

Taking Action: Implementing Effective Teaching Practices in Grades 9-12
NCTM/NCSM 2017 Annual Meetings
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Analyzing Teaching and Learning Activity 6.3

Connecting Representations

- Begin by solving the Squares from Tiles task below. Solve the task using as many different representations as you can. If possible, discuss your solutions, and other representations you predict students might use, with colleagues.
- Imagine that the students in your class produced the solutions to the Squares from Tile task and created posters showing their work, which were hung in the classroom.
 - What questions could you ask to help students make connections between the different representations?
 - In what ways might students benefit from seeing the different ways in which the problem could be represented and solved?

The Squares from Tiles Task

A computer animation program shows a square (called the *main square*) that grows into larger squares by increasing the length of each side by one unit each step of the animation. The main square is made of smaller congruent squares that stay the same size. The main square is created by using green squares (each of which has two edges on the perimeter), blue squares (each of which has one edge on the perimeter), and red squares (with no edges on the perimeter). Every large square must have at least one red square in it.

Write rules that determine how many green, blue, and red squares are needed to make a main square with any given side length.

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*

Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*

Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*

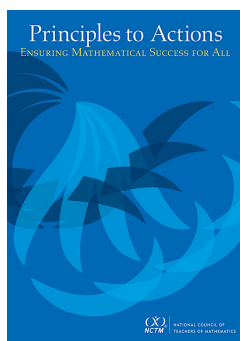
Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*

Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*

Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*

Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*

Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*



National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

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<http://www.nctm.org/principlestoactions>

G	B	G
B	R	B
G	B	G

Side = 3

G = 4

B = 4

R = 1

G	B	B	G
B	R	R	B
B	R	R	B
G	B	B	G

Side = 4

G = 4

B = 8

R = 4

G	B	B	B	B	G
B	R	R	R	R	B
B	R	R	R	R	B
B	R	R	R	R	B
B	R	R	R	R	B
G	B	B	B	B	G

G = 4

B = 16

R = 16

Side	Green	Rate of Change
3	4	$\frac{0}{1}$
4	4	$\frac{0}{1}$
5	4	$\frac{0}{1}$
6	4	$\frac{0}{1}$
7	4	$\frac{0}{1}$

No
Change in
green

Side	Blue	Rate of Change
3	4	
4	8	$4/1 = 4$
5	12	$4/1 = 4$
6	16	$4/1 = 4$
7	20	$4/1 = 4$

$$8/2 = 4$$

$$8/2 = 4$$

Always 4

Side	Red	Rate of Change
3	1	3
4	4	5
5	9	7
6	16	9
7	25	

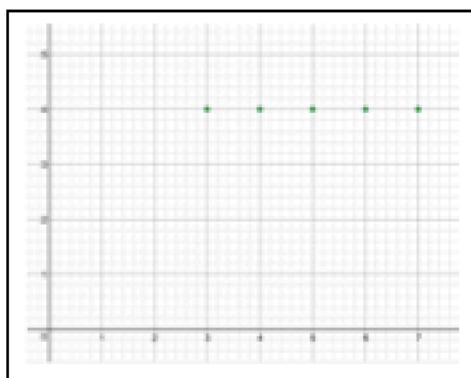
Not the
Same

$G(n) = 4$ is the rule for green tiles used, where n is the number of tiles on one side of the large square.

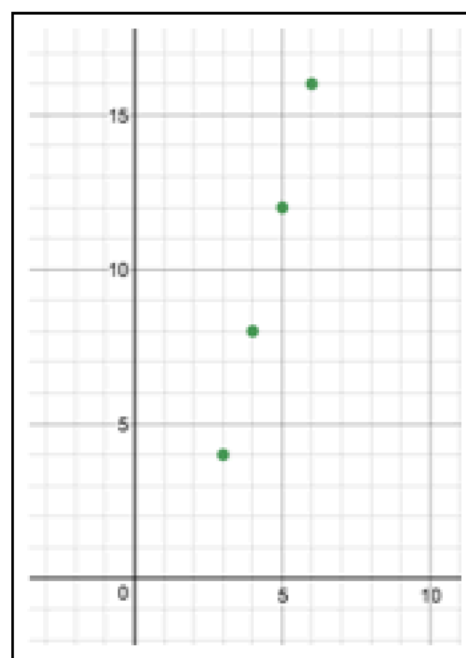
$B(n) = 4(n-2)$ is the rule for blue tiles used because you take 2 away from the number of tiles and then multiply by 4.

$R(n) = (n-2)^2$ is the rule for red tiles used. The shape shows a square of red tiles, but there are two less than the number of tiles on a side of the large square.

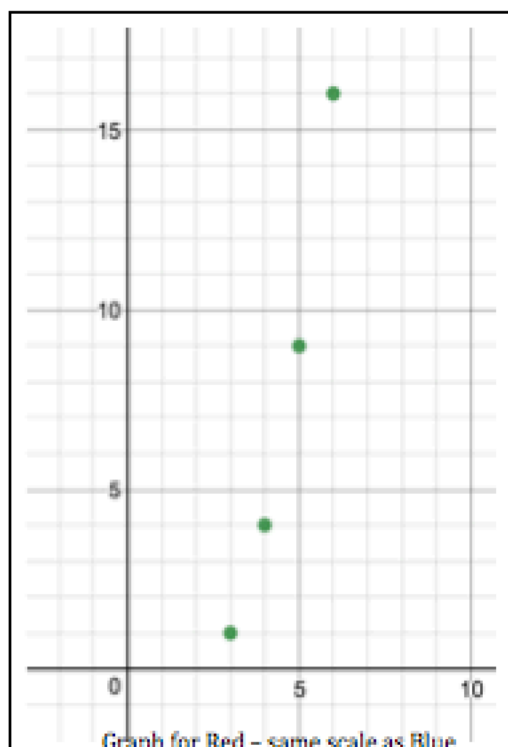
Student D



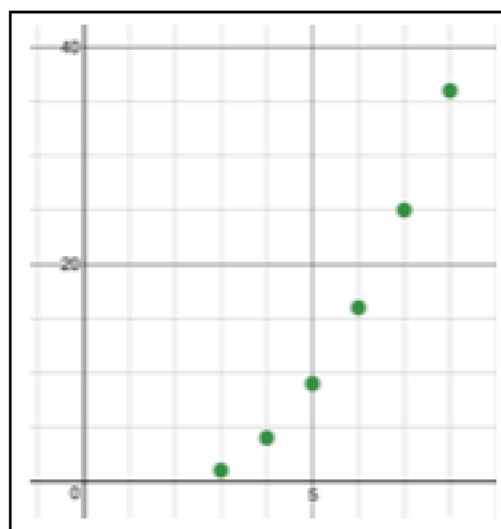
Graph for Green



Graph for Blue



Graph for Red - same scale as Blue



Graph for Red - zoom out

$$(4, 4) \quad (5, 9) \quad (7, 25)$$

$$R(x) = Ax^2 + Bx + C$$

$$N =$$

$$R(N) = AN^2 + BN + C$$

$$R(4) = 16A + 4B + C = 4$$

$$R(5) = 25A + 5B + C = 9$$

$$R(7) = 49A + 7B + C = 25$$

Solve with Systems

$$\begin{aligned} 9A + B &= 5 \\ 24A + 2B &= 16 \end{aligned}$$

$$\Rightarrow \begin{array}{r} -18A - 2B = -10 \\ 24A + 2B = 16 \\ \hline 6A = 6 \\ A = 1 \end{array}$$

$$A = 1$$

$$9(1) + B = 5$$

$$B = -4$$

$$16 + 4(-4) + C = 4$$

$$C = 4$$

$$N^2 - 4N + 4 = R(N)$$

G	B	B	G
B	R	R	B
B	R	R	B
G	B	B	G

$$\text{Green} = 4$$

$$\text{Blue} = 4(n-2)$$

$$\text{Total} = n^2$$

$$\text{So, Red} = n^2 - 4(n-2) - 4$$

$$\text{Test } n=4 \quad 4^2 - 4(4-2) - 4$$

$$= 16 - 8 - 4 = 4 \text{ correct}$$