

It's More than Just a Random Thing!

Analyzing Variability Can Help Us to Make Estimates and Decisions

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The Central Role of Variability

- Statistical reasoning is inherently different from mathematical reasoning, and effective development of it requires distinct exercises and experiences.
- In particular, statistical reasoning involves *focusing on making sense of and reasoning about variation in data in contextual situations.*

Become a Data Detective!—First steps to Analyzing and Quantifying Variability

- Data, especially visual summaries of data, often contain **interesting stories**, sometimes behind the scenes. **Looking ‘behind’ the data** can be an important consideration when investigating variability (What is the context, how might that affect the distribution of data, etc).
- First steps when we analyze a data set can be just *noticing* that there is variability, *describing* it, and *wondering* why its there at all.
- Other first steps can include *anticipating* and *predicting* variability when we gather samples.
- Later steps with students can involve **measuring & quantifying** variability.

Today's goals:

- 1) To generate several sampling distributions, and to *Anticipate, Predict* and *Describe* the variability that will occur in the distributions
- 2) To *Notice, Wonder about, and Describe* the variability in a distribution of data that was gathered from a known context

Introduce yourselves to one another at your table

- Start with the person sitting closest to me at your table
- Each person introduce yourself and what you do, and mention one hobby or activity that you enjoy.
- Go around clockwise.

Two Hospitals

- Two hospitals keep track of the gender of the babies born each day. City Hospital is a large urban medical center. County Hospital is a small regional facility. Many more babies are born each day in City Hospital than in County Hospital.
- Assume that for each birth (in either hospital) the probability that the baby is male is 0.5 and the probability that the baby is female is 0.5.

Two Hospitals—an Open Question

- Would there be more days when at least 80% of the babies born were girls in:
 - The large hospital
 - The small hospital
 - Makes no difference (Kahneman & Tversky, the 1970's)
- Private think time, then at the signal share with an elbow partner

Two Hospitals—a bounded Question

- Which of the following do you think will happen more often:
 - At least 8 of the 10 babies born in a day at City Hospital are female.
 - At least 4 of the 5 babies born in a day at County Hospital are female.
 - Or are these events equally likely to occur?
- Private think time, then at the signal share with an Elbow partner

Generating sample data for the two hospitals

How could we design an experiment to simulate the process of babies being born in both hospitals, and keep track of the number of girls born each day?

Discuss at tables –(2 minutes)

Anticipating and predicting variability

- Make a **predicted dot plot** for 30 trials of the number of girls in 5 births at a time for the small hospital.
- Make a **predicted dot plot** for 30 trials of the number of girls in 10 births at a time for the large hospital.
- Do these individually, then share and compare your predictions with a partner.

Simulating births for the small hospital

- Toss 5 pennies at a time to simulate the birth of 5 babies.
- Let heads represent boys and tails girls.
- Toss the coins and record the number of girls born.
- Repeat the experiment 30 times.
- Create a dotplot of the number of girls born in each of the 30 repetitions of the experiment.
- This is your **actual distribution for the small hospital**.

Simulating births for the large hospital

- Toss 10 pennies at a time to simulate the birth of 10 babies.
 - Let heads represent boys and tails girls.
 - Toss the coins and record the number of girls born.
 - Repeat the experiment 30 times.
 - Create a dotplot of the number of girls born in each of the 30 repetitions of the experiment.
 - This is your **actual distribution for the large hospital.**
-
- Gather the data with elbow partners—each person gather part of the data for each hospital. One can toss, the other record on the dotplot.

Compare your Predicted and Actual distributions of births

- How do your predicted distributions for each of the hospitals compare to your actual distributions from the data gathered?
- Discuss....(1 minute)

Describing and Analyzing Variability

- Analyze the variability in the data in each of your actual distributions.
- Write one sentence that describes the variability in the distribution of data for the small hospital. Same for the large hospital.

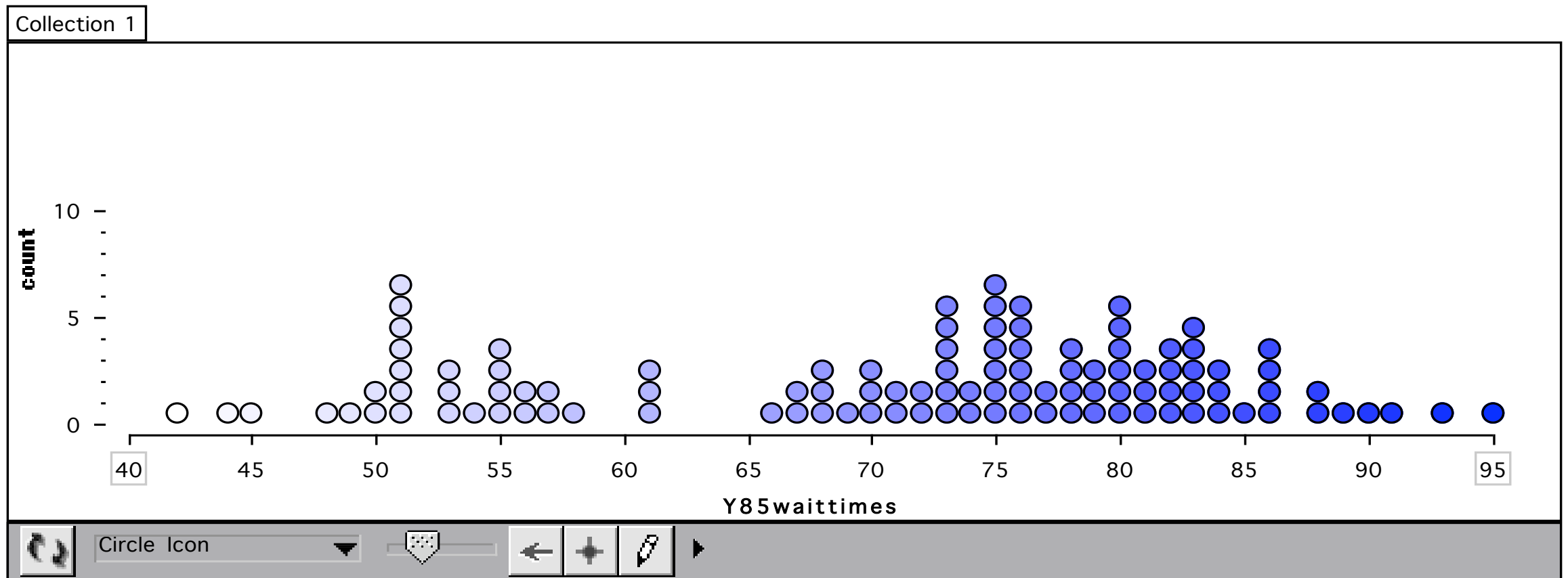
OH Yeah—what about our original question...

- Based on the data for the the two hospitals, what is your answer to the Hospital Question:
- Which of the following do you think will happen more often:
 - At least 8 of the 10 babies born in a day at City Hospital are female.
 - At least 4 of the 5 babies born in a day at County Hospital are female.
 - Or are these events equally likely to occur?

How Faithful is Old Faithful?

- In 1985 the U.S. Geological Survey gathered data on wait times between blasts of the Old Faithful geyser in Yellowstone National Park. They wanted to investigate potential variability in the wait times.
- Here is the distribution of the wait times that they found in their research:

Dot plot of two weeks Old F. data from 1985



Noticing and Wondering about variability

- What do you **notice** in the data for Old Faithful wait times? How would you describe the variability in the data?
- What do you **wonder** about in the data, what questions arise as you analyze the Old Faithful distribution?
- Private think time (2 minutes)—

- About **how long do you think you'd have to wait** for Old Faithful if you went to Yellowstone and had just missed it?
- Private think time—then at the signal share with an elbow partner

From the distribution back to the original data

- What do you think the original data looked like for Old Faithful looked like?
- Share any thoughts at your table.

The Old Faithful --Minutes Between Blasts

- ***–each row represents about 1 days' data***

- 1) 86 71 57 80 75 77 60 86 77 56 81 50 89 54 90 73 60 83
- 2) 65 82 84 54 85 58 79 57 88 68 76 78 74 85 75 65 76 58
- 3) 91 50 87 48 93 54 86 53 78 52 83 60 87 49 80 60 92 43
- 4) 89 60 84 69 74 71 108 50 77 57 80 61 82 48 81 73 62 79
- 5) 54 80 73 81 62 81 71 79 81 74 59 81 66 87 53 80 50 87
- 6) 51 82 58 81 49 92 50 88 62 93 56 89 51 79 58 82 52 88
- 7) 52 78 69 75 77 53 80 55 87 53 85 61 93 54 76 80 81 59
- 8) 86 78 71 77 76 94 75 50 83 82 72 77 75 65 79 72 78 77
- 9) 79 75 78 64 80 49 88 54 85 51 96 50 80 78 81 72 75 78
- 10) 87 69 55 83 49 82 57 84 57 84 73 78 57 79 57 90 62 87
- 11) 78 52 98 48 78 79 65 84 50 83 60 80 50 88 50 84 74 76
- 12) 65 89 49 88 51 78 85 65 75 77 69 92 68 87 61 81 55 93
- 13) 53 84 70 73 93 50 87 77 74 72 82 74 80 49 91 53 86 49
- 14) 79 89 87 76 59 80 89 45 93 72 71 54 79 74 65 78 57 87

Starting from raw data to the O.F. distribution

- Today we started with the distribution of Old Faithful data, considered the variability, and tried to imagine patterns in the original data.
- We could have started with the raw data, and ask students to construct a plot of several days worth of data, and to estimate how long they might expect to have to wait for a blast.
- Here are some responses I've obtained from students....

Group B—Reasoning from ‘middles’ (on stem & leaf plot)

- “We noticed there was a lot of variation in our data—a very wide spread—so we used the **average as a middle point**.

We calculated the average wait time to be about 68 minutes, so we would predict we’d wait about that long—about an hour. “



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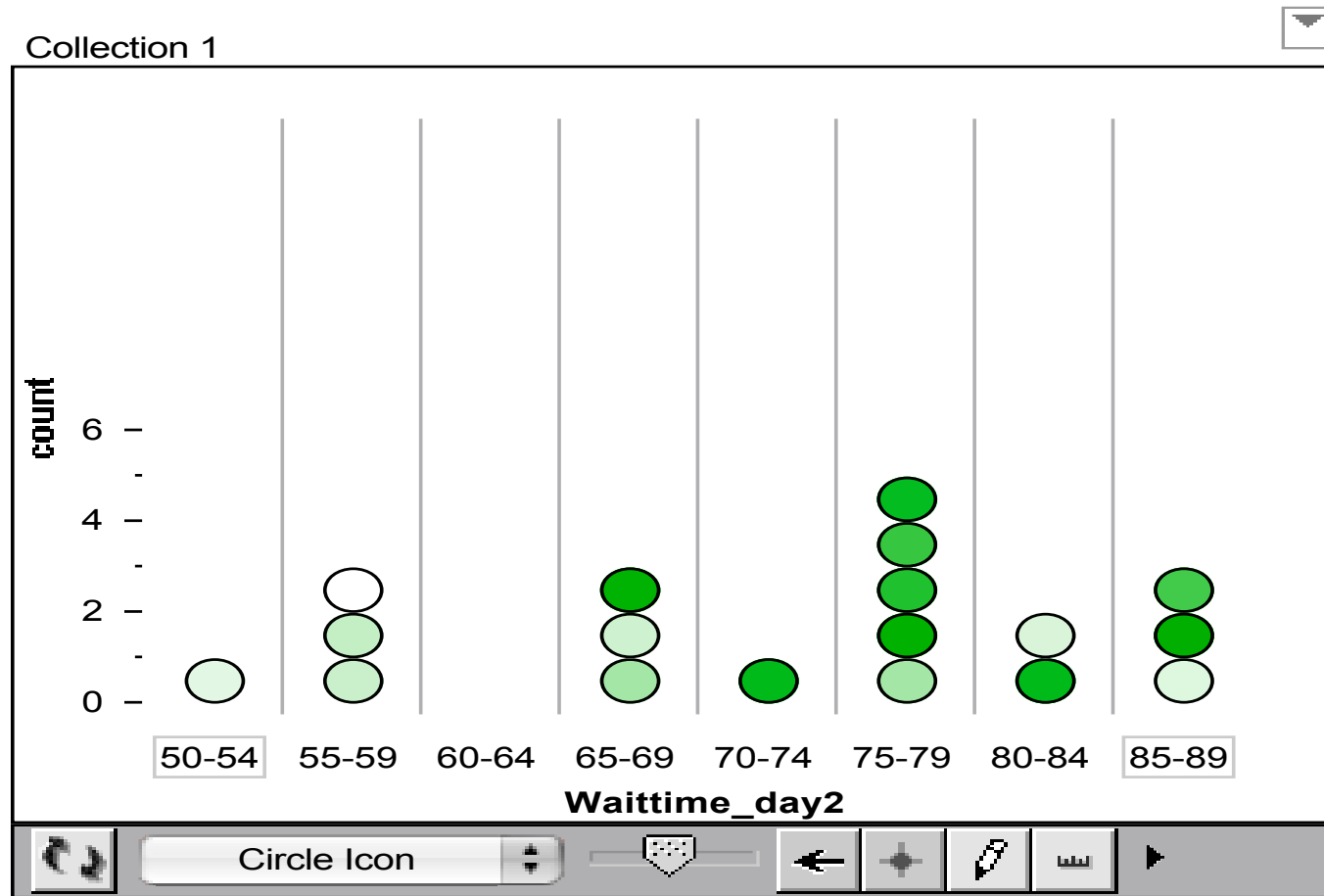
Group C—Reasoning from ‘mosts’

- “On the basis of our first frequency graph we’d expect to wait about 75 minutes, because it shows **most** wait times for the eruption in the 75 to 79 minute range.



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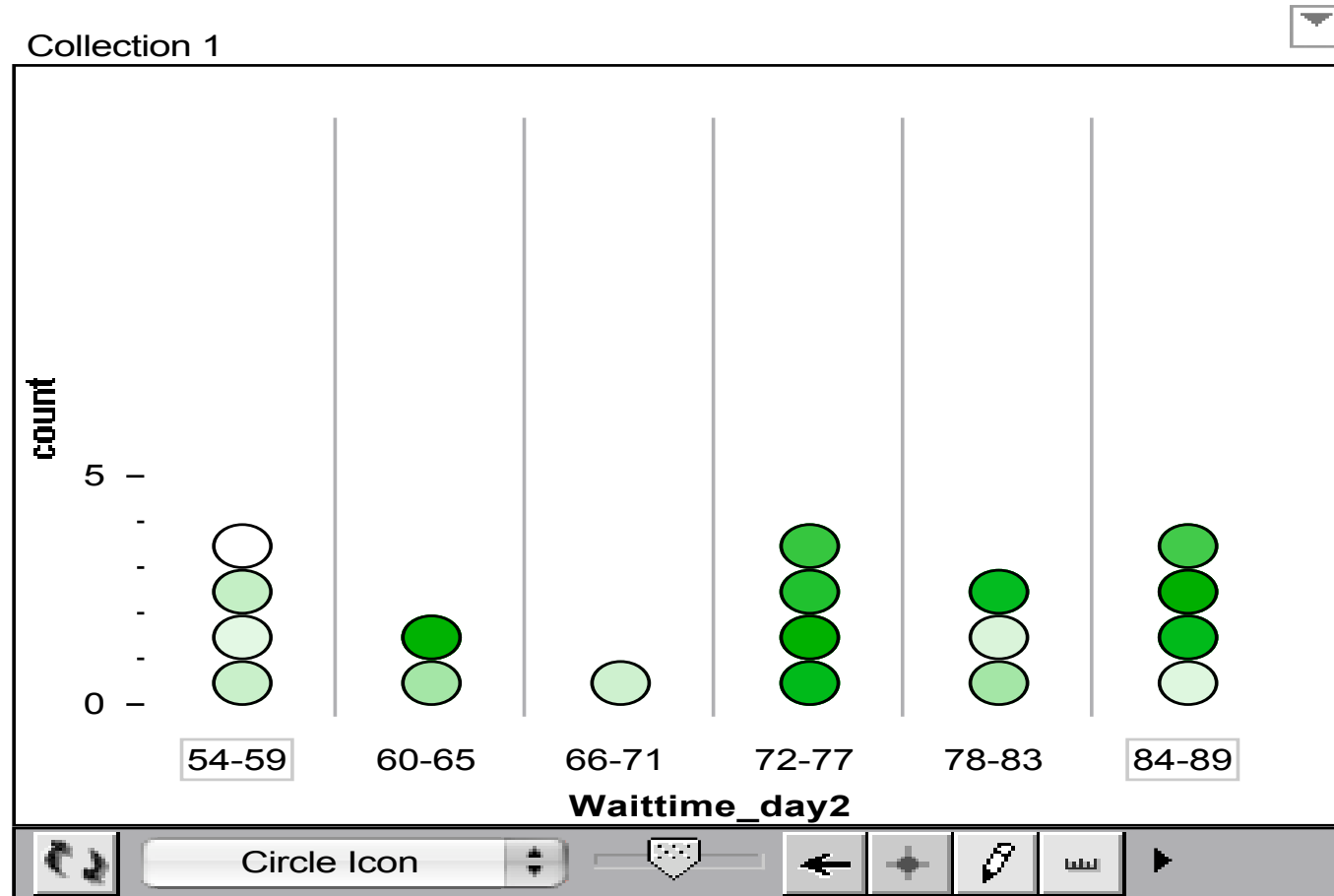
Group C—Graph 1



Group C—more Reasoning from ‘mosts’

- But then we saw that if we chose our intervals in another way we obtained something different. There is no obvious pattern here, and we thought that a person could just as easily wait about 55, or 75, or 85 minutes, because all three of those times were equally frequent in this (second) graph, each occurring 4 times.”

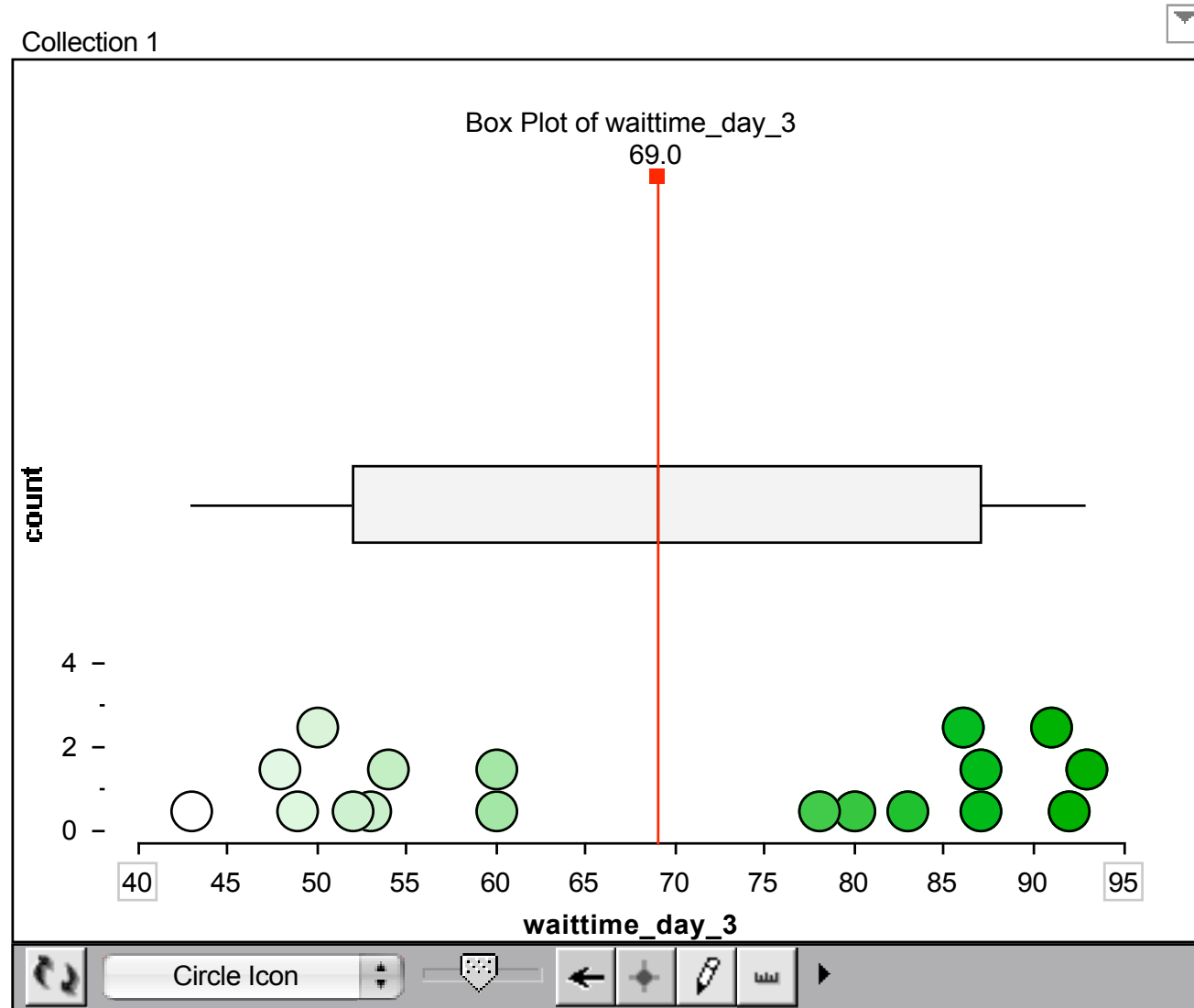
Group C—Graph 2



Group D—Reasoning from ‘Spread’

- “The middle 50 percent goes from 65 minutes to 82 minutes for day 2, and from about 53 minutes to 87 minutes for day 3.
- So, overall from the two days combined we concluded that 50 percent of the time you’d probably have to wait at least an hour, and perhaps as much as an hour and 20 minutes.”

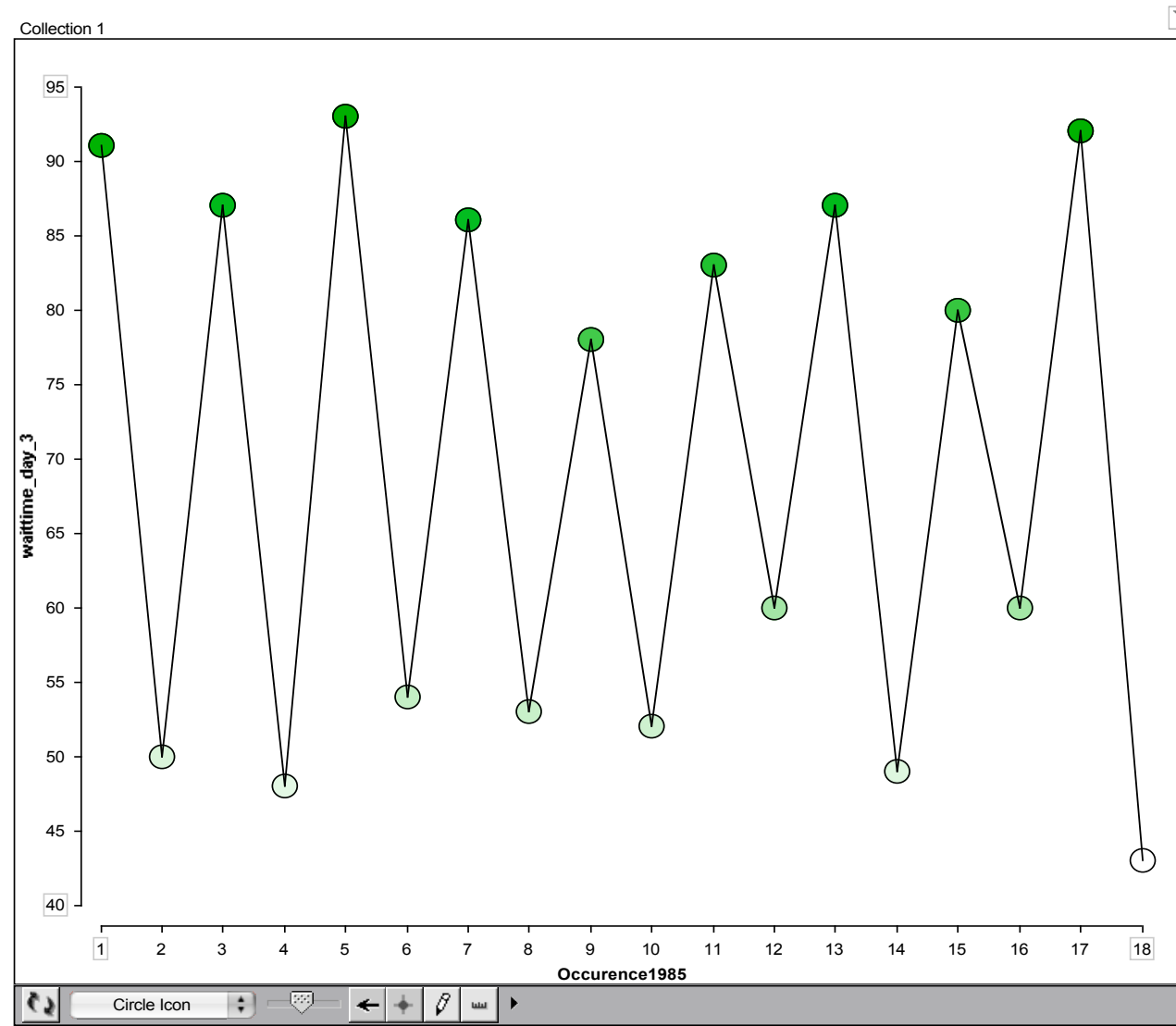
Group D—Graph



Group E—Reasoning from ‘patterns in variability’

- “We think we see a pattern in the data. There seems to be an up-down pattern in the wait times in day 3. It was easier to see when we connected the dots in our plot.
- Then we did the same thing for day 2, and the up-down pattern in wait times appears there, too. It’s not always perfect, but a long wait time is usually followed by a short time, and a short one by a long one.”

Group E—Day 3 graph

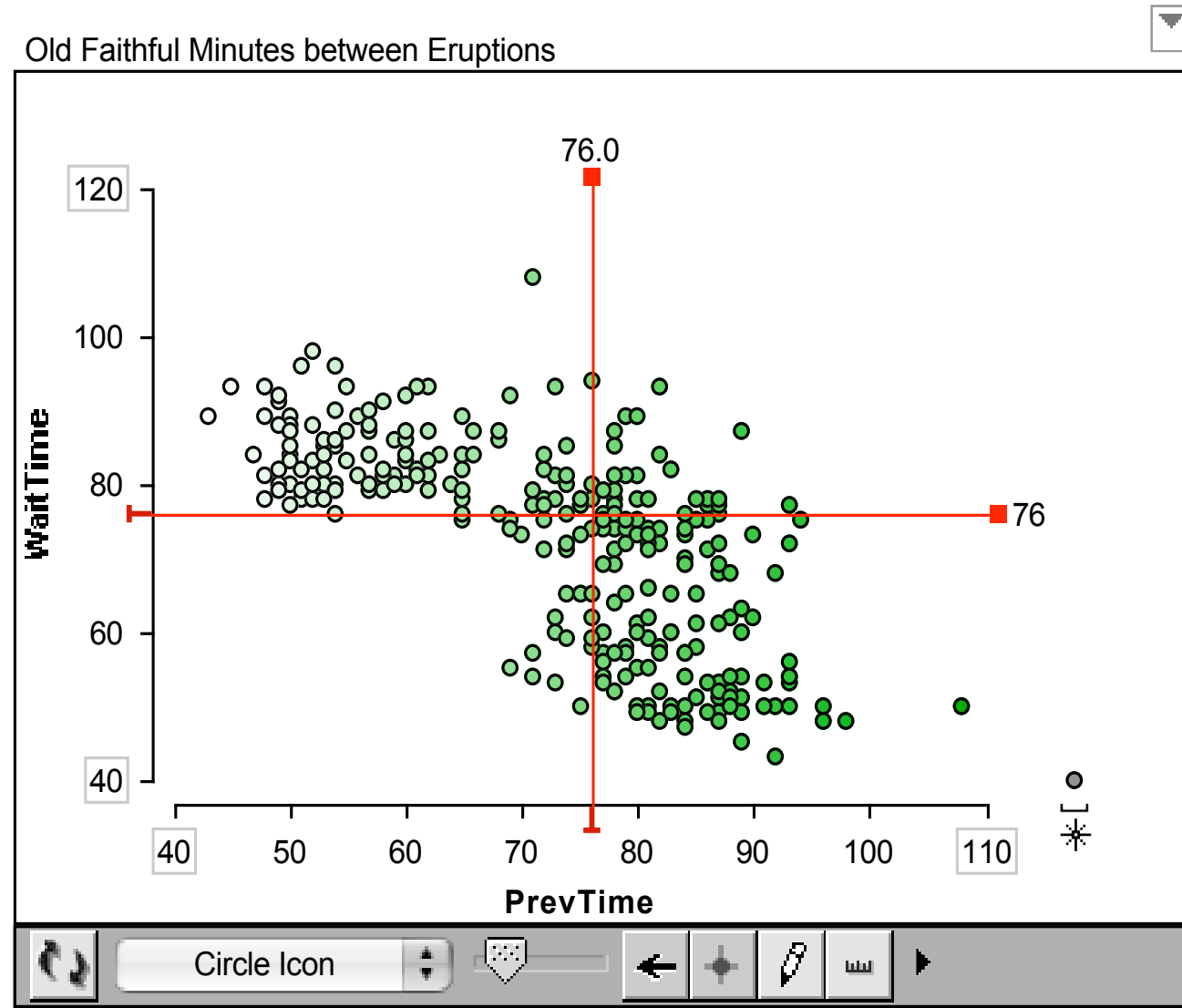


Analyzing the Progression of Student Reasoning about variability in their graphs

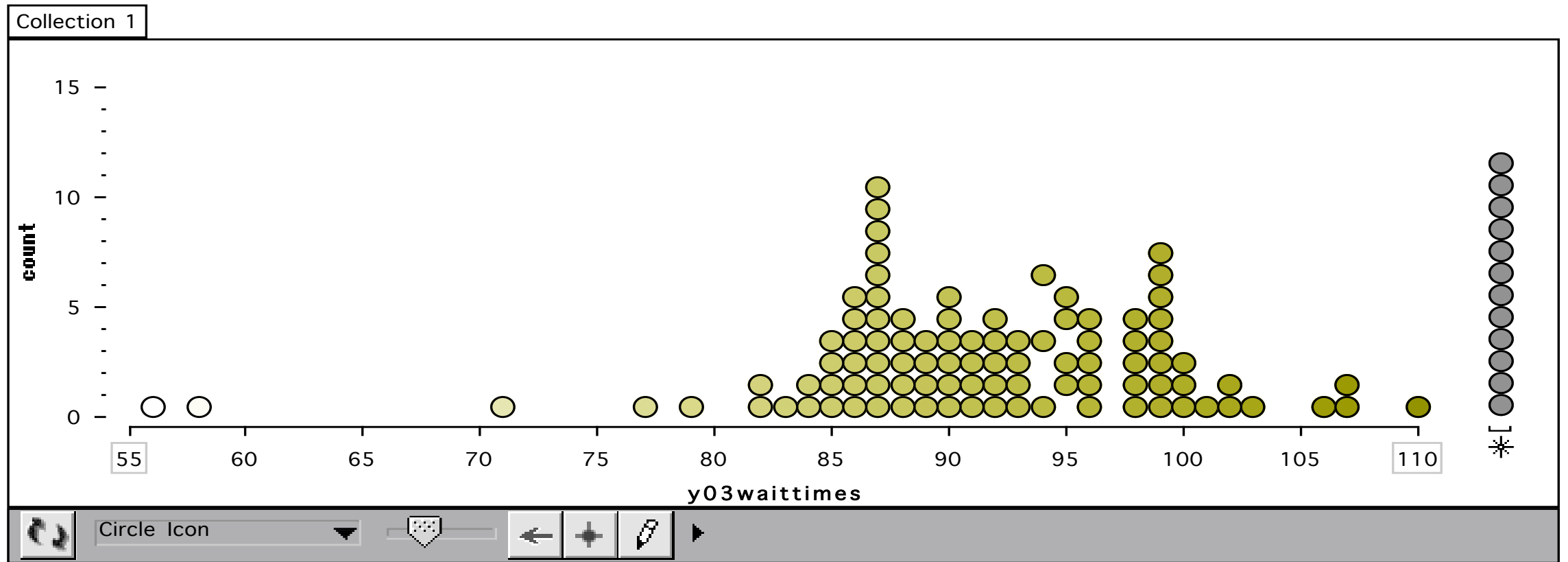
- Case value representations—and clustering (Group C)
- Measures of center—mode, median, mean (Groups B & C)
- Looking at ‘likely range’—a sign of accounting for variation (Group D)
- Closer look at variation over time—it’s not totally ‘random’ in this case—special cause variation vs. common cause variation (Group E)



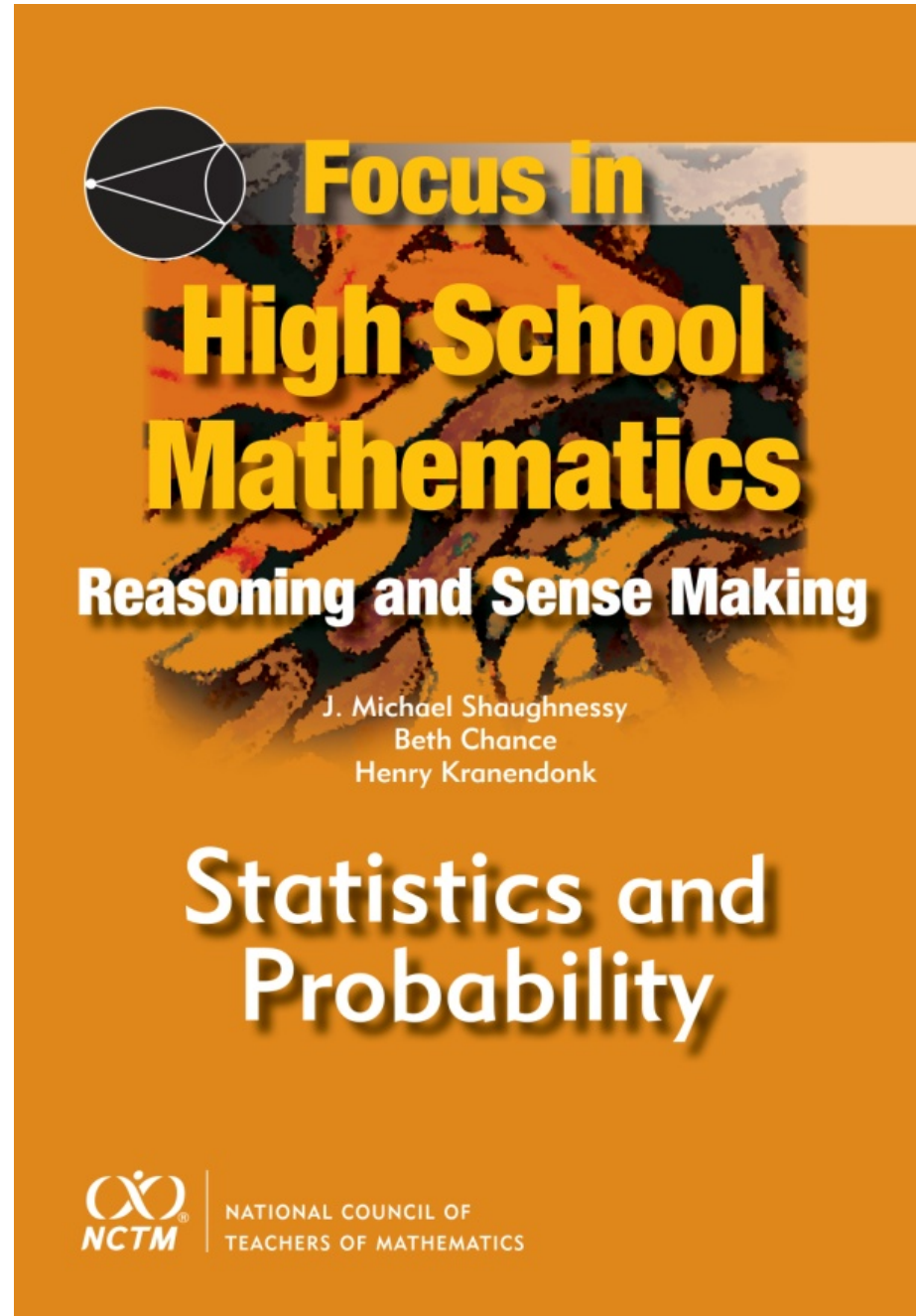
Wait time vs Previous Wait time



Data for same two week period in 2003



Focus on Statistical Reasoning



Statistical Reasoning is Primary

- It may be that the most important quantitative reasoning ability of all is the facility to read and to interpret statistical information, and to make informed inferences based on statistical and probabilistic information.

Variation is the spice of life!!

Without it

- all musical notes would be the same,
- all flavors would taste exactly the same,
- all flowers would be the same color and smell the same
- all people would look exactly the same, think exactly the same, and do exactly the same kind of work

Worst of all

- THERE WOULD BE NO NEED FOR STATISTICS!!

THANK YOU FOR PARTICIPATING!!

Keep an eye out for variability, and the opportunity to investigate it with your students!!

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