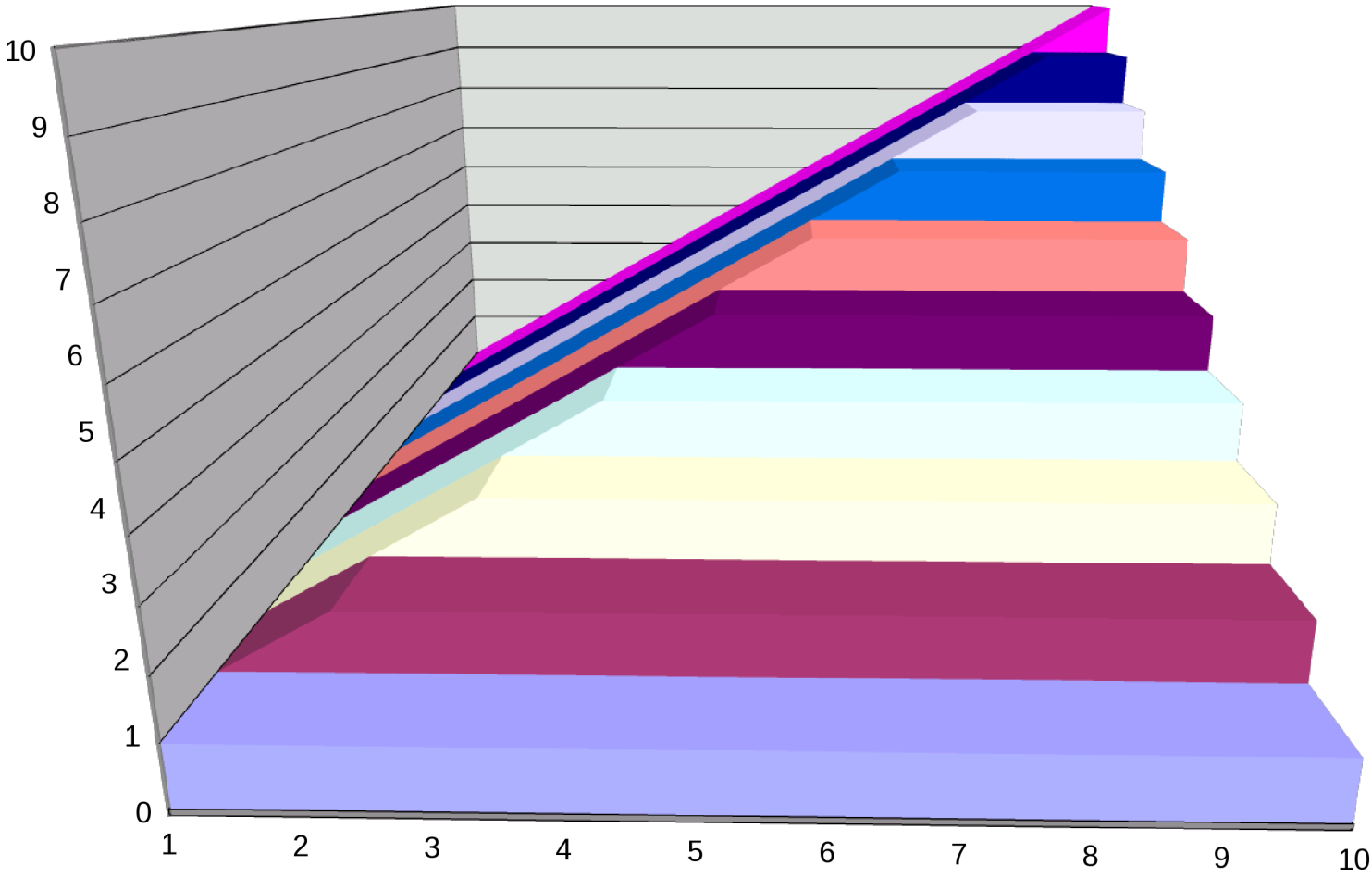
Cost Function with Perfect Substitutes

The two horizontal axes represent the price of the two factors, the vertical axis is the total cost



Multiplant Cost Minimization-2 Plants

Firm's Problem:

$$Min_{y_1,y_2} C_1(y_1) + C_2(y_2)$$
$$Subject\ to:\ y_1 + y_2 = Y$$

Using the constraint to substitute out y_2:

First-order condition:

$$\frac{\partial C_1(y_1)}{\partial y_1} = \frac{\partial C_2(Y - y_1)}{\partial y_1}$$

i.e. the marginal cost of plant 2 is expressed as a function of the output of plant 1, given the output target.

Example: $C_1 = 16 + 6y_1^2$ and $C_2 = 240 + 2y_2^2$

Then

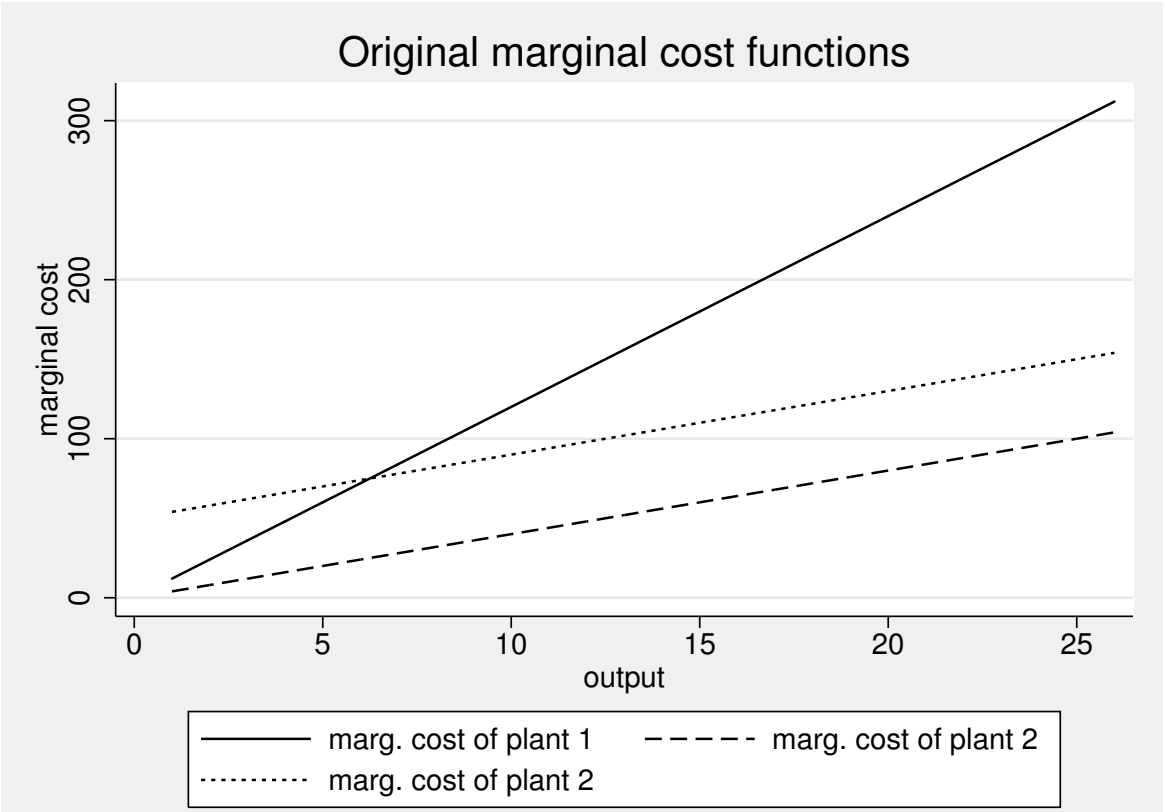
$$MC_1 = 12y_1$$
$$MC_2 = 4(Y - y_1)$$

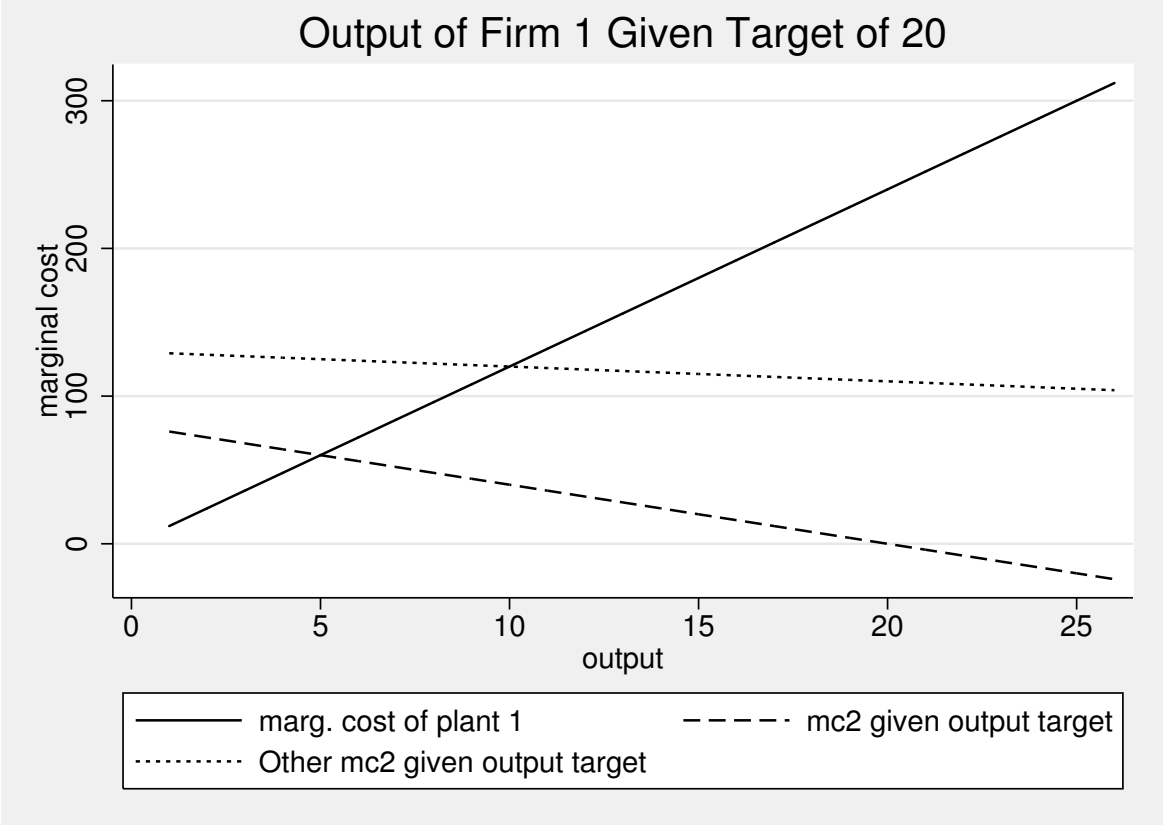
Suppose $Y = 20$.

Then

$$12y_1 = 4(20 - y_1)$$

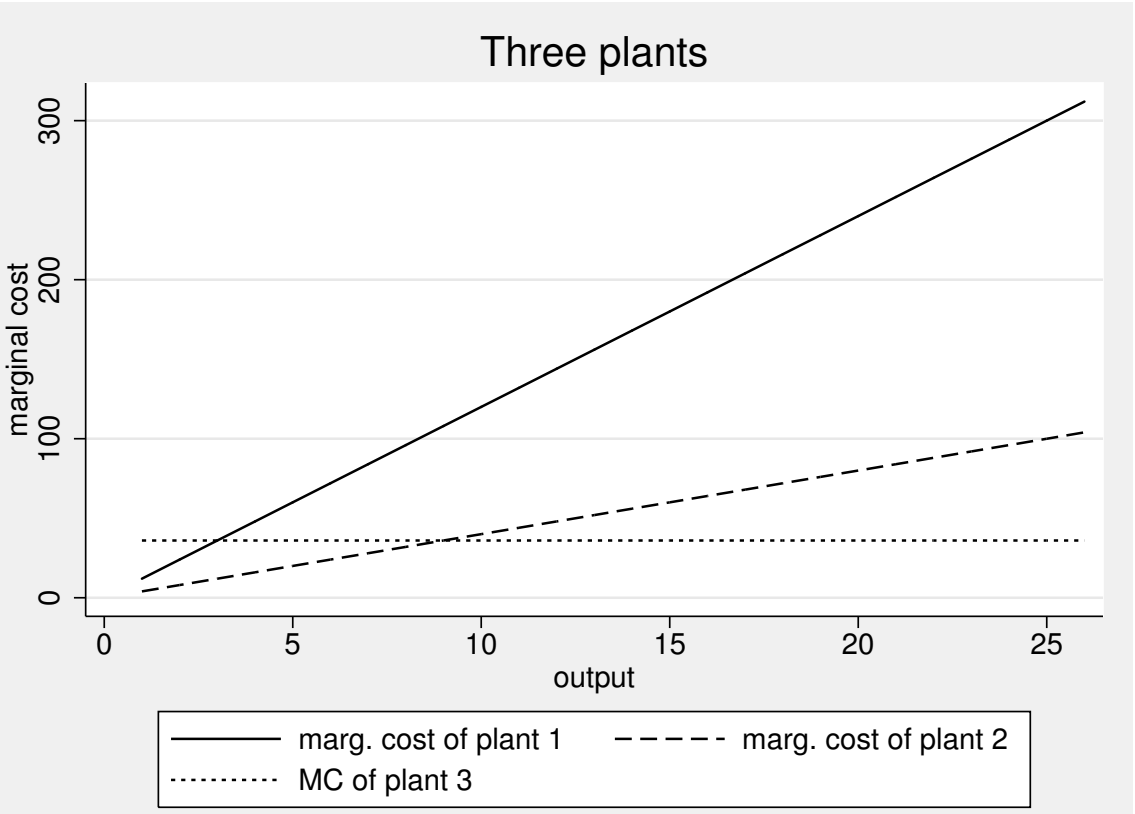
or $y_1 = 5$, hence $y_2 = 15$.

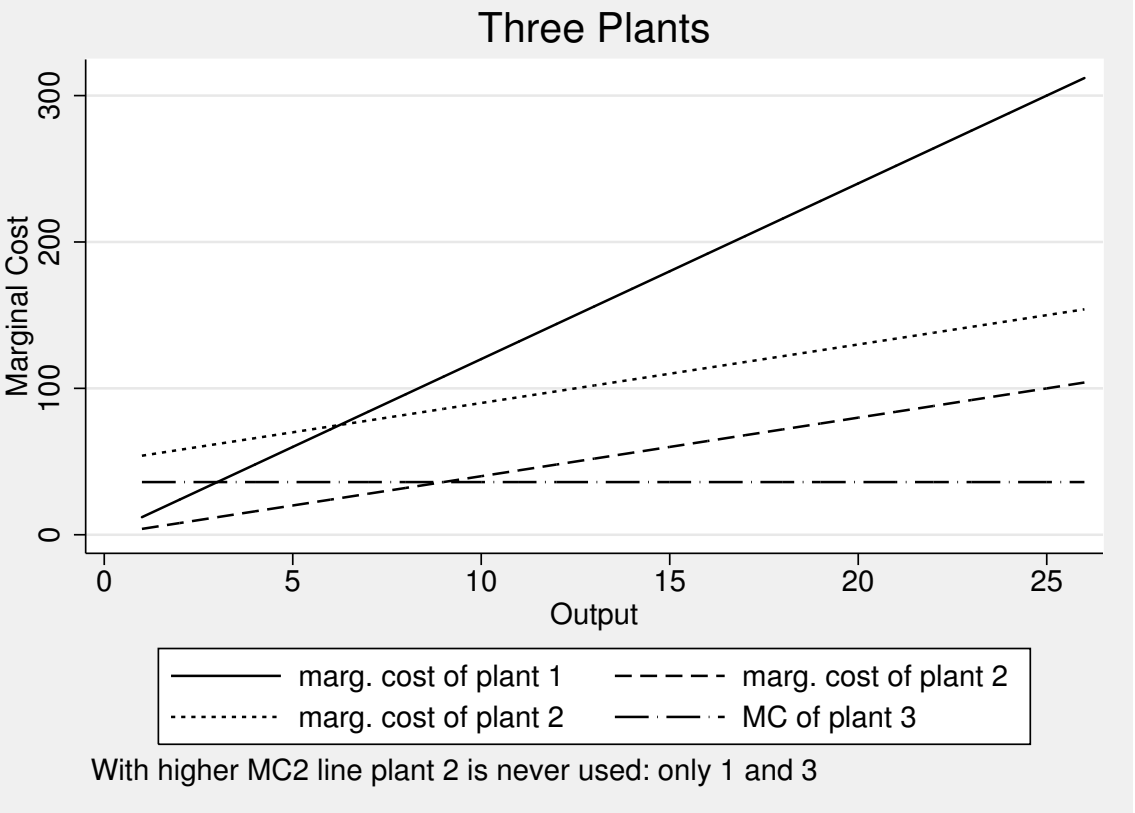

Original marginal cost functions
marginal cost
0
100
200
300
0
5
10
15
20
25
output
marg. cost of plant 1
marg. cost of plant 2
marg. cost of plant 2


Output of Firm 1 Given Target of 20
marginal cost
0
100
200
300
0
5
10
15
20
25
output
marg. cost of plant 1
mc2 given output target
Other mc2 given output target

Multiplant Cost Minimization-3 Plants or More

With N plants, given an output target, we end up with $N-1$ equations with the same number of unknown variables.





With higher MC2 line plant 2 is never used: only 1 and 3

C1	C2	Output	MC1	MC2	Units prod. by plant 1	Units prod. by plant 2	MC2 Conditioned by Target Y=20	MC2 Conditioned by Target Y=12
22	242	1	12	4	4	1	76	44
40	248	2	24	8	8	2	72	40
70	258	3	36	12	12	3	68	36
112	272	4	48	16	16	5	64	32
166	290	5	60	20	20	6	60	28
232	312	6	72	24		7	56	24
310	338	7	84	28		9	52	20
400	368	8	96	32		10	48	16
502	402	9	108	36		11	44	12
616	440	10	120	40		13	40	8
742	482	11	132	44		14	36	4
880	528	12	144	48		15	32	0
1030	578	13	156	52		17	28	-4
1192	632	14	168	56		18	24	-8
1366	690	15	180	60		19	20	-12
1552	752	16	192	64			16	-16
1750	818	17	204	68			12	-20
1960	888	18	216	72			8	-24
2182	962	19	228	76			4	-28
2416	1040	20	240	80			0	-32
2662	1122	21	252	84			-4	-36
2920	1208	22	264	88			-8	-40
3190	1298	23	276	92			-12	-44
3472	1392	24	288	96			-16	-48
3766	1490	25	300	100			-20	-52
4072	1592	26	312	104			-24	-56