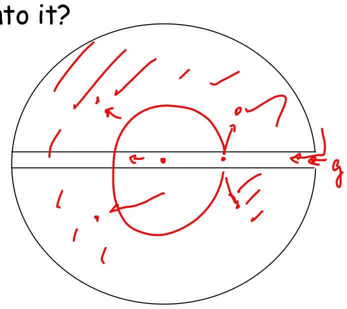
Physics 101 – Lecture 18
Vibrations, SHM, Waves (II)

Reminder: simple harmonic motion is the result if we have a restoring force that is linear with the displacement:

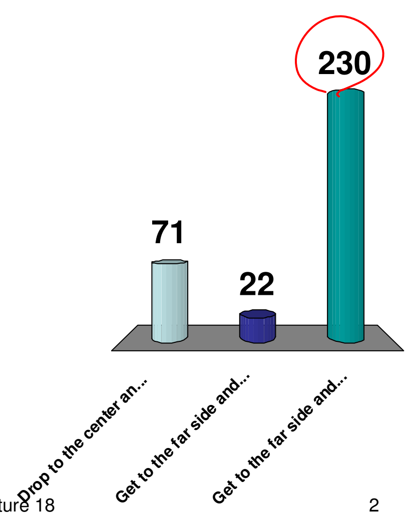
$$F = -k\,x$$

What would happen if you could dig a hole through the center of the Earth and drop something into it?

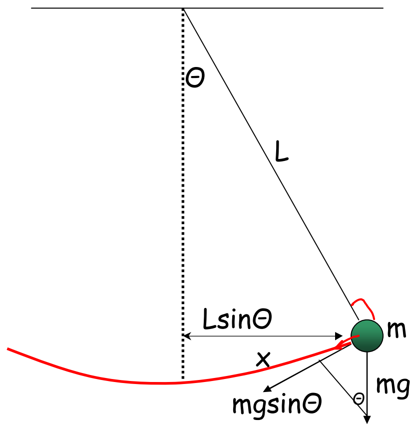


The object dropped into the hole to the center of the Earth (ignore air resistance) would:

1. Drop to the center and stay there
2. Get to the far side and stay there
3. Get to the far side and drop back in



A pendulum represents one of the simplest examples of SHM:



Displacement along arc:

$x = L\Theta$

Restoring force:

$F = -\,mg\sin\Theta$

But for small angles:

$\Theta \approx \sin\Theta$

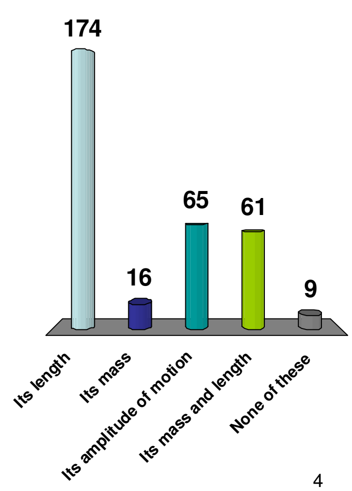
So:

$$F = -\underbrace{(mg/L)}_{\text{“k”}}\,x$$

We will get SHM !

The period of a pendulum depends on:

1. Its length
2. Its mass
3. Its amplitude of motion
4. Its mass and length
5. None of these



From our previous formalism we immediately see:

- the period T will be:

  $T = 2\pi \sqrt{(m/k)} = 2\pi \sqrt{(mL/mg)} = \boxed{2\pi \sqrt{(L/g)}}$

- the frequency is

  $f = 1/T = (1/2\pi) \sqrt{(g/L)}$

- the period does NOT depend on the amplitude, NOR on the mass (first noticed by Galileo in the apocryphal lamp story)

In fact, the amplitude does matter slightly at large amplitudes; remember we made the small angle (ie, small amplitude) approximation in our derivation.

Giancoli 11-29: what length of pendulum is necessary for a swing every second; ie, T = 2.0 sec.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2.0 \text{ sec}$$

$$L = \left(\frac{2.0}{2\pi}\right)^2 g = 9.8\left(\frac{1}{\pi}\right)^2 \simeq \underline{0.997 \text{ m}}$$

(at one point proposed as the definition of the meter !)

Resonance (forced vibrations)

We've seen that SHM such as springs or pendula have natural oscillation frequencies:

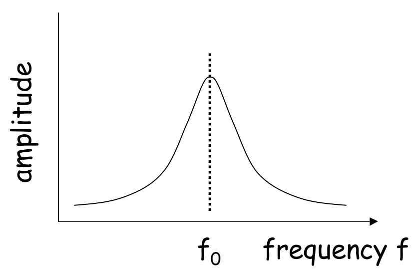
$$f_{spring} = (1/2\pi)\sqrt{(k/m)}$$
$$f_{pendulum} = (1/2\pi)\sqrt{(g/L)}$$

If you nudge such a system we will have SHM at the 'natural' frequency f_0.

What happens if we 'drive' it by applying a periodic impulse or force at some arbitrary frequency f ? Then in general we will see that it will oscillate at that frequency f, even if it is different from f_0.

However, the amplitude of the oscillation will depend on the difference between f and f_0, and will be largest when $f = f_0$.

Graphically:

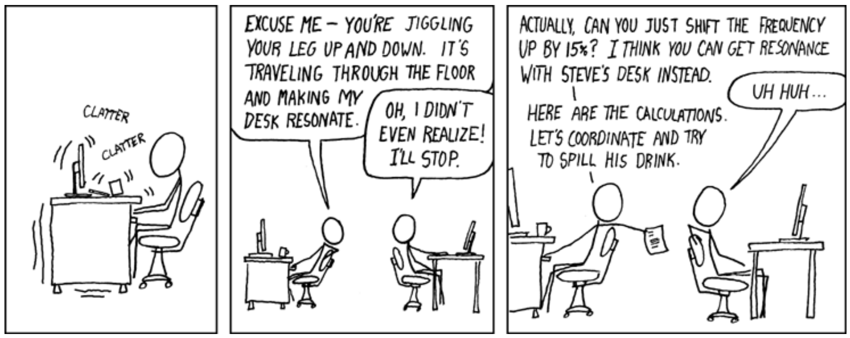


We call this resonance, and the natural frequency f0 is often called the resonant frequency.

Sometimes this phenomenon can be very useful, and sometimes very destructive:

- Tacoma Narrows Bridge, 1940
- feedback in electrical circuits
- resonance in musical instruments

Resonance can have practical uses:



xkcd.com

<u>Summary of SHM relations:</u>

- SHM results in sinusoidal curves for displacement, velocity, acceleration:

$\{x,v,a\}$ = (amplitude) · cos(ωt + phase)

In general:

$\omega = 2\pi f = 2\pi/T$

$T = 1/f$

For a spring:

$T = 2\pi \sqrt{m/k}$ [no dependence on amplitude A]

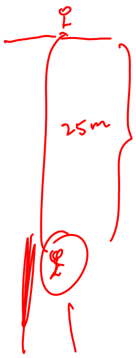
$v_{max} = 2\pi Af = A\omega = A\sqrt{k/m}$

$a_{max} = v_{max}^2/A = \omega^2 A = (2\pi f)^2 A$

For a pendulum at small amplitude:

$T = 2\pi \sqrt{L/g}$ [no dependence on amplitude nor mass]

Giancoli 11-27: Bungee jumper, 65.0 kg. After jumping, he oscillates, hitting low point 8 times in 38.0 s. Finally comes to rest 25.0 m below bridge. What is k, and unstretched length of cord?



$f = 8/38.0\ \text{sec} = \frac{1}{4.75}\ \text{sec}^{-1}$

a) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = (2\pi f)^2 m$

$= (2\pi/4.75)^2\ 65.0$

$k = 114\ \text{N/m}$

b) $kx = mg \quad x = mg/k = \frac{65 \cdot 9.8}{114}\ \text{m} = \underline{5.6\ \text{m}}$

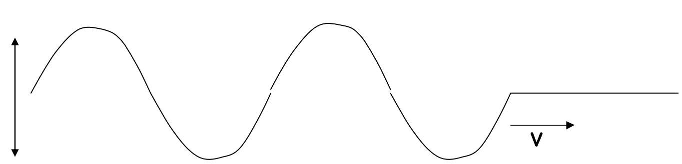
FBD: $\uparrow ky$, $\downarrow mg$

unstretched length : $\underline{19.4\ \text{m}}$

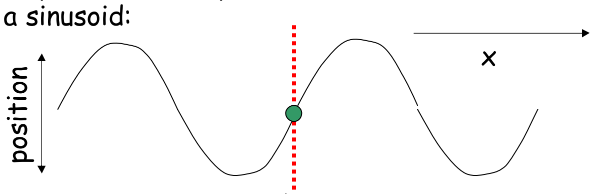
Waves and wave motion

Let's now turn to a discussion of waves, a subject that is linked to oscillations and vibrations. In fact, the sources of waves are always vibrations/oscillations of some sort. If the source is vibrating with SHM, then the wave that is generated will have a sinusoidal shape, both in space and in time.

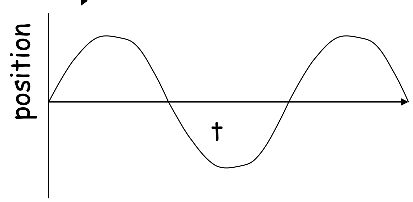
For example, consider a rope that we jiggle up and down in an oscillatory fashion:



The picture represents a snapshot in time, and we see that spatially the wave is a sinusoid:



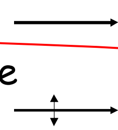
At the same time, we can consider any individual point on the rope over time, we see that it oscillates: and this is again SHM with a sinusoidal form:

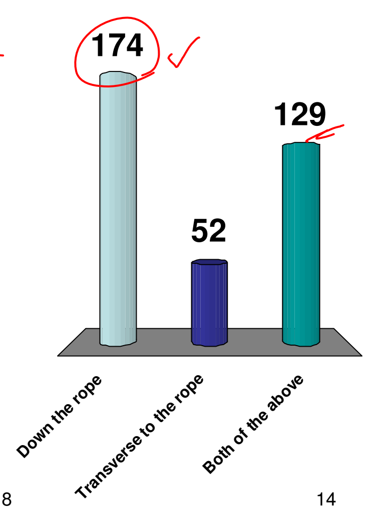


In a wave on a rope,
which way is the wave moving?

129 93 153

1. Down the rope
2. Transverse to the rope
3. Both of the above





In a wave on a rope, which way is the rope moving?

1. Down the rope
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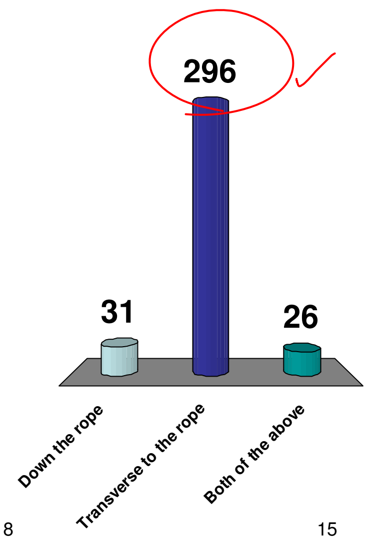


Note the key difference here between the motion of the wave, and the motion of the medium:

The wave propagates (moves) along the rope, but the rope moves side-to-side (tranversely). This is called a transverse wave.

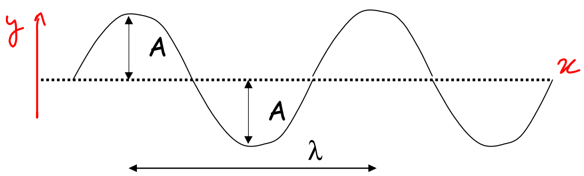
The medium does not move in the same way as the wave !

The same thing can be seen in water waves, where the wave moves across the water surface (horizontally) but a cork floating in the wave bobs vertically. Waves like this do NOT transport matter/mass (although they can transport energy).

The wave velocity in a stretched string or cord depends on only two quantities: the string tension F_T and the mass-per-unit-length of the string, m/L;

$$v = \sqrt{(F_T/[m/L])}$$

A tighter cord has a higher wave velocity, and a light cord has a higher wave velocity than a heavy cord.



We describe waves with the same language we've already seen for oscillations:

T = period
= time for two successive crests to pass the same point
f = frequency = 1/T
= nb. of crests that pass a point in a given time
A = amplitude = distance to peak from equilibrium point

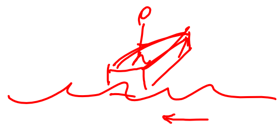
and also:

λ = wavelength = distance between two crests

The wave velocity is

$$\boxed{v = \lambda f}$$

Example: Giancoli 11-36: a fisherman notices that waves pass his boat every 3.0 seconds and the distance between crests is 6.5 m. How fast are the waves moving?



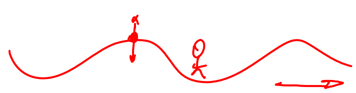
$v = \lambda f$ $\qquad f = \frac{1}{3.0\ sec}$

$= (6.5 m)\frac{1}{3.0\ sec}$

$\underline{v = 2.2\ m/s}$

Example #2: Giancoli 11-45: Approximate a surface wave produced by an earthquake as a sinusoid wave. If the frequency is 0.5 Hz, what amplitude is needed so objects leave the ground ?

$\underline{f = 0.5\ Hz}$



$a_{max} = \omega^2 A = (2\pi f)^2 A > g$

$A > \frac{g}{(2\pi f)^2} = \frac{9.8}{(2\pi\, 0.5)^2}$

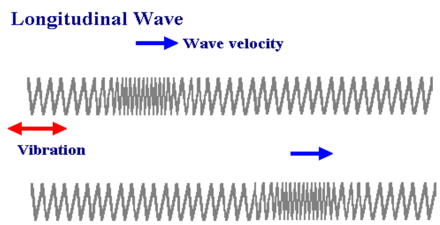
$= 9.8/\pi^2$

$\underline{A \sim 1.0\ m}$

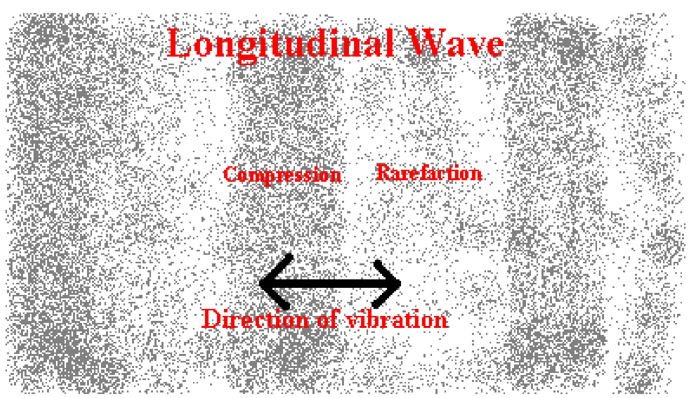
Up to now we have dealt with transverse waves, in which the particle (medium) velocity is transverse to the wave velocity.

A second type of wave is a longitudinal wave, where the particle velocity is in the same direction as the wave velocity. The wave is an alternating series of compressions and rarefactions, corresponding to the peaks and troughs of the wave.

Again, an individual particle or bit of the medium does NOT move at the wave velocity but rather undergoes SHM around an equilibrium position; the SHM is in the direction of the wave's propagation, though.



Sound is a longitudinal wave, in which the molecules of the medium oscillate in the direction of the propagation of the sound, leading to lower and higher air pressure at different parts of the wave:



Energy transport by waves

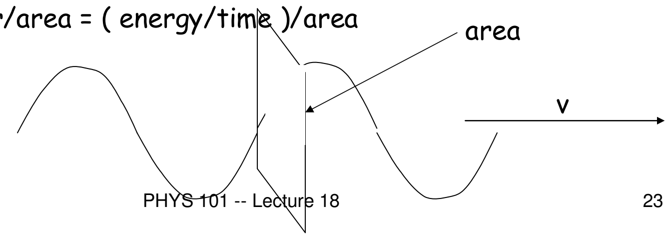
We've seen that waves don't transport particles (the medium), but they can transport energy. Consider the rope example again: each little bit of the rope undergoes transverse SHM; we've seen that its energy is thus

$$E_{bit} = \tfrac{1}{2} kA^2$$

So we immediately see that the energy transported by the wave will depend on the square of the amplitude.

We define the intensity I as the power (energy per time) transported across a certain cross-sectional area perpendicular to the direction of motion of the wave:

$$I = \text{power/area} = (\text{ energy/time })/\text{area}$$



For a one-dimensional wave such as the rope, the intensity doesn't change as we move along the wave (ie, down the rope).

However, for a 3-dimensional wave (ie, a wave spreading out in 3 dimensions from a point source), the 'area' is the surface of a sphere and as the wave spreads, the surface gets larger and the wave intensity diminishes:

$$I = \text{power/area} = P/4\pi r^2$$

ie.

$$I \approx 1/r^2 \qquad \text{(for a constant power source)}$$

This is what happens, for example, with sound waves and light: the intensity drops off as $1/r^2$ as you move away from the source.

Question to ponder on your way home: what happens in 2-D (for example, water waves on the surface of a pond)?

To understand the energy transport in more detail, consider a particle in the medium undergoing SHM as the wave propagates. Its energy is:

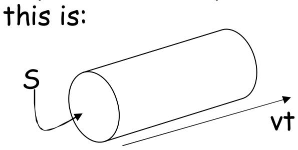
$$E_{particle} = \tfrac{1}{2} kA^2$$

but $\quad k = (2\pi f)^2 m \qquad$ (from $f = (1/2\pi)\sqrt{(k/m)}$)

so: $\quad E_{particle} = 2\pi^2 f^2 m A^2$

Since this is true for all particles, we can replace the 'm' by the total mass affected by the wave. For a time 't' this is:

$$vt\ S\ \rho$$



Where 'vt' is the speed times the time (the distance the wave travels), S is the cross sectional area (so vtS is the volume) and ρ ("rho") is the density of the medium.

So we have:

$$E_{wave} = 2\pi^2 f^2\ vt\ S\ \rho\ A^2$$

Now, the power is the energy per unit time, so

$$P = E/t$$
$$= 2\pi^2 f^2\ vS\ \rho\ A^2$$

and finally, the intensity of the wave is the power per unit area, so

$$\boxed{I = 2\pi^2 f^2\ v\ \rho\ A^2}$$

Thus the intensity of a wave is proportional to the square of the amplitude AND the square of the frequency.

Example: Giancoli Example 11-13. A earthquake wave travels outward from the epicentre and is measured with an intensity of 1.0×10^6 W/m^2 at a distance 100 km from the source. What is the intensity at a distance of 400 km ? What is the energy-rate (ie, power) through an area of 10 m^2 at the 100 km distance?

$I = 1 \cdot 10^6 \ W/m^2$ @ 100 km

$I = P/\text{area} \sim \frac{1}{r^2}$

$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$ $\qquad \frac{I_{400}}{I_{100}} = \left(\frac{100}{400}\right)^2 = \frac{1}{16}$

$I_{400} = I_{100}/16 = 6.3 \cdot 10^4 \ W/m^2$

$P = \text{area} \cdot I = 10 m^2 \cdot 1.10^6 \ W/m^2 = 10^7 \ W$

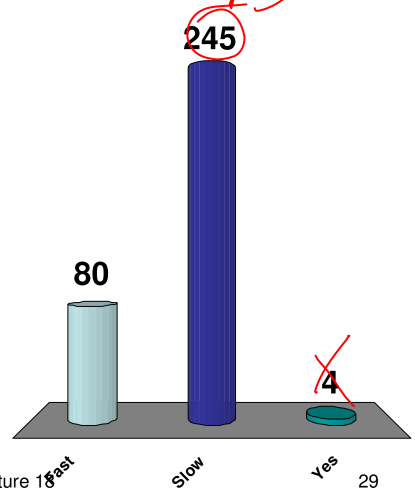
For next lecture:

Read sections 11-10 through 11-15 of Chapter 11
Problems from Giancoli sections 11-1 to 11-9 (except 11-5):
eg. #3, 5, 13, 21, 23, 33, 41, 47, 49

Giancoli Q7: A pendulum clock is accurate at sea level, and is then taken to high altitude. Will it run fast, or slow?

1. Fast
2. Slow
3. Yes

147 189



$T \sim \sqrt{\frac{L}{g}}$

$g \downarrow \Rightarrow T \uparrow$

Giancoli Q11-14: Why do strings for lowest-frequency notes on a piano normally have wire wrapped around them?

$v = \lambda f$

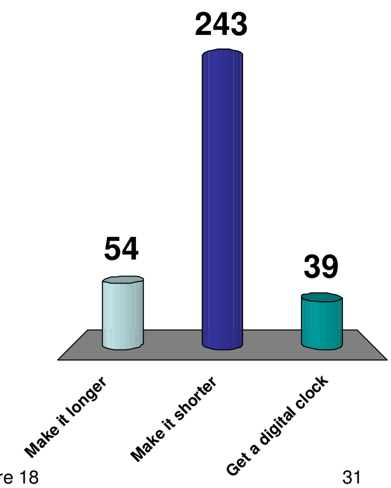
$v = \sqrt{F_T / [m/L]}$

$f\lambda \sim \sqrt{F_T / (m/L)}$

increase m/L so that low f still have "reasonable" λ

Your grandfather's pendulum clock has a length of 0.9930 m. It loses ½ minute a day. How should you adjust the length?

1. Make it longer
2. Make it shorter
3. Get a digital clock



Giancoli 11-P33: Your grandfather's pendulum clock has a length of 0.9930 m. It loses ½ minute a day. How should you adjust the length?

lose 30 sec a day

day = 84400 sec "=" 84370 tick

→ decrease the T

→ $T \simeq 2\pi\sqrt{\frac{L}{g}}$

$$\frac{T_{new}}{T_{old}} = \boxed{\frac{84370}{84400} = \sqrt{\frac{L_{new}}{L_{old}}}}$$

$$L_{new} = L_{old} \cdot \left(\frac{84370}{84400}\right)^2 = 0.9923 \text{ m}$$

ie 0.7 mm shorter

Giancoli 11-P41: A cord of mass 0.65 kg is stretched between two supports 28 m apart. If tension is 150 N, how long does a pulse take to travel between the supports?

28 m

$v = \lambda f$

$v = \sqrt{F_T / [m/L]} = \sqrt{150\,N / (0.65\,kg / 28\,m)}$

$= 80.4\ m/s$

$t = d/v = 0.348$ sec

Ch 11, #71

k_1

max compression 5 m

mass 900 kg @ 22.8 m/s

Energy → k ; then SHM → T