What is Active Learning in Mathematics for Higher Education?

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Introductions

Introduce yourself to the people sitting around you.

CBMS Statement on Active Learning

What is Active Learning and Why is it Important?

What Does Active Learning Look Like In Practice in College and University Math Courses?

How Can We Prepare Students For College and University Math Courses?

CBMS Statement

In August 2016, fifteen presidents of member societies of the Conference Board of the Mathematical Sciences issued a joint statement[1] on active learning containing the following call to action:

- *we call on institutions of higher education, mathematics departments and the mathematics faculty, public policy-makers, and funding agencies to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms.*

Presidents of the following member societies signed the statement:

[1] available at www.cbmsweb.org

- American Mathematical Association of Two-Year Colleges (AMATYC)
- American Mathematical Society (AMS)
- Association of Mathematics Teacher Educators (AMTE)
- American Statistical Association (ASA)
- Association for Symbolic Logic (ASL)
- Association of State Supervisors of Mathematics (ASSM)
- Association for Women in Mathematics (AWM)
- Benjamin Banneker Association (BBA)
- Institute of Mathematical Statistics (IMS)
- Mathematical Association of America (MAA)
- National Association of Mathematicians (NAM)
- National Council of Supervisors of Mathematics (NCSM)
- National Council of Teachers of Mathematics (NCTM)
- Society for Industrial and Applied Mathematics (SIAM)
- TODOS: Mathematics for ALL (TODOS)

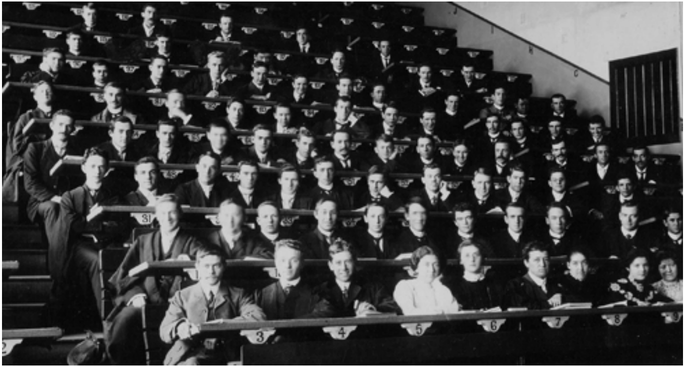
Common questions

1. What is active learning?
2. What does active learning look like in practice in college and university courses?
3. How can we prepare students for college and university math courses (starting from Kindergarten!)?

Discuss with your neighbors

In your personal experience, what happened during class time in your "typical" college/university math courses?

This is a chemistry class at the University of Sydney in 1903.



Source:
http://sydney.edu.au/science/chemistry/history/early-years-chemistry-lectures.shtml

What is Active Learning?

Two sample definitions:

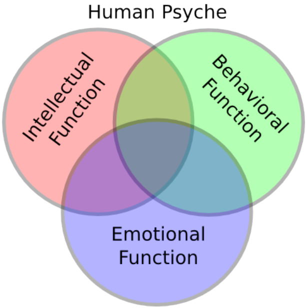
- *Active Learning* (AL) is generally defined as any instructional method that engages students in the learning process during class. In short, active learning requires students to do meaningful learning activities and think about what they are doing during class.
- AL engages students in the process of learning through activities and/or discussion in class, as opposed to passively listening to an expert. It emphasizes higher-order thinking and often involves group work.

There is not a unique definition of AL, either in popular use or in the research literature, and all existing definitions are inherently vague.

We will see later that AL can look *very* different across courses, departments, and universities.

What is Different About AL?

Modern psychology provides a basic framework of the human psyche with three domains.



Many math courses focus primarily on “Intellectual” aspects of student learning. Most AL techniques engage students across two or three of these domains.

What Are Our Goals For Students?

At the college/university level, the goal is typically for students to reach some level of mathematical proficiency by the end of each course.

What does this actually mean? And what role does active learning play here?

We will see that our three-domain psychological framework is reflected in the answers to these questions.

Mathematical Proficiency – NCTM

The National Council of Teachers of Mathematics split proficiency up into two categories in *Principles and Standards for School Mathematics* (2000).

- Content
  - Numbers and operations
  - Algebra
  - Geometry
  - Measurement
  - Data analysis and probability
- Process
  - Problem solving
  - Reasoning and proof
  - Making connections
  - Oral and written communication
  - Uses of mathematical representation

Mathematical Proficiency – Adding It Up

The 2001 NRC report *Adding It Up* emphasized a five "strand" model of proficiency, focused at the K-8 level:

- **Conceptual understanding:** comprehension of mathematical concepts, operations, and relations
- **Procedural fluency:** skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic competence:** ability to formulate, represent, and solve mathematical problems
- **Adaptive reasoning:** capacity for logical thought, reflection, explanation, and justification
- **Productive disposition:** habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy

This framework also works well for courses in high school and the first two years of postsecondary mathematics education.

Common Core State Standards for Mathematics

Other prominent standards/reports/guides make recommendations that reflect a broad vision of mathematical proficiency, e.g.: the Common Core State Standards for Mathematics includes both

- Standards for Mathematical Practice and
- Grade-Level Content Standards.

CCSSM Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

MAA Curriculum Guide Goals

The 2015 Mathematical Association of America *Curriculum Guide to Majors in the Mathematical Sciences* provides:

- four “Cognitive” goals for undergraduate major programs, and
- nine “Content” goals for undergraduate major programs.

Selected MAA Goals for Undergrad Major Programs in Mathematical Sciences

- Students should develop effective thinking and communication skills.
  - e.g. state problems carefully, use and compare analytical, visual, and numerical perspectives in exploring mathematics, assess the correctness of solutions, read mathematics with understanding, communicate mathematical ideas clearly and coherently both verbally and in writing, approach mathematical problems with curiosity and creativity and persist in the face of difficulties
- Students should learn to link applications and theory.
- Students should learn to use technological tools.
- Students should develop mathematical independence and experience open-ended inquiry.
- Students should learn to read, understand, analyze, and produce proofs at increasing depth as they progress through a major.

So. . . What About Active Learning?

Active learning classroom teaching techniques and environments are those that directly support student growth and development across this broad range of learning goals defining mathematical proficiency.

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- When you picture a college or university mathematics course for first-year students, how do you envision students receiving feedback and guidance regarding their "practices" of doing mathematics?

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Active Learning Classroom at University of Minnesota.

Source: https://cei.umn.edu/support-services/tutorials/active-learning-classrooms

This question has many answers!

The key here is to articulate that students will likely encounter one or both of:

- active learning *techniques*, and
- active learning *environments*.

AL Example #1: Procedural computation

In Calculus I: During a lecture, students are provided with one minute to compute the derivative of a polynomial independently.

AL Example #2: Think-Pair-Share

In College Algebra: Students are shown four functions and four graphs. They are provided with two minutes to match each function with its graph.

Students then are provided with two more minutes to discuss their answer with their neighbors.

Students are then asked to share with the class questions that arose during their conversations, and the resolutions that they worked out with their neighbors.

AL Example #3: Extended small group work

In Mathematics for Elementary School Teachers: Students are assigned to small groups. Each group is provided with a short essay justifying the validity of the equation:

$$\frac{2/3}{5/7} = \frac{2}{3} \cdot \frac{7}{5}$$

Each sentence of this justification is written on a small sheet of paper, and these papers are mixed together in a bag. Further, one of these sentences has a minor error in it.

The goal is for each group to work collaboratively to arrange the sentences in the proper order, and to identify and fix the error.

- Next we consider examples of AL environments.

Inverted (Flipped) Classrooms

- In an inverted (or "flipped") classroom environment, instructor presentations of basic definitions, examples, proofs, and heuristics are provided to students in videos or in assigned readings that are completed prior to attending class.
- As a result, class time becomes available for active learning tasks that directly engage students.
- The type of task that instructors use during this time ranges from using think-pair-shares, with complex problems or examples, to having students work in small groups on a sequenced activity worksheet with occasional instructor or teaching assistant feedback.

Photo courtesy of University of Pennsylvania Department of Mathematics.

Math Emporium

- The math emporium model uses a large room filled with computer workstations at which students progress through self-paced online courses.
- Unlike inverted classes, many emporium models do not include a lecture component at all, and most have been developed to handle remediation issues and low-level courses such as developmental mathematics and college algebra.
- An emporium usually has tables at which students can work collaboratively and is staffed by a large number of teaching assistants and tutors.
- Because the work of students is self paced, students spend most of their time actively engaging with course content through a range of tasks.

Photo courtesy of Virginia Tech Department of Mathematics.

Inquiry-Based Learning

- With Inquiry-Based Learning (IBL) courses, class time is spent with students working on problem sets individually or in groups, presenting solutions and/or proofs to the class, and receiving feedback from peers and faculty.
- IBL courses are not based on pure, unguided student discovery; instead, faculty design a series of carefully scaffolded (i.e., sequenced in a structured way) activities, some for individuals, some for pairs, some for small groups, and some for the whole class, including mini-lectures as appropriate.
- Because faculty using IBL need to develop facility with a range of teaching strategies, and need to develop familiarity with many "teaching moves" that are not typically used in lecture environments, IBL is a more ambitious active learning environment.

Photo courtesy of Middlebury College Department of Mathematics.

Photo courtesy of University of Illinois at Urbana-Champaign Department of Mathematics.

Active Learning Can Be Incorporated Into Large Lecture Courses

New lecture hall at University of Kentucky.

A Caveat

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At University of Kentucky,

- 50% or more of the classroom time in *every Calculus I and II course* is based on small group work,
- *every precalculus and college algebra course* incorporates think-pair-shares using "clickers",
- *every math content course for elementary teachers* incorporates small group work and student presentations, and
- *most math majors take at least one inquiry-based learning course* at an advanced level.

This has all developed within the past 10-12 years, and similar changes have happened at many large state universities.

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- Read about mathematics!
- Talk about mathematics with their peers!
- Have opportunities to work on problems that require multiple steps to solve, and which they are expected to get stuck on for a while.
- Experience productive struggle and perseverence through small failures.

Experiencing mathematics as a "sense-making" activity that is collaborative and social, requiring clear communication and discussion, is a key component of preparing students for current math courses in higher education.

Thank you for listening!

Questions? Comments? Ideas?

Please join us to continue this discussion at Reflection Cove #2 from 12:30-1:30.