

Activities to Connect the Mathematical Practices for AP Calculus in Your Classroom

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Link to Materials: <https://goo.gl/qczlyk>

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Differential Equations Group Activities Lesson Plan:

These activities will take place over several class days. Have the students return to the same groups with the same differential equation each time.

Day 1: Slope Field Introduction

1. Model creating a slope field with the differential equation: $y' = -2x$. Use large graph paper and make a grid $[-3, 3] \times [-3, 3]$. Skip at least 2 lines between lattice points. Prompt students to look for patterns in the calculations. (Option: Sketch a solution curve through $(2, 1)$.)
2. Assign student groups DE's to draw their own slope fields on large graph paper. Differentiate instruction by choosing the equation for each group or use random choices.
3. Do a gallery walk. Have students determine the shape of general solution curves for each slope field as an exit slip.

Day 2: Euler's Method

1. Using the slope field for the DE: $y' = -2x$ (or another DE with a drawn slope field), and the point $(-2, -1)$, draw 4-6 tangent line segments with x-increment of $\frac{1}{2}$ or $\frac{1}{3}$ to estimate $y(0)$. At the end of each line segment, recalculate the appropriate slope.
2. Have student groups use their slope fields to create a similar path. Have them begin with any point that has x-value of -2 and take steps the width of one square on their graphs.
3. If time permits, have them repeat the activity (using a different colored marker) for steps of width 1 or steps with width $\frac{1}{2}$ of a grid square.
4. Display the graphs in the classroom.
5. Introduce Euler's Method using their paths as a reference for each step.

Day 3: Separable DE's

1. Determine which of the slope field DE's can be solved using separation of variables and which DE's will need to be solved using technology.
2. Solve the DE's using the initial value chosen for Euler's Method.
3. Compute $y(0)$ for your solution equation. Compare the result with your estimate from Euler's Method.

Variation: Do the Day 1 activity when you introduce anti-derivatives. Have students graph DEs based solely on x and repeat the slope lines vertically from $y = -3$ to $y = 3$. Multiple solution curves reinforce the geometry of anti-derivatives and show the +C.

Partner Volume Project

Determine the volume of your crepe paper decoration in cubic centimeters.

What you will need to do and turn in:

- Trace the outline of your decoration on graph paper.
- Measure key points along your outline to determine x- and y-coordinates in cm.
- Create data tables to represent points on each curve of your outline.
- Compute equation(s) to match your curve(s) well.
- Write integral(s) to represent the volume of your decoration.
- Calculate the volume using your calculator.
- Does your answer seem reasonable? Explain why or why not.

Variation – Solid of Revolution Lesson Plan

Objective: Students will use an object from home as a model solid of revolution. Students will compute the volume of their object using calculus and then compare their result with the volume determined by submerging the object in water.

Advanced Preparation: Once the concept of a solid of revolution has been introduced, students should be told to begin looking for an appropriate object. Remind them every day until you do the project. Arrange to use a lab room for 1-2 periods for this project—the lab tables and access to beakers or graduated cylinders is helpful. (Switch with Chemistry for the day.) Review the regression features on their calculators and how to determine the curve of best fit.

Lab Day: Gather needed supplies. 2 large beakers and/or graduated cylinders. Measuring tools: rulers, calipers, string. Graph paper. Duct tape (to seal over holes in objects).

Student Assignment Sheet:

Find an object of revolution at home and bring it in for this project. Objects should be about the size of your fist and be submersible. The objects cannot be cylinders, cones or spheres.

1. Sketch a graph of the function(s) that could form a region to rotate and create your object. Use the measuring tools in class to find actual lengths on your object and plot points carefully. Be precise!
2. Calculate the appropriate equation(s) for your function(s). A piecewise-defined equation will work best for most objects.
3. Write the integral(s) that determine the volume of your object. Use your calculator to evaluate the integral(s). Round your answer to the nearest cubic centimeter.
4. Submerge your object and determine its actual volume. Compare the actual volume with the result from your calculations. Calculate your percentage of error. Are there any factors that would lead to large error amounts?
5. Repeat for another object.

Closing: Remind the students to bring their work back tomorrow if they are not done. If finished, they should turn in their work at the end of class.