

NCTM 2017 San Antonio, Texas

Develop Meaning by Connecting Multiple Strategies Grades K-2

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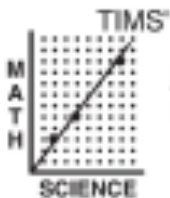
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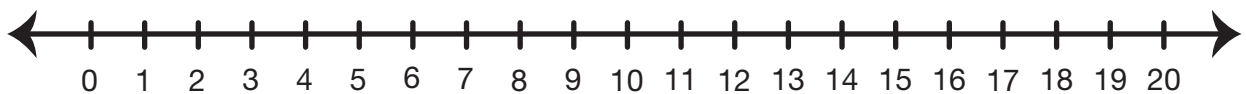
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Nan and Bert Problems

Show or tell how to solve each problem. Write a number sentence. Use cubes, a number line, or ten frames. You may also draw a picture.

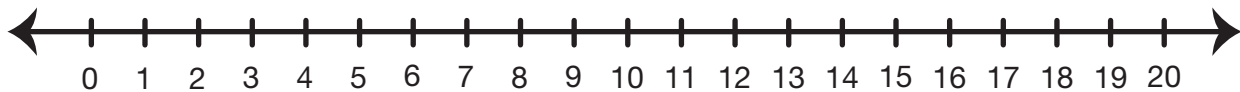
1. Nan and Bert went on a picnic at the lake. Nan ate 9 grapes and Bert ate 5. How many did they eat altogether?

Number sentence _____



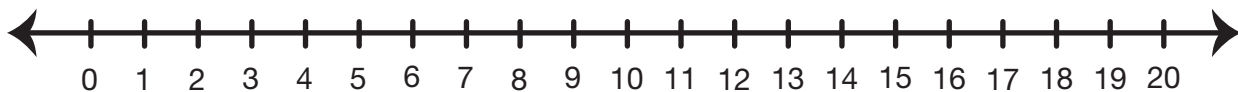
2. Grandma packed 15 baby carrots. Nan and Bert ate them all. If Nan ate 7, how many did Bert eat?

Number sentence _____



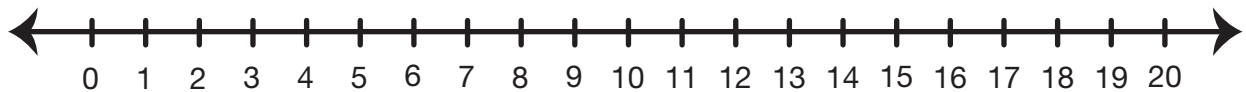
3. Grandma's ticket to the zoo cost \$6. Nan and Bert's each cost \$4. How much did all 3 tickets cost?

Number sentence _____



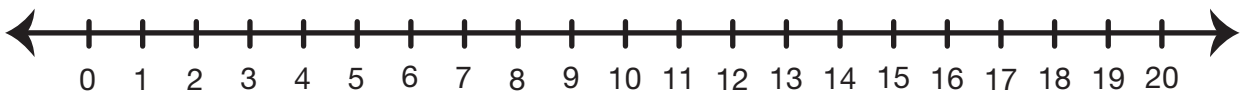
4. At the gift shop, Bert spent \$16 and Nan spent half that much. How much did Nan spend?

Number sentence _____



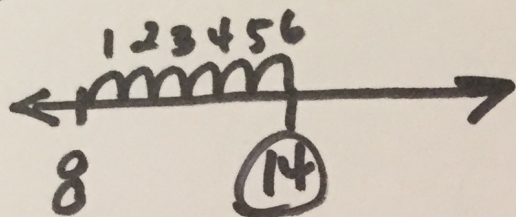
5. Nan and Bert each brought a package of 8 crackers for a snack. How many crackers do they have altogether?

Number sentence _____

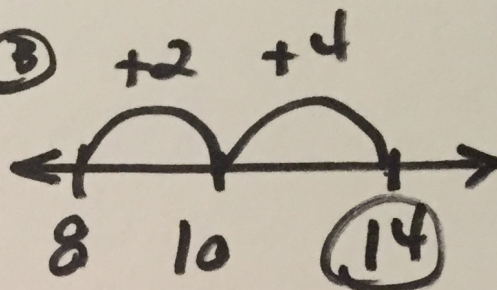


$$8 + 6$$

(A)

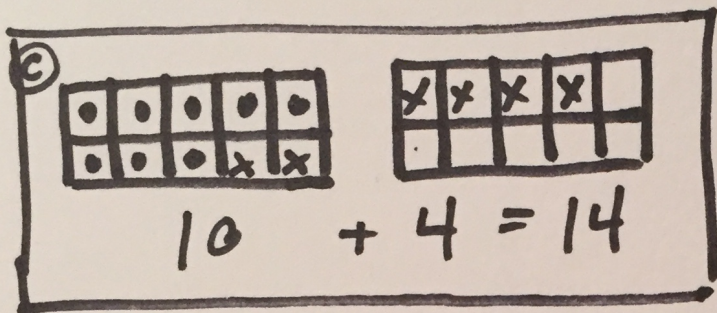


(B)

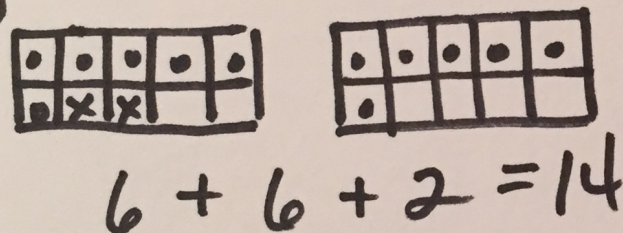


(C)

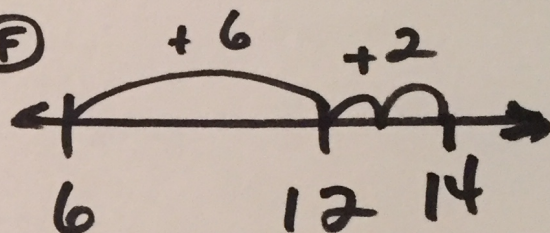
$$8 + 2 + 4 = 14$$



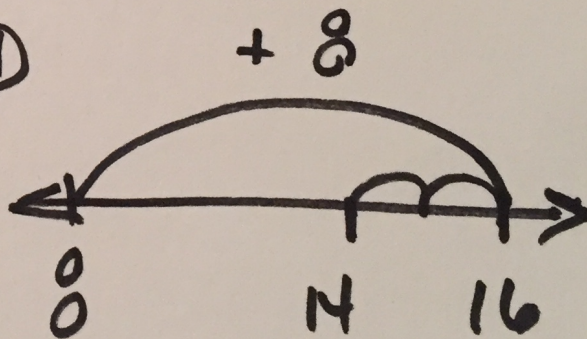
(E)



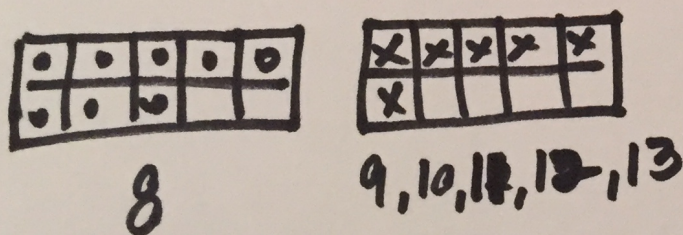
(F)



(H)



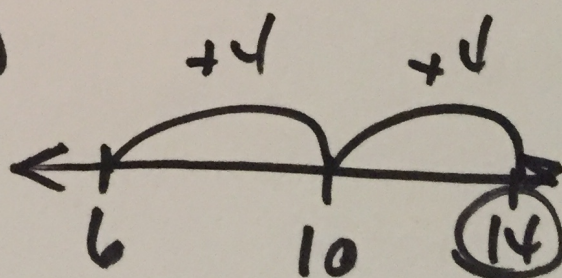
$$8 + 6 = 14$$



(I)

$$8 + 8 - 2 = 14$$

(J)



Teacher Questions from ***Connecting Mathematical Ideas*** (Boaler & Humphries, 2005, p.37)

Teachers' questions play a critical role in establishing the instructional environment as well as the direction of lessons (Boaler and Brodie, 2004).

Question Type	Description	Example
1. Gathering information, checking for a method, leading students through a method	Want direct answer, usually right or wrong Rehearse known facts or procedures Enable students to state facts or procedures	What is the value of x in this equation? How would you plot that point?
2. Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	What is this called in math? How would we write this correctly mathematically?
3. Exploring mathematical meanings and relationships	Points to underlying mathematical relationships and meanings Makes links between mathematical ideas	Where is the x on the diagram? What does probability mean?
4. Probing; getting students to explain their thinking	Clarifies student thinking Enables students to elaborate their thinking for their own benefit and for the class	How did you get ten? Can you explain your idea?
5. Generating discussion	Enables other members of class to contribute and comments on ideas under discussion	Is there another opinion about this? What did you say, Justin?
6. Linking and applying	Points to relationship among mathematical ideas and mathematics and other areas of study or life	In what other situations could you apply this? Where else have we used this?
7. Extending thinking	Extends the situation under discussion, where similar ideas may be used	Would this work with other numbers?
8. Orienting and focusing	Helps students focus on key elements or aspects of the situation in order to enable problem solving	What is the problem asking you? What is important about this?
9. Establishing context	Talks about issues outside of math in order to enable links to be made with mathematics at a later point	What is the lottery? How old do you have to be to play the lottery?

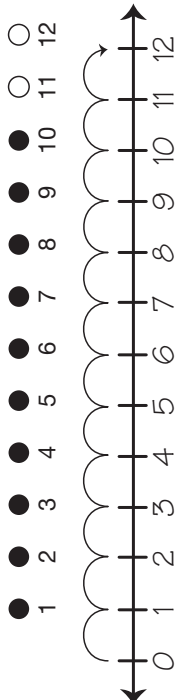
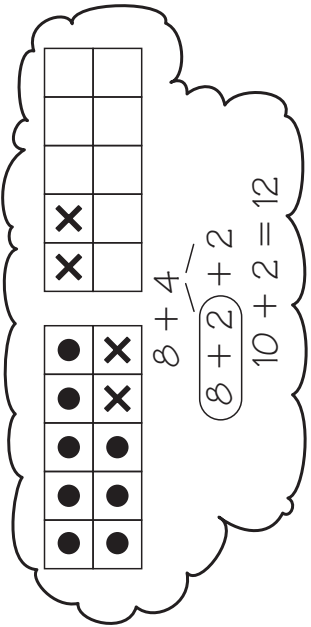
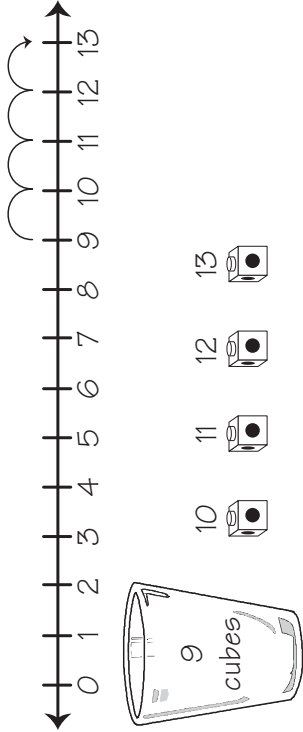
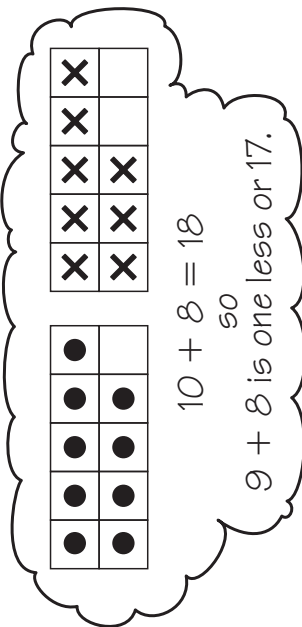
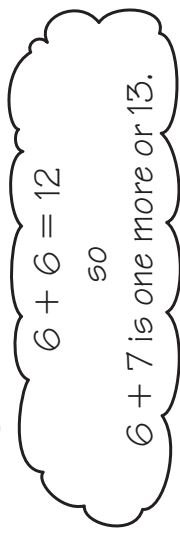
* According to Humphries & Boaler, exploring math meaning and relationships are the most important yet rarely used question type (type 3)

Name _____ Date _____

My Addition Strategies Menu for Larger Numbers

Counting All	Making Ten
Counting On	Using Ten
Another Strategy _____	Using Doubles

Addition Strategies Menu for Larger Numbers

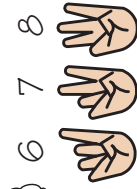
Counting All $10 + 2 = 12$ 	Making Ten $8 + 4 = 12$ 
Counting On $9 + 4 = 13$ 	Using Ten $9 + 8 = 17$ 
Another Strategy _____	Using Doubles $6 + 7 = 13$ 

Subtraction Strategies Menu

Counting Strategies

Counting Up

$$8 - 5 = 3$$



Counting Back

$$9 - 2 = 7$$

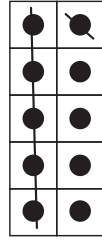


Another Strategy _____

Reasoning Strategies

Using Ten

$$9 - 5 = 4$$



$$10 - 5 = 5$$

so

$$9 - 5 = 4$$

Using Doubles

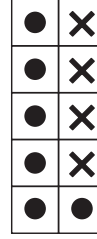
$$8 - 3 = 5$$

$$4 + 4 = 8 \text{ so } 8 - 4 = 4.$$

To solve $8 - 3$, I take off one less
so $8 - 3 = 5$.

Making Ten

$$10 - 6 = 4$$



$$\text{I know } 6 + 4 = 10$$

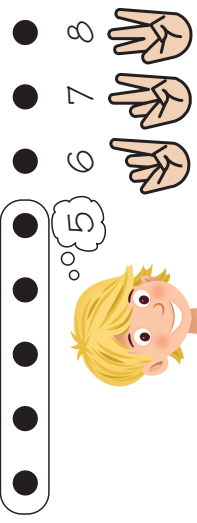
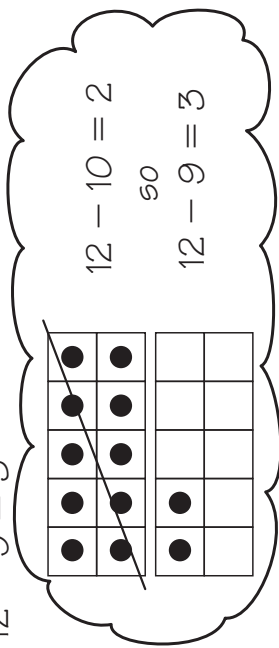
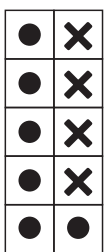
so

$$10 - 6 = 4.$$

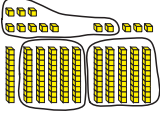
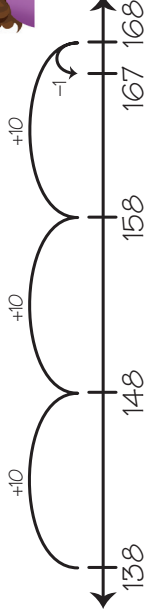
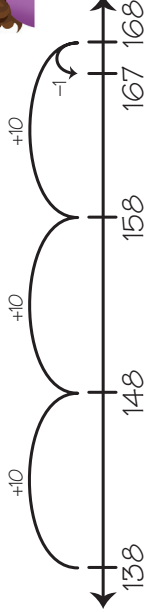
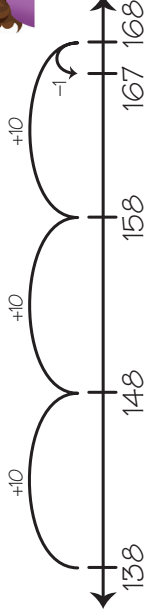
Addition Strategies Menu for the Facts

Counting All $10 + 2 = 12$	Making Ten $8 + 4 = 12$
Counting On $9 + 4 = 13$	Using Ten $9 + 8 = 17$
Another Strategy _____	Using Doubles $6 + 7 = 13$

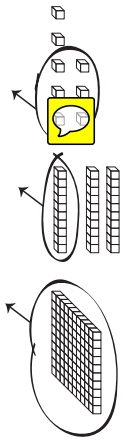
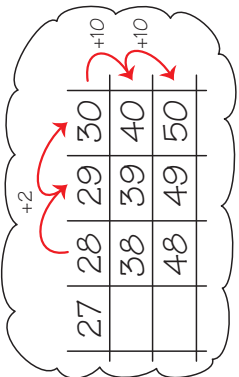
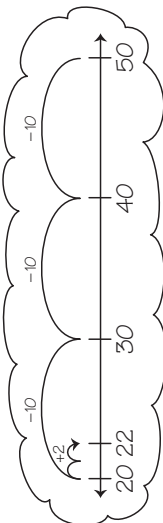
Subtraction Strategies Menu for the Facts

Counting Up $8 - 5 = 3$  Jason	Using Ten $12 - 9 = 3$  $12 - 10 = 2$ so $12 - 9 = 3$
Counting Back $9 - 2 = 7$ 	Using Doubles $13 - 7 = 6$ $7 + 7 = 14$ so $14 - 7 = 7$. To solve $13 - 7$, I start with one less so $13 - 7 = 6$.
Thinking Addition $11 - 3 = 8$ I know $8 + 3 = 11$ so $11 - 3 = 8$.	Making Ten $10 - 6 = 4$  I know $6 + 4 = 10$ so $10 - 6 = 4$.

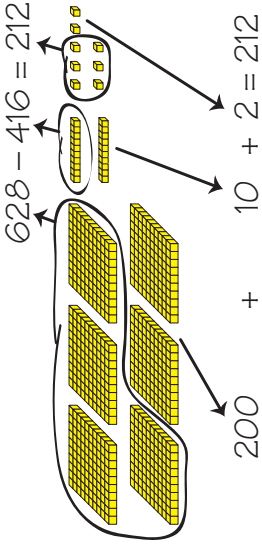
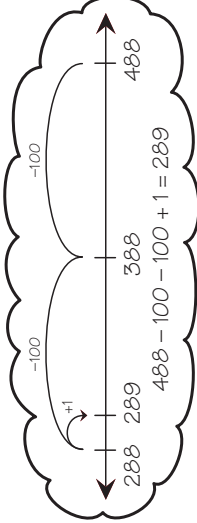
Addition Strategies Menu

Finding Friendly Numbers $138 + 29$ $140 + 30 = 170$ 170 is a reasonable estimate. 	Using Base-Ten Pieces 68 $+55$ 123  Trade 11 skinnies and 13 bits for 1 flat, 2 skinnies, and 3 bits 
Counting On $138 + 29$ $138 + 30 - 1 = 167$  	Using Expanded Form $68 = 60 + 8$ $+55 = 50 + 5$ $110 + 13 = 123$ 
Counting On $138 + 29$ $138 + 30 - 1 = 167$  	Using All-Partials 68 $+55$ 110 13 123 
Counting On $138 + 29$ $138 + 30 - 1 = 167$  	Using the Compact Method 68 $+55$ 123 

Subtraction Strategies Menu

Finding Friendly Numbers $50 - 28$ $50 - 30 = 20$ 20 is a reasonable estimate.  Irma	Using Base-Ten Pieces $138 - 116$  $20 + 2 = 22$  Ana
Counting Up $50 - 28$ $2 + 20 = 22$   Jacob	Using Expanded Form Subtract 39 from 62 $62 = 60 + 2 = 50 + 12$ $39 = 30 + 9 = 30 + 9$ $20 + 3 = 23$  John
Counting Back $50 - 28 = 22$   Michael	Using the Compact Method $\begin{array}{r} 7\ 13 \\ \cancel{6}\ \cancel{2} \\ - 2\ 7 \\ \hline 5\ 6 \end{array}$  Keenya

Subtraction Strategies Menu for Larger Numbers

Finding Friendly Numbers $428 - 179$ $430 - 200 = 230$ 230 is a reasonable estimate. 	Using Base-Ten Pieces  $628 - 416 = 212$ $200 + 10 + 2 = 212$ 
Counting Up $300 - 198 = 102$ $198 + \textcircled{2} = 200$ $200 + \textcircled{100} = 300$ $100 + 2 = 102$ 	Using Expanded Form Subtract 239 from 562. $562 = 500 + 60 + 2 = 500 + 50 + 12$ $239 = 200 + 30 + 9 = 200 + 30 + 9$ $300 + 20 + 3 = 323$ 
Counting Back $488 - 199 = 289$  	Using the Compact Method $\begin{array}{r} 11 \\ 3 \cancel{7} 13 \\ 42\cancel{8} \\ - 165 \\ \hline 258 \end{array}$ 