Faculty of Arts

ECN 154-250D1:Introduction to Economic Theory

Final Exam (4 Pages, including the cover page)

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Instructions: Closed book exam; answer in exam booklet; students are allowed to use calculators; no documentation is allowed; students can keep the exam; translation dictionaries are allowed; regular definition dictionaries are not allowed.

Part One (30%): All Questions Must Be Answered (zero points are awarded for simply answering "true", "false", or "uncertain" without any explanation)

1. **(10%)** Mr. Deming rides yet again. He visits another company which produces a single output y from two inputs x_1 and x_2 with an increasing returns to scale production function $f(x_1, x_2)$. He discovers that when the input prices are $(w_1, w_2) = (2, 5)$, the firm uses the input combination $(x_1, x_2) = (6, 2)$ and produces 100 units of output. When the input prices are $(w_1, w_2) = (3, 6)$, the firm uses the input combination $(x_1, x_2) = (9, 10)$ to produce 300 units of output. Upon learning of these facts, Mr. Deming makes the following remark to management: "I cannot reject the assumption that you are a cost minimizing firm". Why does he say that? Why can he not be more definitive?
   Answer: We have
   $$w_1x_1 + w_2x_2 = 2 \times 6 + 5 \times 2 = 22$$
   with $y = 100$ and
   $$w_1^{'}x_1^{'} + w_2^{'}x_2^{'} = 3 \times 9 + 6 \times 10 = 87$$
   with $y = 300$. If we use the new prices $w^{'}$ with 3 times the original quantities we have
   $$w_1^{'}3x_1 + w_2^{'}3x_2 = 3 \times 18 + 6 \times 6 = 90$$
   With increasing returns to scale, the firm should be able to produce more than 300 units of y when using three times the initial quantities of the factors. Alternatively, the firm should be able to produce 300 units of y with the $x^{'}$ and $w^{'}$ at less than three times $w^{'}x$ when prices are equal to w', which it is doing. Also, $1/3$ of x' at the old prices would give $C = 22.67$, which is greater than 22. Hence there is nothing that can allow us to reject the assumption that the firm is minimizing its cost. However, we do not know for sure whether it is actually minimizing its cost simply because there could be another combination $x^{''}$ such that, at $w^{'}$ the firm achieves an even lower total cost than 87.

2. **(5%)** A commodity bundle lying above an individual's budget line must be superior to all bundles lying on the line. True, False, Uncertain, Explain.
   Answer: It depends on whether the consumer is endowed with the goods or not. If he is, he's actually strictly better off since he can trade off his initial endowment position. If he is not endowed with the goods, then the answer is false in general: it's not because a bundle is above one's budget constraint that it is necessarily preferred to all the bundles in the budget set.

3. **(5%)** A consumer spends all his income on 10 units of X and 20 units of Y. Immediately after he has purchased them, the price of X doubles and he is allowed to readjust his consumption. The consumer will be worse off as a result of the price increase. True, False, Uncertain, Explain.
   Answer: Similar to 2. If endowed with the goods, then a change in relative prices which makes x more expensive could make him better off, depending upon whether the individual is a net seller or a net buyer of x.

4. **(5%)** Do we have
   $$economic\ profits > 0 \Rightarrow (imply) business\ profits > 0$$

or

$$business\ profits > 0 \Rightarrow economic\ profits > 0$$

Answer: Positive economic profits imply positive business profits. Positive business profits need not imply positive economic profits as the costs do not necessarily reflect all opportunity costs. A classic example is the self-employed person leaving a $100,000 salary behind to start her business. That person could make a business profit of $5000 at the end of the year, but when one accounts for her opportunity cost ($100,000) she is making negative economic profits.

5. **(5%)** A firm with a monopoly in two markets with two different marginal costs should always charge a higher price in the market with the higher marginal cost. True, False, Uncertain, Explain.
   Answer: False. $MR_i = MC_i$ is the profit-maximizing condition determining the output and the price in each market. Even if MC_2 is higher than MC_1, the price charged could be lower if demand is sufficiently less elastic in market 1 than in market 2.

Part Two (30%): Do the Following 2 Problems

1. **(15%)** Ann lives on milk and caesar salad. The price of milk is 1 dollar per liter and the price of caesar salad is 2 dollars per pound. Ann allows herself to spend no more than 10 dollars per day. Also, Ann is on diet. Her dietitian restricts her consumption to 3000 calories per day. There are 500 calories in every liter of milk and 300 calories in every pound of salad. Ann's utility function is $U(x, y) = \min\{x, y\}$, where x is the amount of milk (in liters) and y the amount of salad (in pounds).

   (a) **(7%)** Draw Ann's feasible set of bundles (both budget- and calorie-wise) and *on the same diagram*, draw some of Ann's indifference curves. See Figure 1.

   (b) **(8%)** What is Ann's optimal choice?
   Answer: The way to think about this problem is to view it as having two constraints, the standard budget constraint and the calorie constraint. We only need to check which one is binding to get the solution. What makes this easy is that from her preferences we know she wants to consume exactly 1 litre of milk per pound of cheese, or 500 calories from milk with each 300 calories from cheese. Using the calorie constraint one has

   $$500 \times x + 300 \times y = 3000$$

   or, using the fact that $x = y$,

   $$800x = 3000$$

   which gives us $x = y = 3.75$ Similarly, if we were to solve this problem using the budget constraint we would get $x = y = 3.33$. The budget constraint is then the one which is binding as Ann cannot afford to buy 3.75 units of each.

2. **(15%)** Suppose a textbook monopoly can produce any level of output it wishes at a constant marginal cost of $5 per unit. Assume that the monopoly sells its books in two different markets that are separated by some distance. The demand curve in the first market is given by $y_1 = 55 - p_1$and the demand curve in the second market is given by $y_2 = 70 - 2p_2$.

(a) **(5%)** If the monopolist can maintain separation between the two markets, how much should be produced in each market and what price will be charged in each market?
Answer: Solve the following two problems: $Max\ (55 - y)y_1 - 5y_1$ for market 1, and $Max\ (35 - y_2/2)y_2 - 5y_2$ for market 2. This gives us ($y_1^* = 25$, $p_1 = 30$) and ($y_2^* = 30$, $p_2 = 20$).

(b) **(10%)** How would your answer change if it only cost demanders \$5 to mail books between the two markets? What would be the monopolist's new profit level in such a situation? How would your answer change if the mailing costs were 0?
Answer: Since the cost of mailing the books is only \$5, customers in market 2 are willing to accept as low a price as \$25 (the \$20 they paid to buy + the shipping costs) to sell their books to market 1 customers. Basically customers in market 1 can buy up to 30 books at \$25 (see Figure 2). In effect this forces the monopoly to charge \$25 in market 1 instead of \$30 and to sell 30 books instead of 25. If there are no shipping costs, then this means there is really just one market, not two. Just add up the two demand functions and solve in the usual way. Note here that it is not profitable for the firm to simply give up on selling in the "low demand" market and just focus on the high demand one, simply because there is not really a low demand market among the two. The firm is better off selling at one price in the two markets.

Part Three (40%): Do 2 of the Following 3 Problems

1. **(20%)** Mary consumes food and clothes. Her utility function is $U(f, c) = 2\ln f + c$, where f is the amount of food and c is the quantity of clothes Mary consumes[1]. The prices for food and clothes are p_f and p_c (dollars per unit), respectively. Mary has an income of m (dollars).

(a) **(4%)** Find Mary's demand functions for food and clothes.
Answer. We know we must have

$$\frac{MU_f}{MU_c} = \frac{p_f}{p_c}$$

or

$$\frac{2}{f} = \frac{p_f}{p_c}$$

Substituting $f = 2p_c/p_f$ into the budget constraint we have

$$p_f(2p_c/p_f) + p_c c = m$$

or

$$c = \frac{m - 2p_c}{p_c}$$

and

$$f = \frac{2p_c}{p_f}$$

[1] $MU_f(f, c) = \frac{2}{f}$ and $MU_c(f, c) = 1$.

so the demand for clothes does not vary with the price of food and the demand for food does not vary with income. (To be more precise, income does influence f through the budget constraint-if m is low enough then the consumer will not be able to buy $2p_c/p_f$ units. However, for the subsidy schemes considered here, income will never be low enough to prevent buying $2p_c/p_f$ units. So that special case can be ignored.)

(b) **(4%)** Suppose Mary's income m is \$400 and $p_f = 10$, and $p_c = 20$. What is Mary's best affordable bundle?
Just plug those values in the demand functions.
Answer. Suppose the government decides to subsidize Mary's food purchases in the amount of \$5 per unit. What is Mary's best affordable bundle?
Answer. From the demand function for food we have

$$f = \frac{2p_c}{(p_f - 5)} = \frac{40}{5} = 8$$

What is the total amount of subsidy Mary receives from the government?
Answer. $5 \times 8 = 40$

(c) **(4%)** Let T be the total amount of subsidy you calculated in (b). Suppose the government decides to simply pay Mary a lump sum of T dollars *instead* of the subsidy for food purchases given in (b). What is Mary's best affordable bundle?
Answer: just add the extra \$40 to m in the demand function for c. This yields $c = 20$; $f = 4$.

(d) **(4%)** Which scheme of subsidy does Mary prefer? Give an economic explanation for your finding.
Answer: Lump-sum transfer (simply calculate the utility derived from each). The intuition is that the budget constraint with the lump-sum transfer is everywhere above the constraint with the subsidy, except at $c = 0$; $f = 44$. More generally, people prefer income transfers to subsidies (when the two would allow consumers to purchase the same amount of the subsidized good if they spent all their income on it) because it leaves them the choice of how to spend their extra income the way they prefer.

(e) **(4%)** Use your answers for (b) and (c), find the Hicks income and substitution effect for food when p_f changes from 10 to 5 while $p_c = 20$ and $m = 400$.
Answer: Substitution=4; income=0 (f does not change with m).

2. **(20%)** Suppose there are a fixed number of 1,000 identical firms in the perfectly competitive concrete industry. Each firm produces the same fraction of total market output and each firm's production function for pipe is given by

$$q = \sqrt{KL}$$

where K is capital and L is labour. Suppose also that the market demand for concrete pipe is given by

$$Q = 400,000 - 100,000p$$

where Q is total concrete pipe.

(a) **(5%)** Letting w and r be the price of labour and capital, respectively, if $w = r = 1$, in what ratio will the typical firm use K and L? What will be the long run average and marginal cost of pipe?
Answer: The ratio is the same as the price ratio: 1 (TRS=price ratio). No need to use Lagrange here although of course the answer would be the same. Much simpler to simply isolate either K or L using the production function. Doing this we have

$$L = \frac{q^2}{K}$$

The put back in the cost minimization problem

$$Min\ \ K + \frac{q}{K}$$

This yields $K = q$. Hence, $L = q$. Thus the cost function is $C(1, 1, q) = 2q$, and the long-run average and marginal costs are both equal to 2.

(b) **(5%)** In the long-run equilibrium what will be the equilibrium market price and quantity for concrete pipe? How much will each firm produce? How much labour will be hired by each firm and in the market as a whole?
Answer: We know that the equilibrium price is 2. Just plug in 2 in the demand function to find Q. We know there's 1000 firms and total demand is 200,000. Hence $q = 200$ and $L = 200 \times 1000 = 200000$.

(c) **(5%)** Suppose w rose to \$2 while r remained constant at \$1. What will the long-run market equilibrium be? How much labour will now be hired by the concrete pipe industry?
Answer: Do as in a) above writing the cost minimization problem as

$$Min\ rK + wL$$

instead of plugging in the initial factor prices (\$1 for each). Then recalculate the cost function as in a). This would give you

$$c(w, r, q) = 2(wr)^{.5}q$$

Then plug in $w = 2$ and $r = 1$ to get $c(2, 1, q) = 2(2)^{.5}q$.The marginal cost is thus $MC = 2(2)^{.5} \approx 2.8$. This is the new equilibrium price. Solve to get $Q = 118000$ and then $L(firm) = (1/2)^{.5}118 = 83.4$ and $L(industry) = (1/2)^{.5}118 \times 1000$.

(d) **(5%)** How much of the change in total labour demand from part (b) to part (c) represents the substitution effect (i.e. the effect holding output constant) and how much represents the output effect?
Answer: The demand function for L is

$$L = \left[\frac{r}{w}\right]^{.5} q = \left(\frac{1}{2}\right)^{.5} q$$

Using $q = 200$ (i.e. holding output constant) we get $L = (.5)^{.5}200 = 141.4$. Hence the substitution effect is 200 - 141.4 and the rest (141.4- 83.4) is the scale effect.

3. **(20%)** Farmer George lives for two periods. In the first period, he has 100 bushels of wheat. If he consumes only a portion of the wheat in period 1, he can plant the rest and then harvest and consume wheat in period 2. Each bushel of seeds planted in period 1 yields 2 bushels of wheat in period 2. George's utility function is $u(c_1, c_2) = c_1 c_2$, where c_1 and c_2 are the amount of wheat (in bushels) he consumes in periods 1 and 2 respectively $[MU_{c_1} = c_2$ and $MU_{c_2} = c_1]$.

   (a) **(5%)** How much wheat will George consume in the first period and in the second period?
   Answer: We know that
   $$c_2 = 2(100 - c_1)$$
   This is the constraint. From the utility function we must have, at the optimal choice
   $$\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$
   or
   $$\frac{c_2}{c_1} = 2$$
   Just put $c_2 = 2c_1$in the budget constraint and solve for c_1. This yields $c_1^* = 50$. Hence, $c_2^* = 100$.

   (b) **(7%)** Suppose that a new irrigation system can double the yields (that is, each bushel of seeds planted in period 1 yields 4 bushels of wheat in period 2). What is the maximum amount of wheat George is willing to give up in period 1 in exchange for the new irrigation system?
   Answer: One needs to find the amount George is willing to give up at the new price ratio (4) such that he is just as well off as with the old price ratio (2). At the old price ratio we have $U = c_1^* c_2^* = 50 * 100 = 5000$. Here we need to adjust the constraint to take into account the amount T that George gives up in period 1 to get him back to $U = 5000$. The new constraint is $c_2 = 4(100 - T - c_1)$. Solving as in a) we would get $c_1^* = 50 - T/2$ and $c_2^* = 200 - 2T$. So what we need to find is the amount T of first period consumption that George is willing to give up such that the utility is the same as in a). In other words, find T such that
   $$U() = (50 - T/2)(200 - 2T) = 5000$$

   (c) **(8%)** George's big brother Jim is considering giving George *one* of the following two gifts:

   - Jim pays for the irrigation system;
   - Jim gives Q bushels of wheat to George in period 1.
     At what value of Q is George indifferent between the two gifts?
     Answer: Free irrigation results in George maximizing his utility subject to $c_2 = 4(100 - c_1)$.Solve as in a). This yields $c_1^* = 50$ and $c_2^* = 200$. Hence $U() = 50 \times 200 = 10000$. We need to find the amount Q such that the utility level with the old price ratio of 2 is 10000. The problem is thus to maximize $U = c_1 c_2$ subject to the constraint that
     $$c_2 = 2(100 + Q - c_1)$$

Solving it results in $c_1^* = 50 + Q/2$ and $c_2^* = 100 + Q$. Hence we need to find the Q such that

$$U() = (50 + Q/2)(100 + Q) = 10000$$

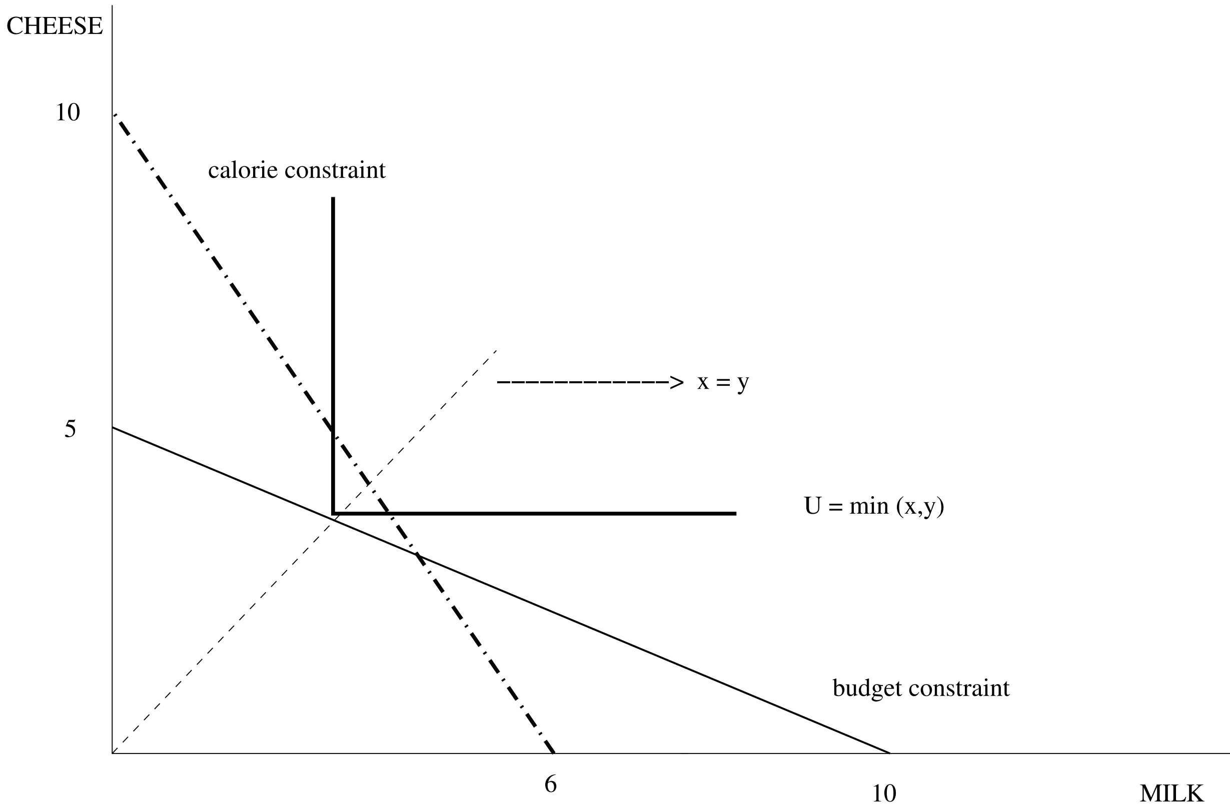

Figure 1

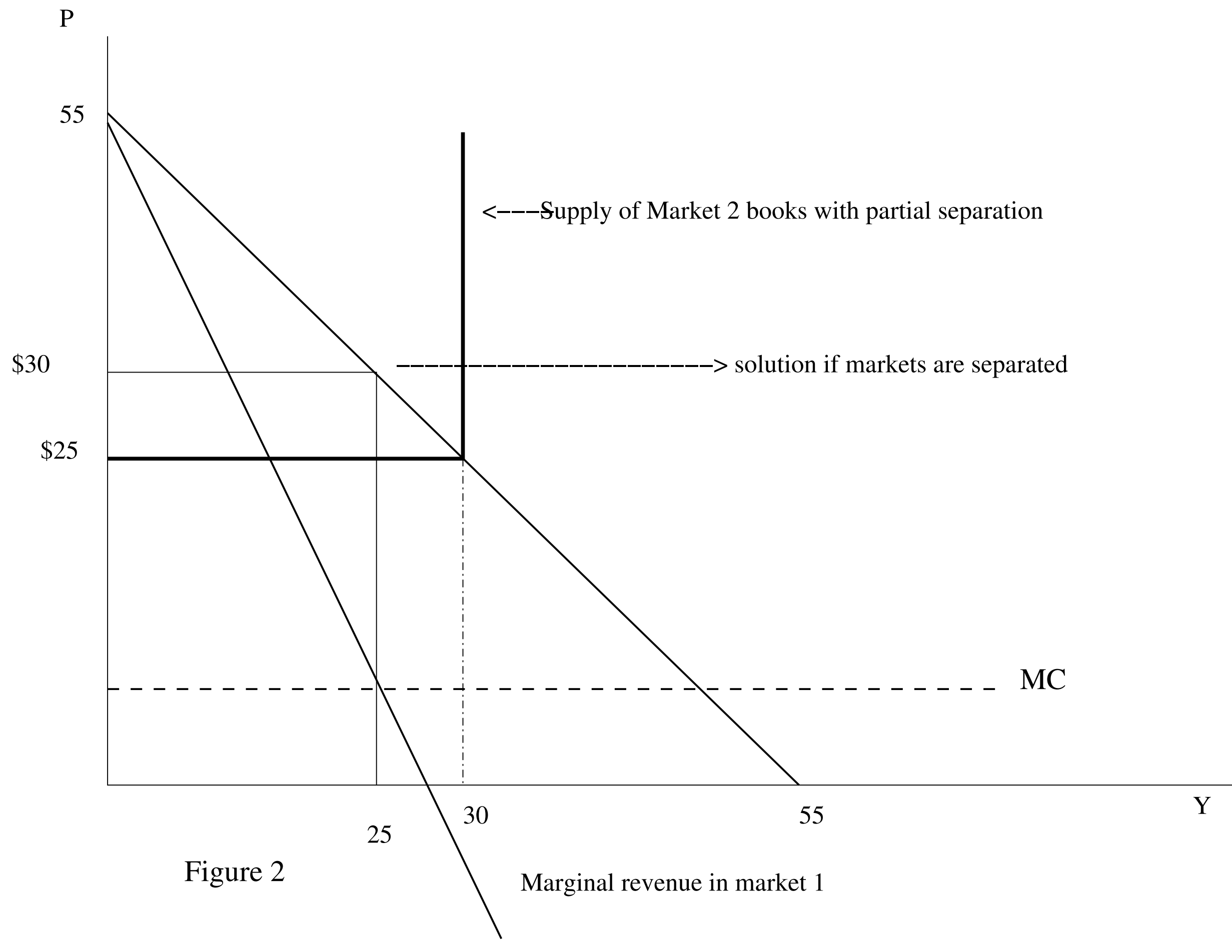
P
55
<---Supply of Market 2 books with partial separation
$30
---------------------------> solution if markets are separated
$25
MC
25
30
55
Y
Marginal revenue in market 1

Figure 2