

Building Fluency During Intervention with a Powerful Pair: Visual Representations and Discourse

Dina Mendola,
US Math Recovery Council ®
Instructional Coach

dina@mathrecovery.org

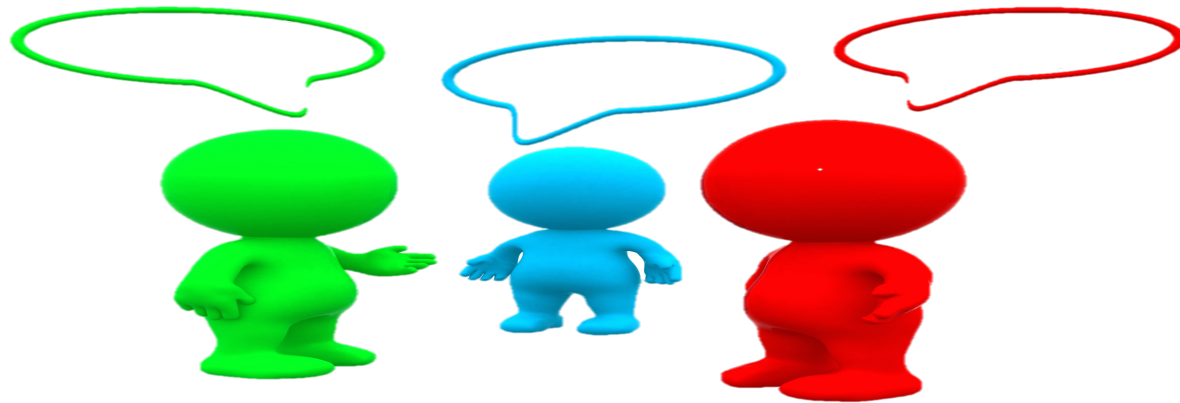
NCTM, 2017



Today We Will Discuss...

- Learn how we can use representations more effectively to build a deeper understanding of mathematics.
- Learn how to build in strong mathematical discourse to the use of instructional strategies to build fluency.

What does it mean to have computational fluency?



Computational fluency refers to having **efficient and accurate** methods for computing. Students exhibit computational fluency when they **demonstrate flexibility** in the computational methods they choose, **understand and can explain** these methods, and **produce accurate answers efficiently**.

Elements of Fluency

EFFICIENCY

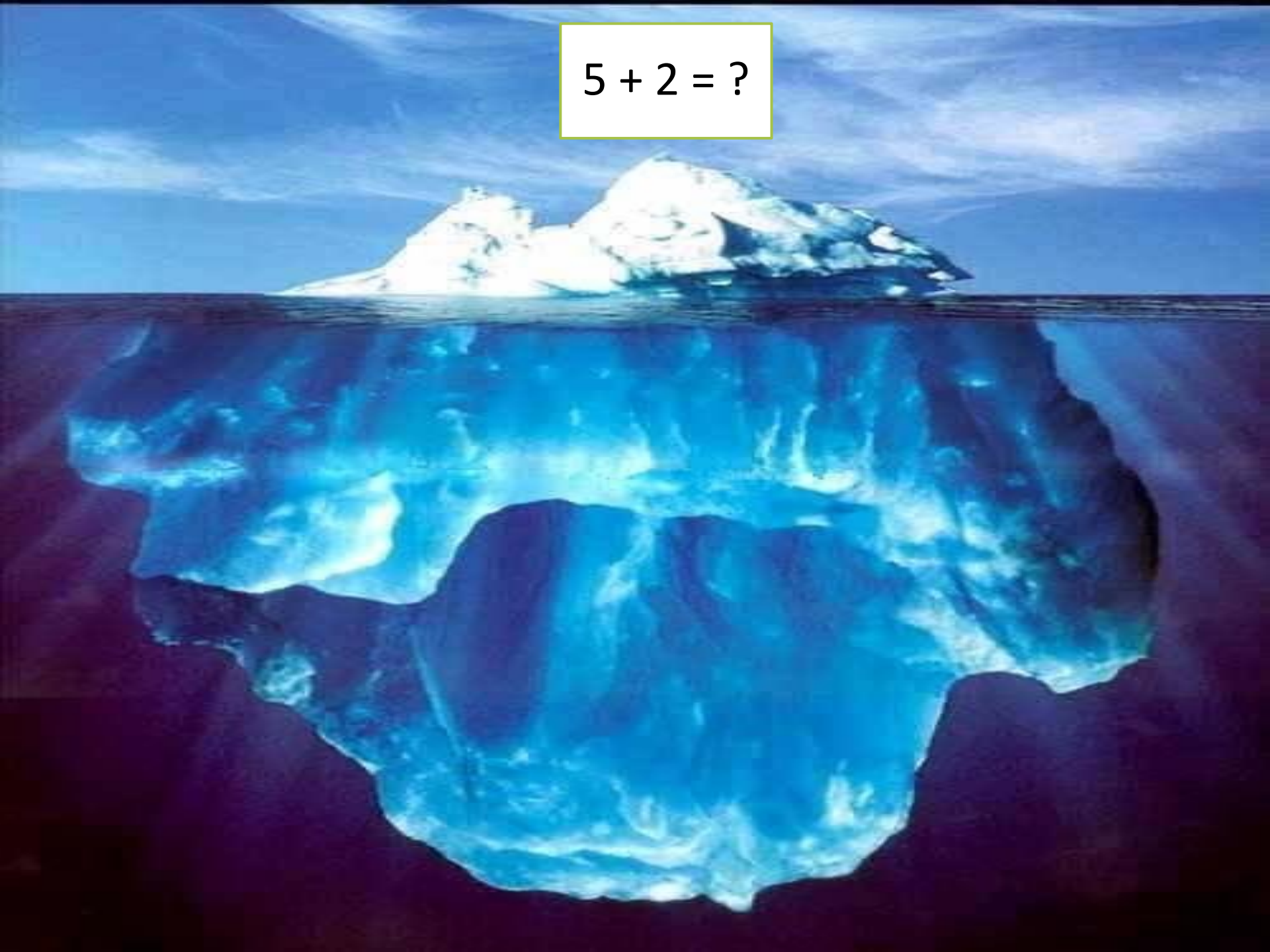
ACCURACY

FLEXIBILITY

The Progression:

Grade Level	Required Fluency
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 Add/subtract within 100
3	Multiply/divide within 100 Add/subtract within 1000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations

$$5 + 2 = ?$$



The Iceberg Metaphor

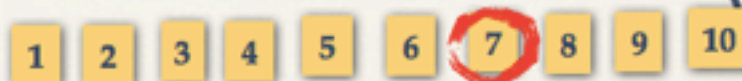
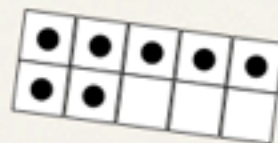
David Webb, Executive Director, Freudenthal Institute USA

the tip of the iceberg

formal

$$5 + 2 = 7$$

pre-formal:
structured



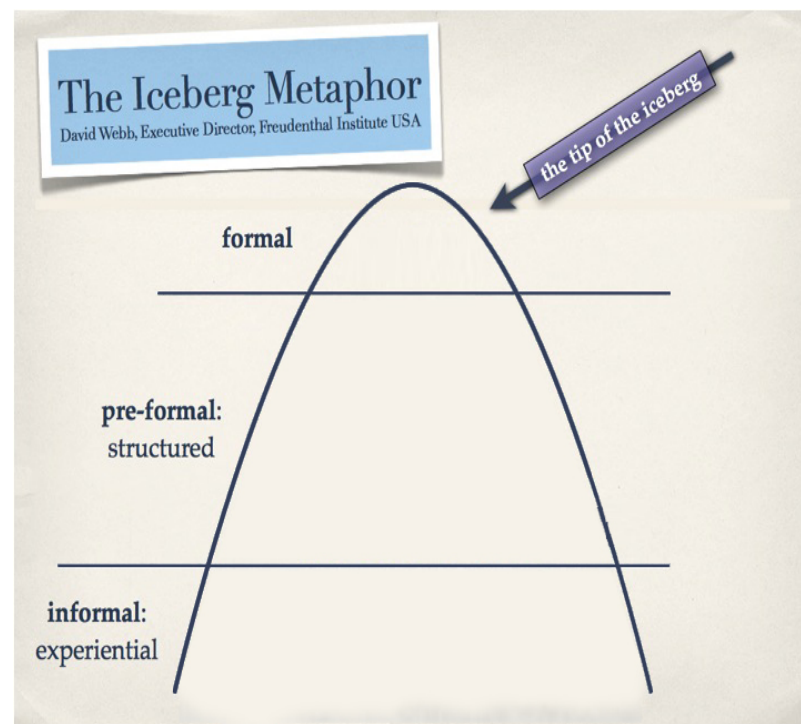
informal:
experiential



What about students that don't make these cognitive connections?



How do we get them there?



Developing Efficient Strategies

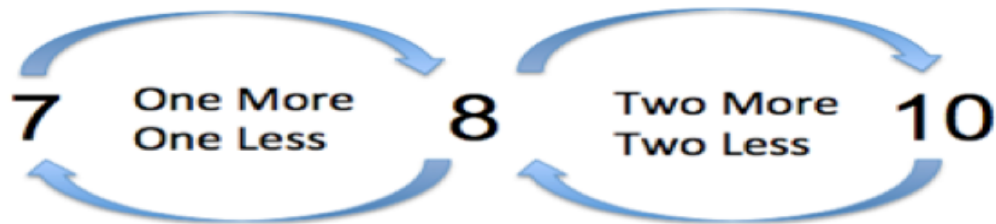
“A strategy is most useful to students when it is theirs, built on and connected to concepts and relationships they already own.”

Van de Walle & Lovin, 2006, p. 96

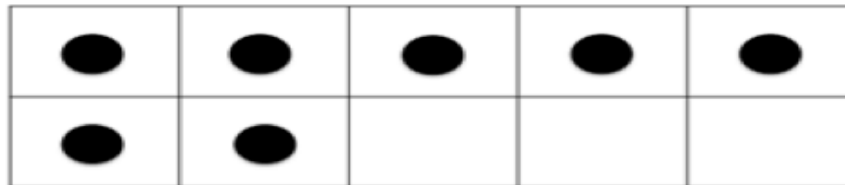
Spatial Relationships



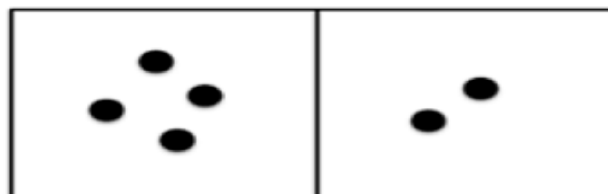
One More, Two More and One Less, Two Less



Benchmarks or Anchors to 5 and 10

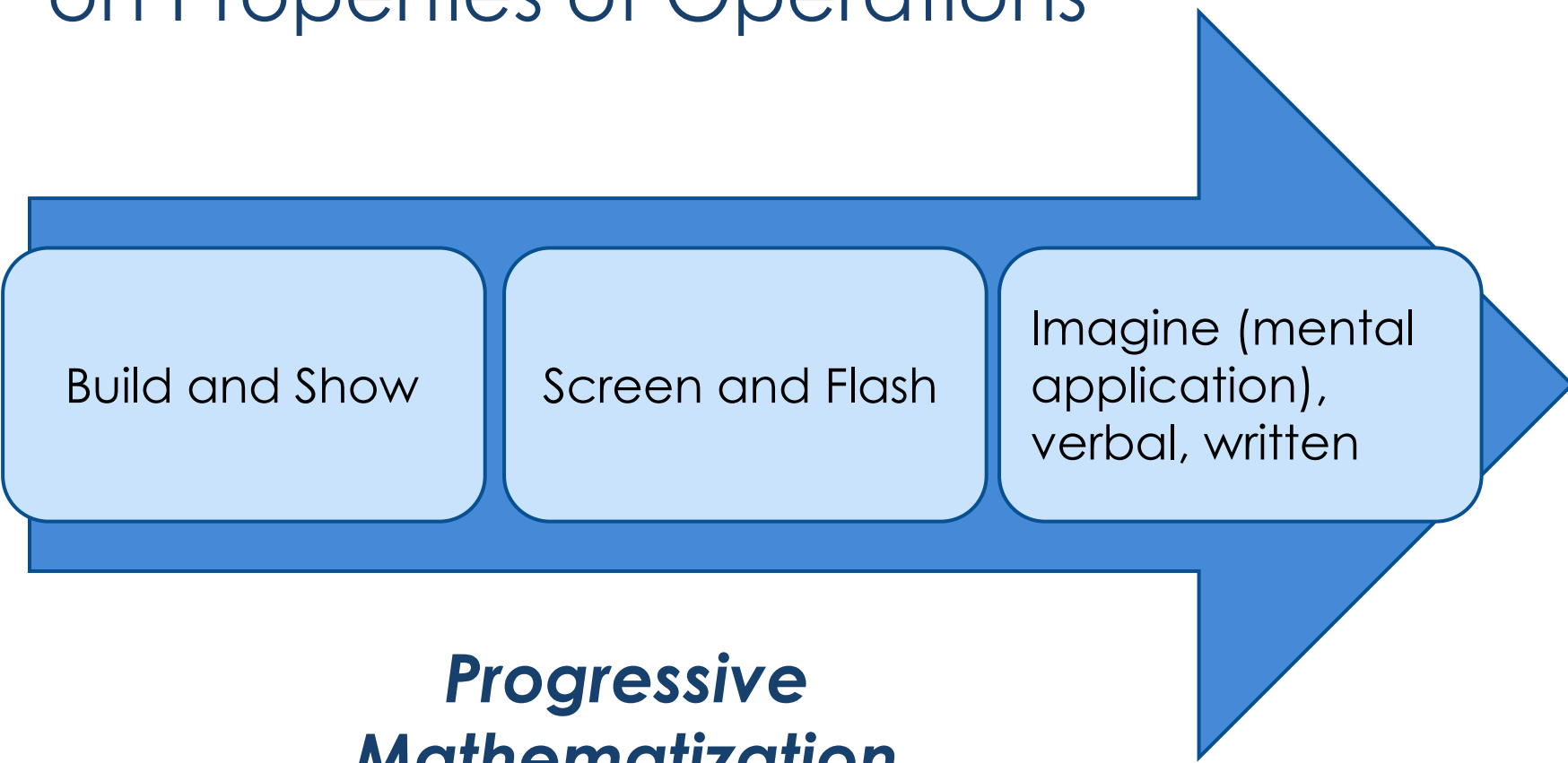


Part-Part-Whole



Goal:
*Moving
beyond
counting by
ones*

Develop and Use Strategies Based on Properties of Operations



Build and Show

Screen and Flash

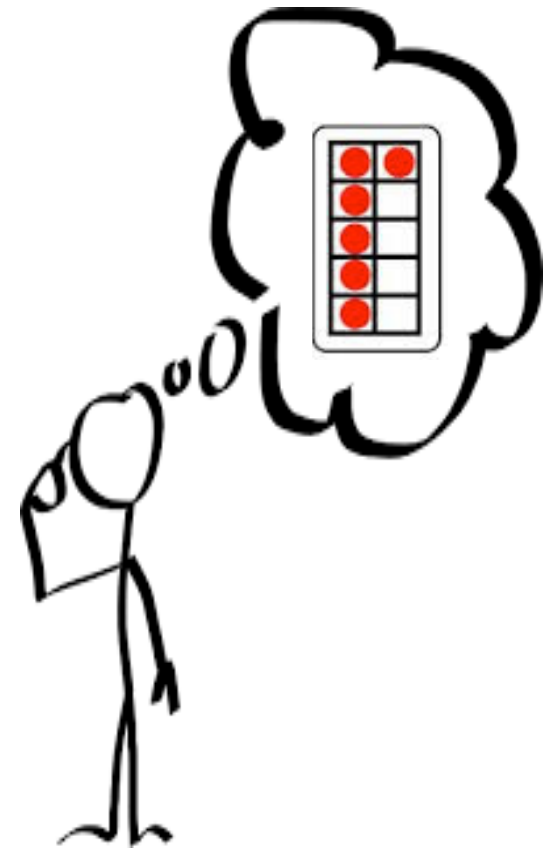
Imagine (mental application),
verbal, written

***Progressive
Mathematization***

Figurative Thinking

$$6 + ? = 10$$

- Mental replay
- Means of finding the patterns that give meaning to experience
- Visual process of seeing patterns within shape, function, or operations



It is about pushing thinking in order to
develop relational thinking!!

*Facile mental computation develops
by enriching math knowledge
and mathematizing students'
own mental strategies.*



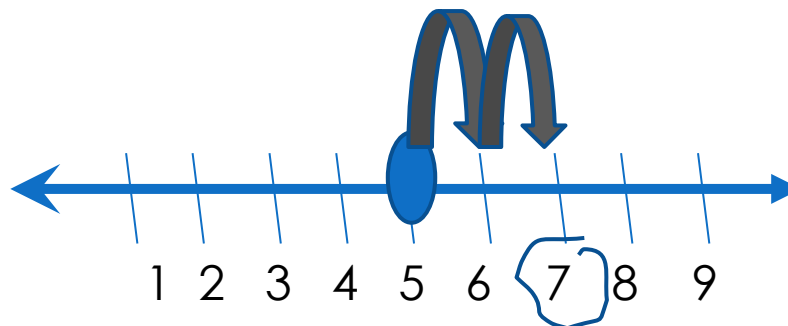
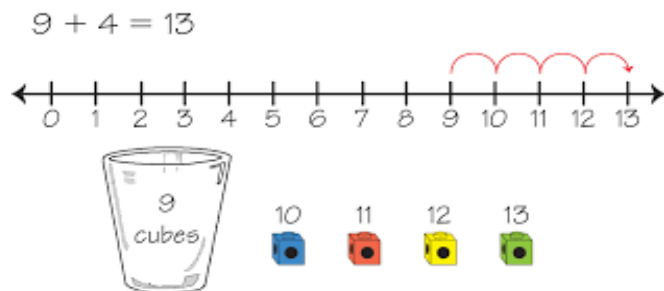
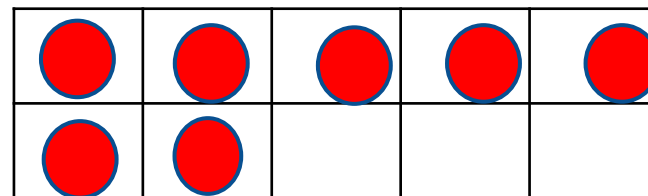
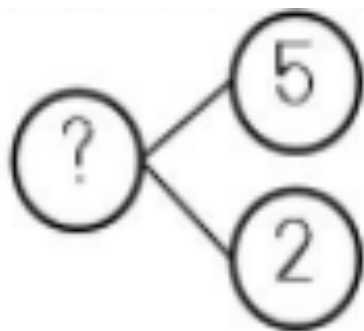
Wright, Ellemor-Collins, Tabor, *Developing Number Knowledge: Assessment, Teaching & Intervention with 7-11 year olds*, pg. 99.

K-2 Visual Representations

- What visual representations do we use in K-2?
- Where and how does discourse occur?
- Are kids making the connection?
What stumbling blocks do you see?

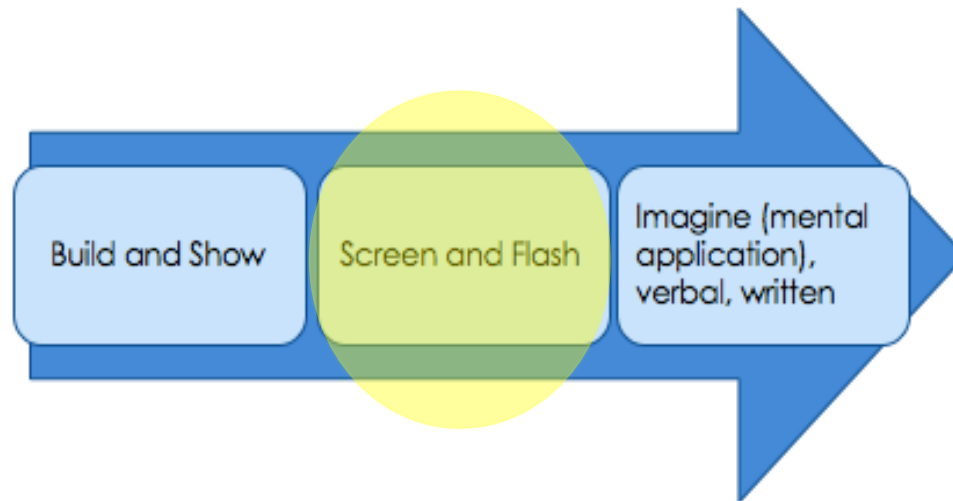


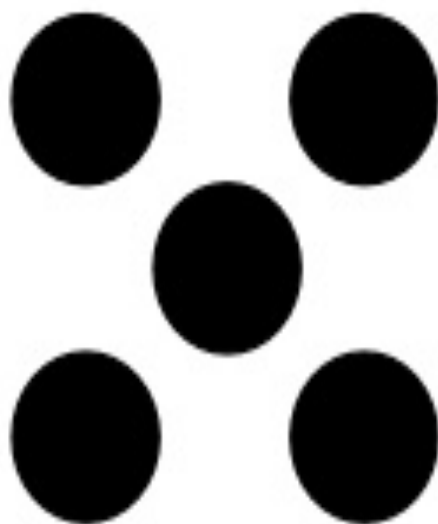
K-2 Visual Representations



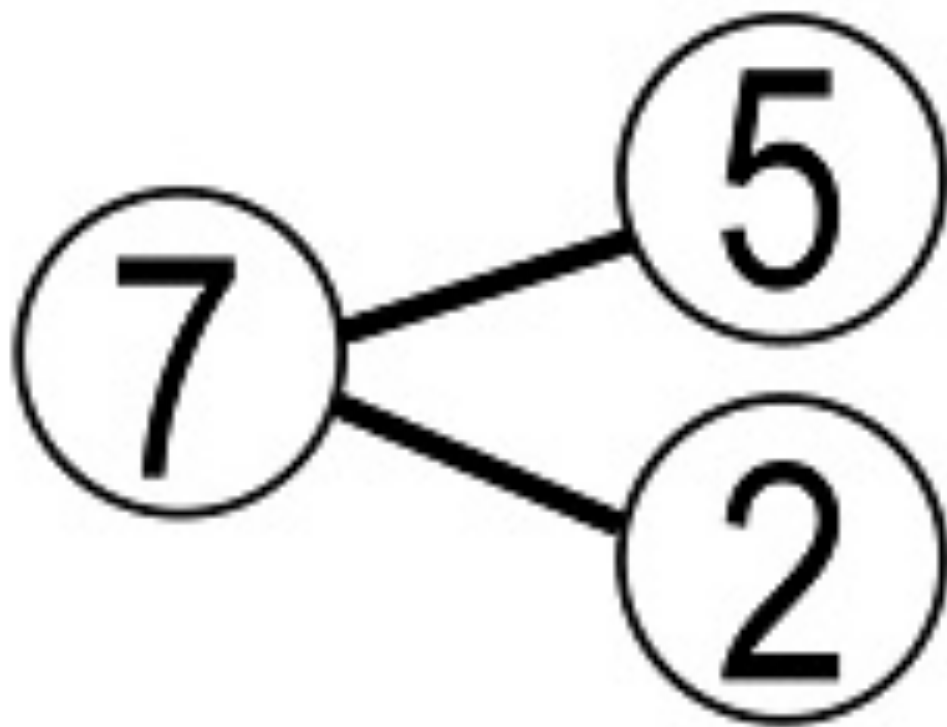
A visual equation demonstrating the commutative property of addition. On the left, there are 6 blue dots followed by a plus sign and 3 orange dots. This is followed by an equals sign. On the right, there are 3 orange dots followed by a plus sign and 6 blue dots. Below the first group of dots is the equation $6 + 3$, and below the second group is the equation $3 + 6$.

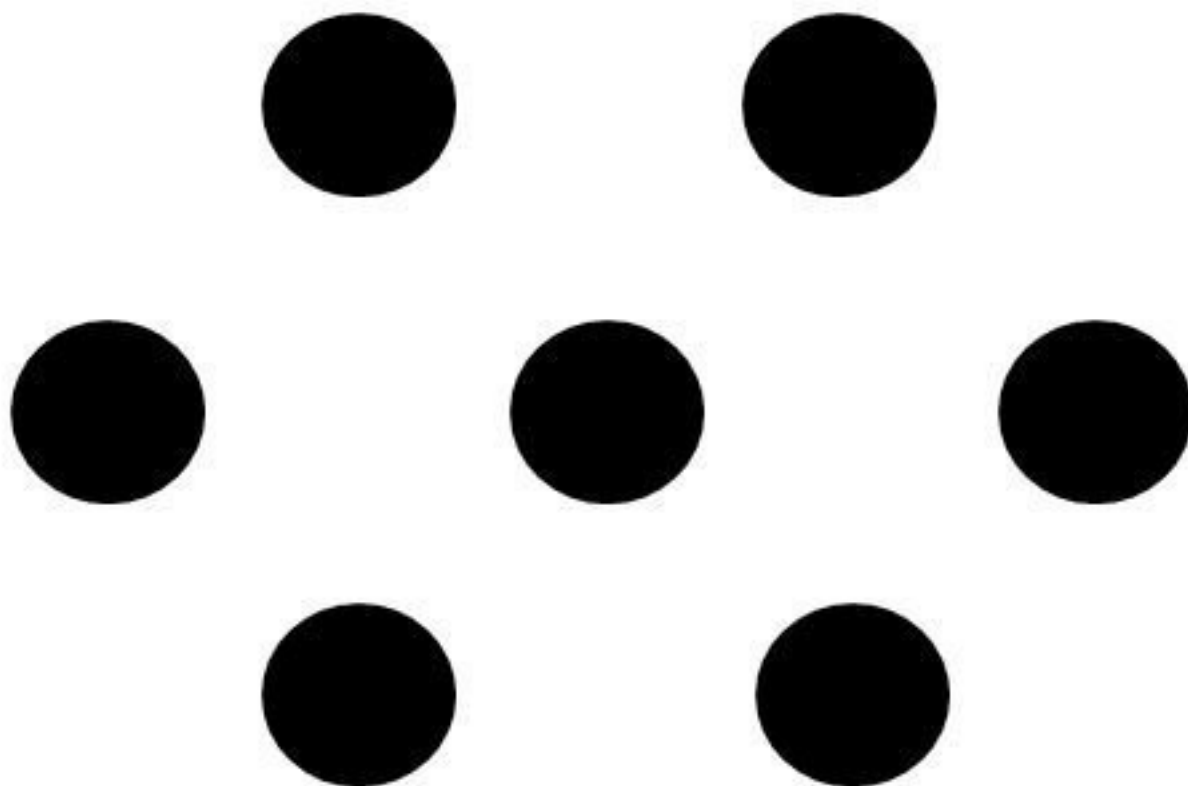
Number Bonds / Dot Images

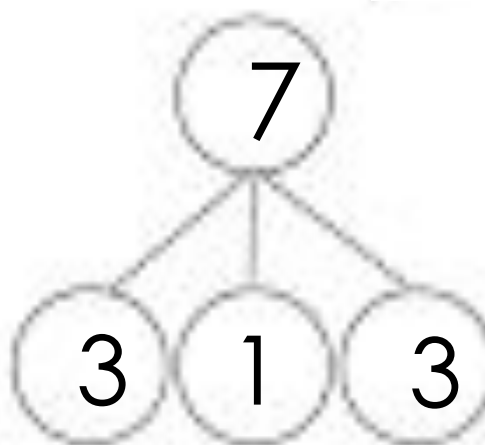
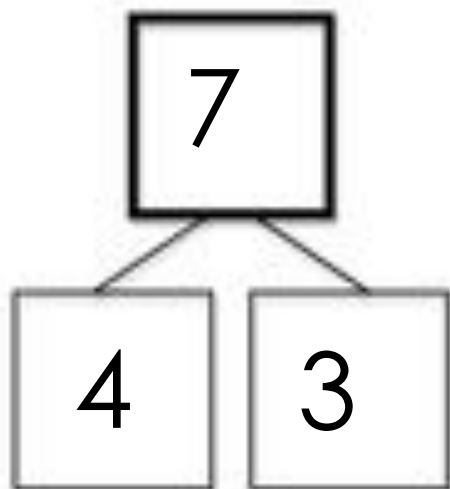
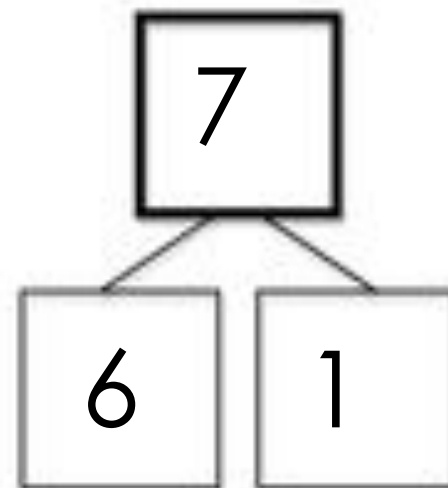
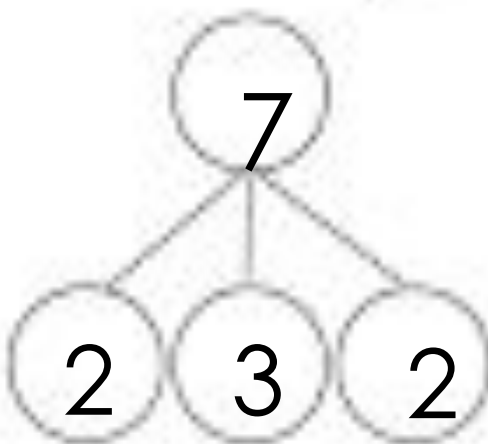
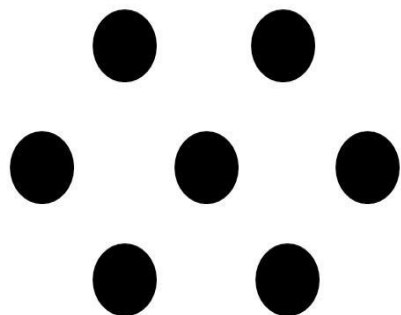


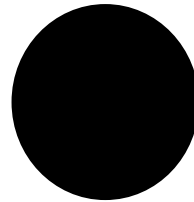
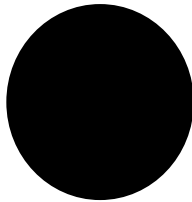
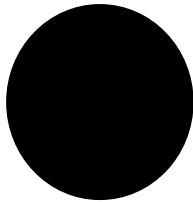
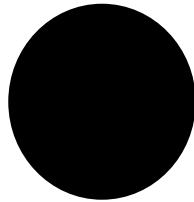
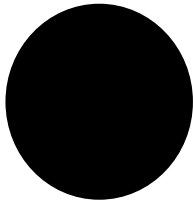
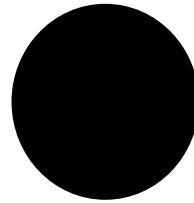
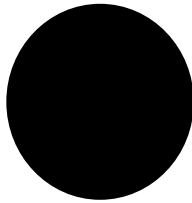
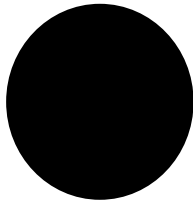
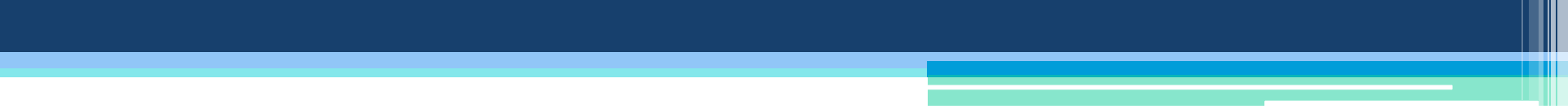


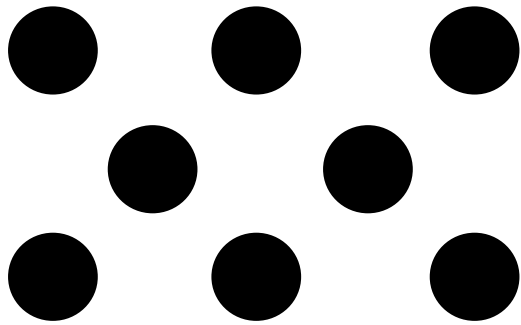
Teacher Notates Student Thinking During Discourse



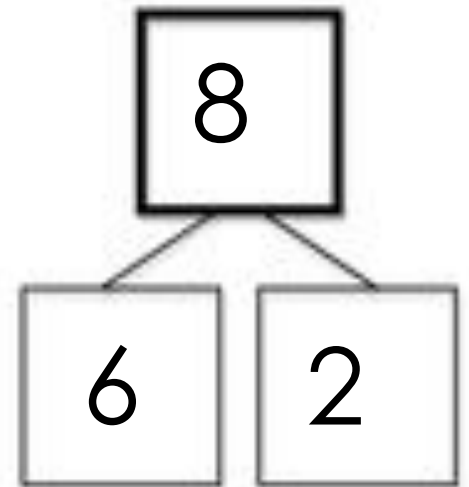
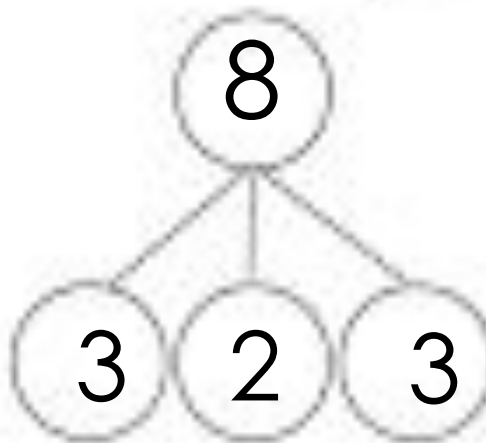
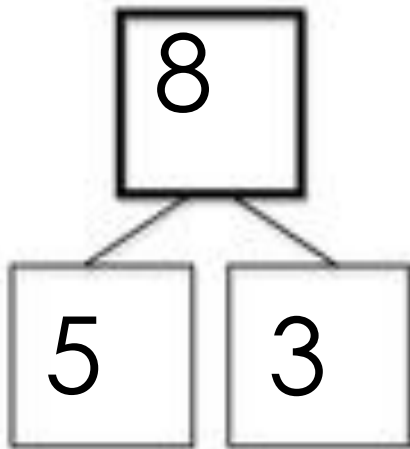




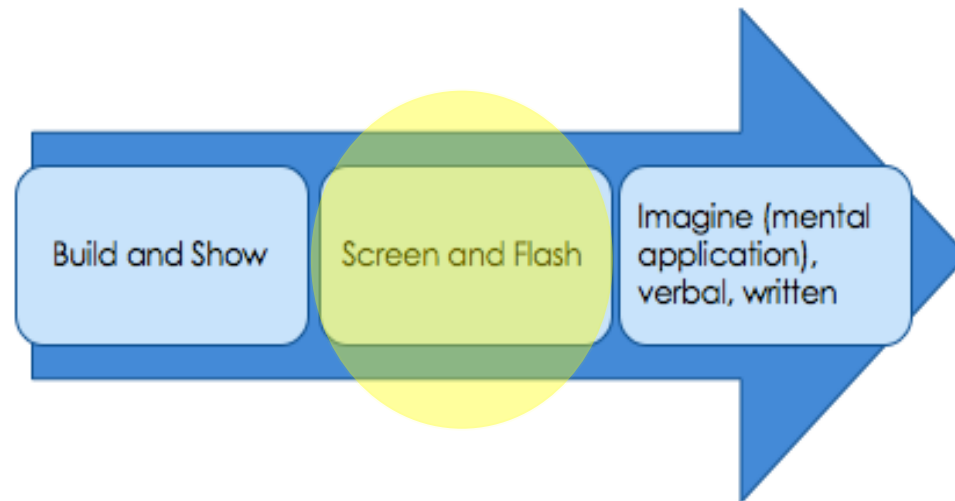


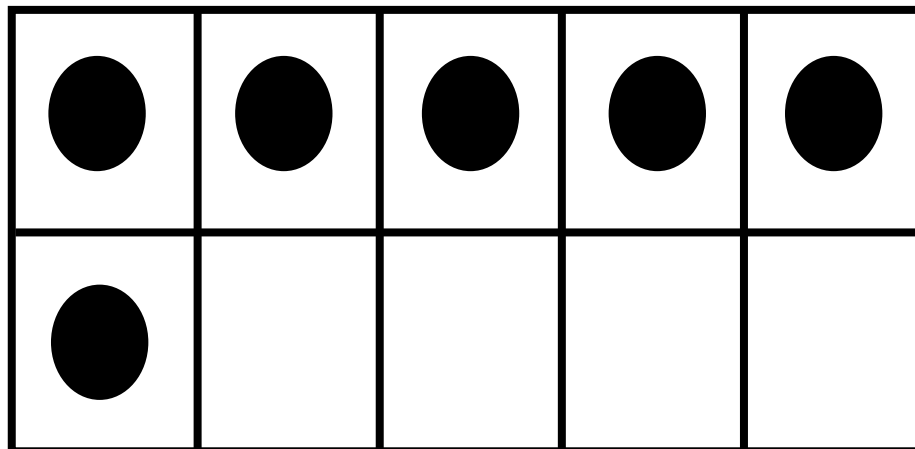
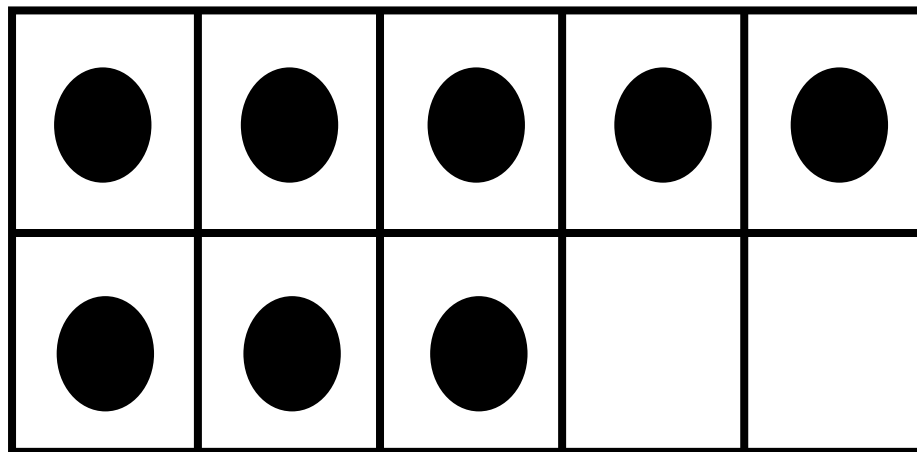


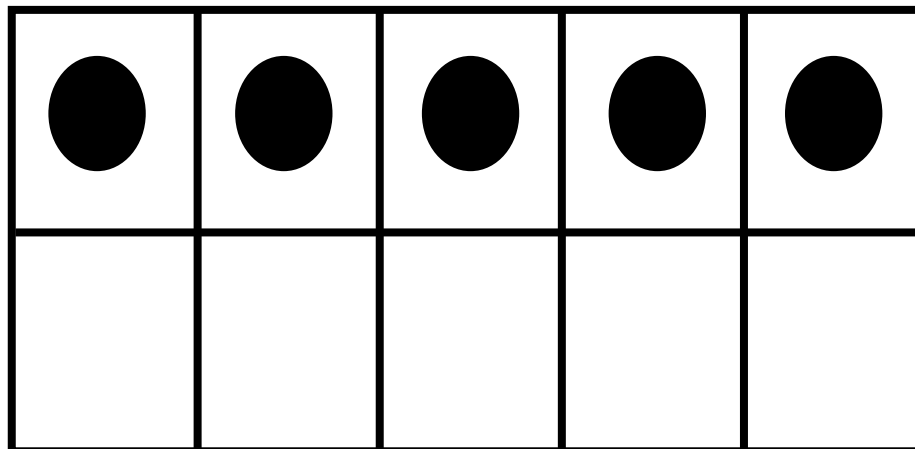
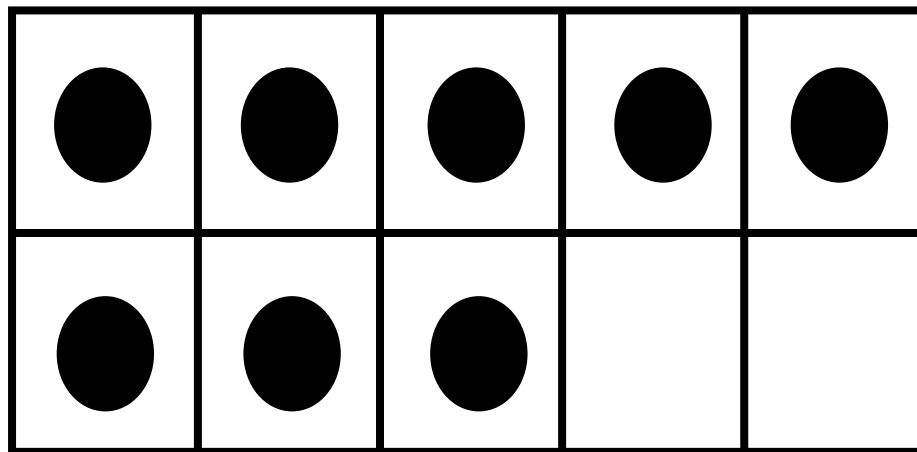
$$(5 + 5) - 2$$



10-frames







How Many Dots?

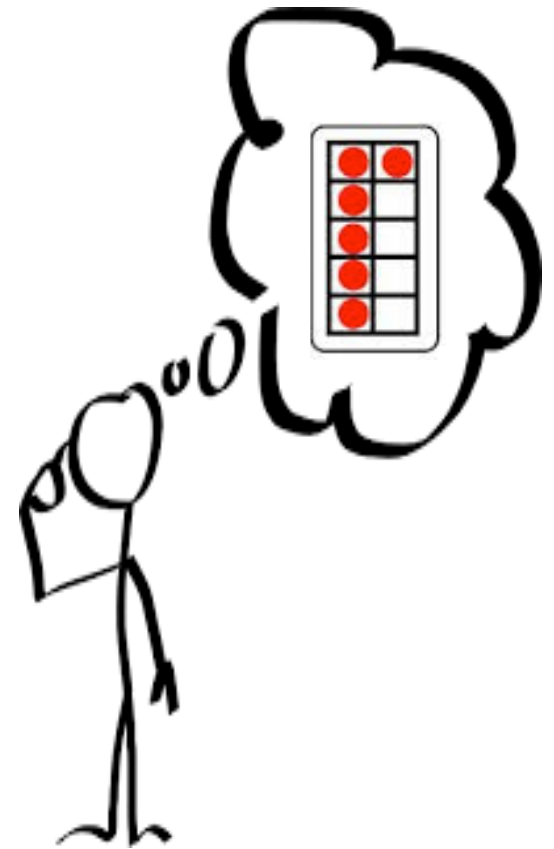
Now take away 6....

How many now?

How do you know?

What would that look like?

When do we
stop using
frames?

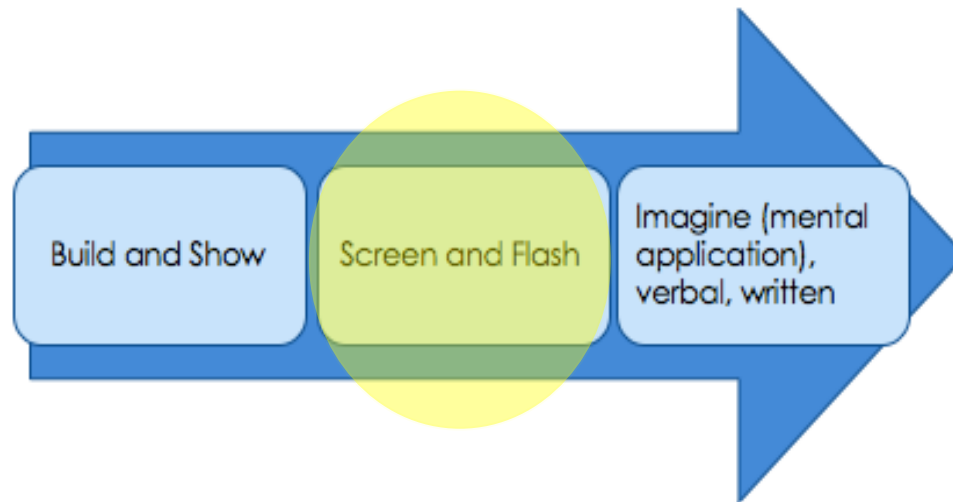


Professional Noticing

What teacher movements support this student's mathematical development?

How can this promote fact fluency and conceptual understanding?

Sticks and Dots



$$35 - 12 = \square$$

~~11~~ ~~X~~ // ()

X //

23

$$41 - 26 = \square$$

~~41~~ ()

↓ borrow
~~///~~ ()

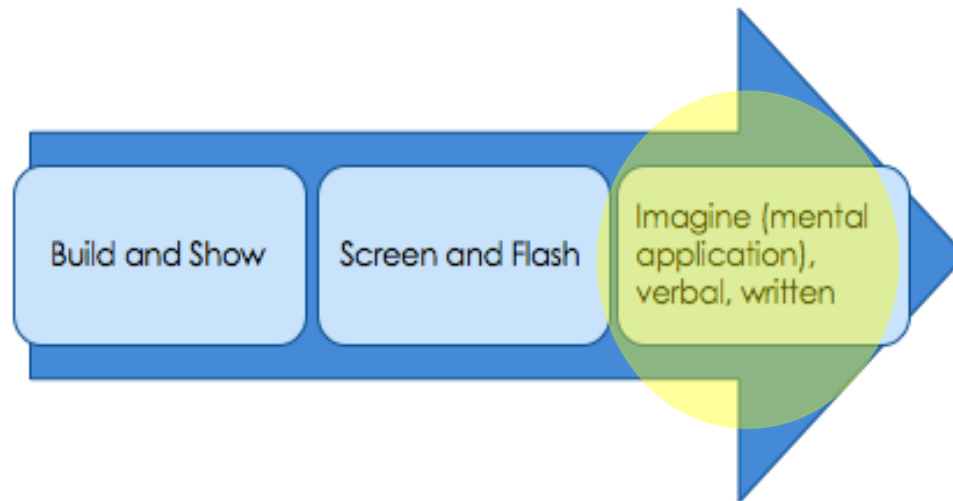
15

Professional Noticing

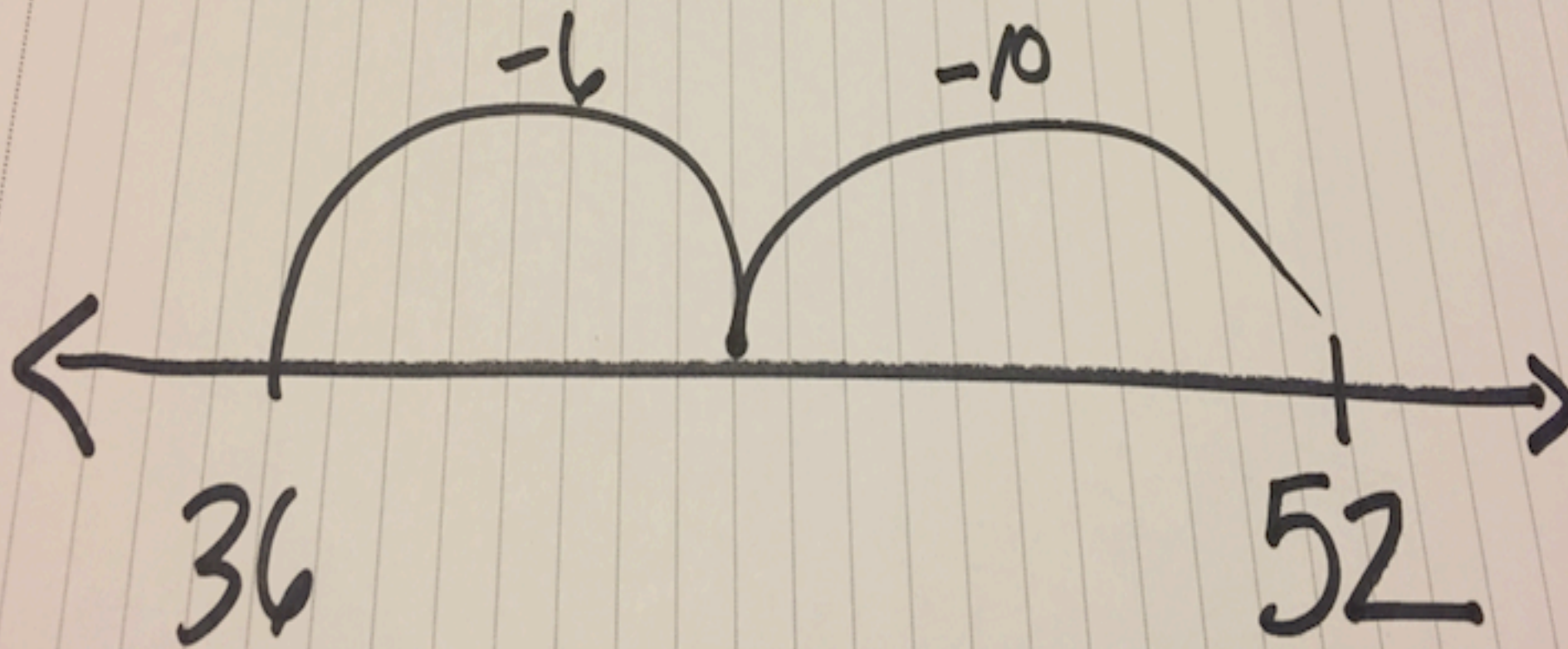
What teacher movements support this student's mathematical development?

How can this promote fact fluency and conceptual understanding?

Empty Number Lines



$$52 - 16 = \boxed{36}$$



**Mental
Calculation:**

$$52 - 16 = ?$$

*The purpose of
ENL is to notate
thinking...NOT to
be used as a
procedure!*

-

36

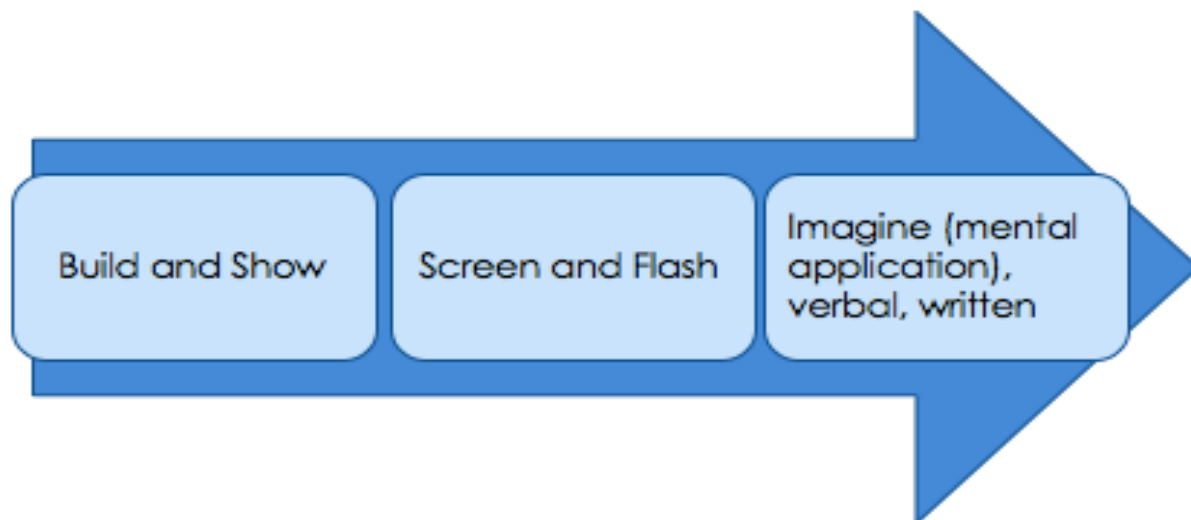
40

42

52

“It’s not what is poured into a student that counts but what is planted.” Linda Conway

It is our job to connect plant meaningful discussion seeds to help students make connection to visual representations and build application on properties of operation to build fluency.





Dina Mendola
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Resources / References:

