

Productive Strategies to Support Students' Engagement in Productive Struggle

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FYI

Electronic copies of slides and handouts are available by request dbriars@nctm.org





Overview of the Session

- Define productive struggle.
- Watch video clips and discuss what a teacher does to support his students productive struggle in solving a mathematical task.
- Discuss the effective mathematics teaching practice of support productive struggle and its relationship to other teaching practices.





21st Century Competencies

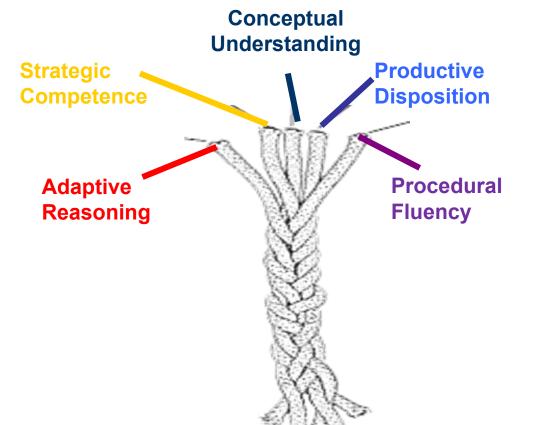
... "deeper learning" [is] the process through which an individual becomes capable of taking what was learned in one situation and applying it to new situations (i.e., transfer). . . . The product of deeper learning is transferable knowledge, including content knowledge in a domain and knowledge of how, why, and when to apply this knowledge to answer questions and solve problems. We refer to this blend of both knowledge and skills as "21st century competencies."

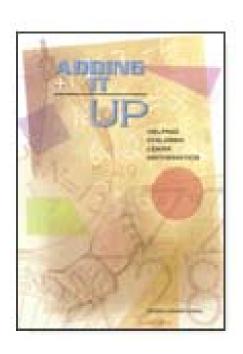
Education for Life and Work, NRC, 2012





Strands of Mathematical Proficiency





NRC (2001). *Adding It Up.* Washington, D.C.: National Academies Press.





Common Core State Standards

- Conceptual Understanding
- Procedural Fluency
- Applications
- Standards for Mathematical Practice

We need instructional practices that support each and every students' attainment of proficiency in all aspects of mathematical knowledge.





We Must Focus on Instruction

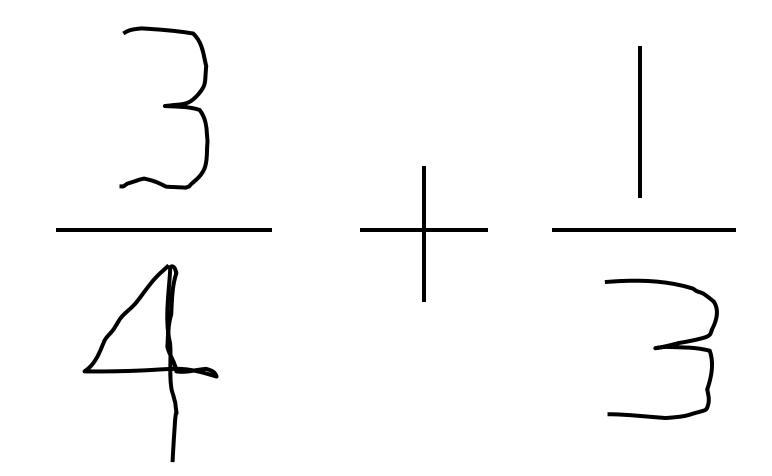
 Student learning of mathematics "depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum."

(Ball & Forzani, 2011, p. 17)

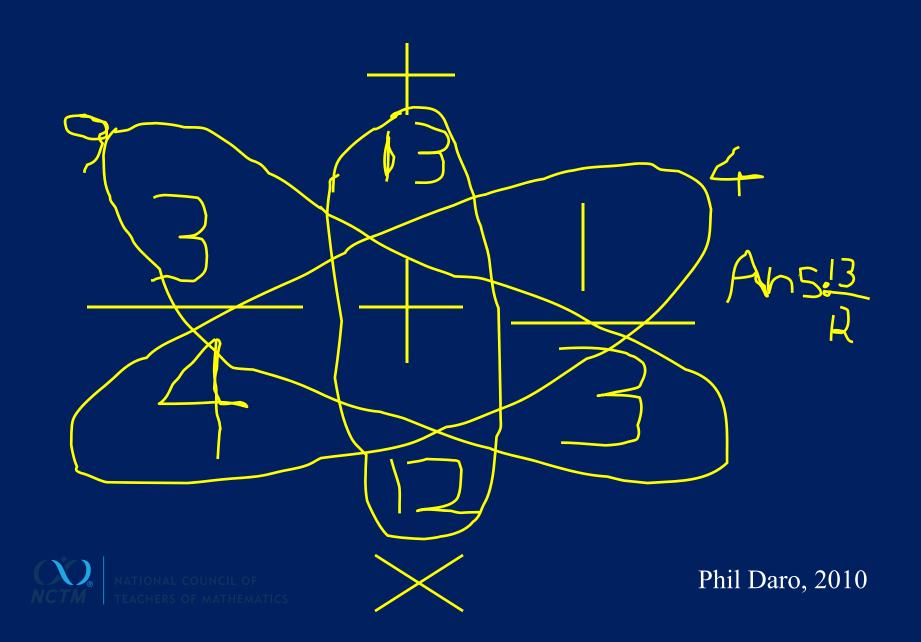
 "Teaching has 6 to 10 times as much impact on achievement as all other factors combined ... Just three years of effective teaching accounts on average for an improvement of 35 to 50 percentile points."

Schmoker (2006, p.9)











Other "Butterflies"?

- FOIL
- Cross multiplication
- Division of fractions: Keep-change-flip, KFC, etc.
- $a b = a + \bar{b}$: Keep-change-change
- Division algorithm: Does McDonalds Sell Cheese Burgers?
- Key words





	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	Addend Unknown	Both Addends Unknown ¹
Put Together/ Take Apart²	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1
			5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare ³	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?
Compare	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

OF EMATICS



Key Instructional Shift

From emphasis on:

How to get answers

To emphasis on:

Understanding mathematics





Principle on Teaching and Learning

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.





Effective Mathematics Teaching Practices

- 1. Establish mathematics **goals** to focus learning.
- 2. Implement **tasks** that promote reasoning and problem solving.
- 3. Use and connect mathematical representations.
- 4. Facilitate meaningful mathematical discourse.
- 5. Pose purposeful **questions**.
- 6. Build **procedural fluency** from conceptual understanding.
- 7. Support **productive struggle** in learning mathematics.
- 8. Elicit and use evidence of student thinking.





Support Productive Struggle in Learning Mathematics

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.





Support Productive Struggle in Learning Mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.





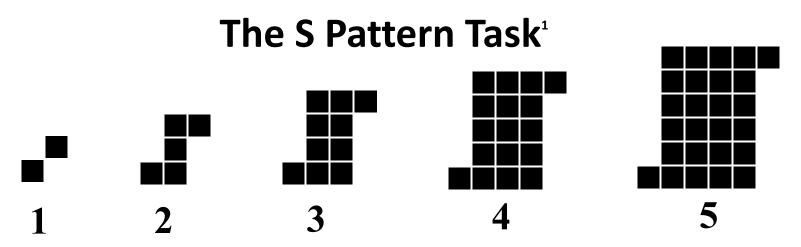
Support Productive Struggle in Learning Mathematics

The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed Hiebert et al., 1996

By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated.

Hiebert & Grouws, 2007, pp. 387-388



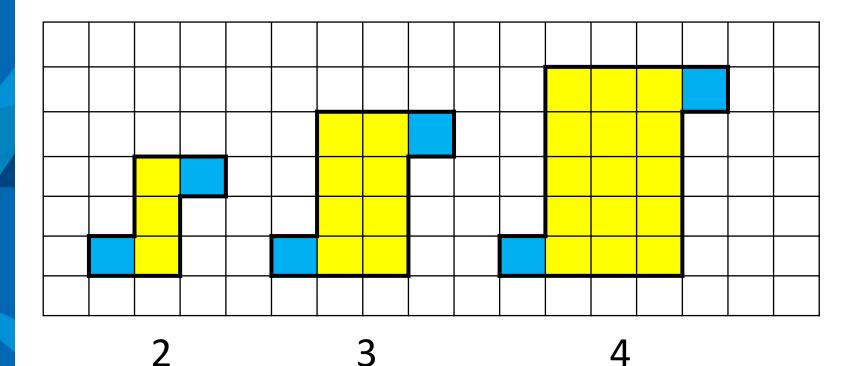


- What patterns do you notice in the set of figures?
- 2. How many square tiles are in figure 7? Write a description that could be used to determine the shape of and total number of square tiles in figure 7. Your description should be clear enough so that another person could read it and use it to think about another figure.
- 3. Determine an equation for the total number of squares in any figure. Explain your rule and show how it relates to the visual diagram of the figures.
- 4. Find a second way to describe the pattern and write the equation that matches the description. Compare the two equations and show in the visual representation how one equation is equivalent to the other.
- 5. If you knew that a figure had 9802 squares tiles in it, how could you determine the figure number? Explain.
- 6. Does the pattern describe a linear relationship between the figure number and the total number of squares? Why or why not?

This task was adapted from *Visual Mathematics Course II, Lessons 1-10* published by The Math Learning Center, Salem, OR.



$$(n-1)(n+1)+2$$

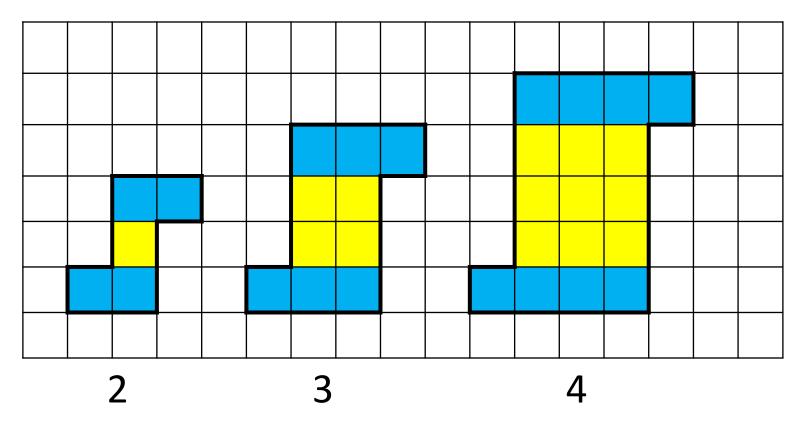


Briars, April 2017





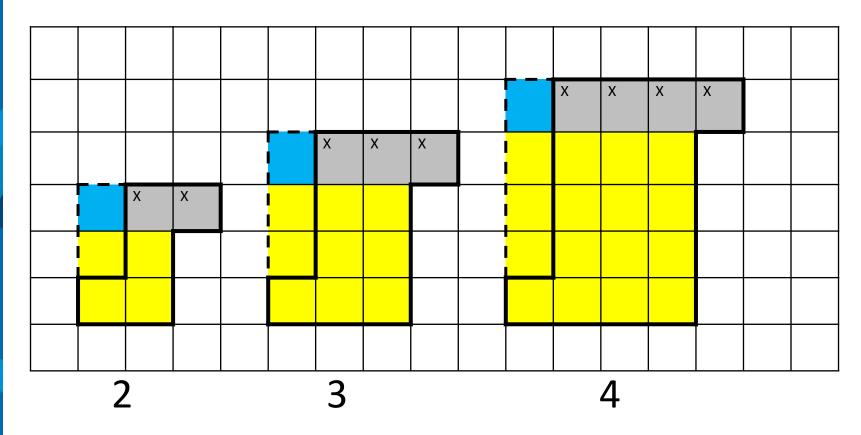
$$(n-1)^2 + 2n$$







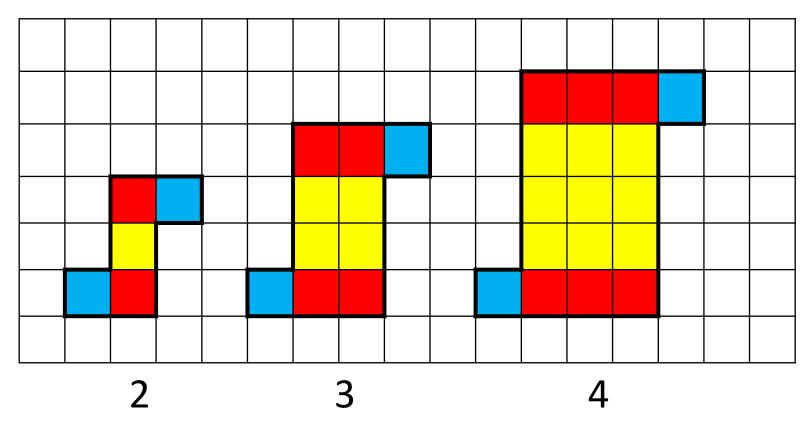
$$n^2 + 1$$







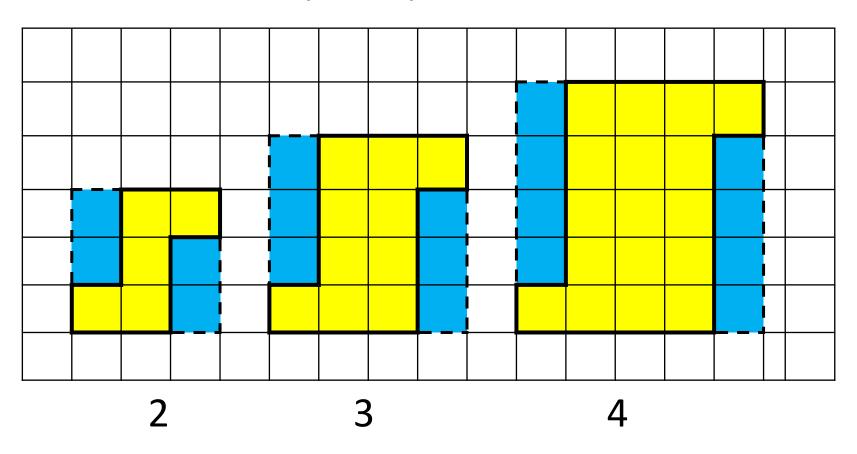
$$(n-1)^2 + 2(n-1) + 2$$







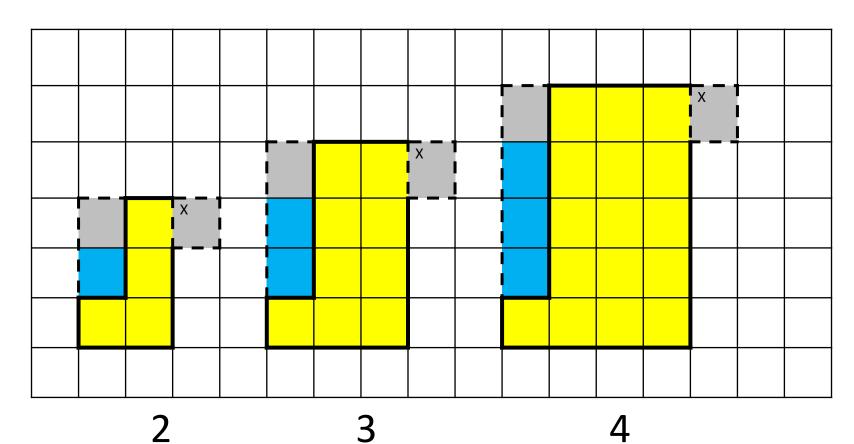
$$(n+1)^2 - 2n$$







$$n(n + 1) - (n - 1)$$



NATIONAL COUNCIL OF

TEACHERS OF MATHEMATICS

Briars, April 2017



Effective Mathematics Teaching Practices

THE CASE OF JEFF ZIEGLER HIGH SCHOOL

Developed by Margaret Smith and Victoria Bill at the University of Pittsburgh. Video courtesy of Pittsburgh Public Schools and the Institute for Learning.

INSTITUTE for LEARNING



The S-Pattern Task Video Context

School: Langley High School, Pittsburgh Public Schools

Teacher: Mr. Jeffrey Ziegler

Principal: Linda Baehr

Class: 11th-12th Grade Students

At the time the video was filmed, Jeffery Ziegler was a coach at Langley High School in the Pittsburgh Public School District. The students are 11th and 12th grade and struggle with mathematics.

(Jeffrey Ziegler is currently a curriculum supervisor for grades 6-12 mathematics in the Pittsburgh Public School district.)

INSTITUTE for LEARNING





Mr. Ziegler's Mathematics Learning Goals

Students will understand that:

- 1. An equation can be written that describes the relationship between 2 quantities;
- 2. Different but equivalent equations can be written that represent the same situation; and
- 3. The symbolic and pictorial representations can be connected.





Connections to the CCSS Content Standards

Creating Equations*

A-CED

Create equations that describe numbers or relationships.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors.





Norms for Watching Video

- Video clips are examples, not exemplars.
 - To spur discussion not criticism
- Video clips are for investigation of teaching and learning, not evaluation of the teacher.
 - To spur inquiry not judgment
- Video clips are snapshots of teaching, not an entire lesson.
 - To focus attention on a particular moment not what came before or after
- Video clips are for examination of a particular interaction.
 - Cite specific examples (evidence) from the video clip, transcript and/or lesson graph.





The S-Pattern Task The Context of Video Clips

The lesson begins with Mr. Ziegler engaging students in a brief discussion of the task. They establish the fact that this was a growth pattern that is growing in two dimensions, getting both "taller" and "bigger". Before they begin their work, Mr. Ziegler tells students:

"Now there are 6 prompts... Kind of the first one, the second one, third one is to kind of get you started but it is on you guys to work with your groups to come up with a way to find the patterns. You don't necessarily have to word-for-word answer these questions, but they're there to help you maybe get started.

The clip begins as small groups begin to work on the task and Mr. Ziegler visits Groups 1 and 2.





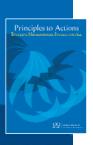
Lens for Watching the Video Clip 1

As you watch the video, make note of what the teacher does as he interacts with groups 1 and 2.

In particular, identify any of the *Effective Mathematics Teaching Practices* that you notice Mr.

Ziegler using.





Video Clip 1



..\..\P2A\PtA Toolkit\Ziegler final\5-VideoClip1-HS-Ziegler.mp4





Effective Mathematics Teaching Practices

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Lens for Watching the Video Clip 2

In the second video clip Mr. Ziegler visits Groups 1 and 2 for a second time.

Considering the teacher's actions and interactions with Groups 1 and 2 in both clips, identify what the teacher does to support his student's productive struggle.







Video Clip 2



..\..\P2A\PtA Toolkit\Ziegler final\6. ZieglerClip2.mp4





Effective Mathematics Teaching Practices



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Support Productive Struggle In Learning Mathematics: Teacher and Student Actions

What are teachers doing?

- Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.
- Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. • Persevering in solving problems and
- Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.
- Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.

What are students doing?

- Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.
- Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.
- realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up.
- Helping one another without telling their classmates what the answer is or how to solve the problem.





Five Practices for Orchestrating Productive Discussions



- Anticipating likely student responses
- Monitoring students' actual responses
- Selecting particular students to present their mathematical work during the whole class discussion
- **Sequencing** the student responses
- Connecting different students' responses—to each other and to key mathematical ideas.





Planning with the Student in Mind

- Anticipate solutions, thoughts, and responses that students might develop as they struggle with the problem
- Generate questions that could be asked to promote student thinking during the lesson, and consider the kinds of guidance that could be given to students who showed one or another types of misconception in their thinking
- Determine how to end the lesson so as to advance students' understanding

Stigler & Hiebert, 1997





Thinking Through a Lesson Protocol (TTLP) Planning Template

Learning Goals (Residue) What understandings will students take away from this lesson?	Evidence What will students say, do, produce, etc. that will provide evidence of their understandings?	
Task What is the main activity that students will be working on in this lesson?	Instructional Support—Tools, Resources What tools or resources will students have to use in their work that will give them entry to, and help them reason through, the activity?	
Task Launch How will you introduce and set up the task to ensure that students understand the task and can begin productive work, without diminishing the cognitive demand of the task?		
Task Enactment What are the various ways that students might complete the activity?	Instructional Support—Teacher What questions might you ask students that will support their exploration of the activity and bridge between what they did and what you want them to learn?	
	To be clear on what students actually did, begin by asking a set of assessing questions such as: What did you do? How did you get that? What does this mean? Once you have a clearer sense of what the student understands, move on to appropriate set of questions below.	
Sharing and Discussing the Task		
Selecting and Sequencing Which solutions do you want to have shared during the lesson? In what order? Why?	Connecting Responses What specific questions will you ask so that students - make sense of the mathematical ideas that you want them to learn - make connections among the different strategies/solutions that are presented	



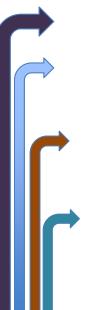


Planning with the Student in Mind

Solution	Questions	Students	Order



Effective Mathematics Teaching Practices

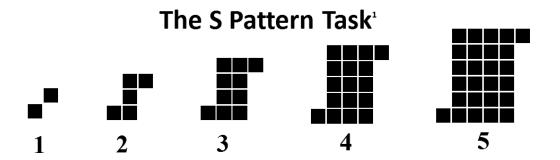


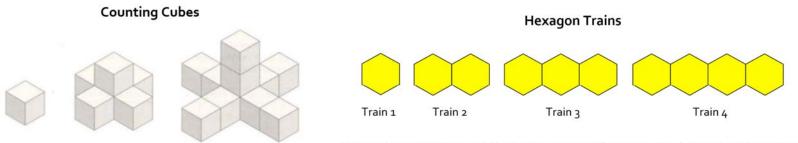
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Use Pattern Tasks to Establish Classroom Climate





Write a description that could be used to compute the perimeter of any train in the pattern.



Building 2

Building 3

Building 1



Starting the Year with Pattern Tasks

...all students can do something mathematical when presented with a geometric pattern. One teacher noted that regardless of your background, you can fly into the task anywhere. You can have the brightest kid in your class and the one who is struggling feel success from the first two weeks. 'So it makes everybody feel kind they're on kind of an even playing ground'...

Smith, M.S., Hillen, A.F., & Catania, C. (2007). Using pattern tasks to develop mathematical understandings and set classroom norms. *Mathematics Teaching in the Middle School*, 13 (1), 38-44. [pp.39-40]





Starting the Year with Pattern Tasks

Establishing Classroom Culture:

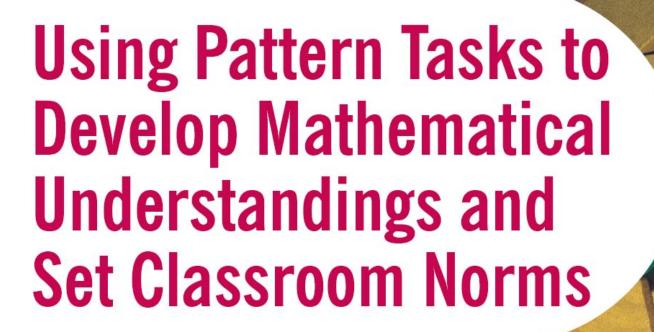
- Pattern tasks accessible to all
- Context for discussing multiple solution strategies
- Developing classroom norms and practices
 - Working in partners/groups
 - Presenting work--clarity
 - Being a good audience member—accountable for understanding work of others
 - Respect
- Basis for teacher discussion/collaboration

Smith, Hillen, Catania, MTMS, 2007 Briars, April 2017





MTMS, August 2007



Margaret S. Smith, Amy F. Hillen, and Christy L. Catania





Students' Mathematics Identities

Are how students see themselves and how they are seen by others, including teachers, parents, and peers, as doers of mathematics.

Aguirre, Mayfield-Ingram & Martin, 2013





Students' Mathematics Identities

- Mathematics identity includes:
 - beliefs about one's self as a mathematics learner;
 - one's perceptions of how others perceive him or her as a mathematics learner,
 - beliefs about the nature of mathematics,
 - engagement in mathematics, and
 - perception of self as a potential participant in mathematics (Solomon, 2009).





Identity & Motivation

- Understanding the strengths and motivations that serve to develop students' identities should be embedded in the daily work of teachers.
- Mathematics teaching involves not only helping students develop mathematical skills but also empowering students to seeing themselves as being doers of mathematics.





Students' Beliefs about Their Intelligence Effects Their Willingness to Struggle

- Fixed mindset:
 - Avoid learning situations if they might make mistakes
 - Try to hide, rather than fix, mistakes or deficiencies
 - Decrease effort when confronted with challenge
- Growth mindset:
 - Work to correct mistakes and deficiencies
 - View effort as positive; increase effort when challenged

Dweck, 2007 Briars, April 2017





Students Can Develop Growth Mindsets

- Teacher praise influences mindsets
 - Fixed: Praise refers to intelligence
 - Growth: Praise refers to effort, engagement, perseverance
- Explicit instruction about the brain, its function, and that intellectual development is the result of effort and learning has increased students' achievement in middle school mathematics.
- Reading stories of struggle by successful individuals can promote a growth mindset (Lin et al, 2016)





Messages?

super light, superhero tough

FeatherLights reduce their school load by trimming weight off the pack.

But what makes them really amazing is they do it without sacrificing durability, thanks to strategically placed 420D and 600D pack cloth.

Lighter registry Same appearance durability.

Lighter weight. Same awesome durability.



light as a feather, tough as long division

FeatherLights reduce their school load by trimming weight off the pack. But what makes them really amazing is they do it without sacrificing durability thanks to strategically placed 420D and 600D pack cloth.



National Alliance for Partnerships in Equity, NAPE, 2016





Micromessages

Small, subtle unconscious messages we send and receive when we interact with others. Negative micromessages cause people to feel devalued, slighted, discouraged or excluded. Positive micromessages cause people to feel valued, included, or encouraged.

National Alliance for Partnerships in Equity, 2016





Micromessages

What are some examples of micromessages that might influence students' mathematics identity?





What Messages Are We Sending About Mathematical Identity?

- "What don't you understand? This is so simple."
- "It is immediately obvious that "
- Which students participate in class?
- How do they participate? What kind of questions are they asked? Wait time? Opportunities for elaboration/explanations?
- Responses to students' questions? To their answers?
- Feedback: Effort-based vs intelligence-based praise.
- Non-verbal communication—smiles, nods, etc.





Peer Observations to Uncover Micromessages and Unintentional Bias

Observe the classroom experiences of students--

- Number of interactions
- Amount of wait/think time
- Nature of questions—higher vs lower level
- Nature of feedback
- Amount of eye contact
- Use of language





Building Positive Mathematics Identities

I had the students write their math autobiography. So they had to talk about what their math identity is. So, you know, where are you now with mathematics? Why do you think you are there? What do you think has happened from the time that you can remember with mathematics until now? When did you find yourself shaping into the person that says, "You know, I really love math. I think math is exciting. I think math is fun." Or that person that says, "You know, I really kind of get that pit in my stomach when I have to come to math class every day." But there's a reason for that. Something happened along the way. Something happened multiple times that got you to this point. What was that?

Jeff Ziegler, 2016





Building Positive Mathematics Identities

Because, you know, it's my responsibility to continue to shape their math identity. So if they really love math and they really enjoy math, so it's my responsibility to challenge them, but to hopefully keep them in that space. And students that come in struggling or really disliking it, how do I create an environment, a space, a year in which helps reshape that or form it in other ways that hopefully when they leave, you know, they're not just saying, "I love Mr. Ziegler's class," or, "I thought this class was great," but that, "Math, I really got a new insight on what mathematics is and how I really do mathematics and how much fun it could be."

Jeff Ziegler, 2016





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Key Take-Aways

- Engaging students in productive struggle is essential to developing conceptual understanding in mathematics
- Tasks that promote reasoning and problem solving are most likely to promote struggle and the need for perseverance
- Teachers need to support productive struggle by providing probing guidance and affordance without taking over the thinking for students by telling or providing too much direction.
- Productive struggle can be accomplished by asking questions, encouraging reflection, providing time, and acknowledge effort.





Taking Action in Your Classroom

Supporting Student Struggle on a Challenging Mathematical Task

Choose a task that promotes reasoning and problem solving that you plan to implement in your classroom. For that task:

- Describe what you will see students doing or hear students saying that would represent productive struggle with the task;
- Describe what you will see students doing or hear students saying that would represent unproductive struggle with the task; and
- Identify the ways in which you will support students productive struggle so that they can make positive progress towards the mathematical goals of the lesson.





Taking Action in Your Classroom

Reflecting on Supporting Productive Struggle

Over the course of a week, note how you interact with students who are struggling. Perhaps take turns with a colleague observing each other's classrooms, videotape, or audiotape if possible:

- How did you respond when a student could not get started?
- What advice did you give when students were stuck?
- How did you encourage students who had found an answer, but did not engage with the mathematical goals of the task?





The Title Is Principles to Actions

Your Actions?





http://www.nctm.org/PtAToolkit/

Principles to Actions Professional Learning Toolkit



These grade-band specific professional learning modules are focused on the Effective Teaching Practices and Guiding Principles from *Principles to Actions: Ensuring Mathematical Success for All.*

Presentation, presenter notes, and required materials are provided in each module to support professional learning with teachers through analyzing artifacts of teaching (e.g., mathematical tasks, narrative and video cases, student work samples, vignettes) and abstracting from the specific examples general ideas about how to effectively support student learning.

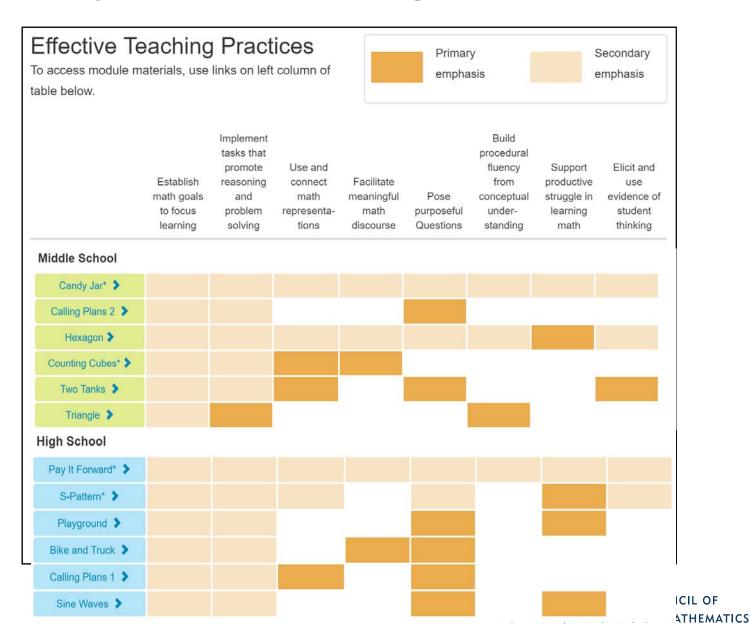
The Teaching and Learning Modules were developed in collaboration with the Institute for Learning (IFL) at the University of Pittsburgh.

Learning modules are available exclusively to NCTM members. Limited open examples are provided for each grade level (denoted with *).





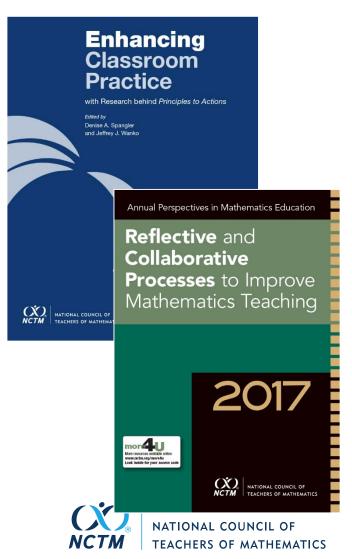
http://www.nctm.org/PtAToolkit/





Taking Action with Principles to Actions





NCTM Interactive Institutes www.nctm.org



Facilitating Meaningful Mathematical Discourse (Pre-K-Grade 12)

July 17–19, 2017 Baltimore, MD Supporting Students'
Productive Struggle
(Pre-K-Grade 12)

July 20–22, 2017 Baltimore, MD



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April 25–28, 2018 Washington, D.C.



Thank You!

Diane Briars dbriars@nctm.org

