Taking Action: PtA Tools for High-Leverage Mathematics Teaching in Elementary Education

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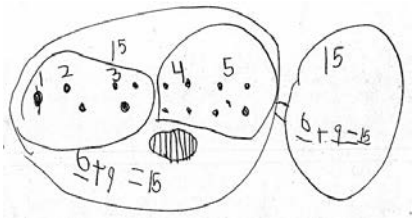

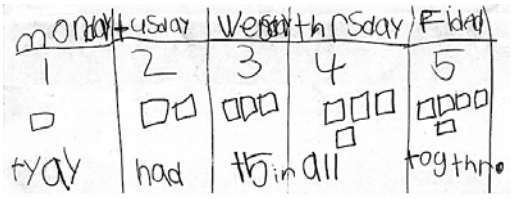

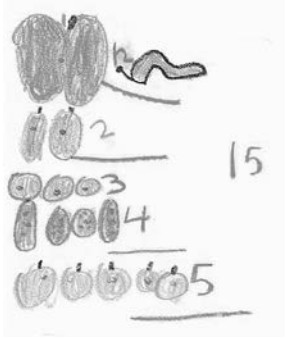

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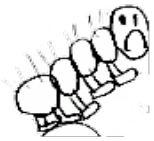

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http://www.nctm.org/PtA/

NCTM® NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*

Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*

Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*

Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*

Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*

Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*

Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*

Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM.

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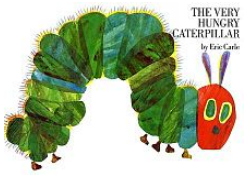
The Caterpillar Task



On Monday the hungry caterpillar ate through one apple, but he was still hungry. On Tuesday he ate through two pears, but he was still hungry. On Wednesday he ate through three plums. On Thursday he ate through four strawberries. On Friday he ate through five oranges. How many pieces of fruit did the hungry caterpillar eat during the week?

Make a picture or diagram that might be used by Grade 1 students.

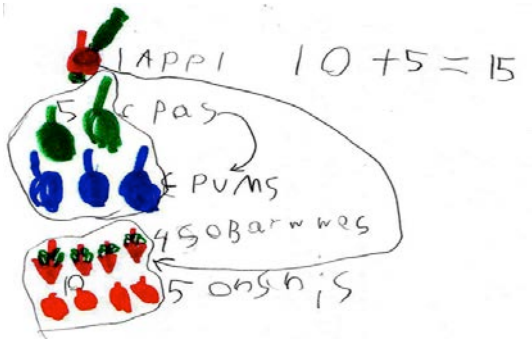
Ponder potential approaches and student struggles.

Types of Questions in Mathematics Teaching

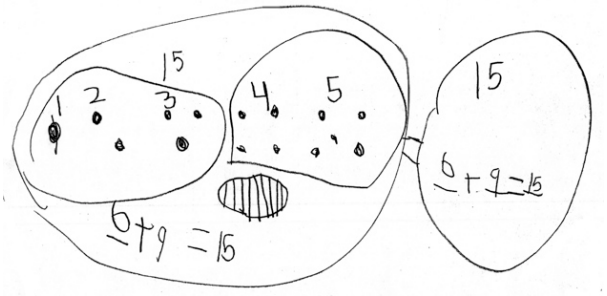
Question Type	Purpose	Examples
Gathering information	These questions ask students to recall facts, definitions, or procedures.	• How many pieces of fruit did the caterpillar eat on Friday? • Can you show me how you counted the fruit?
Probing thinking	These questions ask students to explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or completion of a task.	• I see you wrote 10 + 5 on your paper. Where did the ten come from? • Tell me about your picture. I see you wrote the days of the week and then drew squares.
Making the mathematics visible	These questions ask students to discuss mathematical structures and make connections among mathematical ideas and relationships.	• Marisa wrote 1 + 2 + 3 + 4 + 5 = 15. Is that okay to write an equation with all those plus signs? • What pattern do you see in the equations 10 + 2 = 12, 10 + 3 = 13, 10 + 4 = 14, and 10 + 5 = 15?
Encouraging reflection and justification	These questions reveal deeper insight into student reasoning and actions, including asking students to argue for the validity of their work.	• I see you put a circle around the 1, 4, and 5. Why did you put these pieces of fruit together? • What makes 10 + 6 equal to 9 + 5?
Engaging with the reasoning of others	These questions help students gain understanding of each other's solution paths and thinking, and lead to the co-construction of mathematical ideas.	• Who understands Shyanne's explanation and can say it back in your own words? • Can you add on to what Nate's said? • Do you agree or disagree with Anne? Why?

Source: Huinker, D., & Bill, V. (2017, in press). *Taking action: Implementing effective mathematics teaching practices in grades K-5*. Reston, VA: National Council of Teachers of Mathematics.

Ms. Bouchard's Class Discussion of Solution Paths for the Caterpillar Task



Cole



Evan

Segment #1

We enter the whole group discussion after Cole shares his solution path with the class.

Ms. B: How did you solve for the total number of pieces of fruit?

Cole: I counted the 4, 5, and 1 because that makes 10. Then I added five more.

Ms. B: How many students understood what Cole did? *(Five students raise their hands.)* Rosa, can you say back what Cole did in your own words?

Rosa: He knows that 4 + 5 and 1 makes 10, and he likes to make ten, because it is Easier, and then Cole adds five more.

Ms. B: Can anyone else say back what you heard?

Earth: 10 + 5 is 15 pieces of fruit.

Sengi: When you make ten, you don't even have to count five more because it just goes on the end.

Ms. B: What does Sammy mean when he said, "It [the ones] just go on the end"?

Julia: Like 10 + 5 = 15, 10 + 4 = 14, and 10 + 2 is 12. The ten is always there and then you add the ones on.

Ms. B: *(The teacher records 10+5, 10+4, 10+3, and 10+2.)* So the equations show us that with total amounts such as 15, 14, 13, or 12. We see that we have one ten, in the ten's place, and then some extra ones, in the one's place. *(Mrs. B underlines the tens and circles the ones.)*

The same pattern of talk is used to discuss Evan's solution path of 6 + 9.

Segment #2

Next we share the discussion of the comparison between 10 + 5 (Cole) and 6 + 9 (Evan).

Ms. B: Thank you for sharing your way, Cole and Evan. Cole wrote 10 + 5, and he got 15 pieces of fruit. Evan said he solved 6 + 9, and he got 15 pieces of fruit too. How can both Cole and Evan each get 15 pieces of fruit when they each wrote and solved a different equation? (*10 + 6 = 15 and 6 + 9 = 15 is recorded on the board.)*

Evan: I did it like Cole.

Irene: I know, 10 + 5 = 15.

Ms. B: How did you know the sum was 15?

Irene: I did 10 and then went 11, 12, 13, 14, 15. *(Uses her fingers to keep track of counting on.)*

Ms. B: You counted on 5 more from 10 and got 15. 10 + 5 = 15 pieces of fruit. Who can use what you know about 10 + 5 to think about the sum of 6 + 9?

Marcus: Can I show you? *(Marcus proceeds to the display of counters.)* You take one off of 10, and it is 9. Then you put the extra one with the 5, so now it is 6. So instead of 10 + 5, now he wrote 6 + 9. They both equal 15.

Ms. B: Who agrees with and understands what Marcus just said?

Maya: He just moved them around, but he didn't get any more.

Ms. B: Can someone else add on?

George: Marcus starts at 9 and counts 6. If you start at 10, you have to only count 5 because the other one is in the ten.

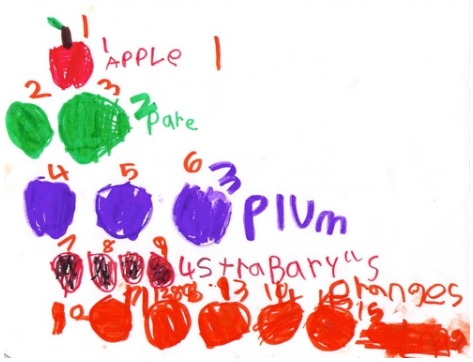
Ms. B: You can add either 10 + 5 or 6 + 9. Cole counted 10, so he only had to add on five more. Evan counted 9, so he had to add on 6. Can we write 10 + 5 = 6 + 9? Why or why not? Turn and talk with your partner.

Juan: Both are 15, so they are the equal.

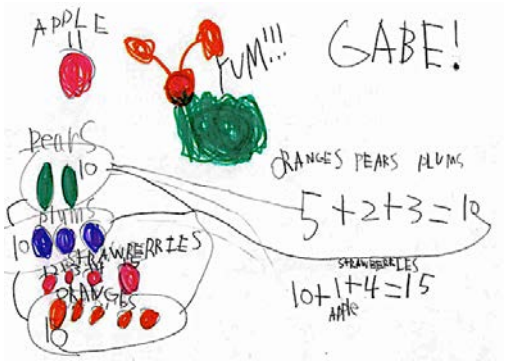
Ms. B: Who agrees or disagrees with Juan?

Source: Huinker, D., & Bill, V. (2017, in press). *Taking action: Implementing effective mathematics teaching practices in grades K-5*. Reston, VA: National Council of Teachers of Mathematics.

Ms. Chong's Class Discussion of Solution Paths for the Caterpillar Task



Sophia



Gabe

Segment #1

We enter the whole group discussion as Keisha shares her solution path with the class.

1 Ms. C: Sophia, how many total pieces of fruit did the caterpillar eat?
2 Sophia: 15
3 Ms. C: How did you figure out the answer was 15?
4 Sophia: I counted.
5 Ms. C: Let's all try that. Everyone count with me. *(Teacher points to each piece of fruit*
6 *as the students count in unison.)*
7 Ss: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
8 Ms. C: Nice job. So does everyone understand how Sophia figured out the answer was 15?
9 Ss: (Some students nod their heads.)
10 Ms. C: Thumbs up if you get what Sophia did.
11 Ss: (Some students put up their thumbs.)
12 Ms. C: Okay let's hear from Landon.

The same pattern of talk is used to discuss Landon's solution who also counted the fruit by ones.

Segment #2

Next we share the discussion of Gabe's solution.

Ms. C: Tell us how you solved it, Gabe.

Gabe: I counted 5 + 2 + 3, and that's 10. Then I added 1 + 4.

Ms. C: So you first added 5, 2, and 3, and got 10. Then you added on 1 + 4, which was 5 more?

Gabe: Yeah.

T: So how many total pieces of fruit did you get?

Gabe: 15.

Ms. C: So everyone count with me to check Gabe's work. He had 10, and then added on 5 more. So, let's start counting at 10. *(Teacher holds up one hand showing how to keep track of the counts as the students count on in unison.)*

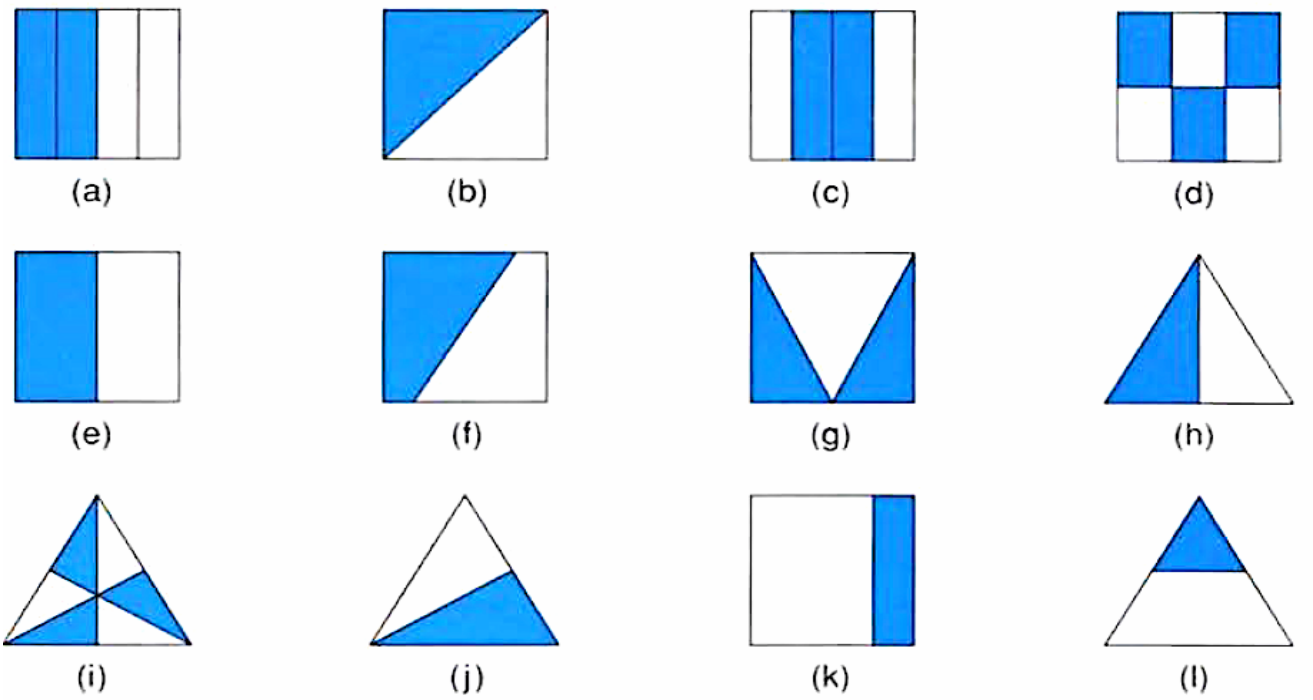
Ss: Ten… 11, 12, 13, 14, 15. *(Most students also use one hand to track their counting.)*

Ms. C: Nice job. So we can use counting all to find the answer, or a faster way would be to use counting on to find the answer.

Source: Huinker, D., & Bill, V. (2017, in press). *Taking action: Implementing effective mathematics teaching practices in grades K-5*. Reston, VA: National Council of Teachers of Mathematics.

Half-of-a-Whole Task

Identify all of the figures that have one-half of it shaded and be prepared to explain and justify how you know that one-half of the figure is or is not shaded. Write a written description giving your reasons why each figure is or is not showing halves.



Task adapted from Watanabe, T. (1996, April). Ben's understanding of one-half. *Teaching children mathematics* (2): 460-464.

Teachers Goals for the Half of a Whole Task

Ms. Brooks established two mathematics goals for the lesson, which she used as a target toward which she advanced student learning.

- Students will understand that one-half of a figure can be represented in multiple ways as long as the areas of each half have equal size, regardless of the location of the pieces that comprise each half
- Students will notice and understand a numeric relationship between the numerator and the denominator for fractions that are equivalent to one half, specifically, when the denominator is twice the value of the numerator the fraction is equivalent to one half.

Analyzing Teaching and Learning 9.1
Student Struggles and Teacher Supports

Watch the video clip of Ms. Brooks interacting with a small group of students.

- What are the students struggling with as they work on the Half-of-a-Whole task?
- What progress do students make in their understanding of the mathematics?
- How does the teacher support students in struggling productively in understanding the mathematics toward the intended learning goals?

Source: Huinker, D., & Bill, V. (2017, in press). *Taking action: Implementing effective mathematics teaching practices in grades K-5*. Reston, VA: National Council of Teachers of Mathematics.

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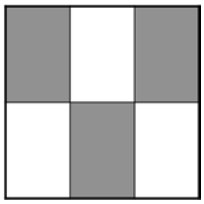
Handout 8

Lesson: Half-of-a-Whole
Teacher: Millie Brooks
District: Paterson Public Schools
Grade: 3

Javier: He said it was like 5 times bigger than the…

Teacher: Tell me whether or not this (*points to Figure d*) shows me halves.



[d]

Mayah: It doesn't show you halves.

Teacher: Prove it.

Mayah: It doesn't show you halves …

Teacher: I say it does. Prove it to me.

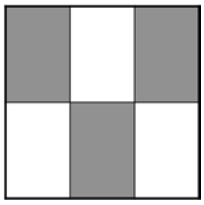
Mayah: Because…

Teacher: Well, prove me wrong.

Mayah: 2. It shows you 2. It's 2.

[Crosstalk]

Mayah: It doesn't show you halves because one is—because one is not shaded. Three are shaded in this, but then there are 3 that are not shaded. (*Student points to three white parts and three shaded parts*.)



[d]

Teacher: Oh, so you're telling me that 3 are shaded. I want you to convince them now, because you saw it. So now I want you to get them to see what you saw, that this is half. Okay? If you need to cut it up, cut it up. Don't be afraid to cut it up. Okay?

Teacher leaves to monitor other small groups and returns in 2 minutes.

Teacher: When I left here, you were held accountable for what you were saying. You had to prove to your group that that was a half. Okay?

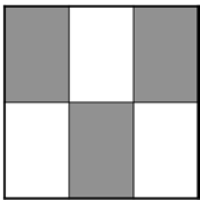
Zaria: I only proved it to them two. I didn't prove it to him because he didn't ...

Teacher: Okay. Mayah, show me why that's a half. All right. Who can tell me what Zaria was talking about before I left? Yes?

Hensley: She was talking about how it was half, but…

Teacher: Show me the shape. Show me the shape. Okay.

Hensley: But this one was half. *(Student holds up Figure d showing 3/6.)*



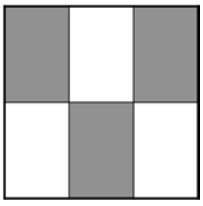
[d]

Teacher: Okay.

Hensley: But we didn't think so, so she said, do you think it's more than 3 or less than 3? It's half. And then we said less than 3. So she was trying to tell us how it was half, but we didn't believe what she said.

Teacher: You still don't believe her? Mayah, why don't you believe her?

Mayah: I kind of don't believe her, because how—let me see this. Because of how there are 3 shaded, but there's not 1 only shaded, like this one. (*Student points to the 3 parts on Figure d and compares it to the one shaded part on Figure e.)*



[d]

[e]

Teacher: Okay.

Mayah: But I know that it is equal, because 3 are shaded and 3 are not.

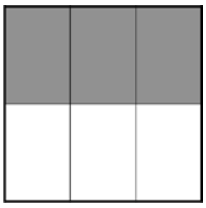
Teacher: Oh, so you know it's equal because 3 are shaded and 3 are not shaded? So we have an equal number that is shaded and an equal number that is not shaded? Okay. Can you write me a fraction for that? Everyone, write me a fraction for this.

Mayah: (*Writes on the paper*.)

Teacher leaves to monitor other small groups and returns in 3 minutes.

Mayah: (*The student cut apart Figure d and rearranged the pieces to look like Figure e.)*

This is—this is equal to half because like this is the same thing. This is the same exact same thing, because we have 3 that are shaded that look just like this, and 3 on the bottom that are not shaded that look just like this.



[Rearranged *Figure d*]

[e]

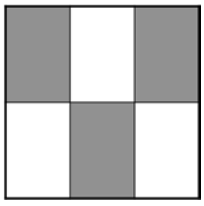
Teacher: Okay. Hensley, where's your shape like that? Hensley, do you agree that that's a half?

Hensley: Yes. That's a half.

Teacher: Okay. Mayah, now prove to him that this is the same as this. (*Teacher points to 1/2.*)

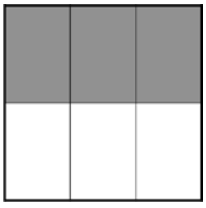
Mayah: Hensley, look, this is the same as this, but it's just into pieces. And this one's not into pieces. See, there's…what…there's 3 ones on the bottom, but this one doesn't have 3 right at the bottom. It has 3 shaded at the top. This one doesn't have 3 shaded at the top. It just has some straight big pieces, which is all into one.

Hensley: But still, but we cut it up diff- like, processes, because like, because this one was here before, so… (*Student moves the sixths around to show the original Figure d*.)

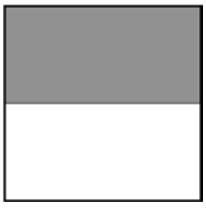


[d]

Javier: Yeah, but then we cut it up, and then this…and then this was equivalent to this one, because this is how I know it fits, because if you go like this with them… *(Student puts the three sixths on top of the shaded one half of Figure e.)*



[Rearranged *Figure d*]



[e]

Teacher: Why don't you do the same? (*Hensley cuts apart his Figure d*.)

Javier: First, the 3 shaded, on the part that's shaded, at the top.

Teacher: Okay. Don't move so fast for him. Hensley? Tell him again.

Javier: You put…so this is…this here is one half, but then if you put the 3 pieces here, then it'll fit in the shaded part, the shaded part.

Mayah: But Hensley was saying that you could put 2 of these in here like that. That's what Hensley said at first. But I told him we have to use all of our shapes.

Teacher: What makes it half?

Hensley: Because like these pieces, like if you put this together, if you put these together, it will be half when you put the shaded parts on the other one. And then if you put these on this one.

Teacher: Okay.

[End of Audio]

Taking Action: Implementing Effective Teaching Practices in Grades K-5

Chapter	Analyzing Teaching and Learning Activities (ATL)	Artifacts	Grade
1	ATL 1.1. Investigating Teaching and Learning in a Grade 3 Classroom	Narrative case	3
2	ATL 2.1. Three Potential Goal Statements for a Multiplication Lesson	Goal statements	3
	ATL 2.2: Observing How Goals Guide Teacher-Student Interactions	Video & transcript	K
	ATL 2.3: Examining How Goals Inform Instructional Decisions	Video & transcript	5
	ATL 2.4: Relating Goals to the Other Teaching Practices	Video & transcript	K
3	ATL 3.1. Comparison of Two Multiplication Tasks	Tasks	3
	ATL 3.2. Sorting Tasks by Cognitive Demand	Set of tasks	K-5
	ATL 3.3. Comparing Instruction in Two Classrooms	Narrative cases	3
	ATL 3.4. Exploring a Sequence of Tasks	Set of tasks	1
4	ATL 4.1: Mathematical Affordances of a Task	Student work	3
	ATL 4.2: Using an Equation Sequence to Build toward Fluency	Video & transcript	1
	ATL 4.3: Considering Sequences of Tasks	Sequence of tasks	4
	ATL 4.4: Student Learning Related to the Task Sequence	Student work	4
5	ATL 5.1: Noticing the Effect of Teacher Questions	Class discussion	1
	ATL 5.2: Noticing How Teachers Respond to Students	Class discussion	1
	ATL 5.3: Posing Assessing and Advancing Questions	Student work	3
6	ATL 6.1: Assessing Knowledge of Representations	Student work	3
	ATL 6.2: Flexibility in Using Representations	Student work	2
	ATL 6.3: Shifting Instruction to a Focus on Representations	Vignettes	1 & 4
7	ATL 7.1: Discourse in the Case of Robert Harris	Narrative cases	3
	ATL 7.2: What is the Teacher Doing? What are Students Doing?	Lesson transcript	1
	ATL 7.3: Anticipate Student Approaches	Task	4
	ATL 7.4: Select, Sequence, and Connect	Student work	4
8	ATL 8.1: Examining the Potential of a Task	Task	1
	ATL 8.2: Examining the Details in Student Work	Student work	1
	ATL 8.3: Building on Students' Mathematical Ideas	Video & transcript	3
	ATL 8.4: Writing as a Window into Students' Thinking	Student work	3
	ATL 8.5: Structuring the Class Discussion	Narrative case	5
9	ATL 9.1: Student Struggles and Teacher Supports	Video & transcript	3
	ATL 9.2: Student Impasse Scenarios	Scenarios	5
	ATL 9.3: Intentional Teacher Actions	Class discussion	1
10	ATL 10.1. Reflecting On and Improving Teaching	Narrative case	4

Source: Huinker, D., & Bill, V. (2017, in press). *Taking action: Implementing effective mathematics teaching practices in grades K-5*. Reston, VA: National Council of Teachers of Mathematics.