Develop Meaning by Connecting Multiple Strategies

NCTM 2017 San Antonio, Texas
K-2 Workshop
Session 371

Solve Nan and Bert Problems on pages 2-3.

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UIC LEARNING SCIENCES RESEARCH INSTITUTE
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METRO CHICAGO MATH INITIATIVE

Math Trailblazers

Develop Meaning by Connecting Multiple Strategies

NCTM 2017 Annual Conference San Antonio, Texas
K-2 Workshop

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METRO CHICAGO MATH INITIATIVE

Math Trailblazers

Nan and Bert Problems (pg. 2-3)

Solve and then discuss with a partner:

- Did you use the same strategy for each problem? Why?
- What strategies would you expect from your students?
- What strategies would you hope for?
- How might the tools support reasoning?

Goals for this Session

- Briefly establish a rationale for helping students develop multiple and flexible computation strategies.
- Explore how students develop proficiency with whole number operations.
- Analyze instructional strategies that encourage students to develop meaning and proficiency with those computation strategies.

Math Trailblazers
Research and Revision Study

2003–2006	Research on implementation of 2nd edition in classrooms
2006–2009	Revision and field test of new materials in grades 1–5
2008–2009	Student Achievement Study
2010–2014	Final revision of materials for publication

National Science Foundation
WHERE DISCOVERIES BEGIN

Math Trailblazers

Research Studies

- Whole Number Study–UIC & KSU 2003–2008
- Implementation Study– –2006
- Fractions and Ratios– –2006
- Video Study–UIC 2003–2006
- Field Test Study–UIC 2006–2010
- Student Achievement Study–UIC 2009–2011
- Embedded Assessment Study–UIC 2010 - 2014

How can instruction support that learning?

How do students learn?

Research Studies

- Whole Number Study–UIC & KSU 2003–2008
- Implementation Study–UIC 2003–2006
- Fractions and Ratios–UMN 2004–2006
- Video Study–UIC 2003–2006
- Field Test Study–UIC 2006–2010
- Student Achievement Study–UIC 2009–2011
- Embedded Assessment Study–UIC 2010 - 2014

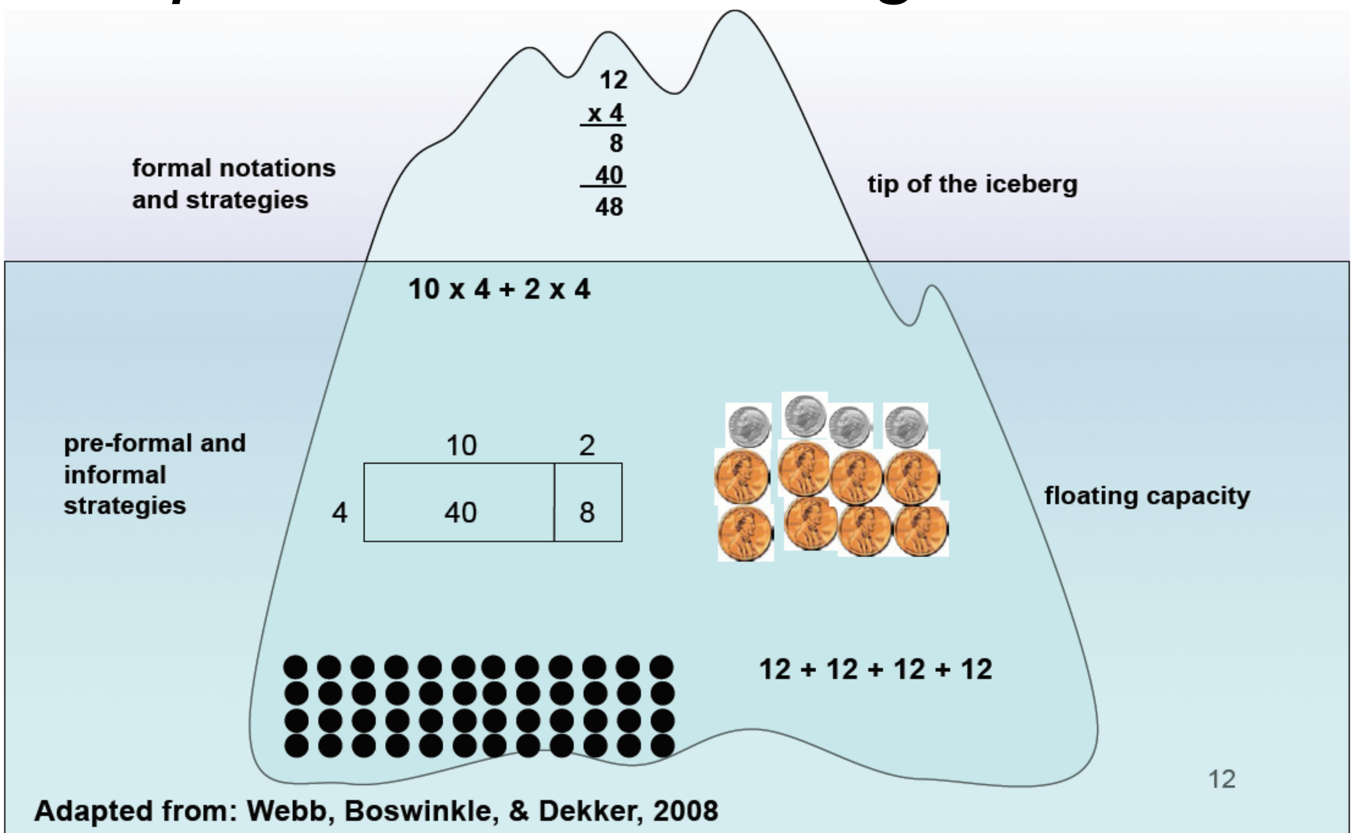
"To find one's way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar, and how they are different. The degree of students' conceptual understanding is related to the richness and extent of the connections they have made."

- National Research Council, 2001

Why Multiple Strategies?

Why Multiple Strategies?

Rationale #1 A range of strategies allows for *sense-making* and development of *conceptual understanding*.



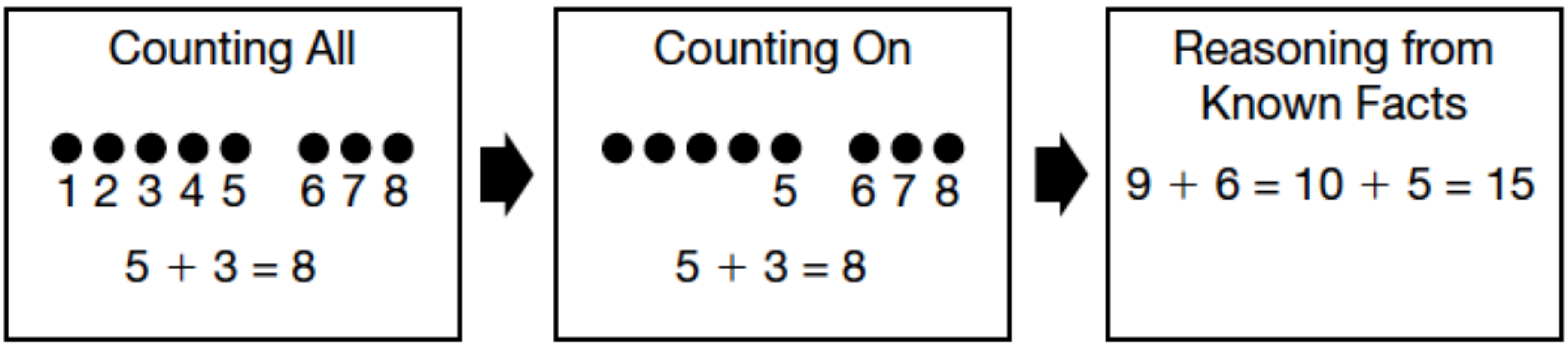
Stages of Conceptual Development

Direct Modeling	Counting Strategies	Reasoning from Known Facts

Carpenter, 1999; National Research Council, 2001

Stages of Conceptual Development

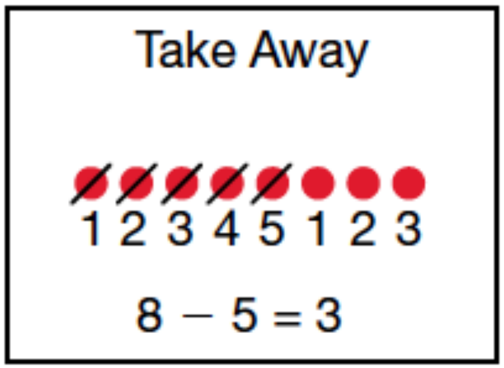
Direct Modeling	Counting Strategies	Reasoning from Known Facts

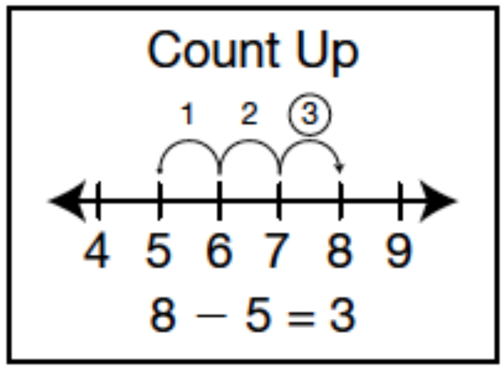


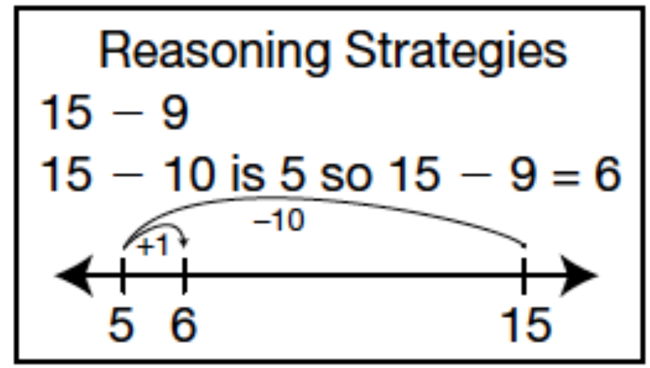
Carpenter, 1999; National Research Council, 2001

Stages of Conceptual Development

Direct Modeling	Counting Strategies	Reasoning from Known Facts







Carpenter, 1999; National Research Council, 2001

Why Multiple Strategies?

Rationale #1 A range of strategies promotes computational fluency.

Flexibly
Accurately
Efficiently
Appropriately

-National Research Council

Fluency: Flexibility

9 + 5	8 + 7	8 + 6	8 + 8

Would you use the same strategy?

9 + 2

Fluency: Appropriateness

What strategy would you use?

104 – 89 =

Is the strategy efficient?

Why Multiple Strategies?

Rationale #3 A range of strategies helps students *access and respond to* mathematical contexts.

There are 5204 Chocos. A customer came in and bought 565. Another customer came in and wanted to buy 4859 pieces of candy. Was there enough candy in the store so that he could buy that much?

Estimation

Mental Math

Paper and Pencil

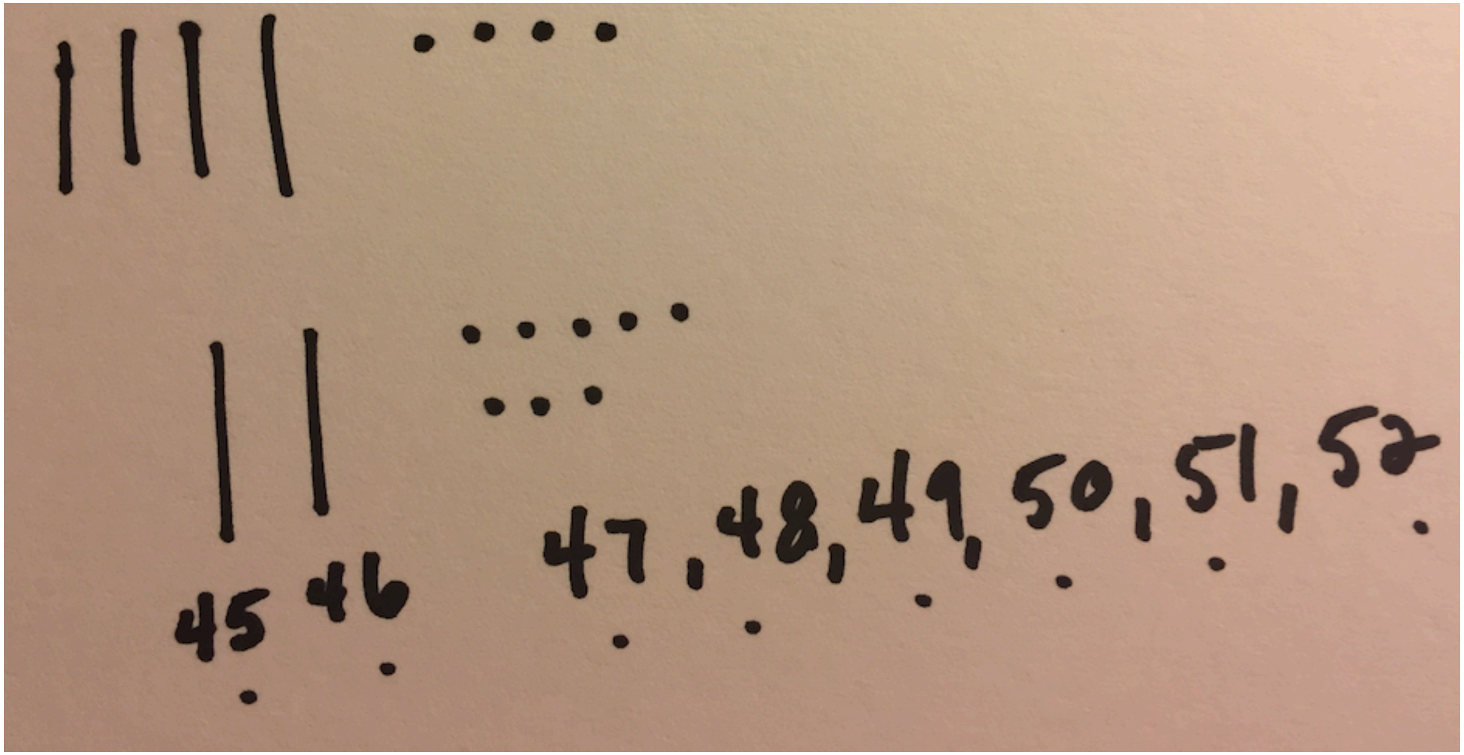
Why Multiple Strategies?

Rationale #4 A range of strategies supports a range of student *identities* and *needs*.

Chris's group made 28 hats. Julia's group made 44 hats. How many hats did both groups make altogether?

Why Multiple Strategies?

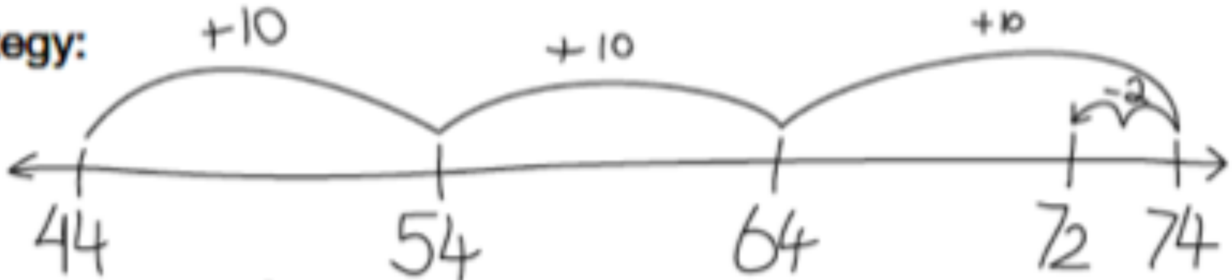
Rationale #4 A range of strategies supports a range of student *identities* and *needs*.



Chris

I can think about it better if I make a number line in my head. I think about starting at 44, moving forward 30 and then back 2, since 28 is 2 less than 30. I can write it like this.

Chris's Strategy:



I start at 44 and then add on 30, going by tens: 54, 64, 74. Subtract 2 and it is 72. 72 hats.

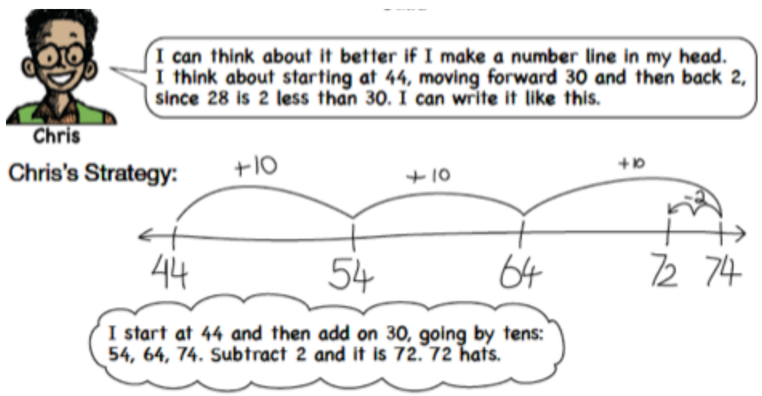
Julia's Strategy:

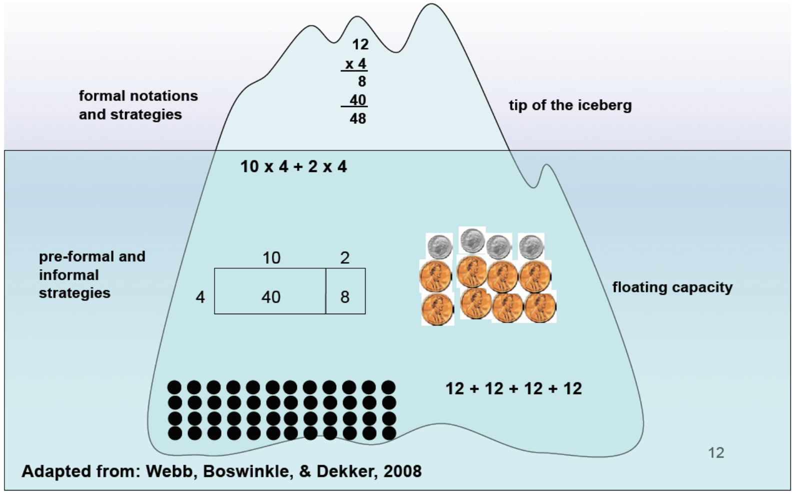
Altogether we made 72 hats. I broke the numbers into tens and ones: 20 + 40 is 60, 8 and 4 is 12, 60 + 12 is 72. We made 72 hats.

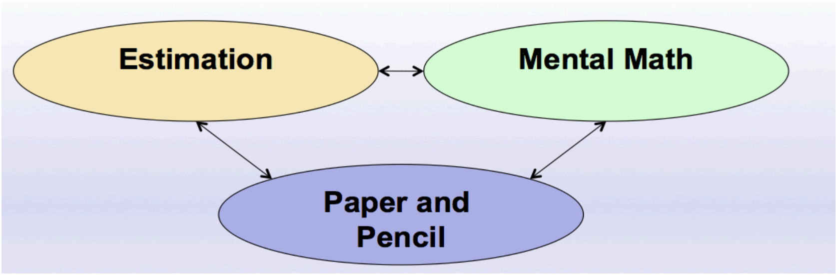
Julia

$$\begin{array}{r} 28 = 20 + 8 \\ +\ 44 = 40 + 4 \\ \hline 60 + 12 = 72 \text{ hats} \end{array}$$

Why Multiple Strategies?







Flexibly

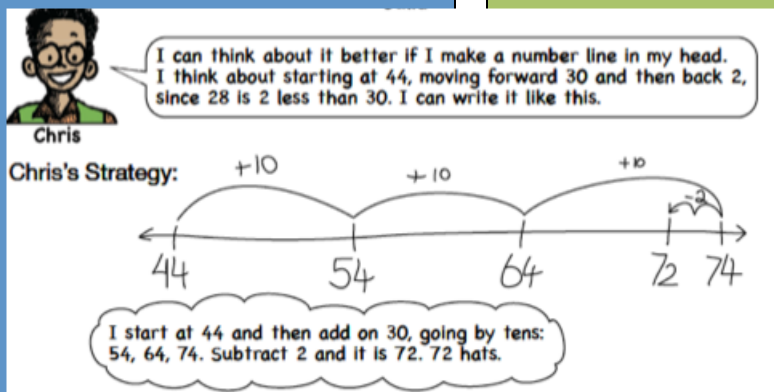
Accurately

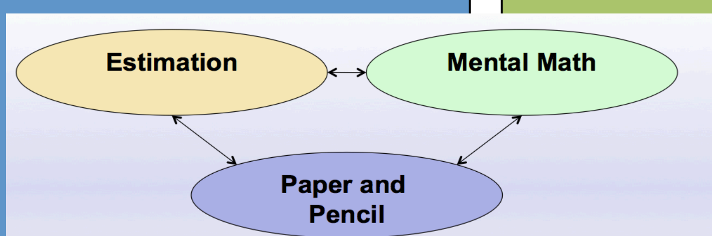
Efficiently

Appropriately

Why Multiple Strategies?

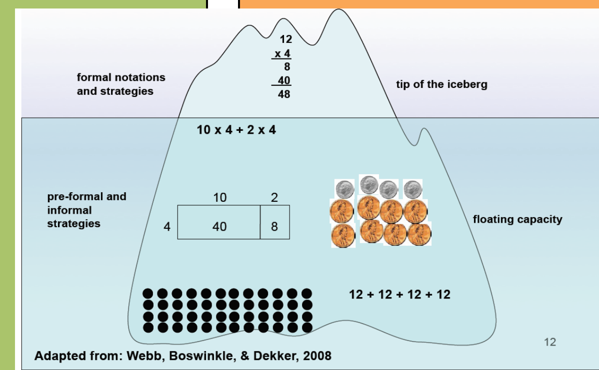
Direct Modeling





Counting Strategies

Reasoning from Known Facts



Flexibly

Accurately

Efficiently

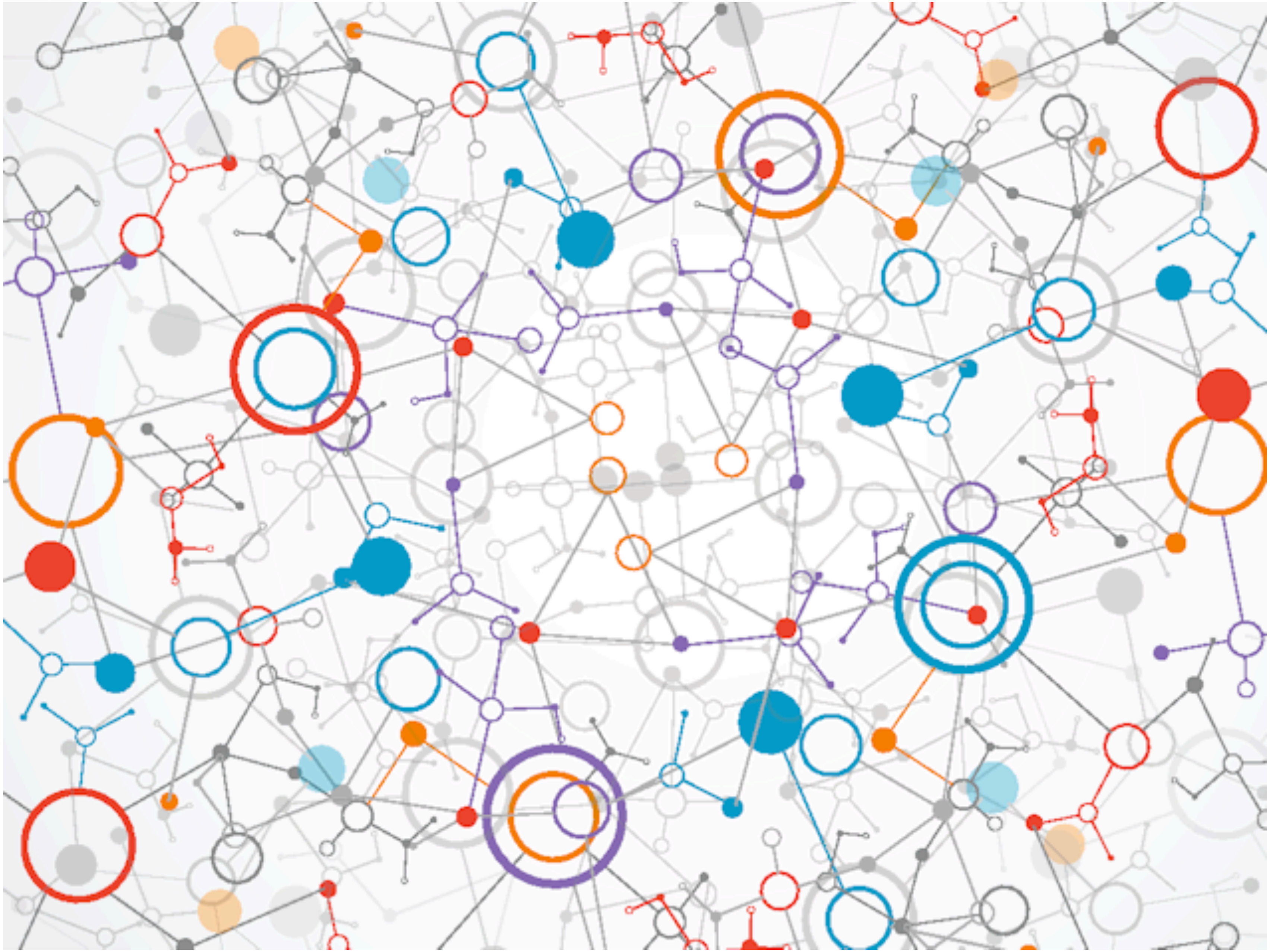
Appropriately

Carpenter, 1999; National Research Council, 2001

Why Multiple Strategies?

Why Multiple Strategies?

- Too many.
- Keep using the same inefficient strategy.
- Found a favorite.
- Some students do not connect to other strategies.



Stages of Conceptual Development

Direct Modeling	Counting Strategies	Reasoning from Known Facts

Look at the student work, what phase of reasoning is evident?

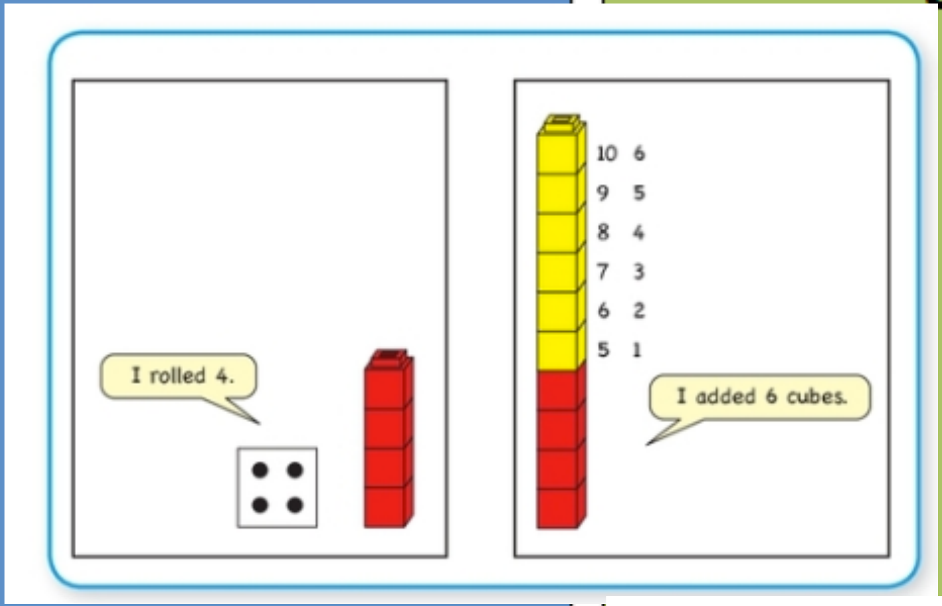
Carpenter, 1999; National Research Council, 2001

Promote Phases Reasoning

Direct Modeling

Counting Strategies

Reasoning from Known Facts



E. Using Partners (TG p. 25)

Circle the number partners that make ten. Then solve the problems.

A. $9 + 1 + 4 = \square$ B. $3 + 7 + 5 = \square$

C. $5 + 5 + 6 = \square$ D. $4 + 6 + 9 = \square$

E. $3 + 6 + 4 = \square$ F. $6 + 8 + 2 = \square$

Promote Phases Reasoning

Direct Modeling	Counting Strategies	Reasoning from Known Facts

Choose one activity from the folder. Which phases does it support or promote?

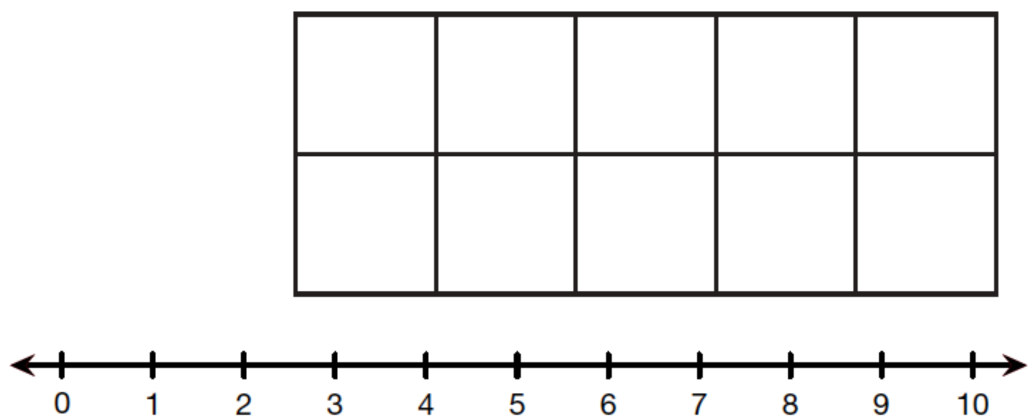
Carpenter, 1999; National Research Council, 2001

Connect to Student's Thinking

Birthday Party

Show how you solve each problem. Use connecting cubes, ten frames, or number lines.

1. John has 5 red balloons and 3 blue balloons. How many balloons does he have?



Invention

Make Thinking Visible

Multiple Ways

Connect to Student’s Thinking

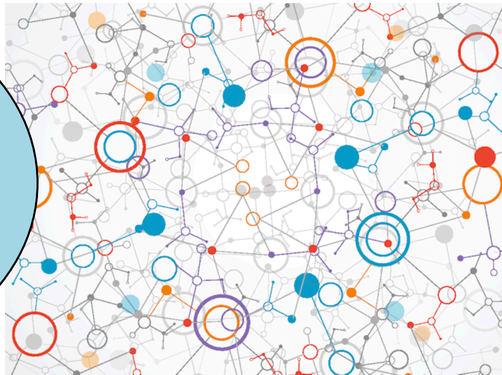
1. Jason solved the following problem. Does his answer make sense? Why or why not?

$$\begin{array}{r} 97 \\ +\ 86 \\ \hline 1713 \end{array}$$

Analyze Other’s Thinking

Invent other ways

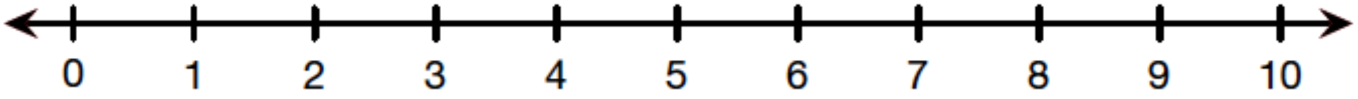
Try Other Strategies



Connect to Student's Thinking

Solve each problem and write a number sentence. Show how you can use the number line to solve each problem.

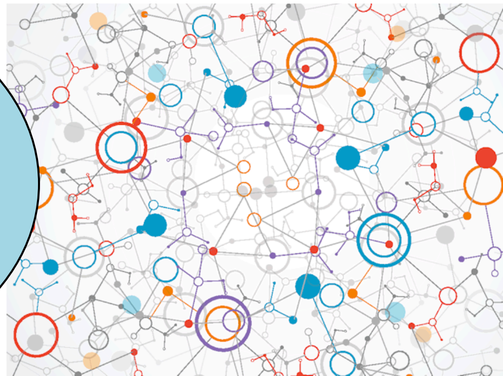
1. Frank had 10 balloons. Four floated away. How many balloons does Frank have now?



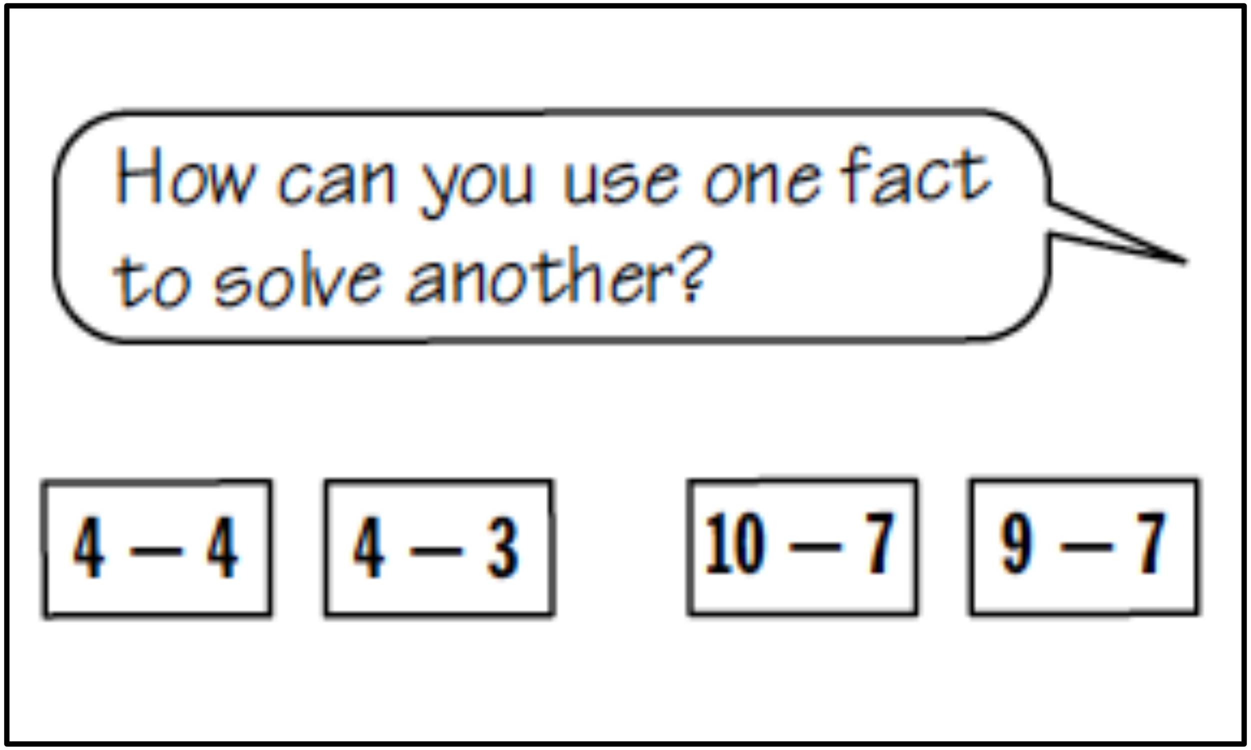
Analyze Other's Thinking

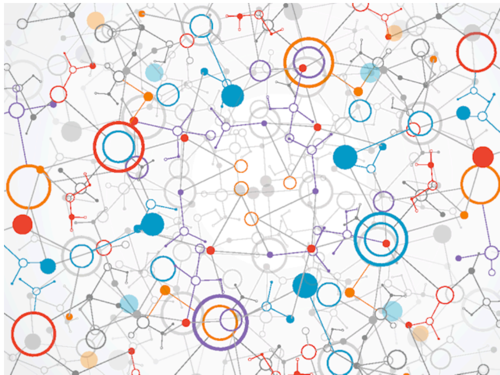
Invent other ways

Try the Strategies of Others



Connect to Student's Thinking





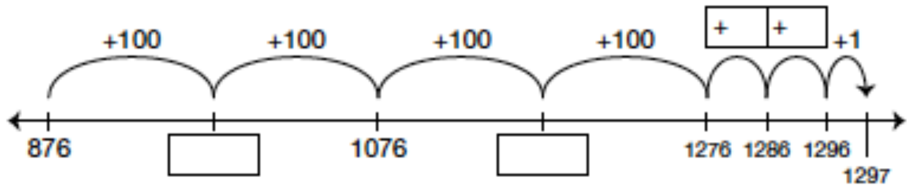
Finish It: Addition

1. Ming, Rosa, Chris, and Levi started solving 876 + 421. They did not finish. Estimate the sum. Help each student finish the problem using the method they chose.

   876 + 421

   Estimate ____________________

   A. Ming used a number line:



   Number sentence ______________________________

   B. Rosa used expanded form:

   876 = 800 + ______ + ______
   + 421 = 400 + ______ + ______

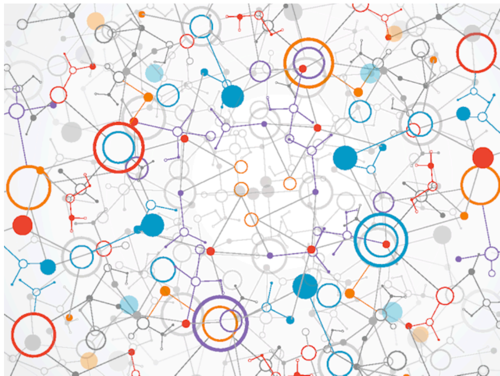
   ______ + ______ + ______ = ______

   C. Chris used all-partials:

   876
   + 421
   1200

   D. Levi used the compact method:

   876
   + 421
   7



Connect to Student's Thinking

Promote Phases Reasoning

Direct Modeling	Counting Strategies	Reasoning from Known Facts

What phases are supported by this activity?

Carpenter, 1999; National Research Council, 2001

"Discussions that focus on cognitively challenging mathematical tasks, namely those that promote thinking, reasoning, and problems solving, are a primary mechanism for promoting conceptual understanding of mathematics."

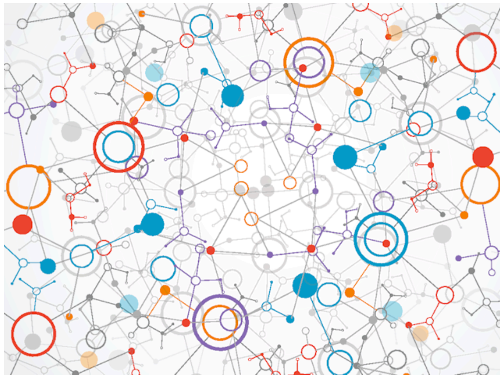
Smith, Hughes, Engle & Stein, 2009, p. 549

Connect to Representations

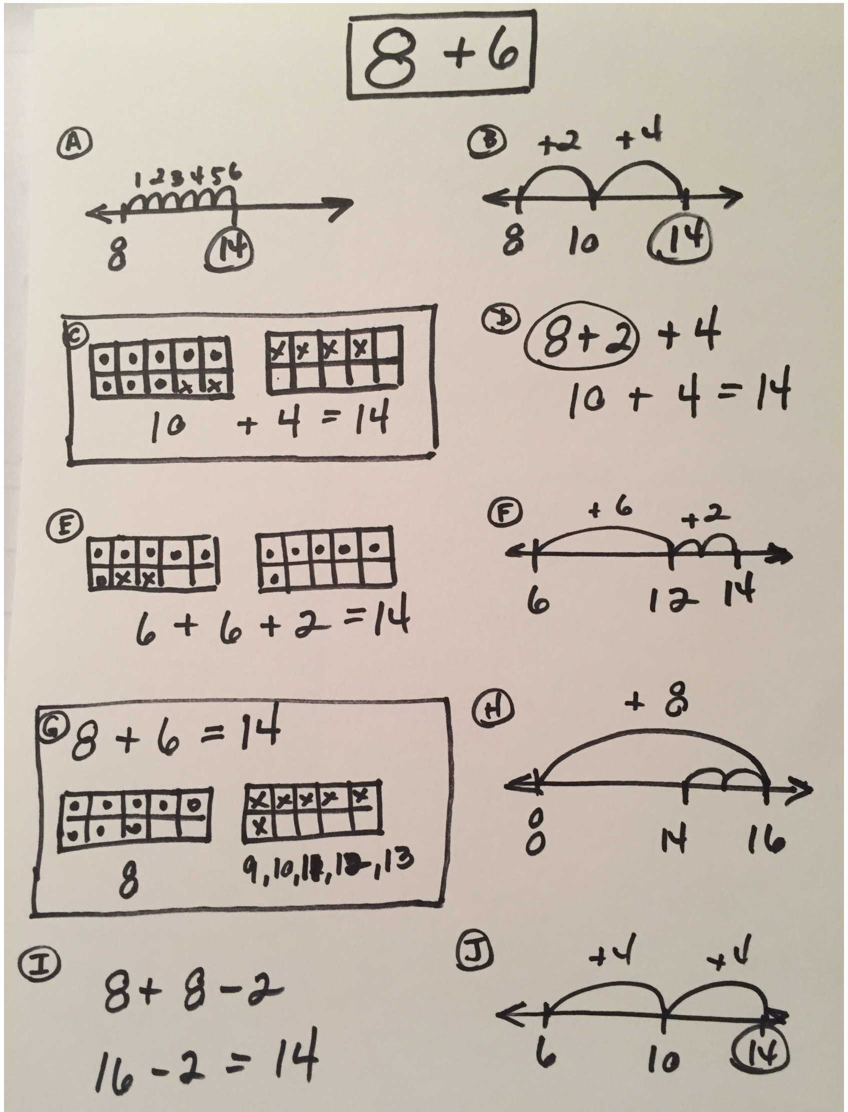
Five frogs on the log.
Three jump off.

There are 2 frogs on the log.

3 minus 5 is 2



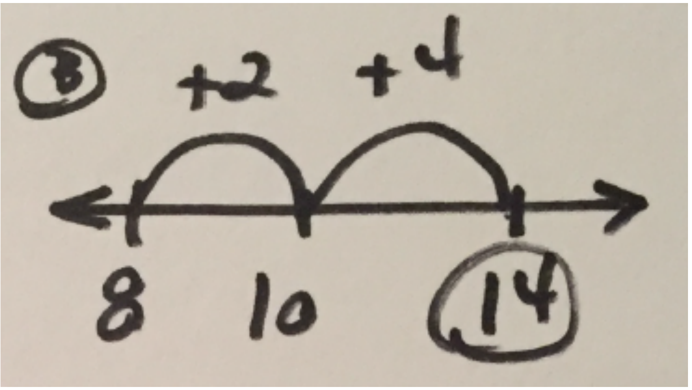
Connect to Representations and Strategies(pg. 5)

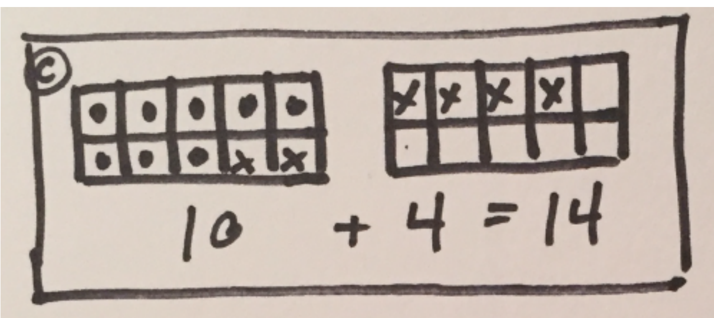


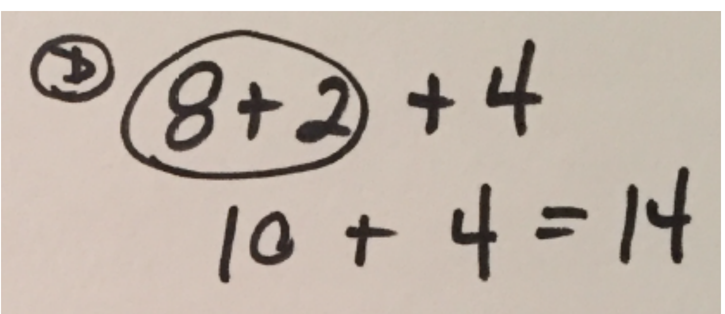
Look at the solutions to 8 + 6

- Which used the same representation?
- Which used the same strategy?

Connect to Representations and Strategies(pg. 5)

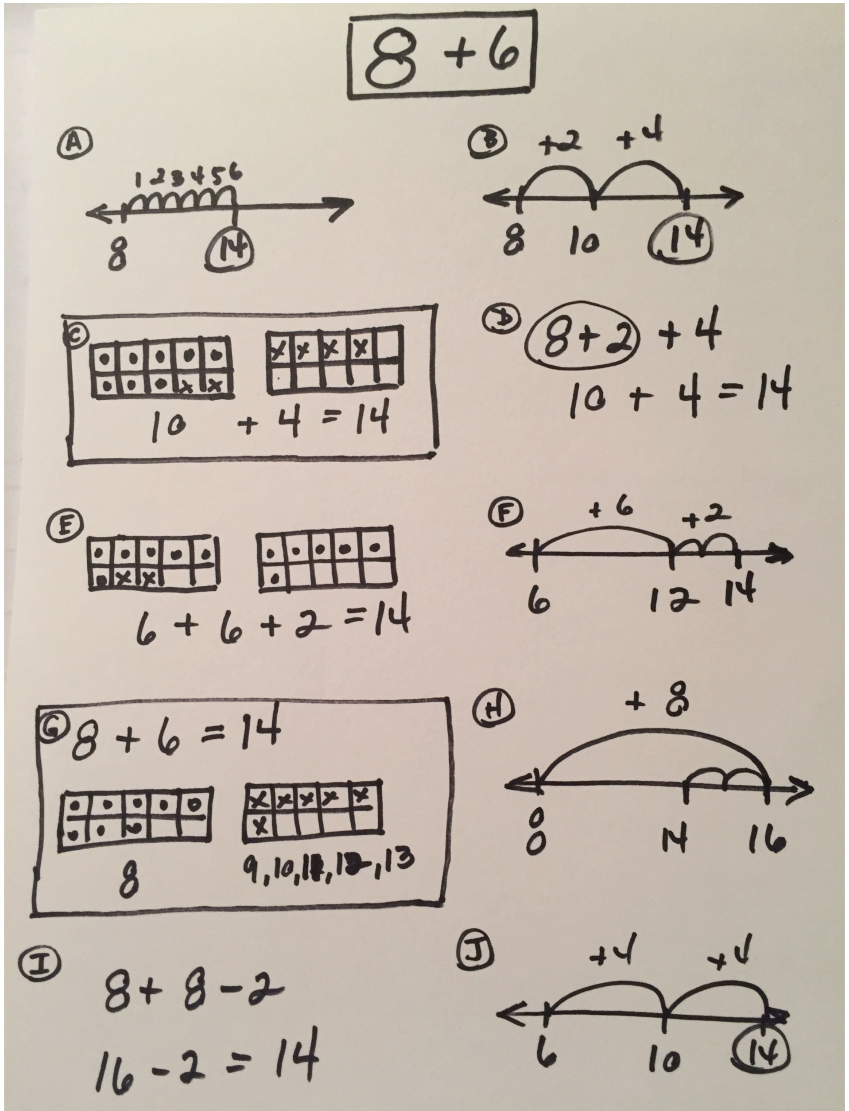






- **Did these three students use the same strategy?**
- **I notice the +2 in Student B solution. Does student C add 2 to 8? How is that shown?**
- **How is the +2 shown in Student D's solution?**

Connect to Representations and Strategies(pg. 5)

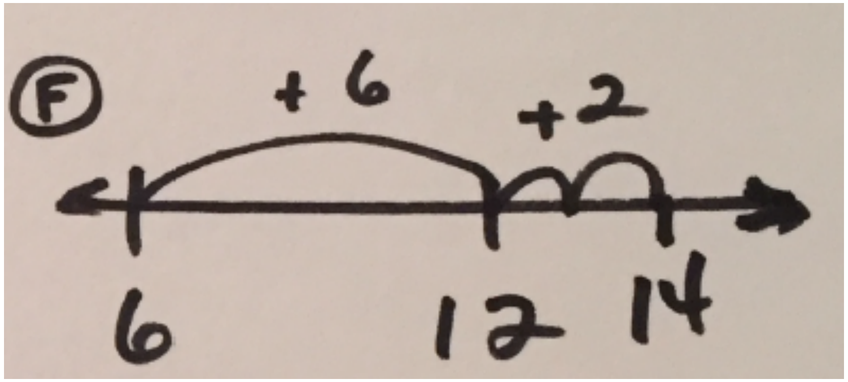


Your turn. Work with a partner.

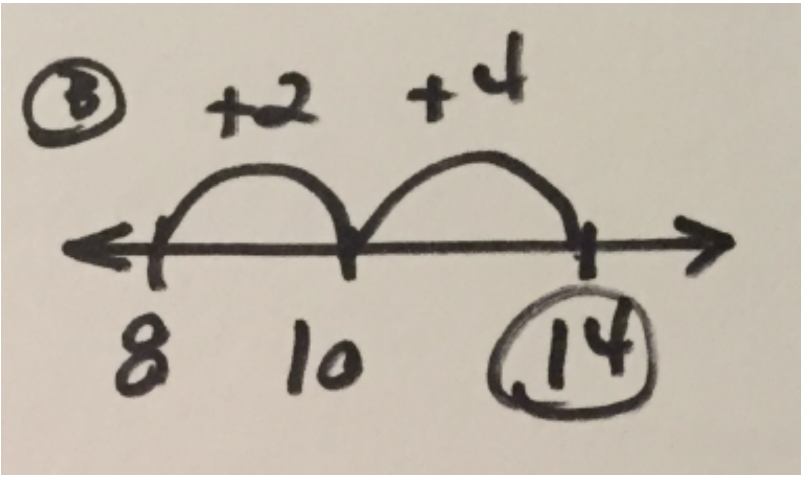
Write questions that help students connect the strategies and representations.

Use Questions on pg. 4 as a guide.

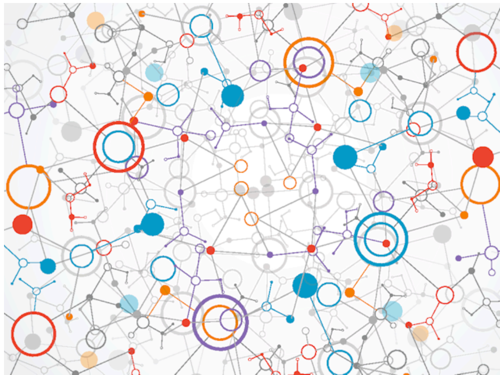
Connect to Representations and Strategies AND Properties or Big Ideas



I notice that both of these students counted on.
Student F started at 6 and Student B started at 8.



Can they do that? Why does that work?



Connect to Representations and Strategies

1. Chris's group made 28 hats. Julia's group made 44 hats. How many hats did both groups make altogether?

Julia's Strategy:

Altogether we made 72 hats. I broke the numbers into tens and ones: 20 + 40 is 60, 8 and 4 is 12, 60 + 12 is 72. We made 72 hats.

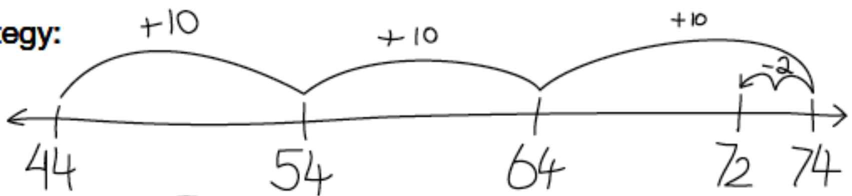
Julia

$$\begin{array}{r} 28 = 20 + 8 \\ +\ 44 = 40 + 4 \\ \hline 60 + 12 = 72 \text{ hats} \end{array}$$

Chris

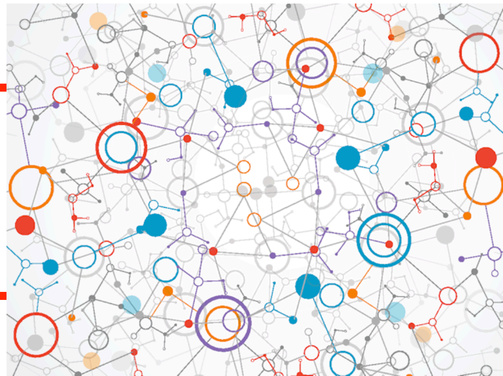
I can think about it better if I make a number line in my head. I think about starting at 44, moving forward 30 and then back 2, since 28 is 2 less than 30. I can write it like this.

Chris's Strategy:



I start at 44 and then add on 30, going by tens: 54, 64, 74. Subtract 2 and it is 72. 72 hats.

2. **A.** How did Julia use tens and ones to add?
   B. How did Chris use tens and ones?

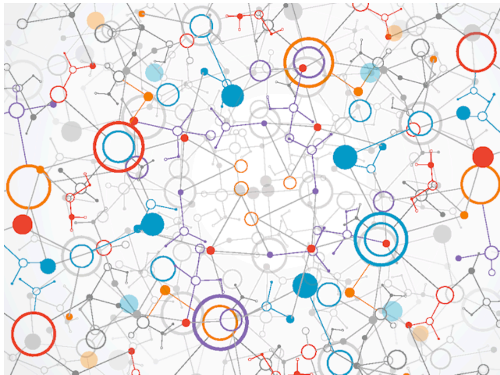


Connect to Context Demands

9 + 5	8 + 7	8 + <u>6</u>	8 + 8

Would you use the same strategy?

9 + 2



Connect to Context Demands

Sort the problems by strategy?

2 – 1

2 – 2

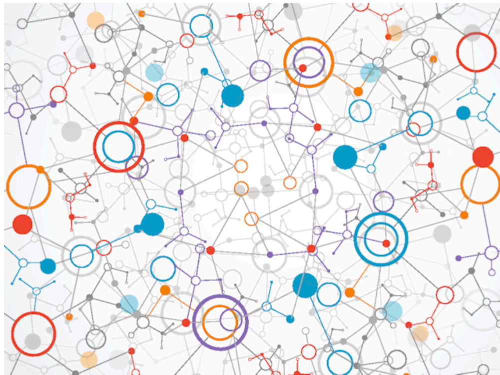
6 – 5

8 – 7

7 – 5

7 – 6

3 – 1



Connect to Context Demands

Sort the problems by strategy?

Counting Back	Doubles +1	Sarah's Strategy Doubles -1
2 – 1	8 – 7	7 – 5
2 – 2	<u>6</u> – 5	7 – <u>6</u>
3 – 1		

Connect to Context Demands

My Addition Strategies Menu for Larger Numbers

Counting All	Making Ten
Counting On	Using Ten
Another Strategy ________________	Using Doubles

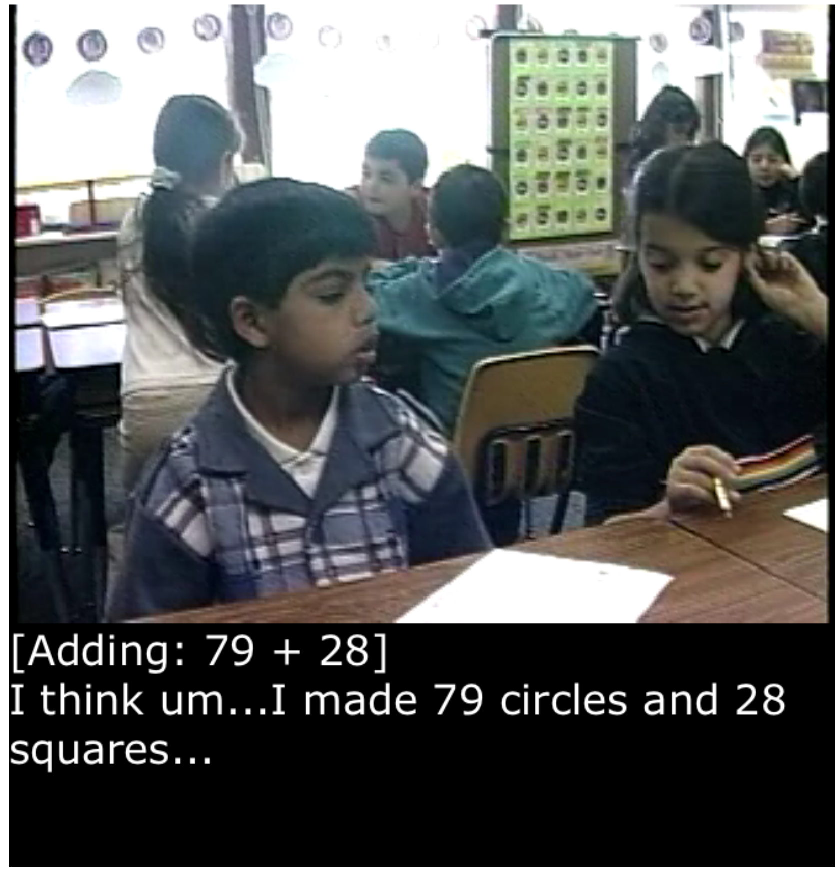
Stages of Conceptual Development

Direct Modeling	Counting Strategies	Reasoning from Known Facts

Solve 79 + 28.

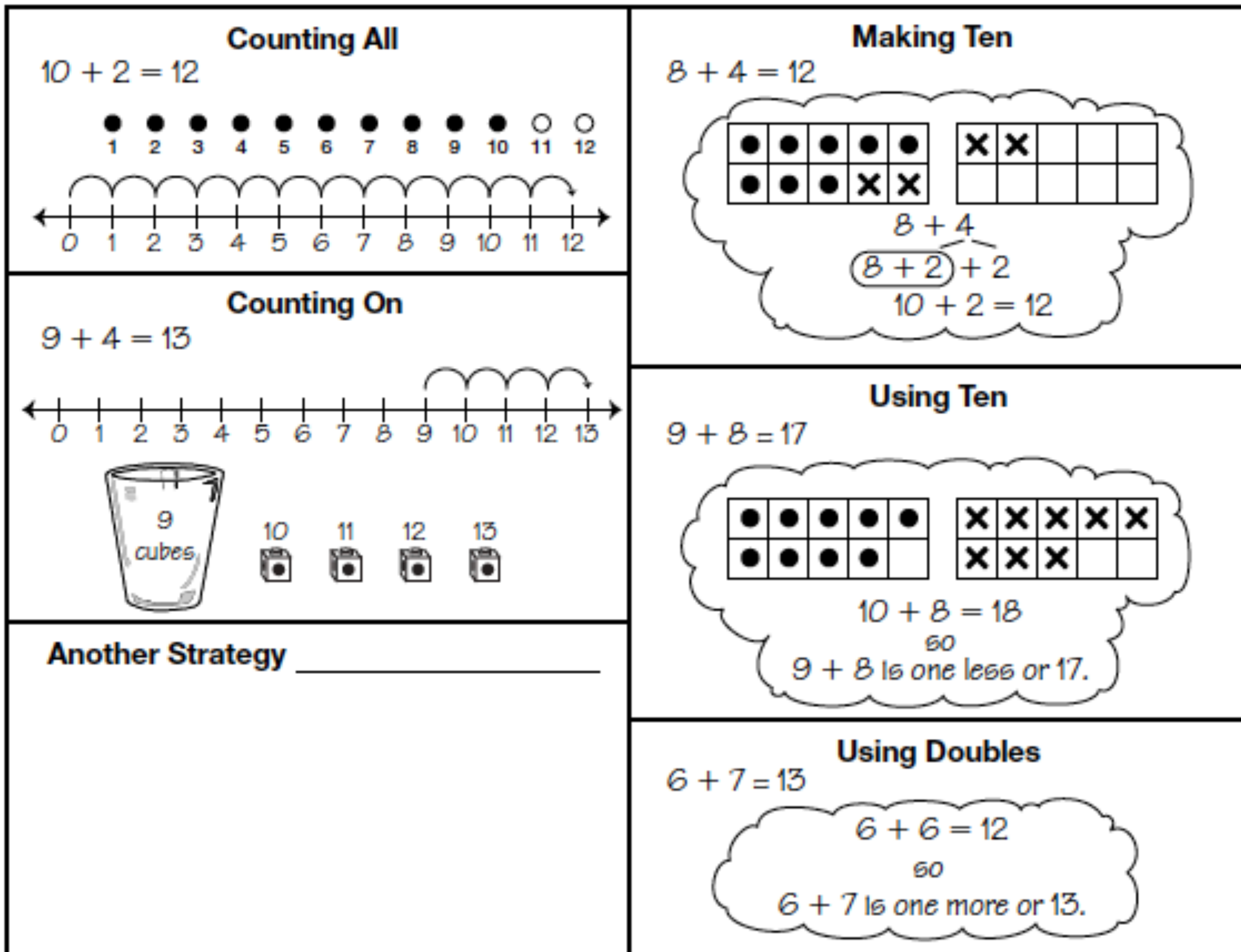
Carpenter, 1999; National Research Council, 2001

Connect to Student's Thinking



Are all students in the same place?

Addition Strategies Menu for the Facts



Addition Strategies Menu

Finding Friendly Numbers

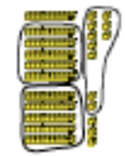
138 + 29

140 + 30 = 170

170 is a reasonable estimate.

Using Base-Ten Pieces

$$\begin{array}{r} 68 \\ +\,55 \\ \hline 123 \end{array}$$

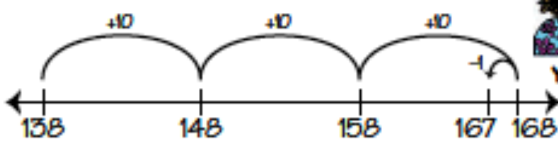


Trade 11 skinnies and 13 bits for
1 flat, 2 skinnies, and 3 bits

Counting On

138 + 29

138 + 30 − 1 = 167



137	138	139	140
147	148	149	150
157	158	159	160
167	168	169	170

138
↓
148
↓
158
↓
167 ← 168

Using Expanded Form

$$\begin{array}{r} 68 = 60 + 8 \\ +\,55 = 50 + 5 \\ \hline 110 + 13 = 123 \end{array}$$

Using All-Partials

$$\begin{array}{r} 68 \\ +\,55 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$$

Using the Compact Method

$$\begin{array}{r} {}^{1}68 \\ +\,55 \\ \hline 123 \end{array}$$

Use Menus to prompt. . .

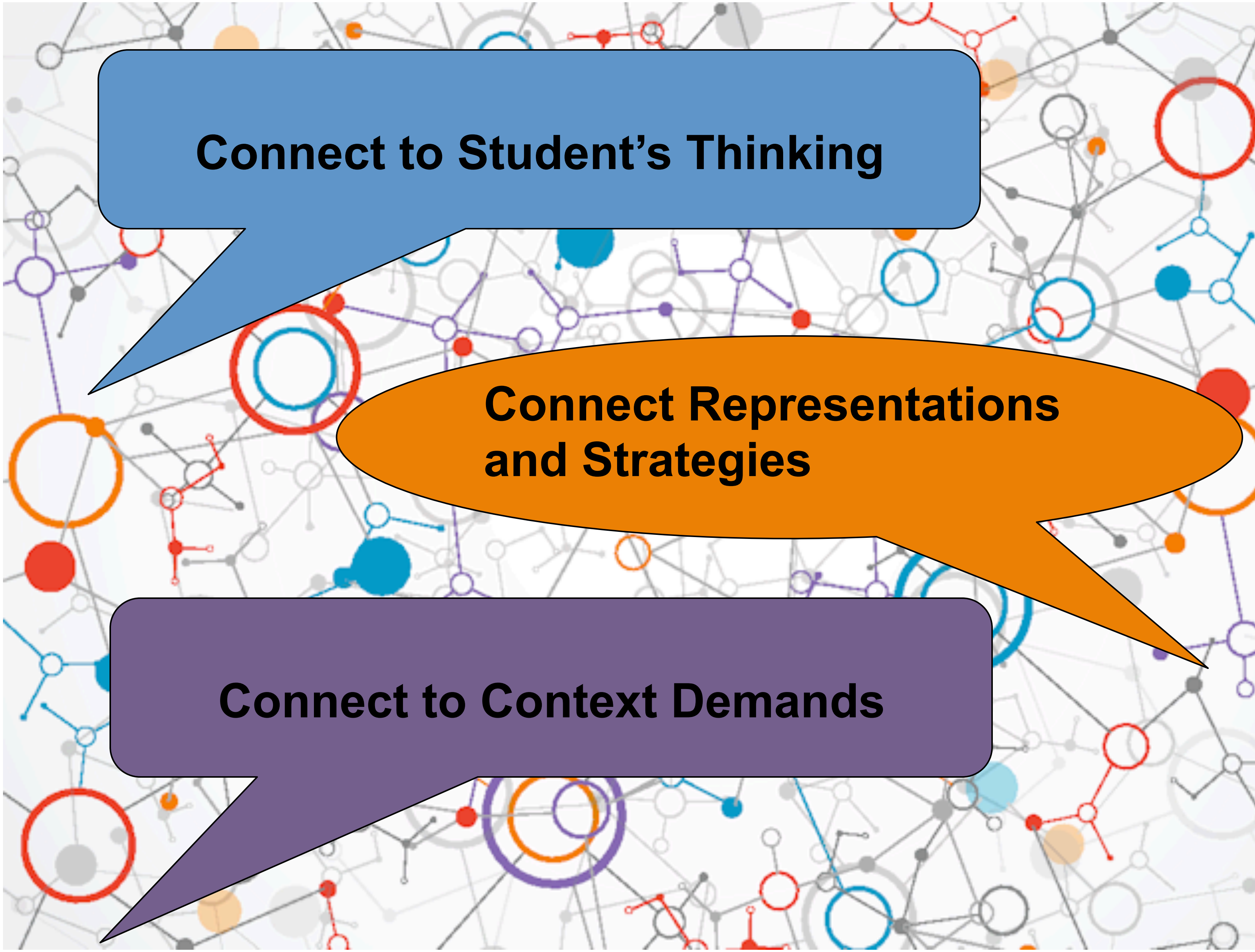
Try a method you hardly ever choose.

Show Tanya's method using a number line instead.

Which strategy do you think is best?

Carols got stuck. . .what strategy do you think will help him?

Is your strategy similar to . . ..


Connect to Student’s Thinking
Connect Representations and Strategies
Connect to Context Demands

Develop Meaning by Connecting Multiple Strategies

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Tell a Story

There are 10 pieces of fruit. Some are in the bowl and some are out of the bowl.

Show and tell a story with the story mat and 10 cubes.

Write a number sentence for your story.

Towers of Ten Game

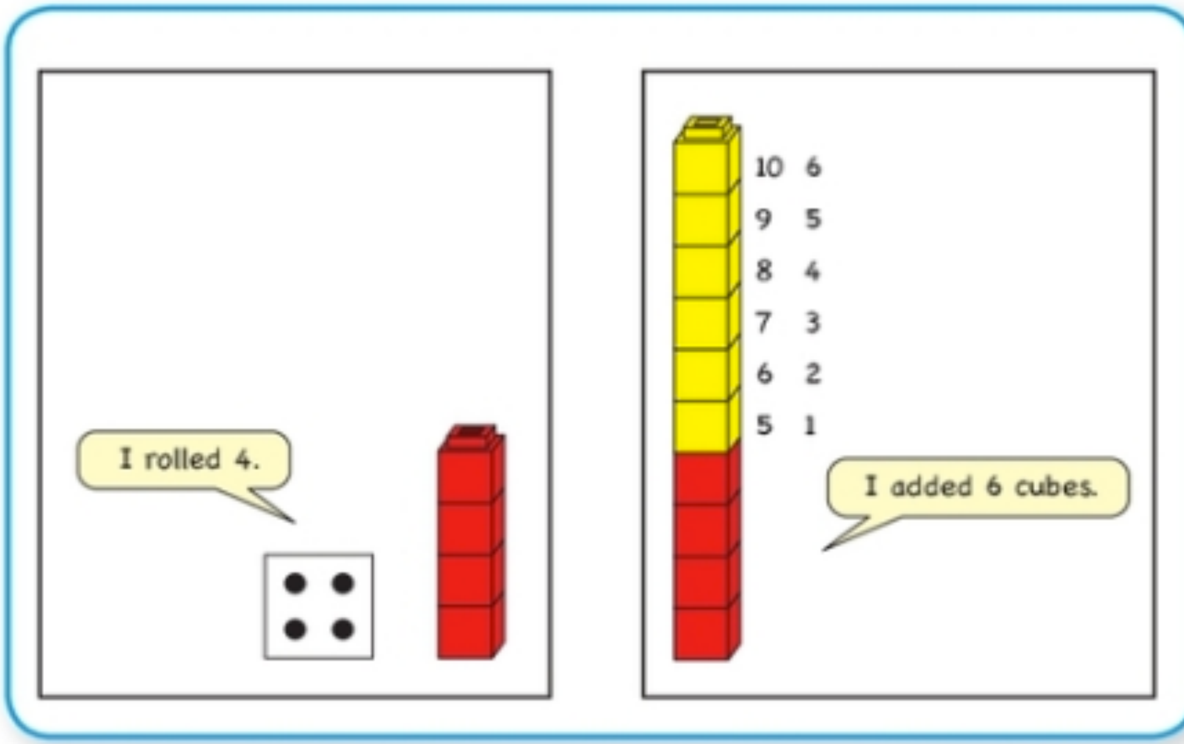


Figure 1: A sample round of Towers of Ten

Flip All 10

- Place 1 chip above each number on the number path. All the chips should have the same color showing.
- Student 1 rolls the die and places the cube on the number on the number path as a marker. He flips over the chip above the number. For example, the number is 6 and the chip above the 6 is turned over. Once a chip is turned over during a game, it cannot be flipped over again.
- Student 2 rolls the die. He can go right or left on the number path. Student 2 starts at the number where the marker is and counts up or counts back. For example, he rolls a 2. He starts at 6 where the marker is and counts up 2 to 8. He flips that chip over.
- Student 1 rolls the die and follows the same procedure. If he or she cannot make a move, he skips a turn.
- The game ends when all the counters are flipped to the other color or there are no more moves.

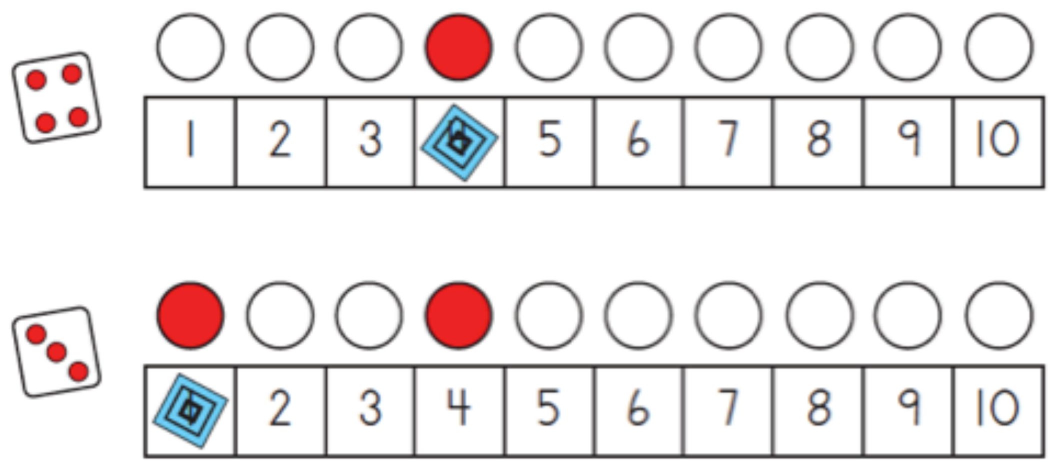


Figure 12: *Game progression of Flip All 10*

Find Ten Matches

Put two sets of 0-10 ten frame cards on the table, face down or face up.

Players take turns finding the partners to ten.

Goal, match all cards.

Make 5 (or 10)

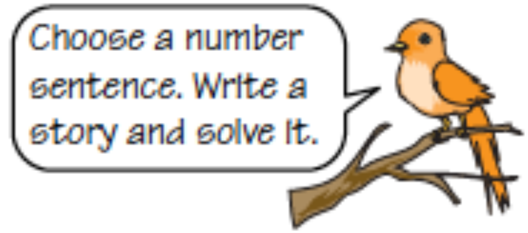
Roll 10 (0-5) dice. (6 is a zero.)

Look for ways to make of 5 (or 10). Pull those to the side.

Roll the remaining dice and look again.

Repeat until all dice are paired to show partners to 5.

Addition Stories



$0 + 2$

$5 + 1$

$3 + 2$

$2 + 2$

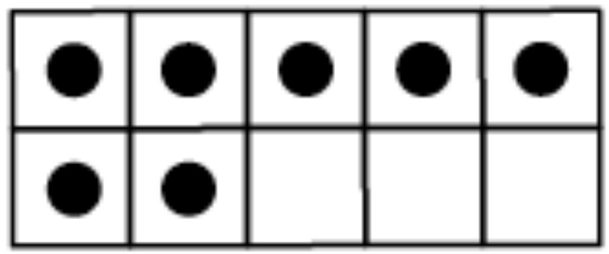
$4 + 3$

$7 + 3$

$4 + 5$

$3 + 3$

Play Ten Frame Flash: Subtract from Ten

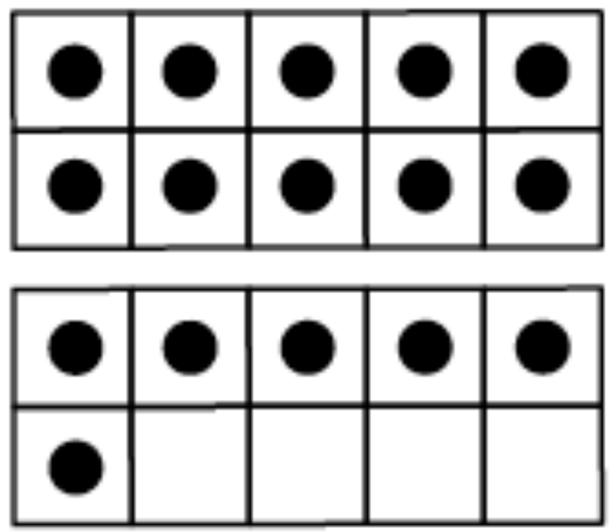


$$10 - 3 = 7$$

or

$$10 - 7 = 3$$

Play Minus Five



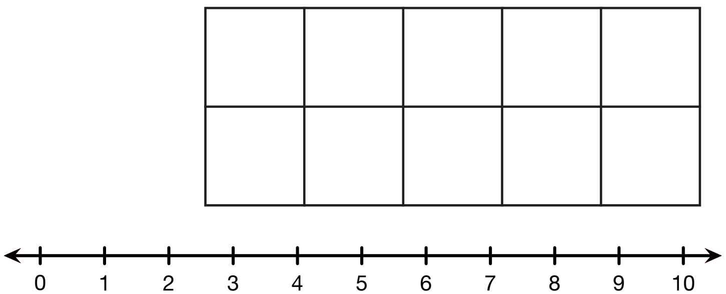
$$16 - 5 = 10 + 1 \textit{ or } 11$$

Name ______________________ Date ____________

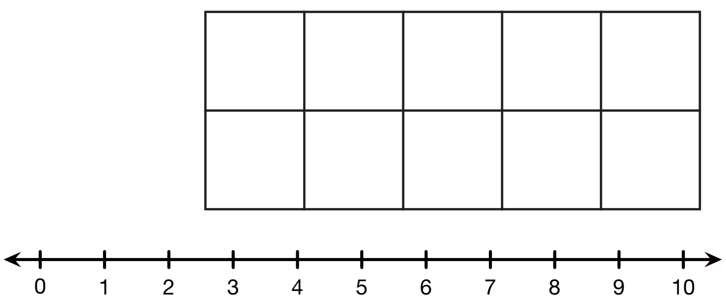
Birthday Party

Show how you solve each problem. Use connecting cubes, ten frames, or number lines.

1. John has 5 red balloons and 3 blue balloons. How many balloons does he have?

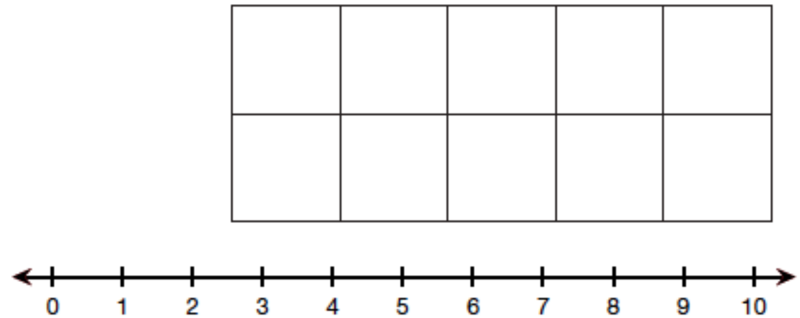


2. There are 9 children at the party. Four of the children are girls. How many are boys?

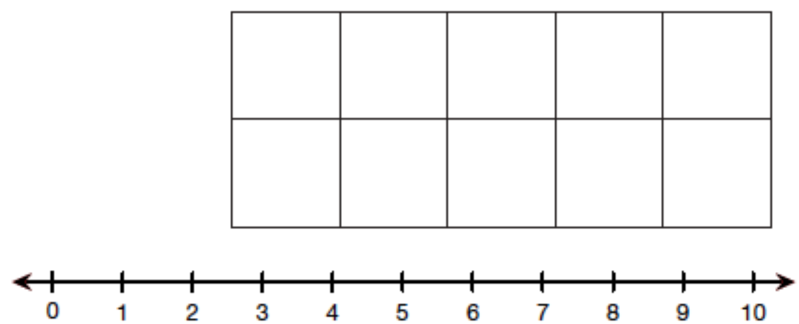


Name ______________________ Date ____________

3. There are some candles on the cake. John put 4 more candles on the cake and now there are 7 candles. How many candles were on the cake?



4. The children played 5 games at the party. They played 2 games before lunch and some games after lunch. How many games did they play after lunch?

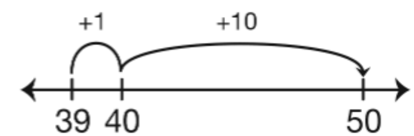
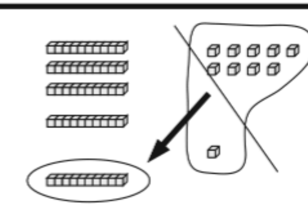


Start By

Solve the addition problems. Start each solution a different way. Circle the strategy you like best.

One Strategy	Another Strategy
1. A. 8 + 9 Start by adding 8 + 8.	**B.** 8 + 9 Start by splitting 8 into 7 + 1.
2. A. 15 + 6 Start by adding 15 + 5.	**B.** 15 + 6 Start by adding 5 + 6.

Possible Strategies for 39 + 11

Start by adding 39 + 10	39 + 10 = 49 49 + 1 = 50
Start by adding 39 + 1	 39 + 1 + 10 = 50
Start by adding 9 + 1	 5 skinnies or 50