Developing an Understanding of the Number Line Through Measurement Concepts

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What is a number line? (In 3-4 words)

What do your students know about the number line?

Common Core State Standards - Number Line

2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the number 0,1,2,..., and represent whole-number sums and differences within 100 on a number line diagram.

3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

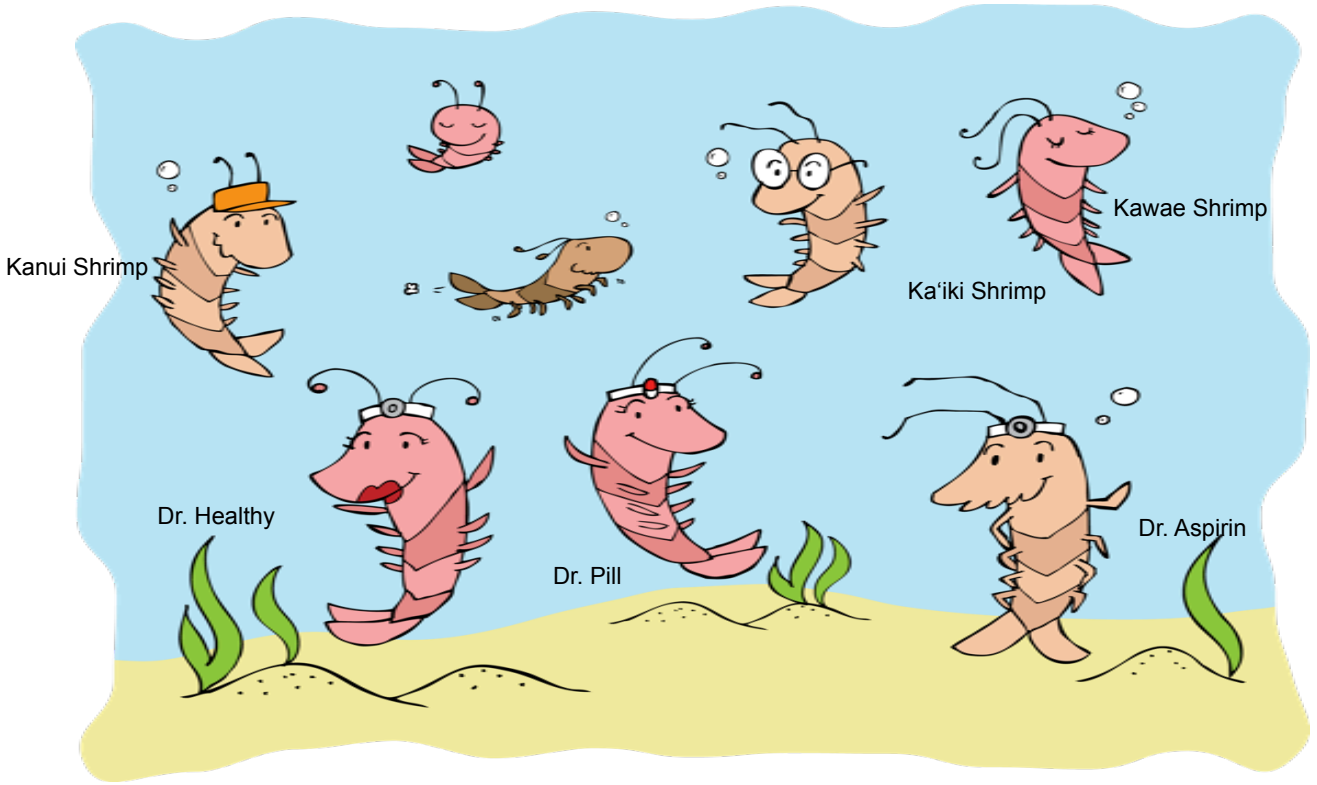
3.NF.A.2.A Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

3.NF.A.2.B Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off *a* lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

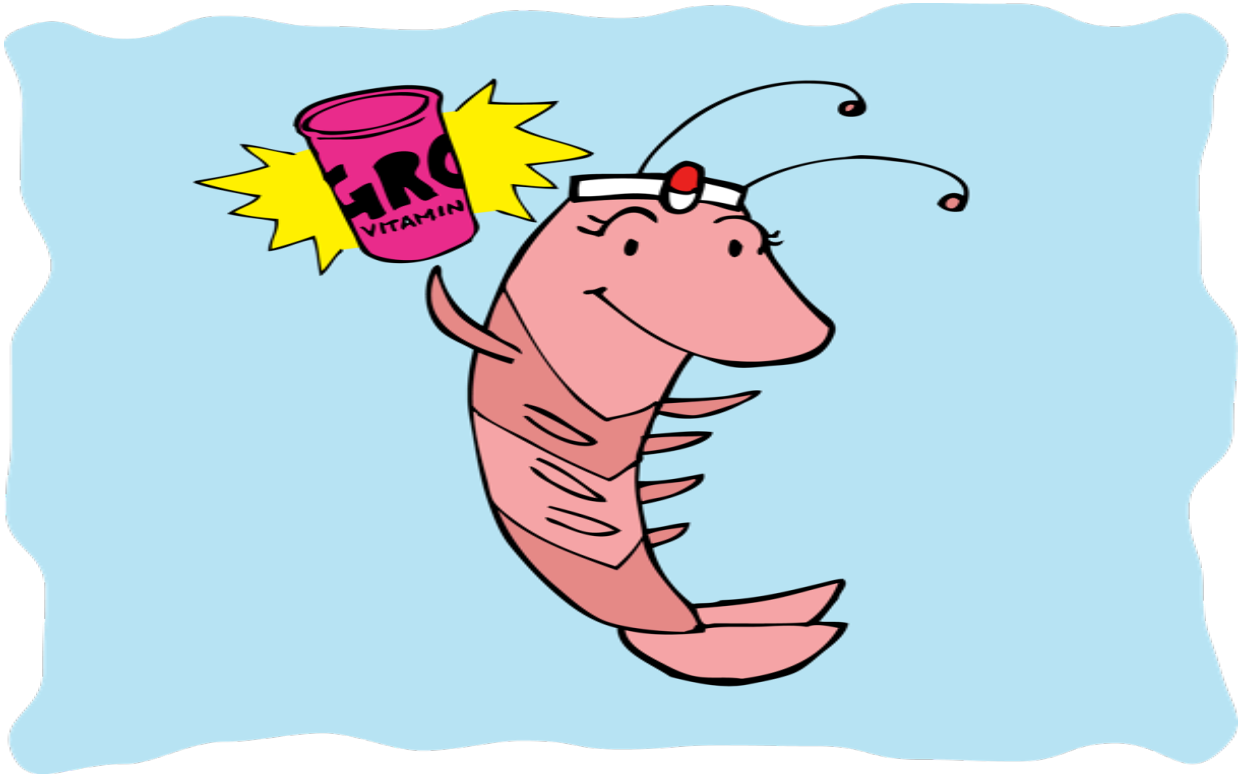
The Measure Up (MU) Curriculum Research and Development Project (2001)

- Based on mathematics developed by a team of Russian mathematicians, educators, and psychologists, led by Davydov and El'konin (1975)
- Adapted by the University of Hawai'i's Curriculum Research & Development Group for research and development purposes
- Uses measurement as a context for learning quantitative relationships
- Use of continuous quantities, such as area, volume, mass and length, and generalized measurement concepts

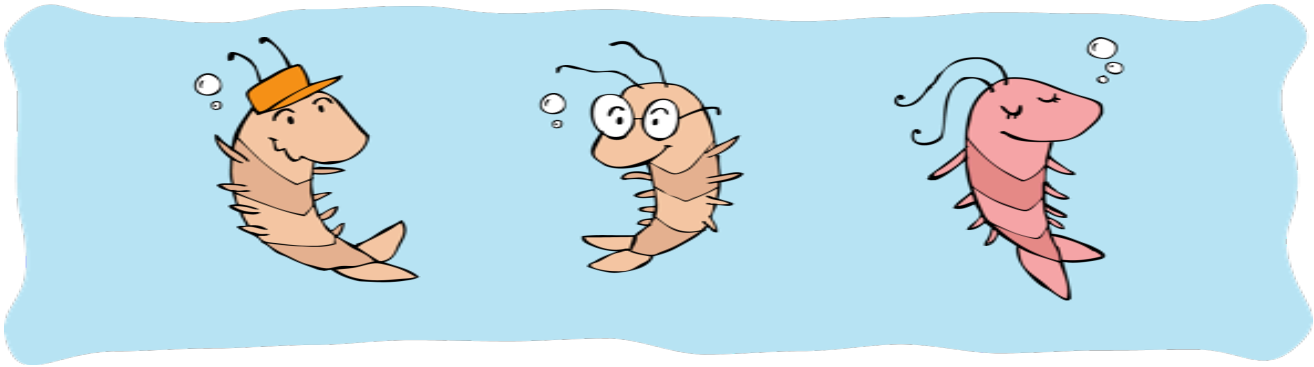
Akamai ‘Opae:
An Introduction to Measurement



Welcome to the City of ‘Opae where all of the citizens are shrimps!



One of the City of ‘Opae’s most famous citizens is Dr. Pill, known for inventing a special growth vitamin.

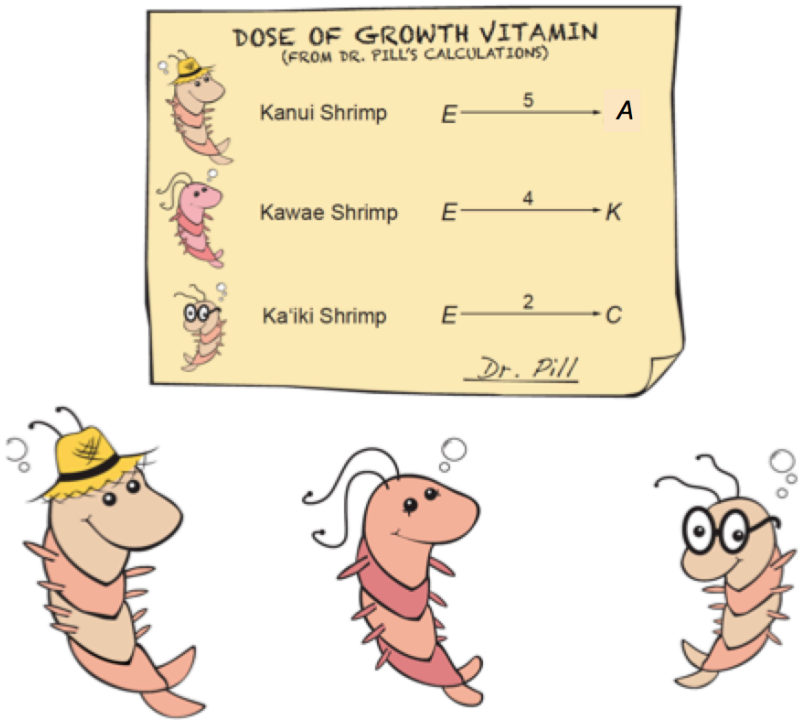


Three best friends – Kanui Shrimp, Kawae Shrimp, and Ka‘iki Shrimp – wondered if Dr. Pill’s vitamin could help them grow big and strong, so they gave her a call.

Dr. Pill was excited to help the young shrimps, and she began calculating daily doses of the vitamin for Kanui, Kawae, and Ka‘iki

Each day, her colleague, Dr. Aspirin gave the three friends their growth vitamin.

Dr. Pill wrote the number of volume-unit *E*s of vitamin to give each of the three shrimps each day. She labeled each dose on the chart. Kanui ("big") shrimp needed a larger number of volume-unit *E*s because he was bigger than Kawae ("middle") and Ka'iki ("small"). Ka'iki needed a smaller number of volume-unit *E*s because he was smaller than Kawae and Kanui.



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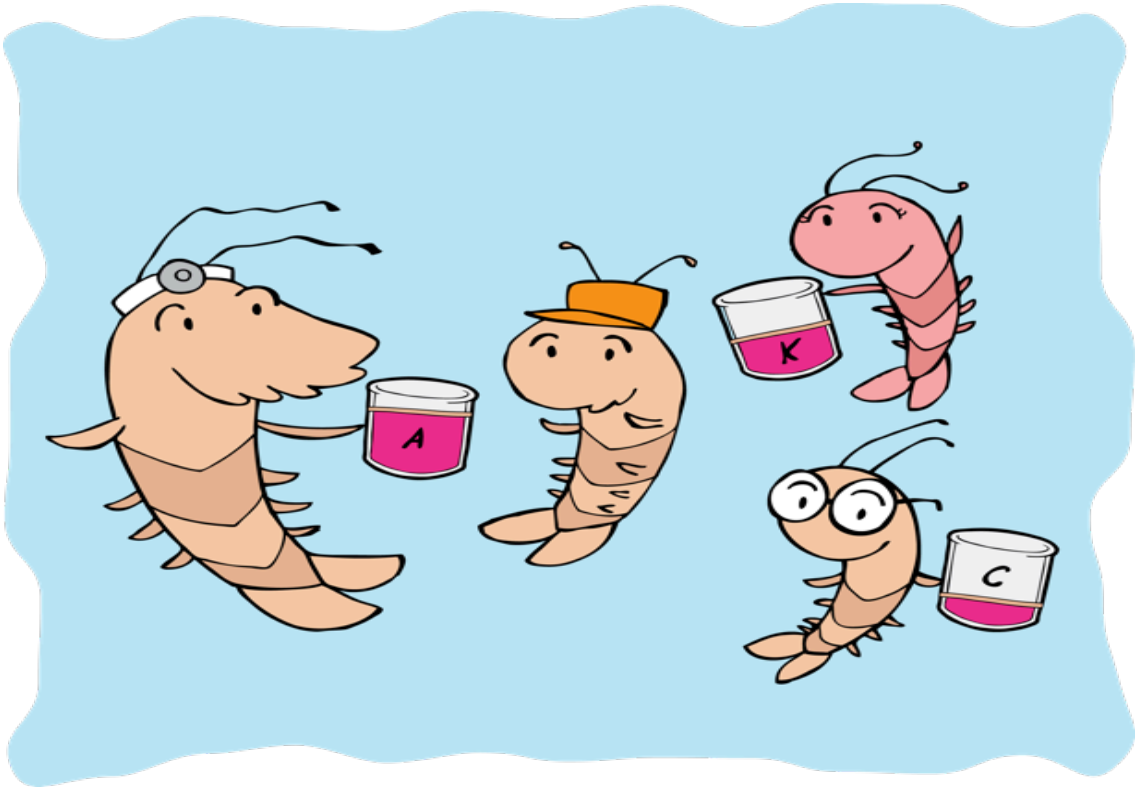
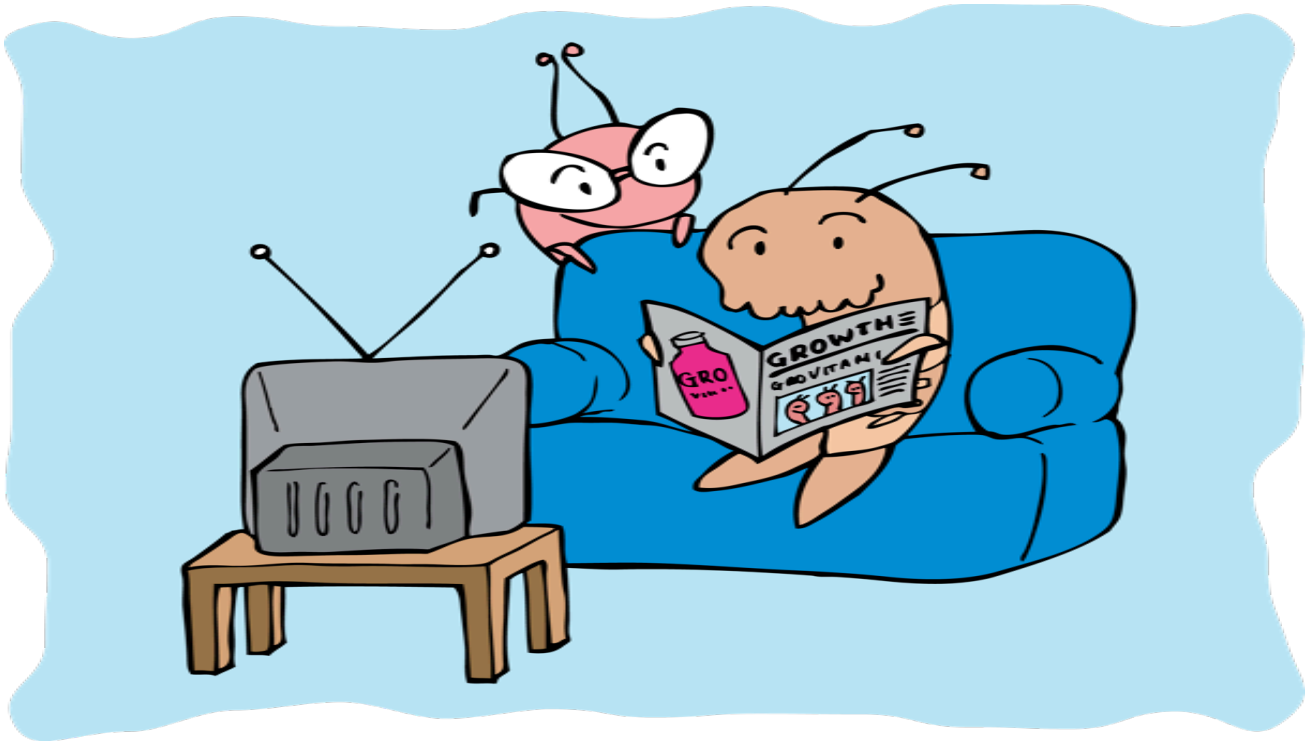
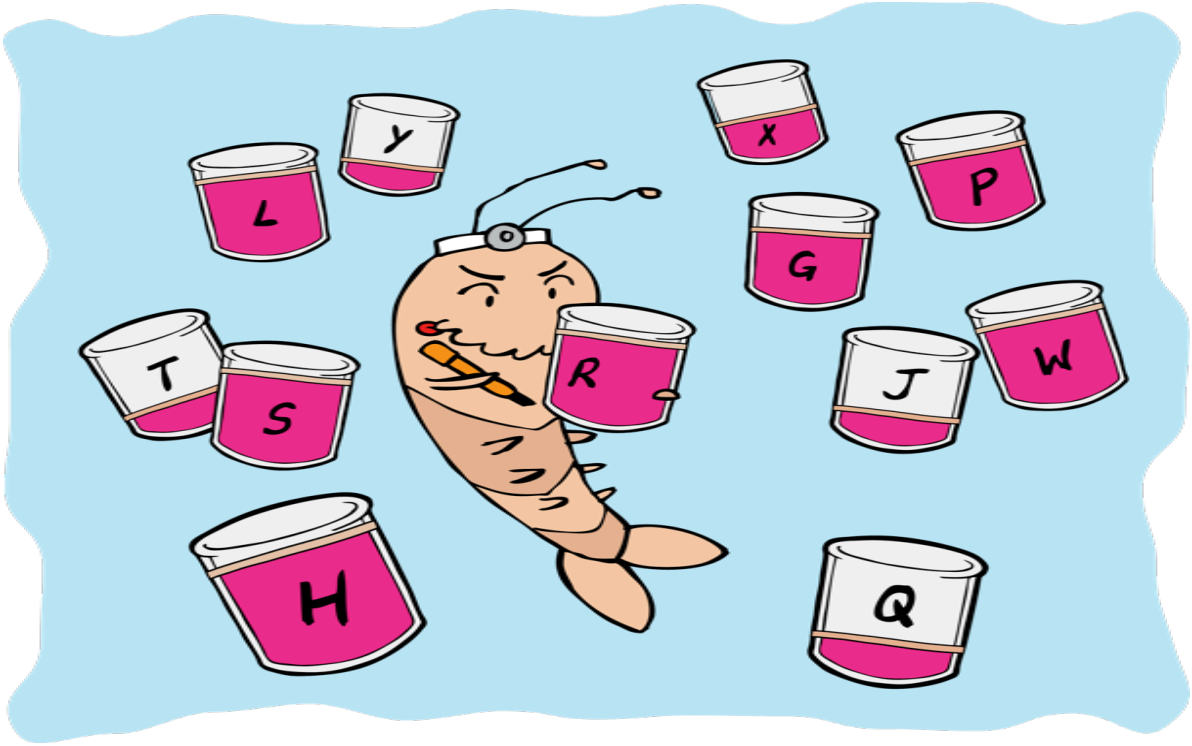


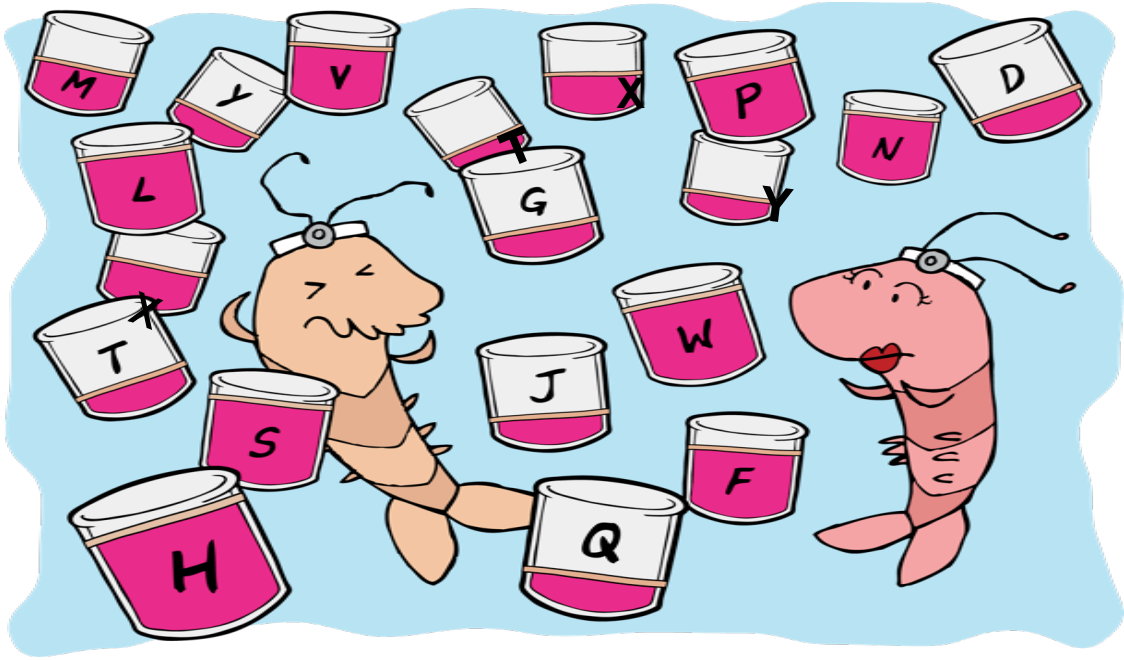
Dr. Aspirin was happy! He can now pour the vitamin directly into the special container for each shrimp.



News of Dr. Pill's growth vitamin spread throughout the City of 'Opae. Other citizens wanted to grow like Kanui Shrimp and his friends, too.



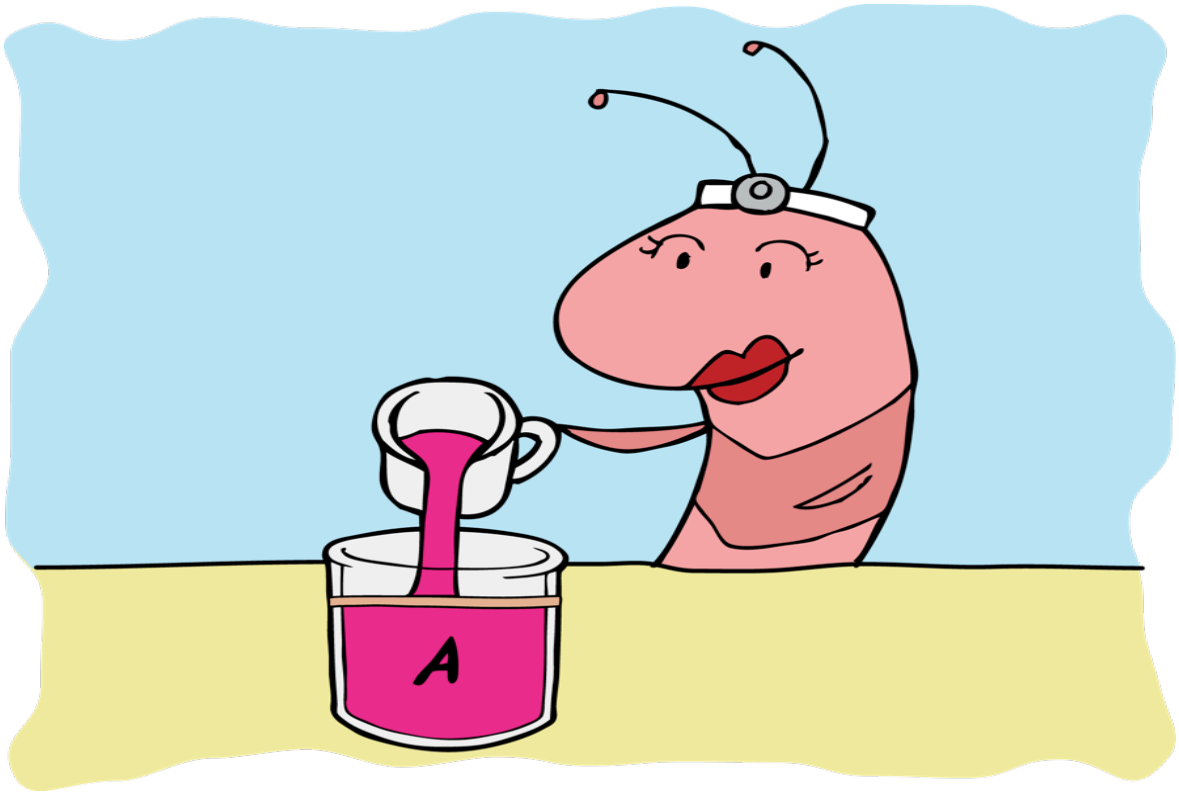
Dr. Aspirin busily marked containers for each new patient. There were more and more containers!



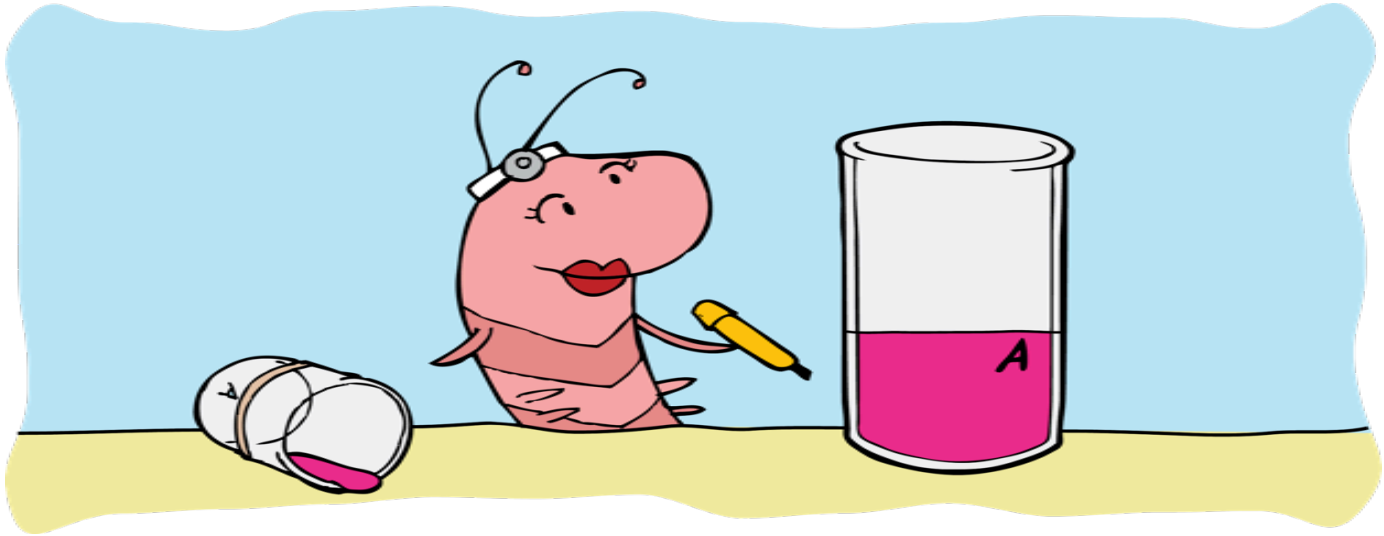
In fact, Dr. Aspirin had so many containers that he couldn't keep track. He had a headache!

Luckily for him, Dr. Healthy dropped by to see how Dr. Aspirin was doing.

Dr. Healthy took one look at all of the containers and knew that Dr. Aspirin could use her help again.



Dr. Healthy found Kanui Shrimp's container and filled it with the correct volume of Dr. Pill's Gro Vitamin.



She poured the vitamin from Kanui's volume into a large container, then marked and labeled it volume *A*.

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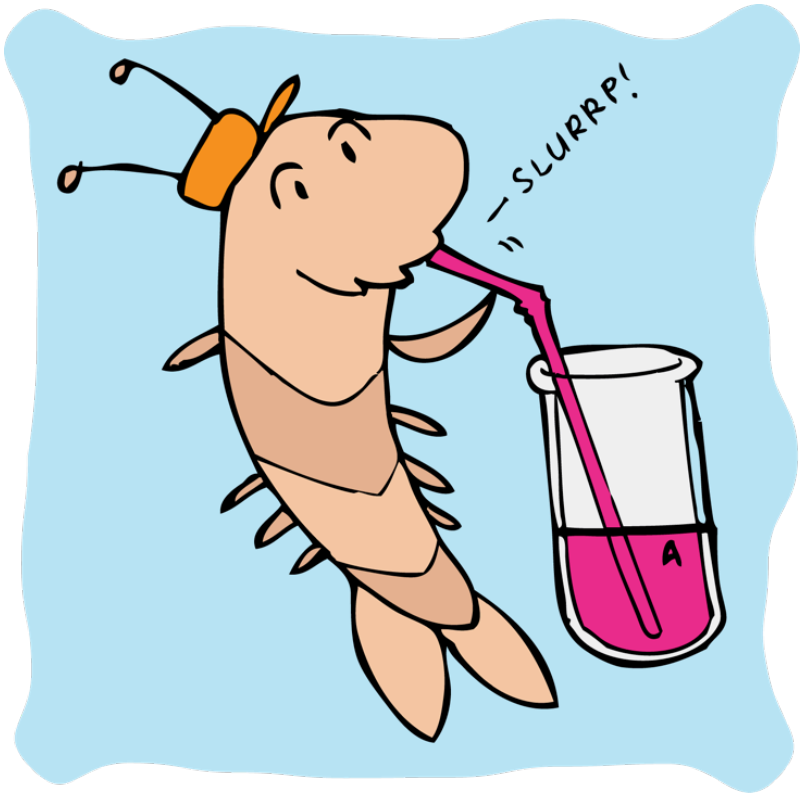
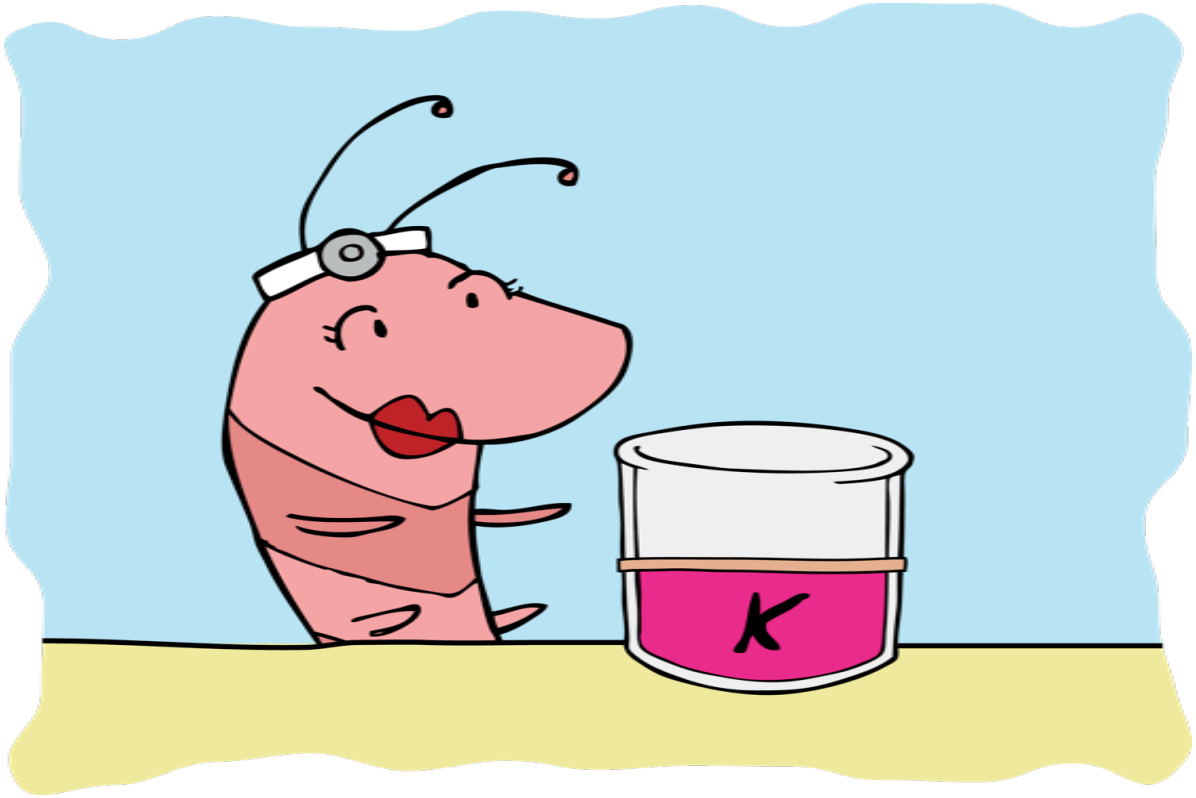
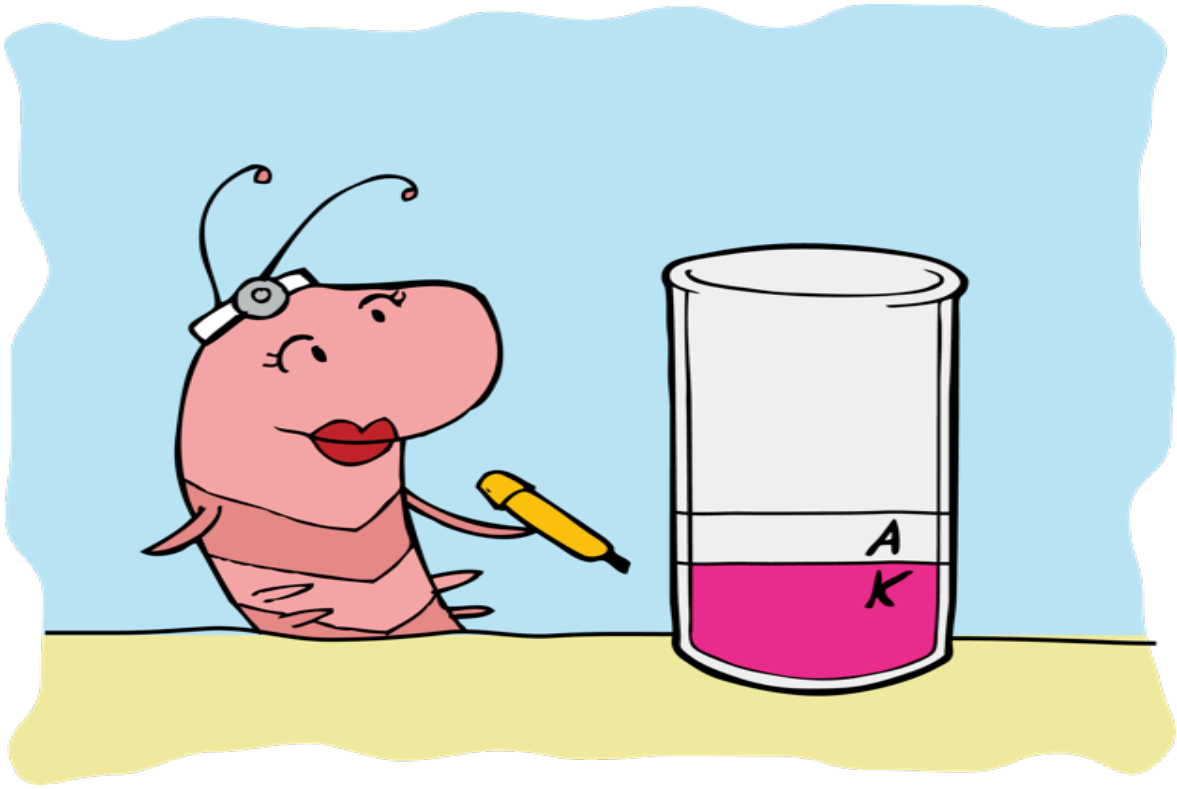


Kanui Shrimp drank his vitamin.



Next, Dr. Healthy found Kawae Shrimp's container and filled it with the correct volume of Dr. Pill's Gro Vitamin.



She poured Kawae's volume of vitamin into the same large container. Using a pen, she marked the volume, and labeled it volume *K*.

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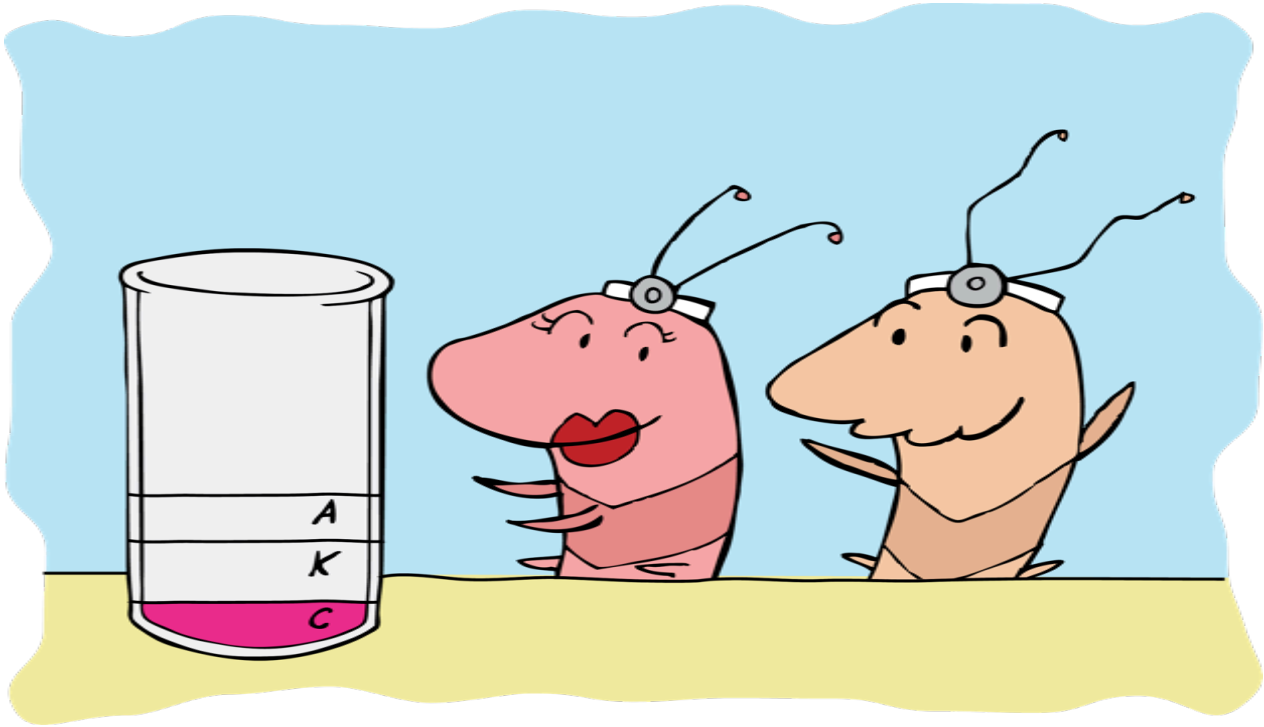

After Kawae Shrimp drank the vitamin, Dr. Healthy found the container marked with Ka‘iki Shrimp’s volume. She poured Ka‘iki Shrimp’s volume into the large container. Now the large bottle was marked with volumes *A, K*, and *C*.

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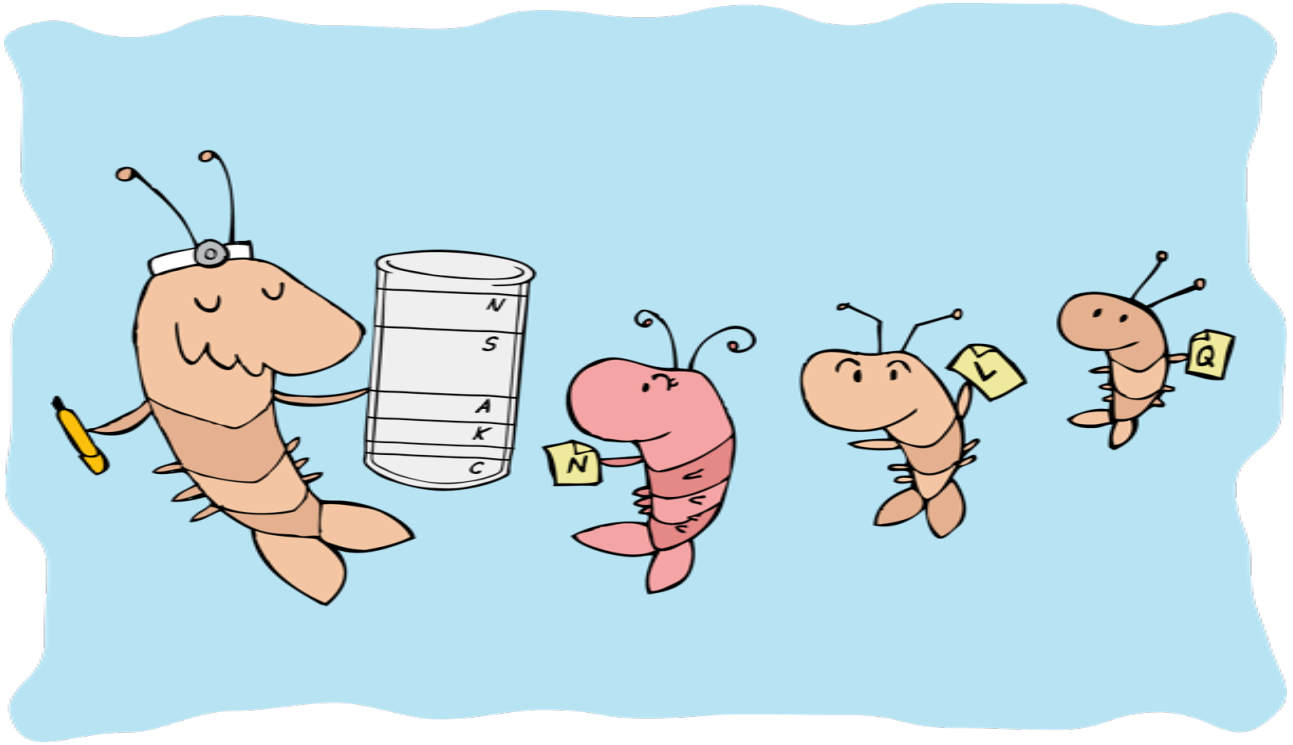
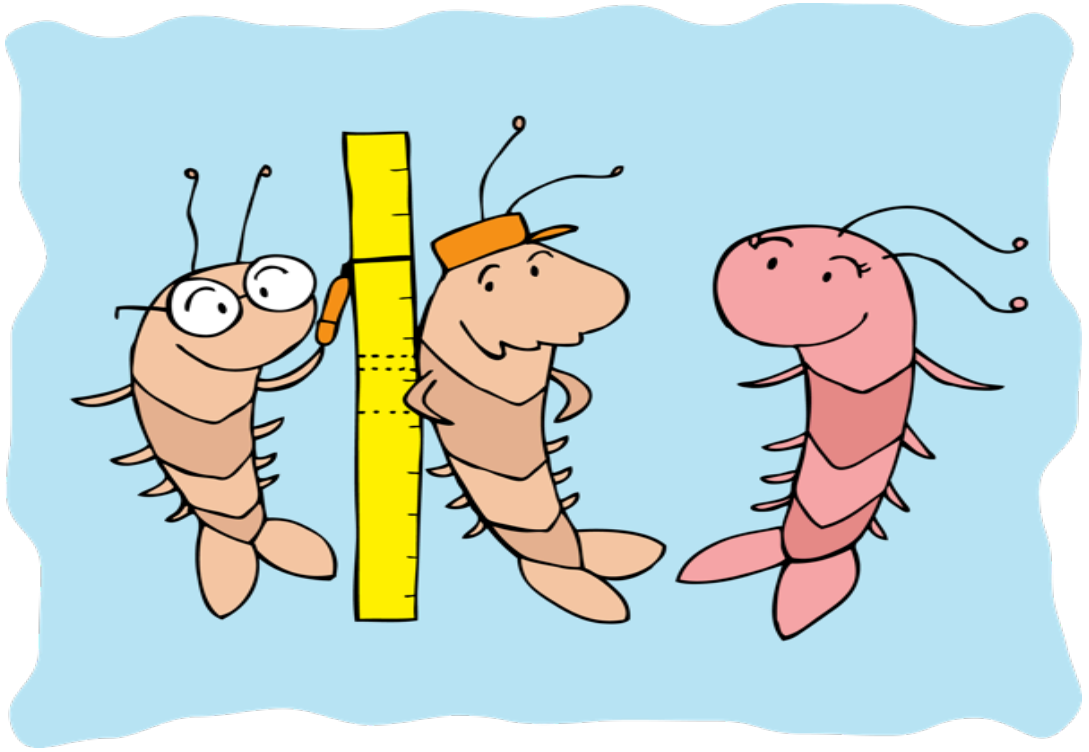
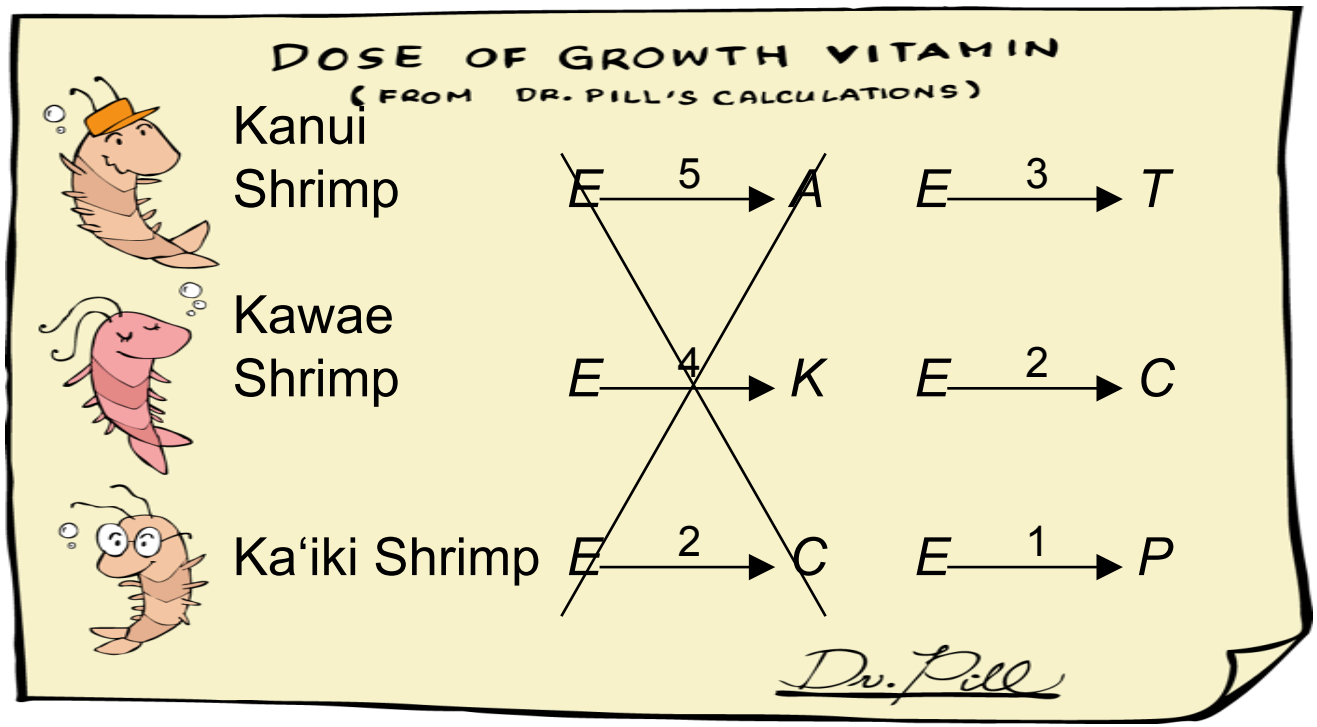


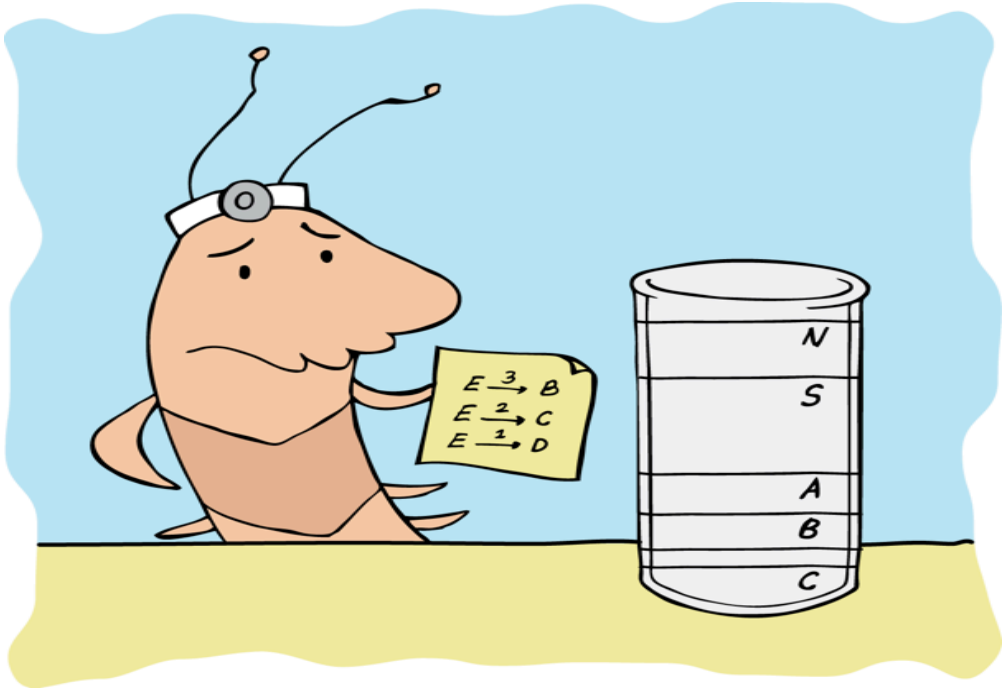
Dr. Aspirin was happy. Each time a new shrimp patient came for vitamins, he could use the marked container to give them their volume.



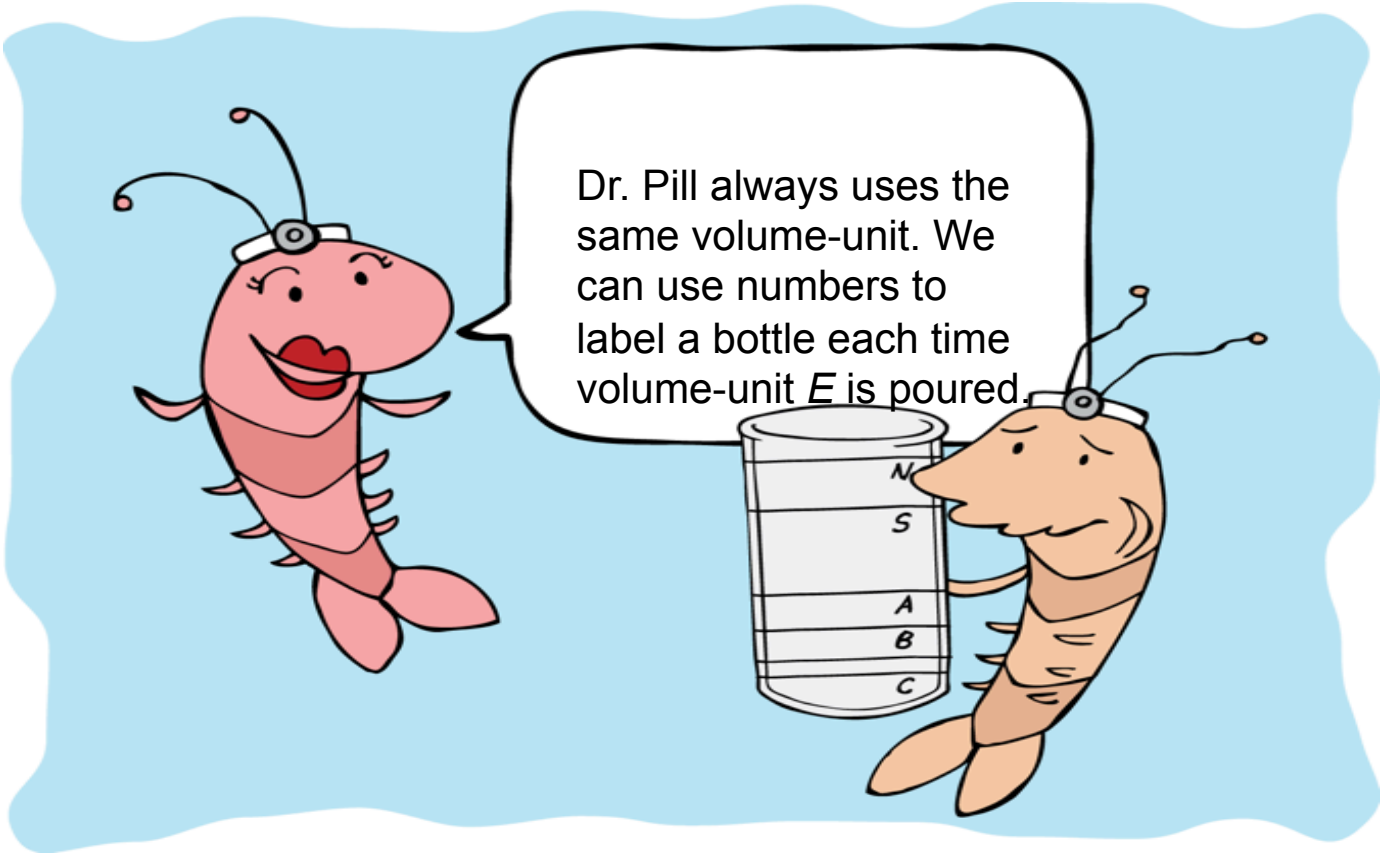
Dr. Pill was amazed by how well her vitamin worked. The citizens of the City of ‘Opae were healthy and growing, so they did not need to take as much of the vitamin.



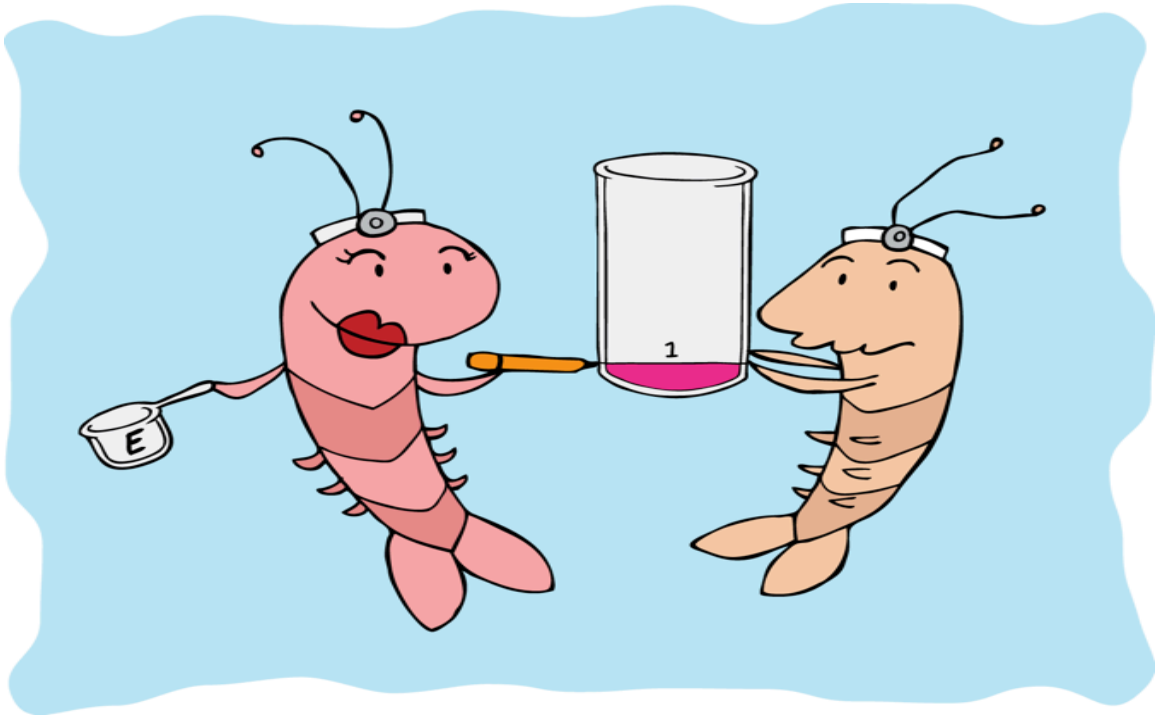
Dr. Pill then decided to lessen the amounts of vitamin the growing citizens were taking.



Poor Dr. Aspirin. The large container marked for all of the patients was not helpful anymore.



Luckily, Dr. Healthy had another idea for a new container.



Dr. Healthy poured 1 volume-unit *E*, marked the volume on the container, and labeled it “1”.

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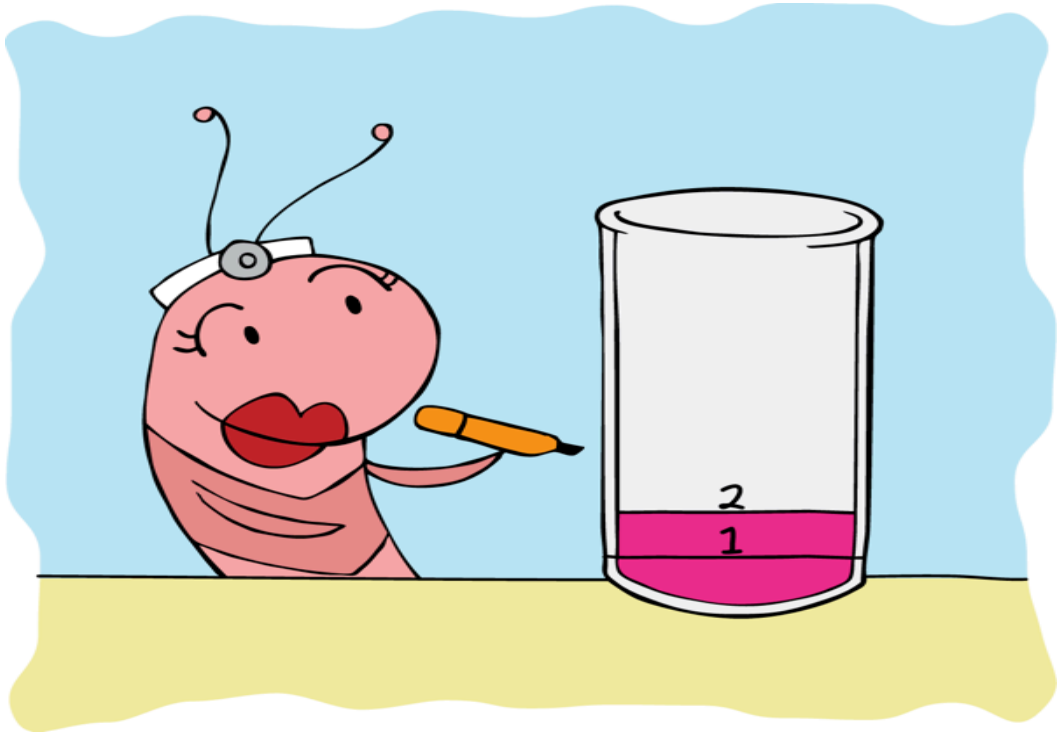

She poured a second volume-unit *E*, marked the total volume on the container and labeled it “2”.

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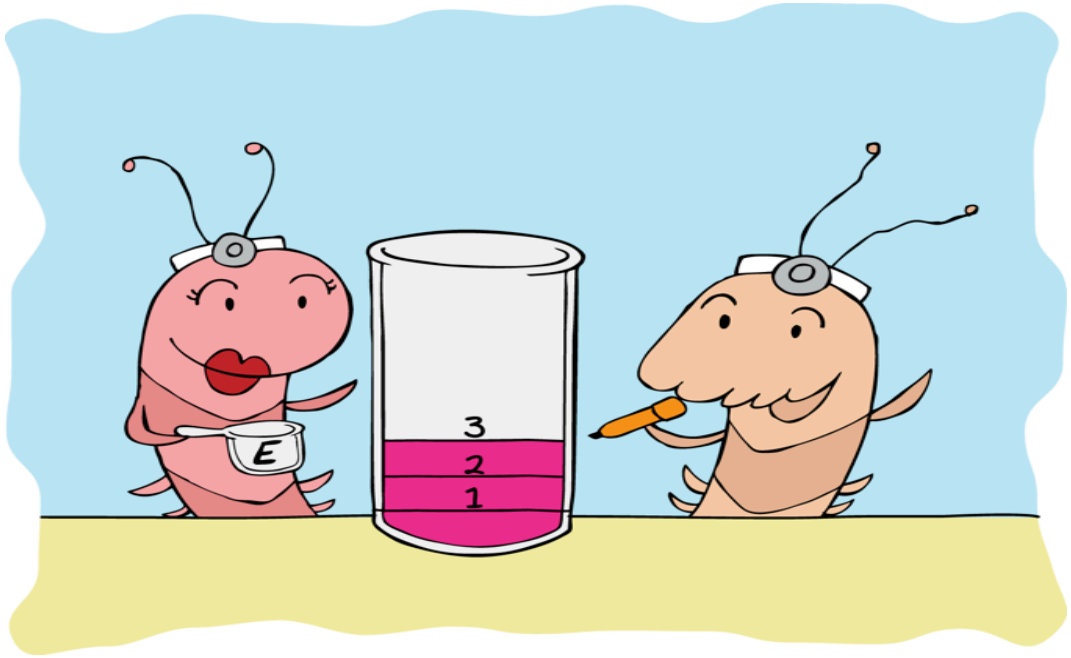

Dr. Aspirin said, “I see!” He helped Dr. Healthy pour and mark more volume-unit *E*s.

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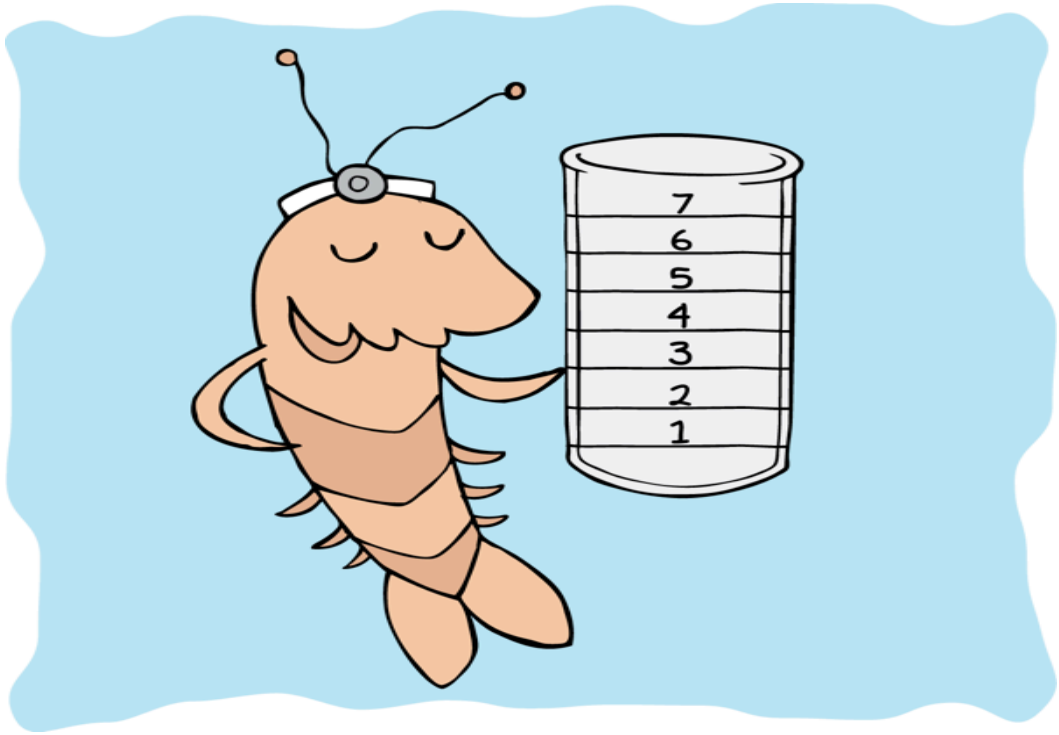
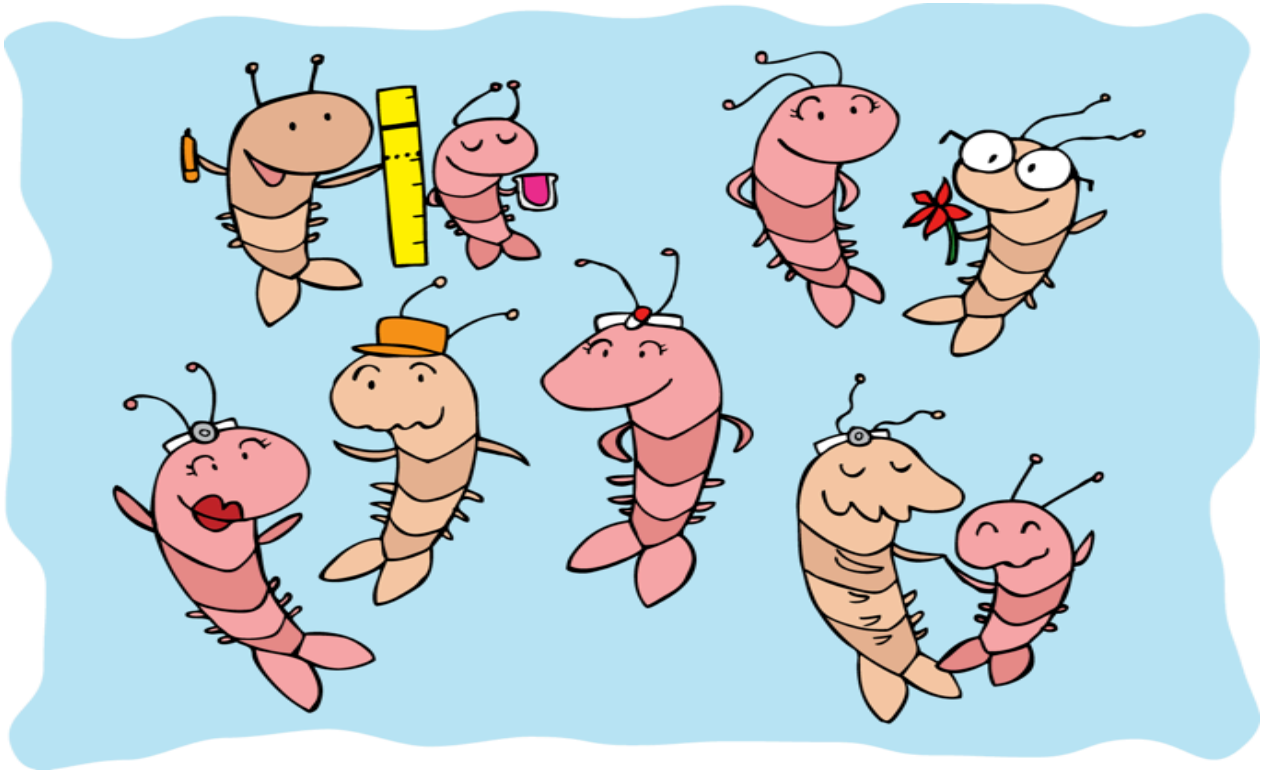


Dr. Aspirin exclaimed, “Now I have a tool to measure doses of vitamin for anyone Dr. Pill helps!”



Dr. Pill, Dr. Aspirin, Dr. Healthy, and the citizens of the City of ‘Opae grew happily ever after!

STOP here

What would happen if we marked volume-unit *Es* on tape attached to these containers?

1

2

3

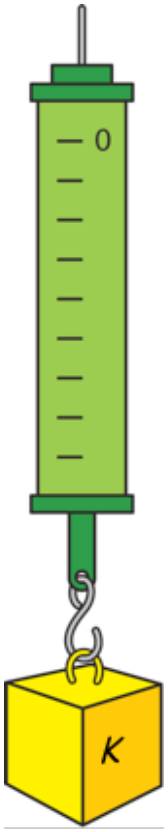
Investigating mass with a spring scale

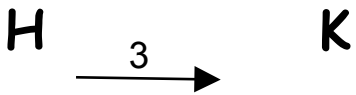
- Explore how the spring scale works.
- Holding the handle of the spring scale, make a note of the indicator attached to the spring when no weights are attached. What number corresponds with this position?
- Place one mass-unit on the scale. Mark and label the tape on the scale to keep track of each additional unit placed on the scale.
- Remove the tape from the spring scale. What do your markings represent?

Concepts inherent to the number line

These brothers are not feeling well. A doctor prescribed medicine and recorded an arrow notation for each brother. The marks on the medicine bottles indicate volume-units *U*. Where should the marks be for volume *A* and volume *B* of medicine?

Concepts inherent to the number line





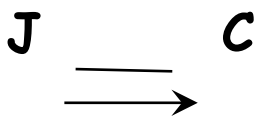
The marks on the spring scale indicate mass-units **H**. Show the mark for mass **K** on the scale.

Concepts inherent to the number line

Before

After

Carlo drank volume **C** of orange juice from his bottle. How many volume-units **J** of orange juice did he drink?

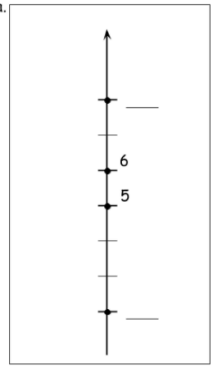


Locating points for numbers on a number line

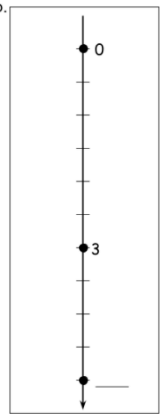
Student Page 1

On each number line, label the number for each point marked.

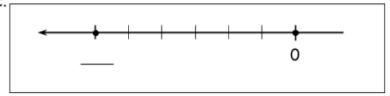
a.



b.

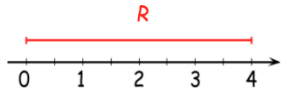


c.



Problem 1

Kim used a number line marked with length-units *F* to measure length *R*.



How long is length-unit *F*? On the segment below draw and label length-unit *F*.

Complete the right-arrow notation for length *R*.

F _____ *R*

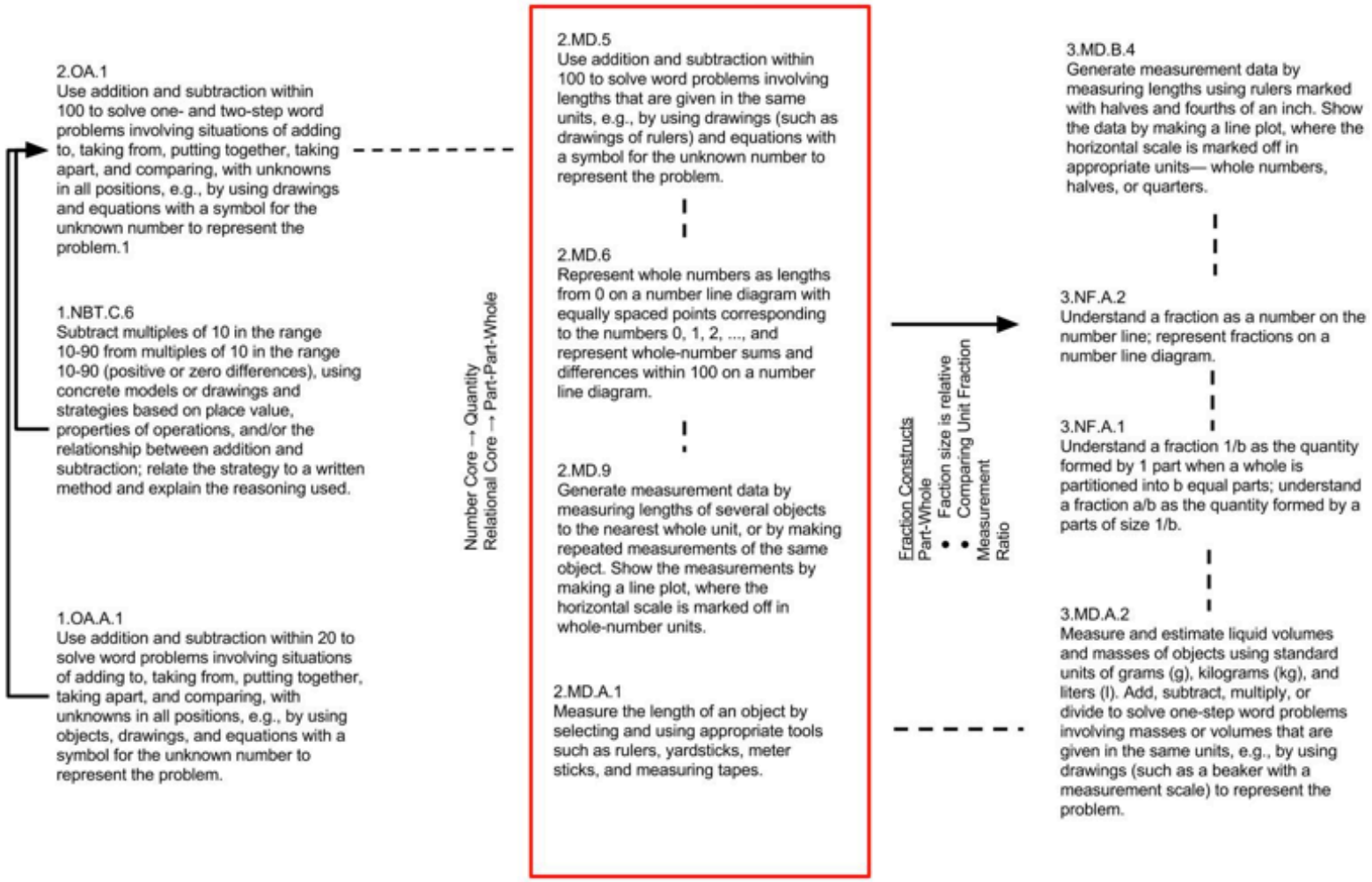
Common Core State Standards - Number Line

2.MD.B.6 Represent whole numbers as ***lengths from 0*** on a ***number line diagram*** with ***equally spaced points*** corresponding to the number 0,1,2,..., and represent whole-number sums and differences within 100 on a number line diagram.

3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.A.2.A Represent a fraction $\frac{1}{b}$ on a number line diagram by ***defining the interval from 0 to 1*** as the whole and partitioning it into *b* equal parts. Recognize that ***each part has size*** $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

3.NF.A.2.B Represent a fraction $\frac{a}{b}$ on a number line diagram by ***marking off a lengths*** $\frac{1}{b}$ from 0. Recognize that the resulting **interval** has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.


2.OA.1
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1
1.NBT.C.6
Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
1.OA.A.1
Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
Number Core → Quantity
Relational Core → Part-Part-Whole
2.MD.5
Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
2.MD.6
Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
2.MD.9
Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
2.MD.A.1
Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
Fraction Constructs
Part-Whole
Faction size is relative
Comparing Unit Fraction
Measurement
Ratio
3.MD.B.4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
3.NF.A.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.
3.NF.A.1
Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
3.MD.A.2
Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Mahalo!

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