

Discovering Rules: Making Sense of Fractions Through Investigations

Sue O'Connell

@SueOConnellMath

www.qualityteacherdevelopment.com

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Our goal is for our students to develop a comprehensive, multi-faceted knowledge of fractions, not simply to memorize a series of procedures.

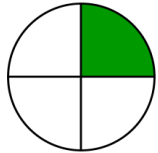
Exploring Fraction Models

Area Models

What part of this circle is green?

"a fourth"

What do these numbers represent?

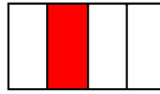


$$\frac{1}{4}$$

What does the 1 represent?
What does the 4 represent?

Try More Examples

Think about the colored sections of each shape.



Write the fraction that shows the part that is colored?

What does each number in your fraction show?

Extending Understanding: Moving Beyond Circles and Triangles

- *Let's explore fractions with pattern blocks.*
- *Can you talk about any of the blocks using fractions?*
- *If a yellow hexagon is a whole, what fraction of the whole would the other pieces represent? Why?*
- *If the red trapezoid was a whole, what fraction of the whole would the other pieces represent? Why?*

Considering Context

Katie walked $\frac{1}{2}$ mile to school.

How might you show that?

Creating Fraction Number Lines:

- Fold a paper circle to show fourths.
- Tell your partner what you know about fourths.
- Wrap a wikki stick around the circle and place clips to show each fourth.
- Unwrap the wikki stick and stick it in a line on your desk.
- Use sticky notes to label the sections of the line.

What do you notice?

How is the fraction number line like the circle model?

Exploring the Concept of the Whole Using Pattern Blocks

Yellow hexagon = 1

Think about the blue rhombi.

Can you tell me about the parts and the whole ?

"A third + a third + a third = a whole"

"3 thirds is a whole."

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

How would you show the parts and whole using red trapezoids? Green triangles?



What do you notice?

$$\frac{1}{2} + \frac{1}{2} = 1$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

What do you notice?

Does it make sense? Why?

Is there another way to say "1 whole"?



Is there another way to say "1 whole"?

$$\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3}$$

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6}$$

What do you notice?

Does it make sense? Why?

Is $\frac{5}{5}$ the same as a whole? Explain.



Exploring Equivalence

Form teams of 4.

Count off 1-4.

Each person takes a piece of colored paper, folds it in half, and traces the fold line.

Persons 2, 3, 4: Fold your paper in half again. What would you call the $\frac{1}{2}$ now? Why?

Persons 3, 4: Fold your paper in half again. What would you call the $\frac{1}{2}$ now? Why?

Person 4: Fold your paper in half again. What would you call the $\frac{1}{2}$ now? Why?

Record the data on the board.

What do you notice?

Discussions Following Paper Folding:

Has the size of $\frac{1}{2}$ of your paper changed?

What has changed about the half of your paper?

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{1}{2} = \frac{4}{8}$$

$$\frac{1}{2} = \frac{8}{16}$$

Can you use your data to develop a definition of equivalent fractions?

Can you use your data to develop a rule for generating equivalent fractions?

Further Investigations on Equivalence

Equivalent Fractions



- Draw a rectangle.
- Split it into fourths with vertical lines.
- Shade $\frac{1}{4}$.
- Divide the rectangle again with a horizontal line.
- *What fractional part is shaded?*
- *Write an equality to show it.*

$$\frac{1}{4} = \frac{2}{8}$$

Equivalent Fractions



Record on the board.

$$\frac{1}{4} = \frac{2}{8}$$

$$\frac{1}{4} = \frac{3}{12}$$

$$\frac{1}{4} = \frac{4}{16}$$

$$\frac{1}{4} = \frac{5}{20}$$

What do you notice?

What do you wonder?

Will it always happen?

And More Investigations

Try it with a different unit fraction (e.g., $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$). Model your fraction and find some equivalent fractions.

Be ready to present your findings.

What do you notice?

Why is that happening?

Predict...

What is the rule? Is there a way you could find the equivalent fractions without creating the models? Explain.

Give each team a non-unit fraction to model and then find equivalent fractions (e.g., $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{3}{8}$)

Have teams present their findings.

What do you notice?

Does it make sense?

How can you tell if fractions are equivalent? What is always true for equivalent fractions?

What is the rule for generating equivalent fractions?

Talk About It/Write About It

What does it mean for fractions to be equivalent?

Agree or disagree?

$$\frac{1}{5} = \frac{5}{20}$$

Justify your answer.

Subtracting Fractions

In April it rained $1\frac{1}{6}$ inches.

In May it rained $\frac{5}{6}$ inch.

How much more rain fell in April than May?

What operation would you use? Explain.

What tool would you use?

How many $\frac{1}{6}$ s are in $1\frac{1}{6}$?



Show me $1\frac{1}{6}$ using all sixths.

$$\frac{6}{6} + \frac{1}{6} = \frac{7}{6}$$



What if the mixed number was $2\frac{1}{6}$?

$$\frac{6}{6} + \frac{6}{6} + \frac{1}{6} = \frac{13}{6}$$



What if the mixed number was $2\frac{2}{3}$?

$$\frac{3}{3} + \frac{3}{3} + \frac{2}{3} = \frac{8}{3}$$

What do you notice?

What is the rule?

Back to Our Subtraction Problem

Solve it with a model and record the result.

$$\frac{7}{6} - \frac{5}{6} = \frac{2}{6}$$

Try some more and record the results.

$$\frac{8}{6} - \frac{3}{6} = \frac{5}{6}$$

$$\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$

What do you notice?

Does this make sense? Why?

Predict the sum of $\frac{4}{6} - \frac{1}{6}$

Check your prediction.

4 sixths – 1 sixth = 3 sixths

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