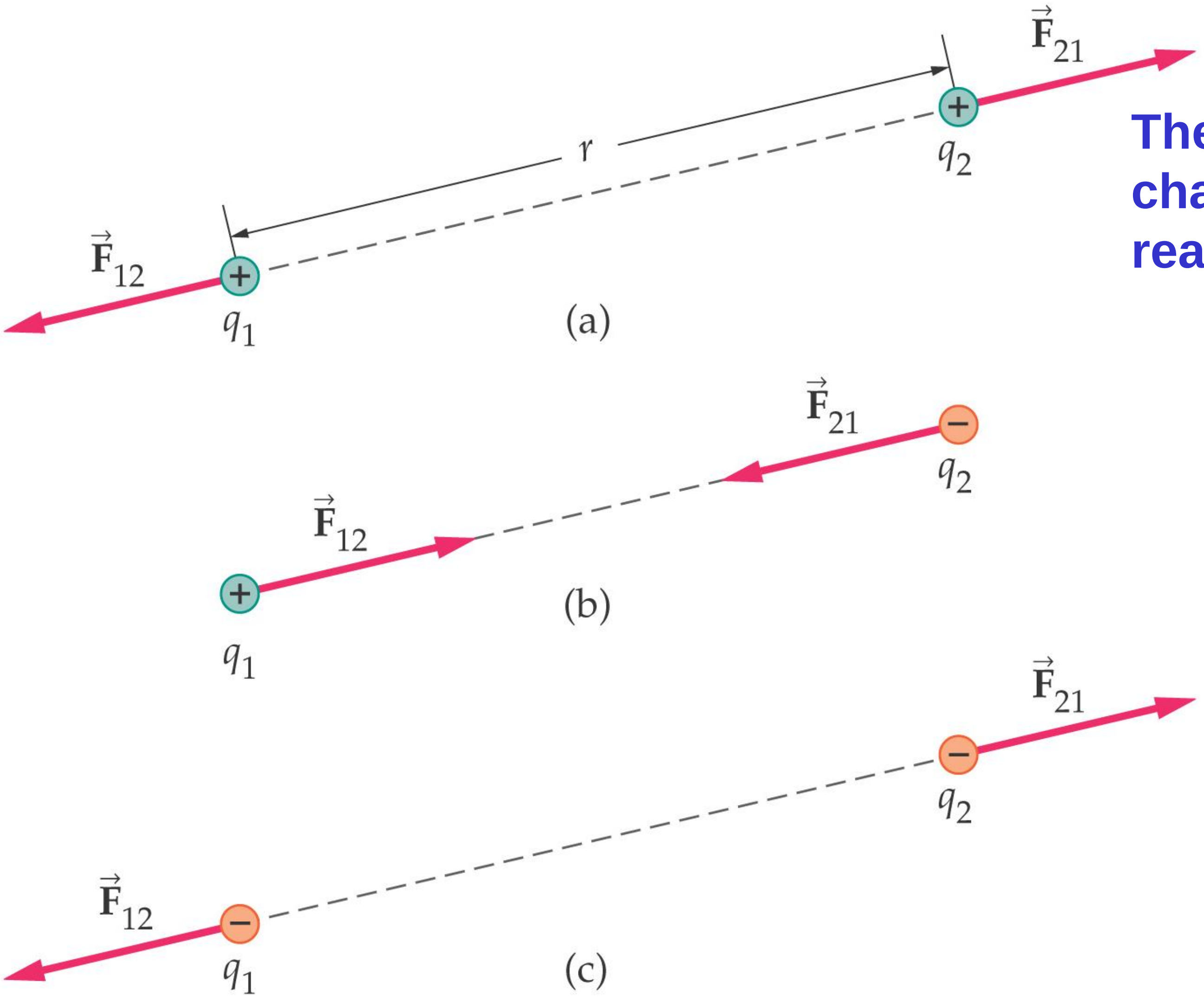
$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2} \qquad \textbf{12–2}$$

The gravitational force is always attractive, and points along the line connecting the two masses:



Coulomb's Law

The electrostatic force may be Attractive or Repulsive depending on the sign of the two charges:



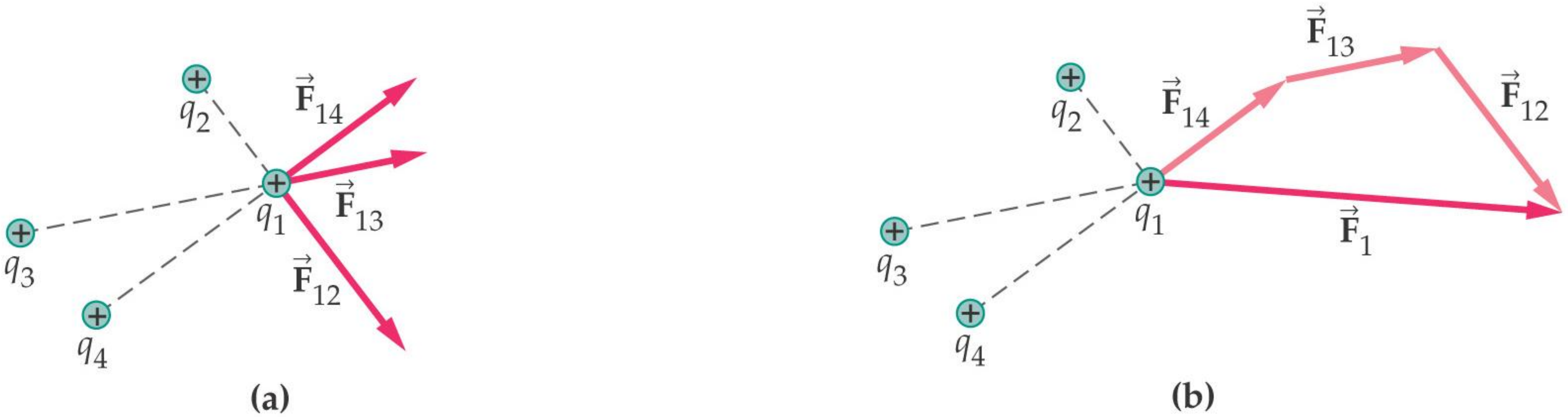
The forces on the two charges are action-reaction forces.

Forces have directions and thus are vector quantities.

The are added and subtracted like vectors

Coulomb's Law

If there are multiple point charges, the forces add by superposition (F_{net} is the vector sum of individual forces) [Join head to tail as shown in the figure (b)].



Example: Calculate the ratio of the Electrostatic force to the Gravitational force for two protons separated by a distance r.

$$\frac{|F_E|}{|F_G|} = \frac{kq_1q_2/r^2}{Gm_1m_2/r^2} = 10^{36}$$

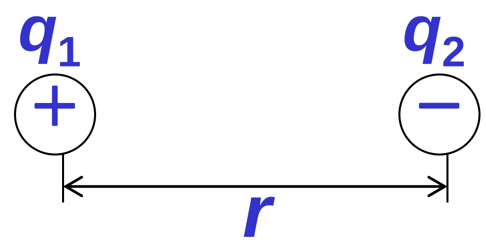
Coulomb's Law

Using Coulomb's Law:

We use Coulomb's law the same way we used the Newton's universal law of Gravitation. Two charges a distance, *r*, away will be stated and the force between them will need to be calculated.

Some examples:

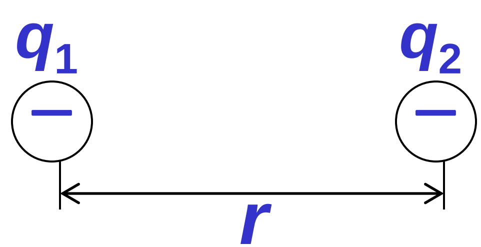
1.



$q_1 = 1$ C, $q_2 = -2$ C, $r = 20$ cm, Find F_{12}

4.5×10^{11} N

2.



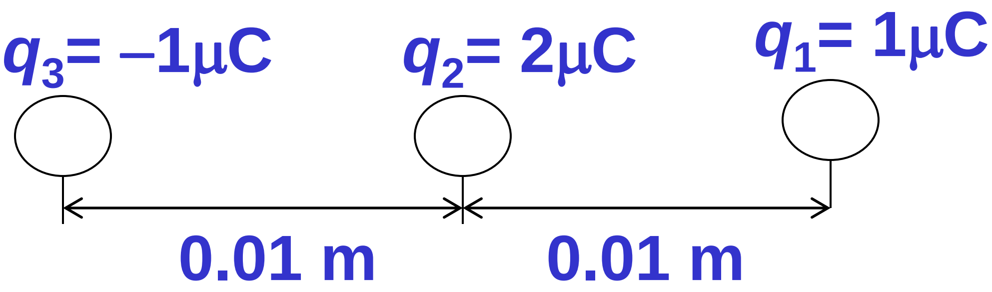
$q_1 = -2$ mC, $q_2 = -5$ mC, $r = 5$ m,

Find F_{21}

3600 N

Coulomb's Law

Example 3



Calculate the net electrostatic force on charge q_1

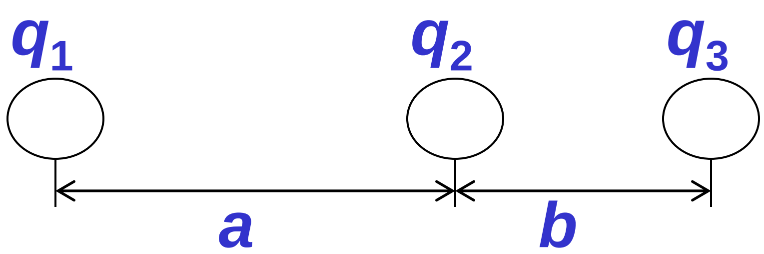
$F_{12} = 180\ N$

$F_{13} = 22.5\ N$

$F_{q1} = 157.5\ N$

Coulomb's Law

4.



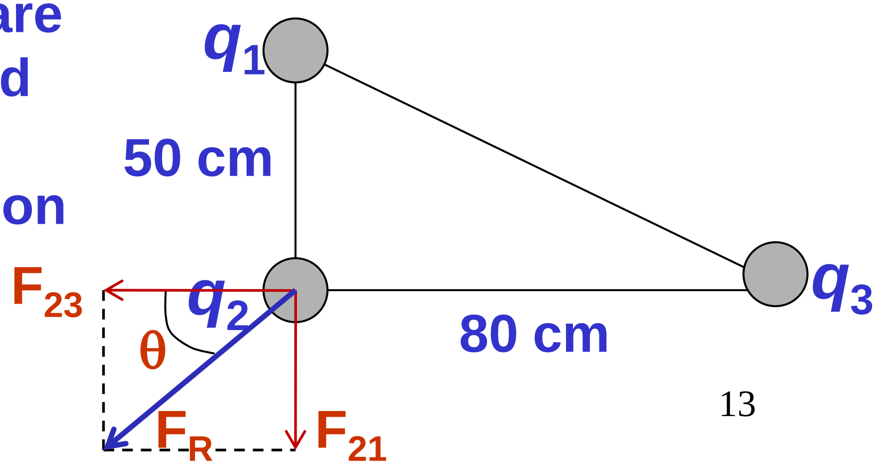
a = 120 cm, b = 90 cm,
q_1 = 15 μC, q_2 = –2 μC and q_3 = 10 μC

Find F_{21} and F_{23}. What is the net electrostatic force on charge q_2?

5. Three charges of magnitude 5 μC are placed at the corners of a right angled triangle as shown.
Find the resultant electrostatic force on charge q_2.

F_{21} = 0.90 N ↓ F_R = 0.97 N

F_{23} = 0.35 N ← θ = 68.7°



The Electric Field

Definition of the electric field Strength: **Electric field strength at any point is defined as the force on a unit positive test charge placed at that point in the field.**

$$\vec{E} = \frac{\vec{F}}{q_0}$$

SI unit: N/C

Compare with Gravitational field:

$$\vec{g} = \frac{\vec{F}}{m}$$

Here, q_0 is a "test charge" – it serves to allow the electric force to be measured, but is not large enough to create a significant force on any other charges in the near vicinity.

To determine electric field strength some distance, *r*, away from a charge, *q*:

(i) We place a small positive test charge, q_0, at that point.

(ii) Use Coulomb's law to calculate the force, *F*, on the test charge.

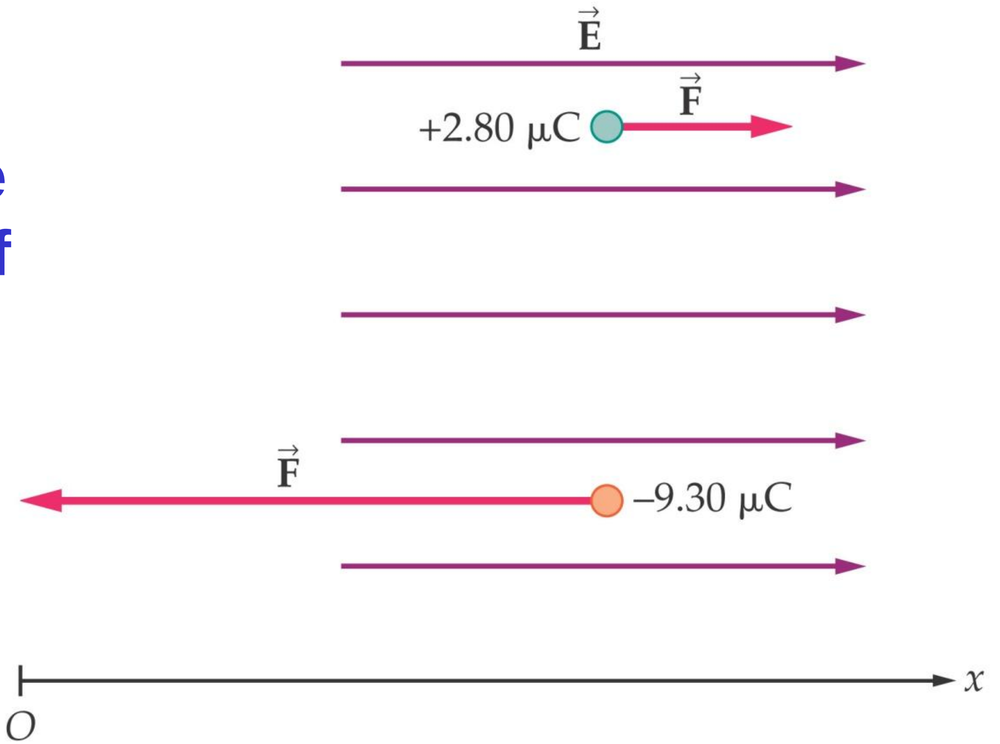
(iii) Use $E = F/q_0$ to calculate the electric field strength.

The Electric Field

If we know the electric field, we can calculate the force on any charge:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

The direction of the force depends on the sign of the charge – in the direction of the field for a positive charge, opposite to it for a negative one.



The Electric Field

The electric field of a point charge points radially away from a positive charge and inwards towards a negative one.

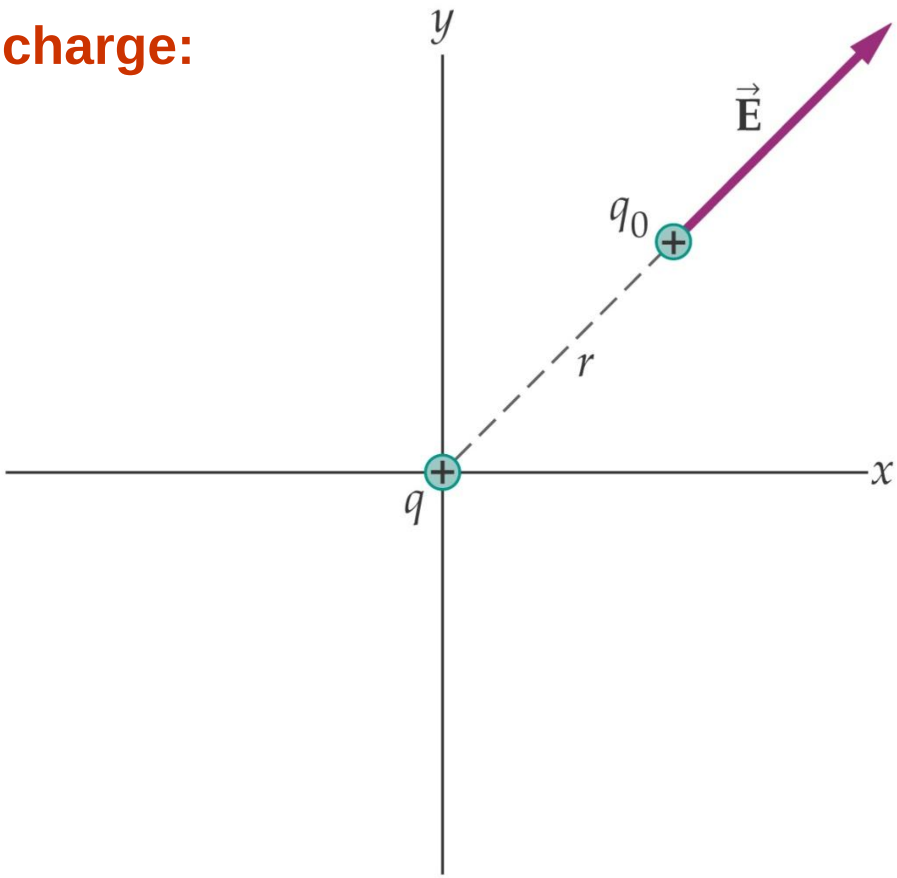
Electric field due to a point charge:

Given:

$$F = \frac{k_e q_1 q_0}{r^2}$$

$$E = \frac{F}{q_0}$$

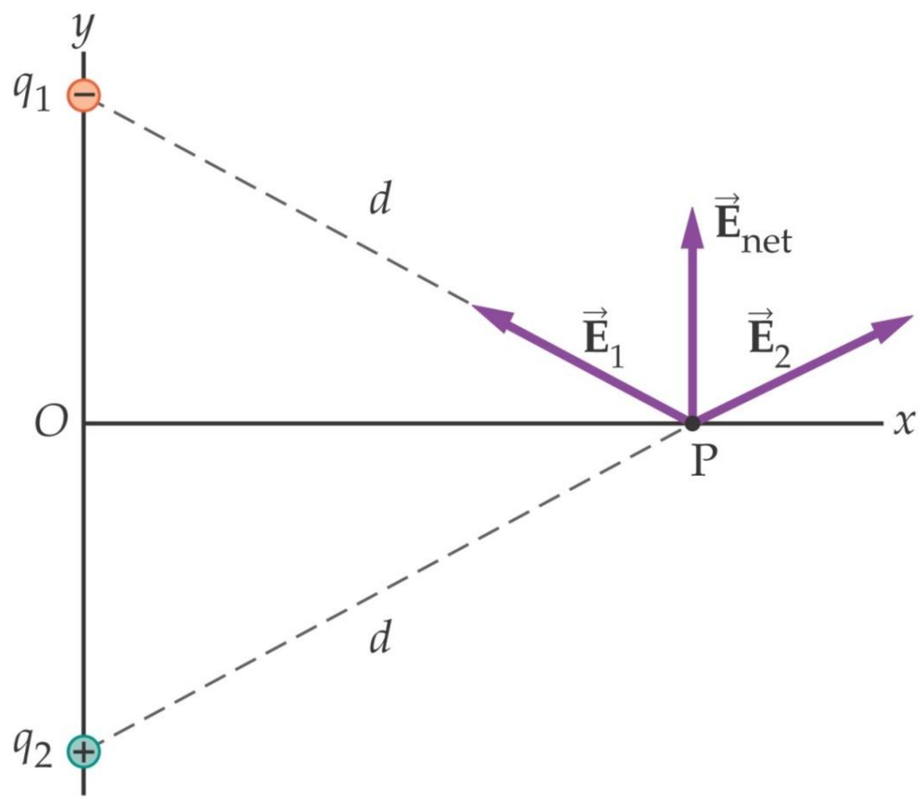
$$E = \frac{k_e q_1}{r^2}$$



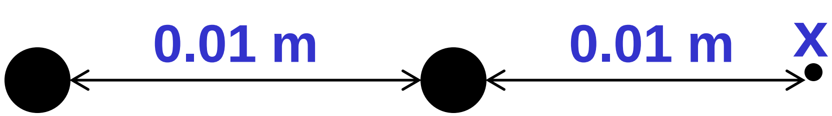
Like Electric force, Electric field is a vector having the same direction as force.

The Electric Field

Just as electric forces can be superposed, electric fields can as well. Net electric, E_{net}, field is the vector sum of the individual fields.



EXAMPLE: Calculate the electric field strength at position x.

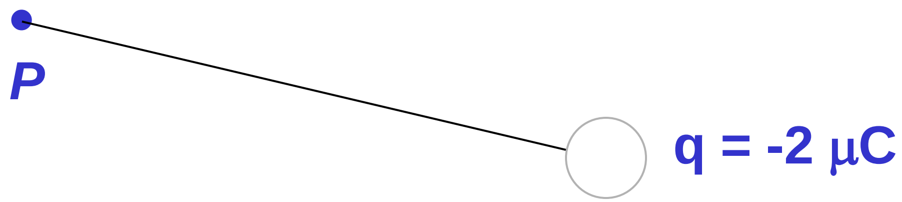


$q_1 = -1\mu C$ $q_2 = 2\mu C$ $E_X = 1.6 \times 10^8$ N/C

The Electric Field

Exercises:

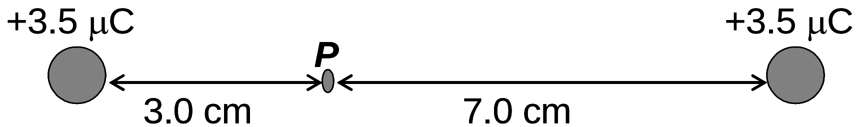
1. Find the electric field at point *P*, which is 0.75 m away from a –2 μC charge as in the arrangement below:



2. A test charge is placed 2 m from a charged particle (*q* = - 3 μC). The test charge has a magnitude of charge of 3 nC.
   (i) Is the test charge a positive or negative charge?
   (ii) What is the force exerted on the test charge?
   (iii) Hence what is the electric field at the test charge?

3. Find the electric force on a proton placed in an electric field of 2×10^4 N/C directed horizontally to the right.

4. (i) Determine the magnitude and direction of the electric field at point *P* shown in the diagram, (ii) Where would the electric field strength be zero?



Electric Field Lines

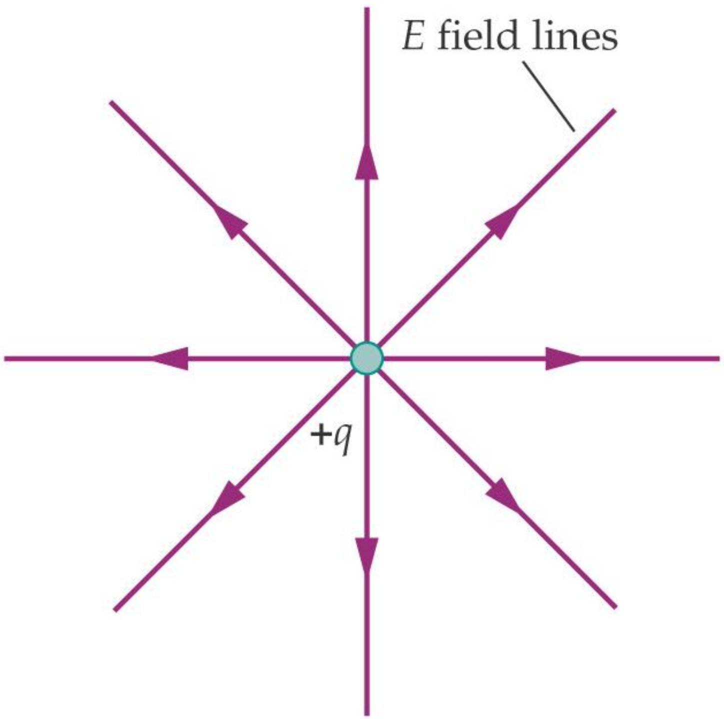
Electric field lines are a convenient way of visualizing the electric field. These are just imaginary lines. The direction of field, *E*, at any point is tangent to the field lines.

Electric field lines:

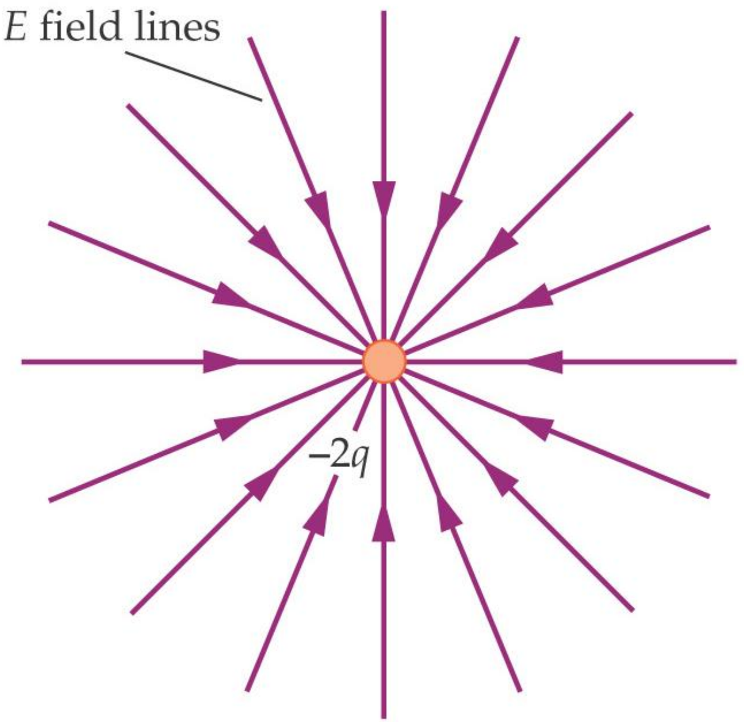
1. **Point in the direction of the field vector at every point**
2. **Start at positive charges or infinity**
3. **End at negative charges or infinity**
4. **Are more dense where the field is stronger**

Electric Field Lines

The charge on the right is twice the magnitude of the charge on the left (and opposite in sign), so there are twice as many field lines, and they point towards the charge rather than away from it for negative charge.



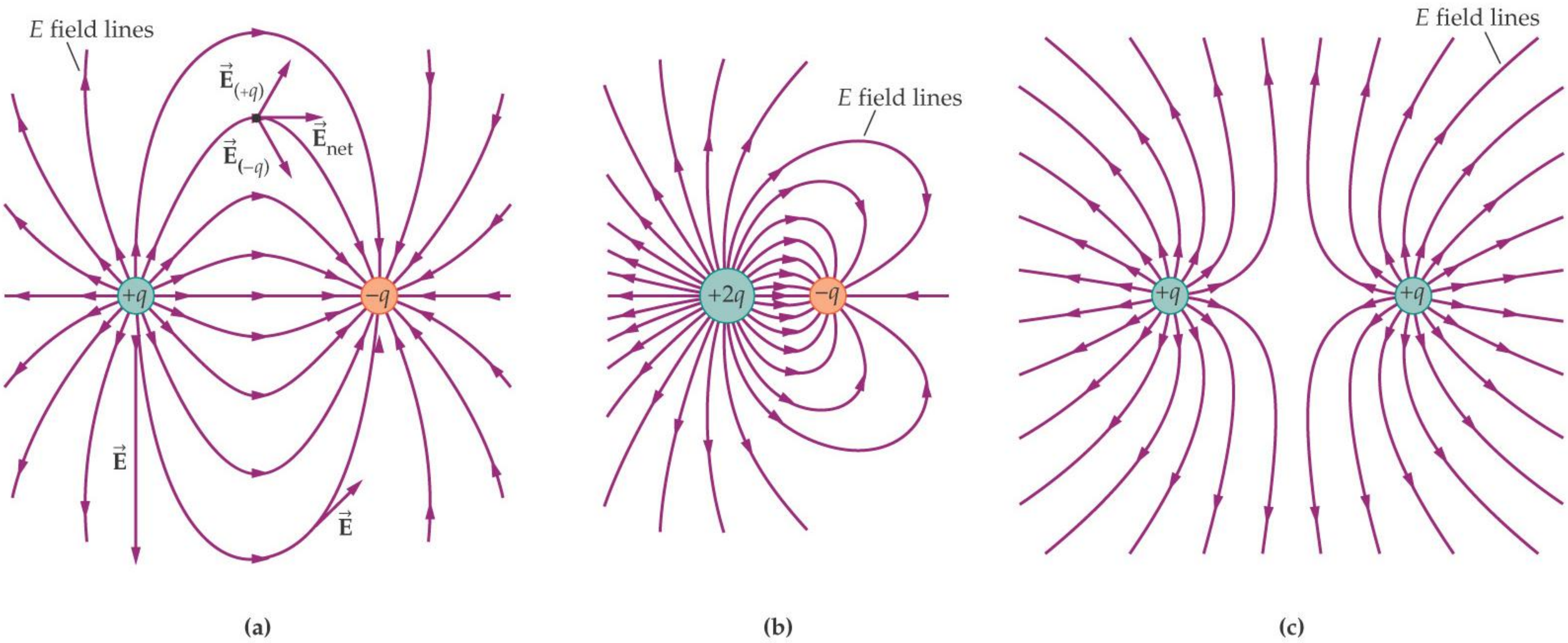
(a)



(b)

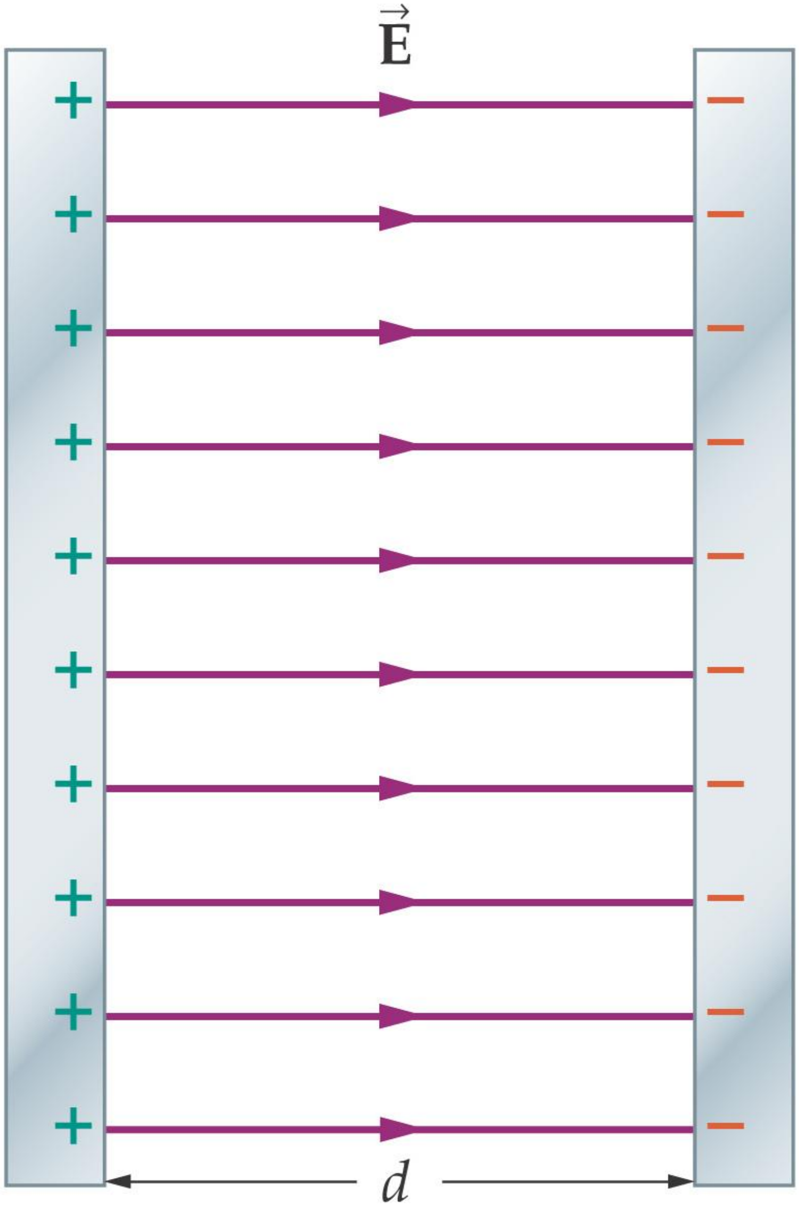
Electric Field Lines

Combinations of charges. Note that, while the lines are less dense where the field is weaker, the field is not necessarily zero where there are no lines. In fact, there is only one point within the figures below where the field is zero – can you find it?



Electric Field Lines

A parallel-plate capacitor consists of two conducting plates with equal and opposite charges on each plate. Shown is the electric field between the plates:

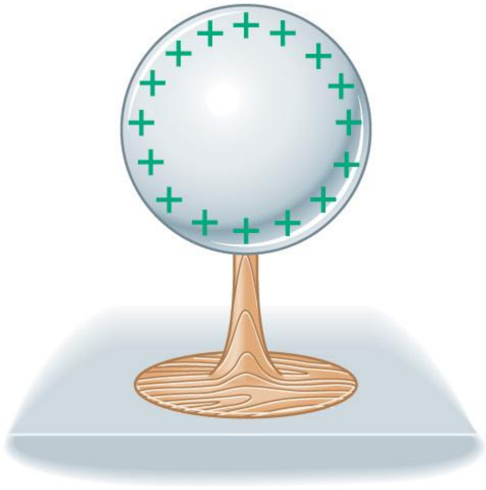


Electric Field and Conductors

Any excess charge placed on a conductor is free to move, the charges will move so that they are as far apart as possible.

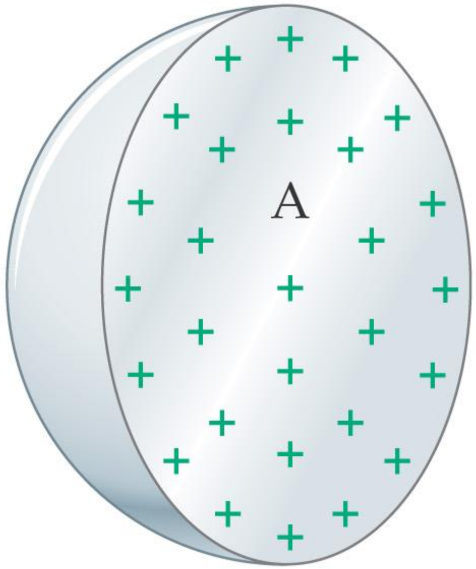
$\Rightarrow$ Electric force (F) and hence Electric field (E) = 0

This means that excess charge on a conductor resides on its surface, as in figure (a) and E = 0 inside a conductor.



(a)

(a) A charge placed on a conducting sphere distributes itself uniformly on the surface of the sphere; none of the charge is within the volume of the sphere.

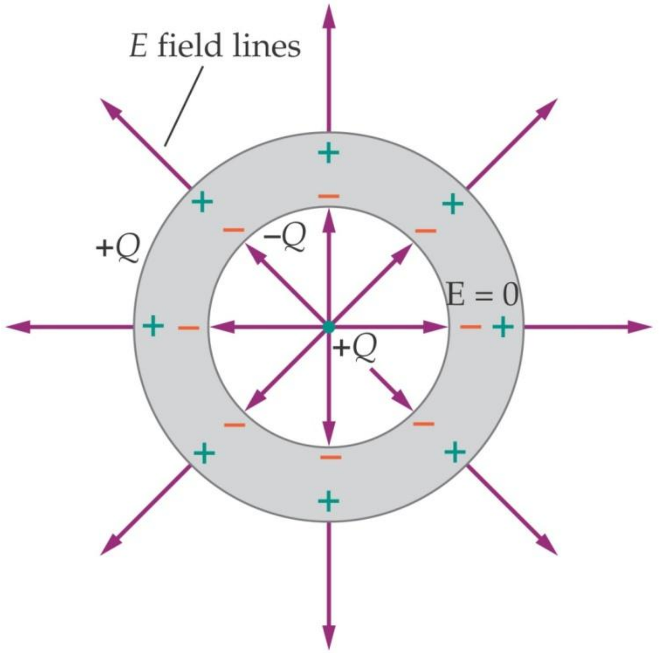


(b)

(b) If the charge were distributed uniformly throughout the volume of a sphere, individual charges, like that at point A, would experience a force due to other charges in the volume. Since charges are free to move in a conductor, they will respond to these forces by moving as far from one another as possible; that is, to the surface of the conductor.

Electric Field and Conductors

When electric charges are at rest, i.e. in the static situation the electric field, *E*, within a conductor is zero.

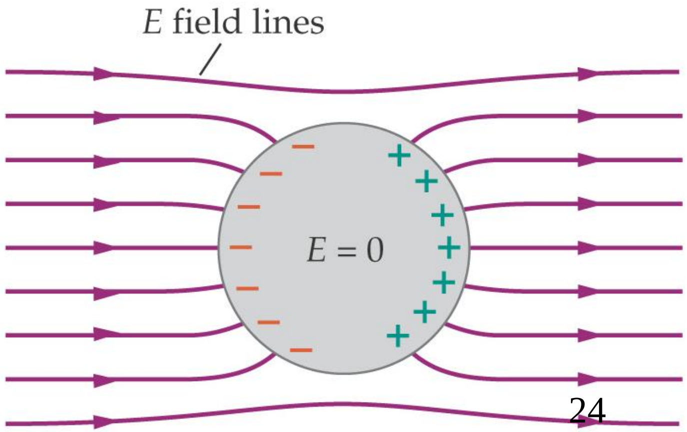


A charge +*Q* placed inside a neutral metal spherical shell induces an equal amount of -ve charge –*Q* on the inner surface of the metal shell

Since the shell is neutral, a +ve charge of magnitude +*Q* will be induced on the outer surface of the shell

Thus although *E* = 0 within the metal shell, an electric field exists outside of it

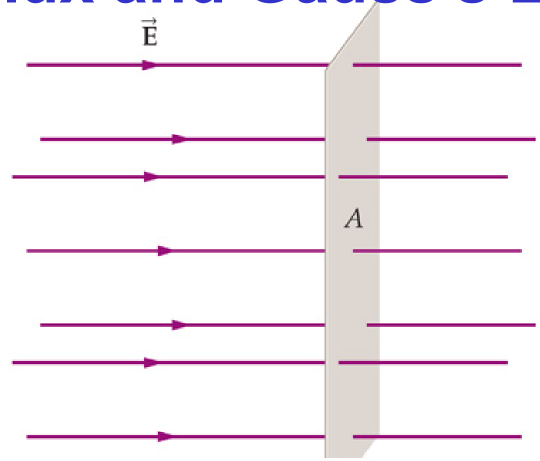
The electric field is always perpendicular to the surface of a conductor – if it weren't, the charges would move along the surface.

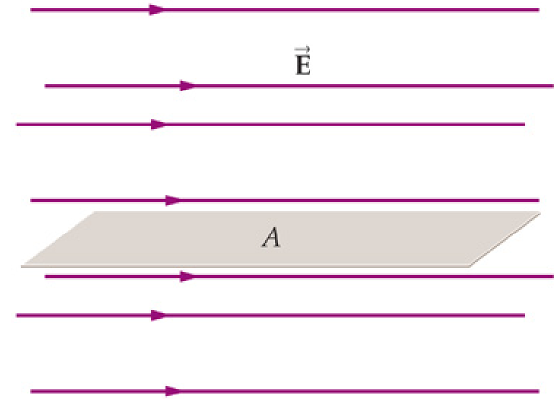


(a)

Electric Flux and Gauss's Law

Electric flux is a measure of the electric field perpendicular to a surface or the electric field passing through a given area .





Electric flux is defined as the product of Electric field perpendicular to surface area, A, and the area, A.

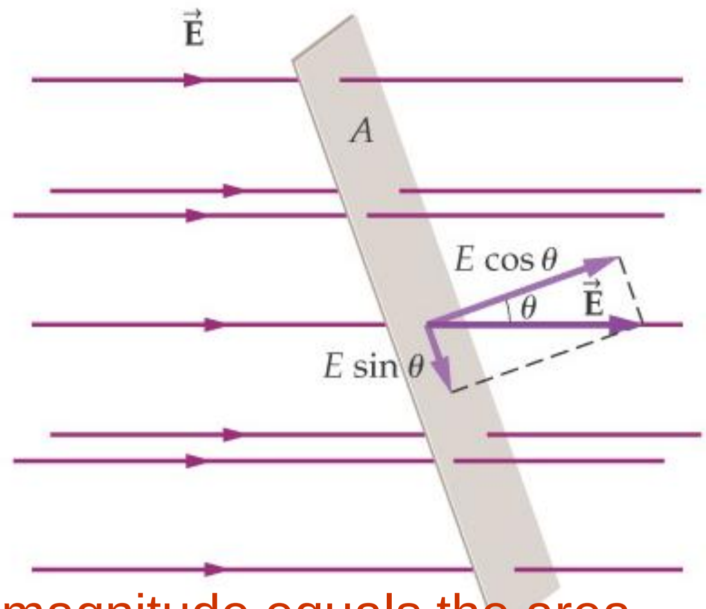
Definition of Electric Flux, Φ

$$\Phi = EA\cos\theta$$

SI unit: $\mathrm{N \cdot m^2/C}$

$$\Phi = E \bullet A$$

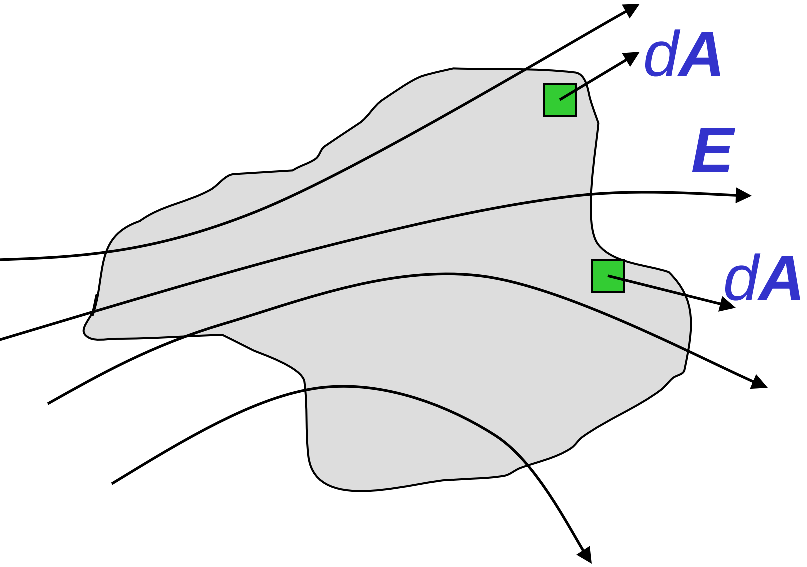
Vector dot product



We define a surface area vector, ***A*,** whose magnitude equals the area of the surface and whose direction is perpendicular to the surface. For small areas, the surface area vector becomes Δ***A*,** and in the limit as Δ***A*** $\rightarrow$ 0, surface area vector is represented by d***A***

Electric Flux and Gauss's Law

What about an Irregular surface in a Non-uniform electric field



We split the surface into tiny areas represented by surface area vector ***dA*, so that *E*** is constant in magnitude and direction within small area ***dA***

$$d\Phi = E \bullet dA$$

$$\Phi = \int E \bullet dA$$

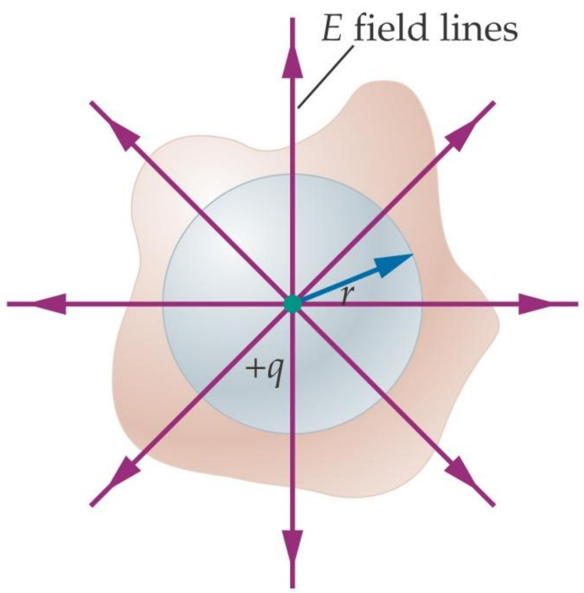
We will consider surfaces which are completely closed. – Gaussian Surfaces: Real or Imaginary surfaces.

$$\Phi = \oint E \bullet dA$$

Flux is positive if there is a net flux out of the Gaussian surface

Flux is negative if there is a net flux into the Gaussian surface

Electric Flux and Gauss's Law



Consider an isolated point charge, q, at the centre of an imaginary sphere of radius r, (the Gaussian surface, i.e the closed surface we choose to apply Gauss law).

Gauss's law states that the electric flux through a closed surface is proportional to the net charge enclosed by the surface.

$$\Phi \propto q$$ **Charge inside G.Surface**

$$\Phi = \frac{q}{\varepsilon_0}$$

$$\Phi = \oint E \bullet dA = \frac{q}{\varepsilon_0}$$ **Gauss's Law**

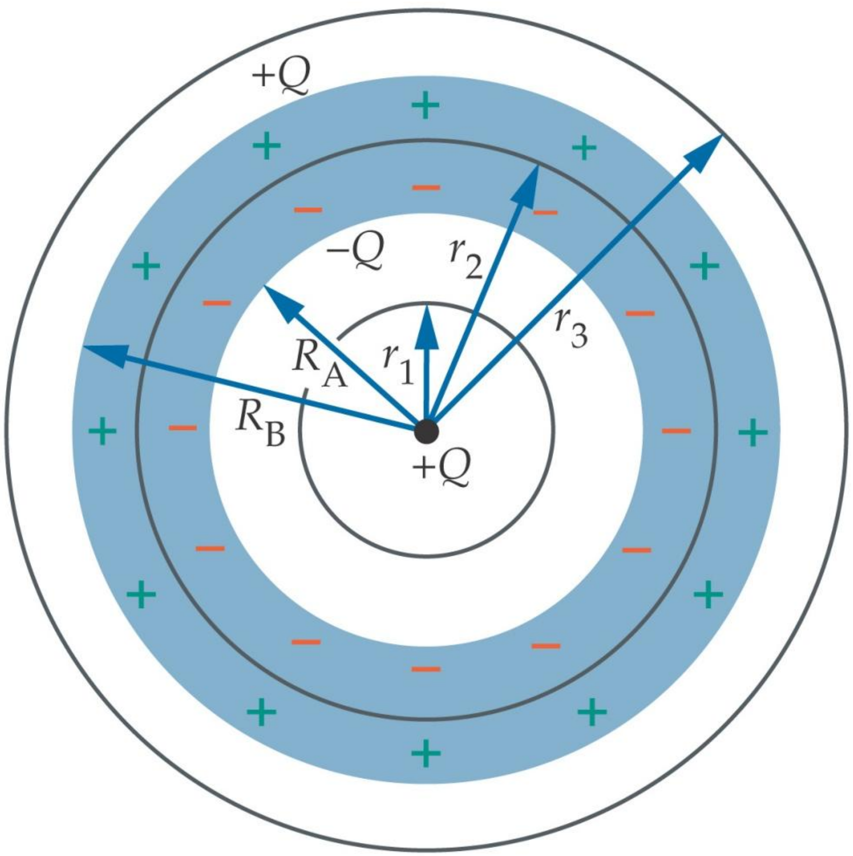
SI unit: N m^2/C

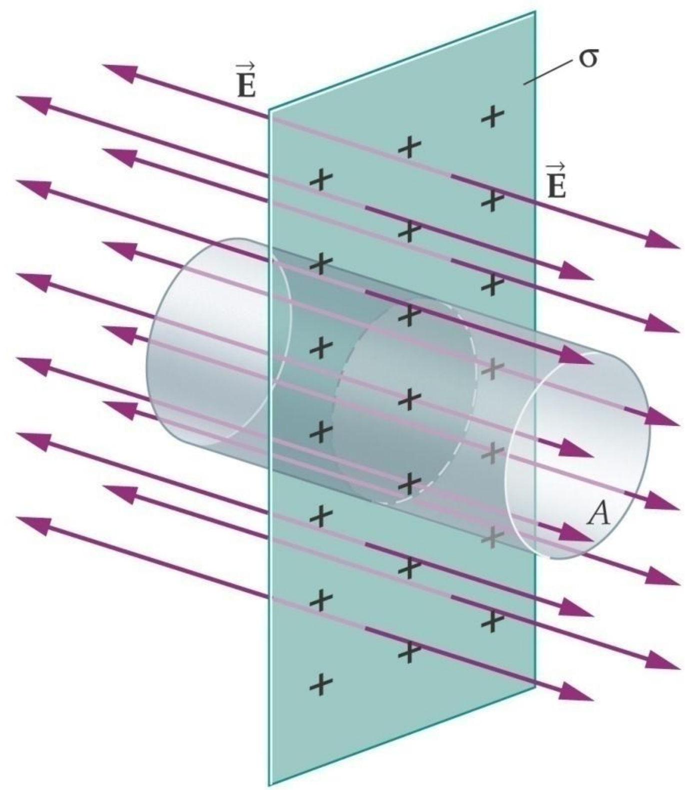
$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

ε_0 = permittivity of free space

Applications of Gauss's Law

Gauss's law can be used to find the electric field in systems with simple configurations. Is useful when charge distribution exhibits symmetry, i.e. Spherical Symmetry, Plane Symmetry and Radial Symmetry.





Applications of Gauss's Law

Gauss's law can be used to find the electric field in systems with simple configurations. Is useful when charge distribution exhibits symmetry, i.e. Spherical Symmetry, Plane Symmetry and Radial Symmetry.

$$\Phi = \oint E \bullet A = \frac{q}{\varepsilon_0}$$

When applying Gauss's Law, we carefully choose the Gaussian surface so that we can simplify the integral on left side of Gauss's Law and determine, electric field, *E*.

The Gaussian surface should be such that:

(i) $\vec{E}$ **and** $\vec{dA}$ **are parallel to each other (or both perpendicular to the surface) at each point, so we can remove the dot product, since cos**θ **=1**

(ii) $\vec{E}$ **is constant in magnitude and direction on the Gaussian surface, so it can be taken outside the integral**

Applications of Gauss's Law

Use Gauss's Law to derive an expression for the Electric field strength, *E*, at a distance, *r*, from a point charge *Q*