NCTM
April 2017

Early Elementary Algebraic Reasoning Development for Students Receiving Intervention Support

Beth L. MacDonald, Jill Ashby, and Kristy Litster

Utah State University

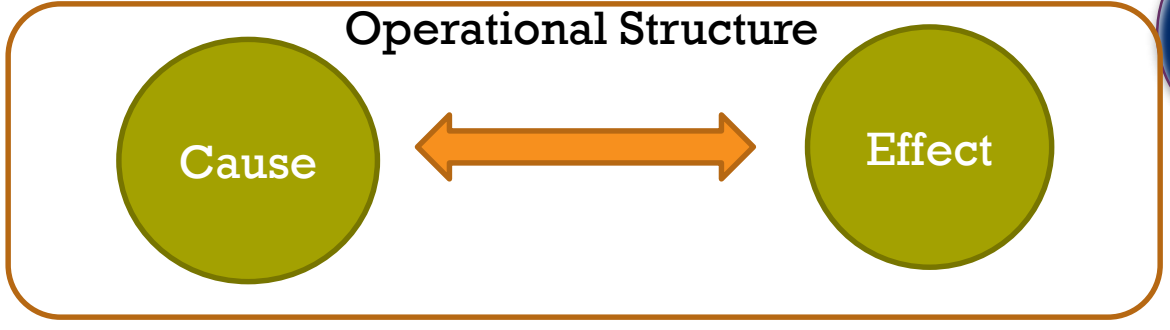
Parking Lot Feedback

There are Post-its in the center of your table. Please feel free to give us honest feedback to place it in the "Parking Lot."

Defining Terms … and Connecting to Early Algebra

What does this have to do with Early Algebraic Reasoning Development?

Mental reversibility:



- **Inverse relationships:** one operation undoes the effect of another

  $4 + x = 7$ and $7 - 4 = x$

- **Compensation relationships:** an operation returns to an equivalent state

  $x + 4 - 4 = 7 - 4$

Methods

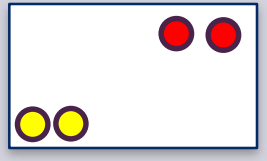
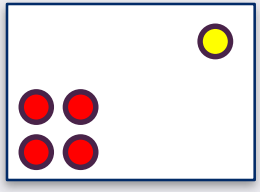
- We conducted in 2016 an **inquiry-based intervention** to understand children's construction of **early mental reversibility development**.
- Participants: **11 students, K-2**, in the West Mountain Region of the U.S.
- We had approximately **28 sessions** with **four kindergarten** students, **four first grade** students, and **three second grade** students.
- Sessions took place **twice a week** after school for first and second grade students and during a middle of the day intervention time in kindergarten.
- From our findings we hope to establish some **Hypothetical Learning Progressions** to justify a longer study.

Utah State University

Pre-Algebraic Tasks

Pre-Algebraic Tasks

- Perceptual Subitizing has **five known variations** and indicates a student's **primary reliance on the arrangement** of the counters when subitizing.
- Conceptual Subitizing has two known variations and indicates a student's **primary reliance on his or her number understanding** when subitizing.
- Addition and Subtraction tasks like "Bears in the Cave" press students **step away from a reliance upon the counters when counting AND grouping**. These tasks also allow **inversion concepts to be explored**.

Type of Task	Visual of Task	Objective of Task and Number Sentence
Perceptual Subitizing		**Obj:** TO Rely on Patterns/Color when **Attending** to Additive Concepts **2 and 2 ≠4**
Conceptual Subitizing		**Obj:** To Rely on Rules of Patterns or Number Concepts when **Attending** to Additive and Subtractive Concepts **4 and 1 = 5**
Additive/ Subtractive Bears in the Cave Task		**Obj:** To **Explore** Inverse Relationships (e.g., addition-subtraction) by Relying on More Abstract Materials (i.e., Fingers, Pattern Rules) **3+2=?**

Inversion Task

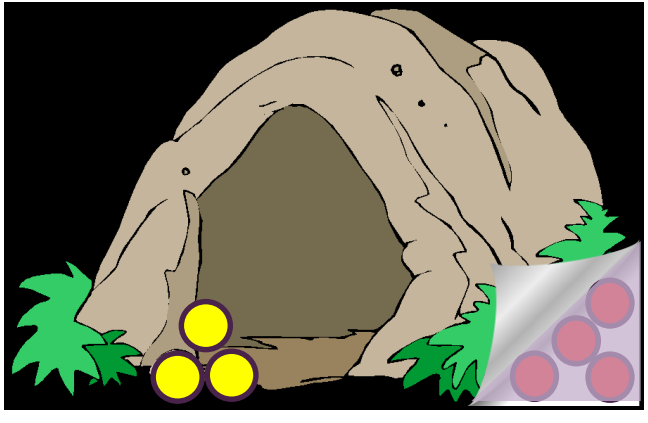
Reversibility Task 1

Inversion Task - Variations

Reversibility Task 1

Bears in the Cave

Some bears are in the cave
3 bears are outside the cave



There are 7 bears altogether. How many bears are inside the cave?

Consider:

How did this task change from the addition task?

Result Unknown:

$4 + 3 = \square$

Start Unknown:

$\square + 3 = 7$

Inversion Task – You Try It!

Reversibility Task 1

Piñata Task:

1. "I'm gonna put some candy in the piñata.
2. Later, I put 3 more pieces of candy in the piñata.
3. Then we hit the piñata. Whack! And 7 pieces of candy fell out.

How many pieces of candy did I put in at the beginning?"

Consider & Discover:

How would children solve these? What strategies would they use?

How are these tasks similar/different?

What effect do these differences have on students and their strategies?

Inversion Task Solutions

Reversibility Task 1

Allen's solution

Francis's Solution

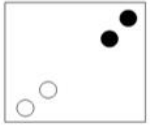
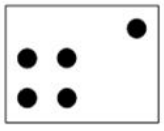
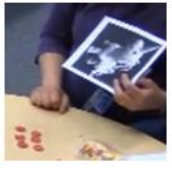
Language and Cultural Changes

BEFORE		AFTER
▪ Vocabulary: turn it around, balanced/equal.	Build Up	▪ Linguistically responsive: rotate, the same number.
▪ Comprehension: abstract language or situations	Adapt	▪ Relevant Situations: use "you language"
▪ Cultural reference: Rocket ship/Bear in Cave.	Replace	▪ Culturally Responsive: piñata, cupcake trays.

Task Progression for Inversion Tasks

As additive and subtractive tasks change over time, students are pressed to **rely on more abstract material** affording them an opportunity to **explore and access inversion tasks**.

As students first engage in inversion tasks, they **revisit concrete manipulatives** in the task **before changing** to a task that presses them again to **rely on more abstract material**.

Type of Task	Visual of Task	Obj. of Task and Number Sentence
Perceptual Subitizing		Obj: To Rely on Patterns/Color when **Attending** to Additive Concepts 2 and 2 ≠ 4
Conceptual Subitizing		Obj: To Rely on Rules of Patterns or Number Concepts when **Attending** to Additive and Subtractive Concepts 4 and 1 = 5
Additive/Subtractive Bears in the Cave Task		Obj: To **Explore** Inverse Relationships (e.g., addition-subtraction) by Relying on More Abstract Material (i.e., Fingers, Pattern Rules) 3 + 2 = ?
Inversion Piñata Task		Obj: To **Solve** for Inverse Relationships by Relying on More Concrete Material (e.g., counters) 6 + ? = 8
Inversion Cookie Jar Task OR Bears in the Cave		Obj: To **Solve** for Inverse Relationships by Relying on More Abstract Material (i.e., Fingers, Pattern Rules) 5 + ? = 7

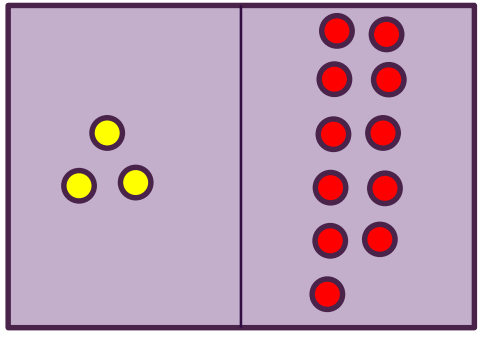
Compensation Task

Reversibility Task 2

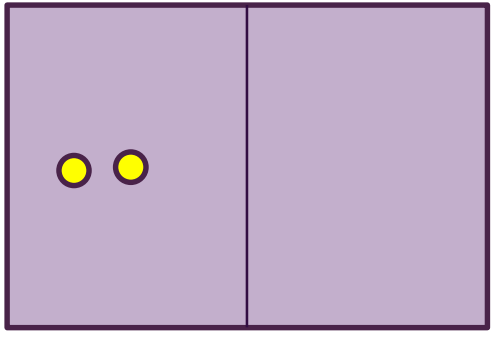
Compensation Task

Reversibility Task 2

Your cupcake tray



My cupcake tray



Problem:

How many cupcakes can we put in my empty slot, to make it so that we have the same number of cupcakes?

Consider:

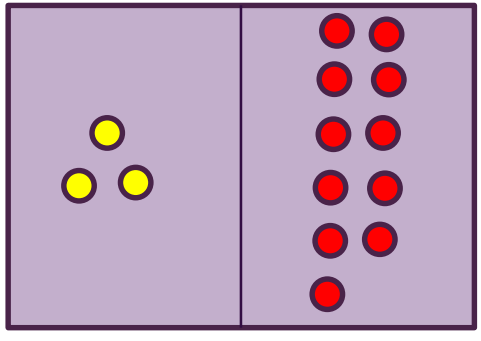
How would students solve this task? Think of at least two strategies.

How does this task help build reversibility?

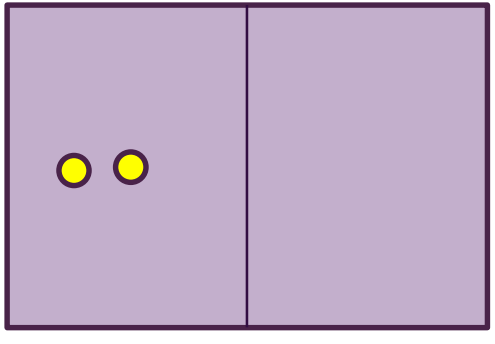
Compensation Task – Strategy 1

Reversibility Task 2

Your cupcake tray



My cupcake tray



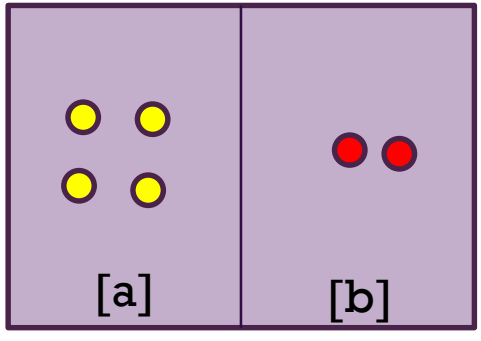
Strategy 1: Counting Totals

"There's 14 cupcakes on my tray, and you have 2. So, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14. You need 12 more cupcakes."

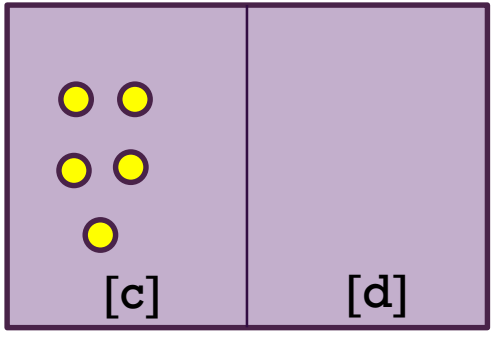
Compensation Task – Strategy 2

Reversibility Task 2

Allen's Tray



Becka's Tray

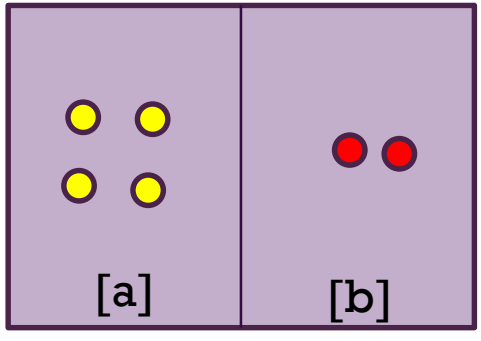


Strategy 2: Match and Move Between Mats

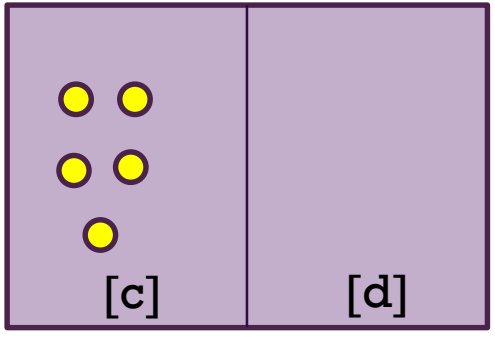
Compensation Task – Strategy 2

Reversibility Task 2

Allen's Tray

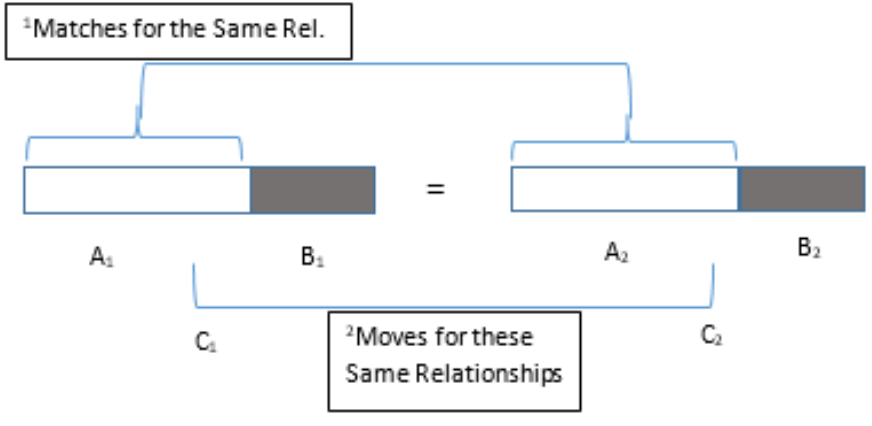


Becka's Tray



Strategy 2: Match and Move Between Mats

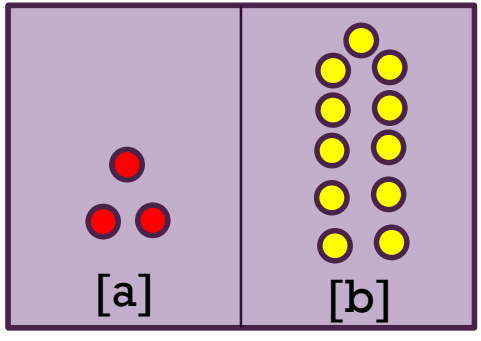
"Two here [d]. . . You take one of those [c]. . . Yeah, take one of those[d]"



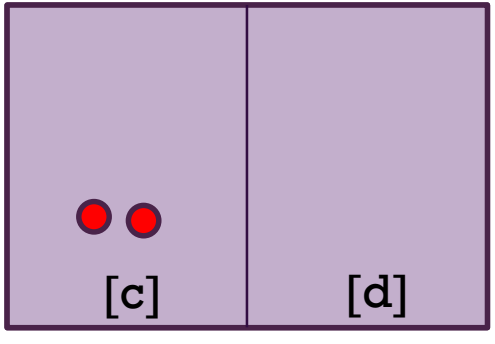
Compensation Task – Strategy 3

Reversibility Task 2

Your cupcake tray



My cupcake tray

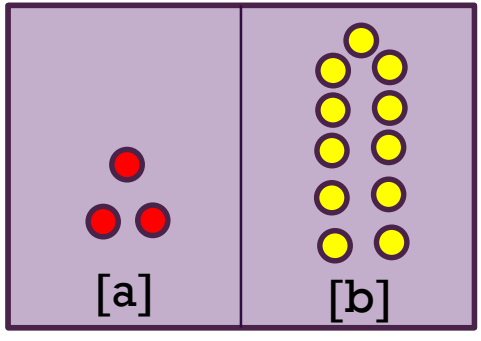


Strategy 3: Match and Move Within Mats

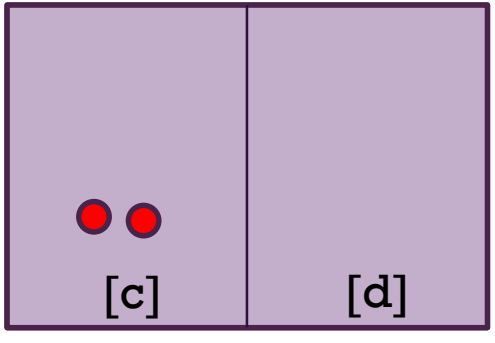
Compensation Task – Strategy 3

Reversibility Task 2

Your cupcake tray

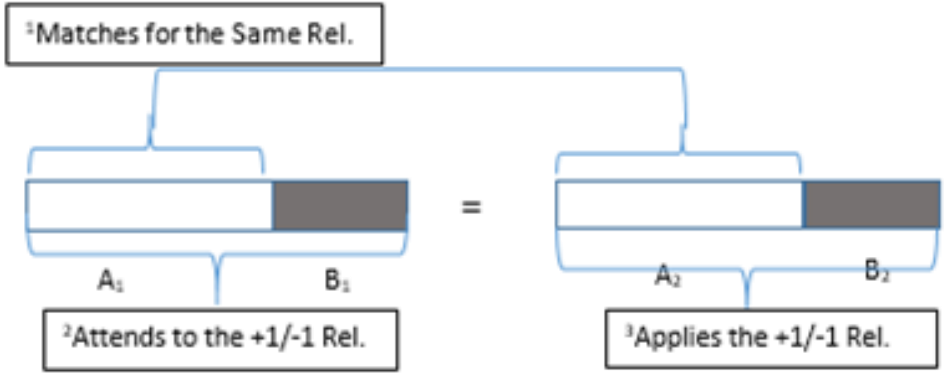


My cupcake tray



Strategy 3: Match and Move Within Mats

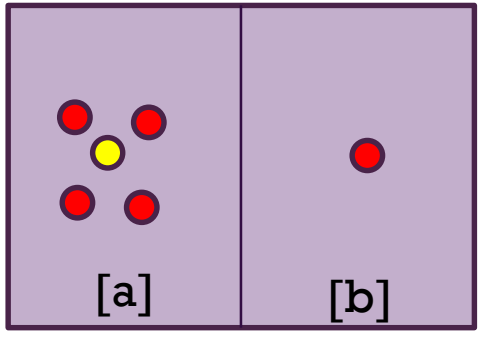
1, 2 [a], 1, 2, 3, . . . 10, 11 [b]
You said 1, 2 [a]and then you stopped.
Why didn't you count this one?
You have 2 [c] and I have 3 [a]. . .
1 [a], 2, 3, 4, 5 . . . 12! [b]



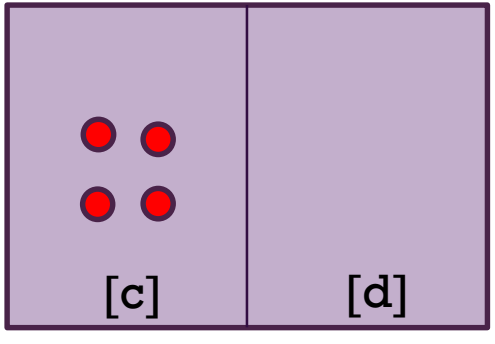
Compensation Task – Strategy 4

Reversibility Task 2

Your cupcake tray



My cupcake tray

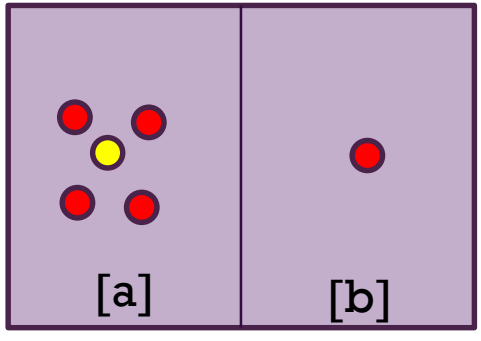


Strategy 4: Imagined Match and Move

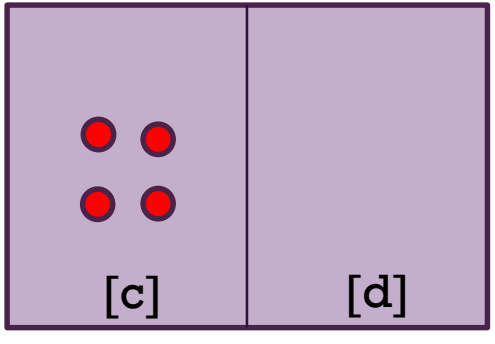
Compensation Task – Strategy 4

Reversibility Task 2

Your cupcake tray

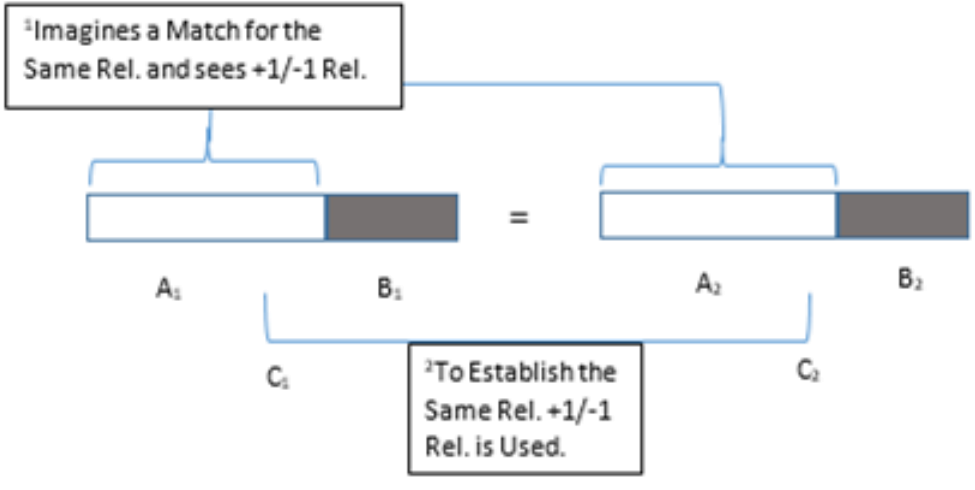


My cupcake tray



Strategy 4: Imagined Match and Move

2! [d]. You would put 1 here [c] and 2 here [d].
You think 1 [c] and 2 [d]?
No, 1[c] and 1[d]. 2[d].



Evaluating Reversibility

Rubric

Now, it is YOUR turn to evaluate Reversibility

Material or Actions Used with Strategy			Strategy to Solve Compensation Tasks
Materials	**Counting**	**Grouping**	
Moving Items	Counting All	Groups Parts Only	Match
Drawing	Counting On	Groups Parts-Compose Decomposes	Counting Totals
Fingers	Counting To		Match and Move
Finger Patterns	Counting +1/-1	Doubling	Counting and Match and Move
Imagined Pattern or Item	Counting Down	Near Doubling	Imagined Match and Move
Represented with other Material	Counting Down To	Parts-Whole-Units	

We are going to show you two videos of students solving Compensation Tasks.

Circle the Materials or Actions the students are using.

Circle the strategy the students are using.

Finally, turn to a neighbor and discuss **what you might do** to promote this student's development.

Evaluating Reversibility

You Try It!

-Use Rubric

-Circle Any Strategies you Notice

Logan

Discuss and Share Out

- What Materials and/or Actions did Logan use?
- What Strategy did the students use?
- What would you do to help these students progress in their Early Reversibility development?

Evaluating Reversibility

You Try It!

-Use Rubric

-Circle Any Strategies you Notice

Henry & Georgia

Discuss and Share Out

- What Materials and/or Actions did Henry and Georgia use?
- What Strategy did the students use?
- What would you do to help these students progress in their Early Reversibility development?

Two First Graders' Use of Strategy

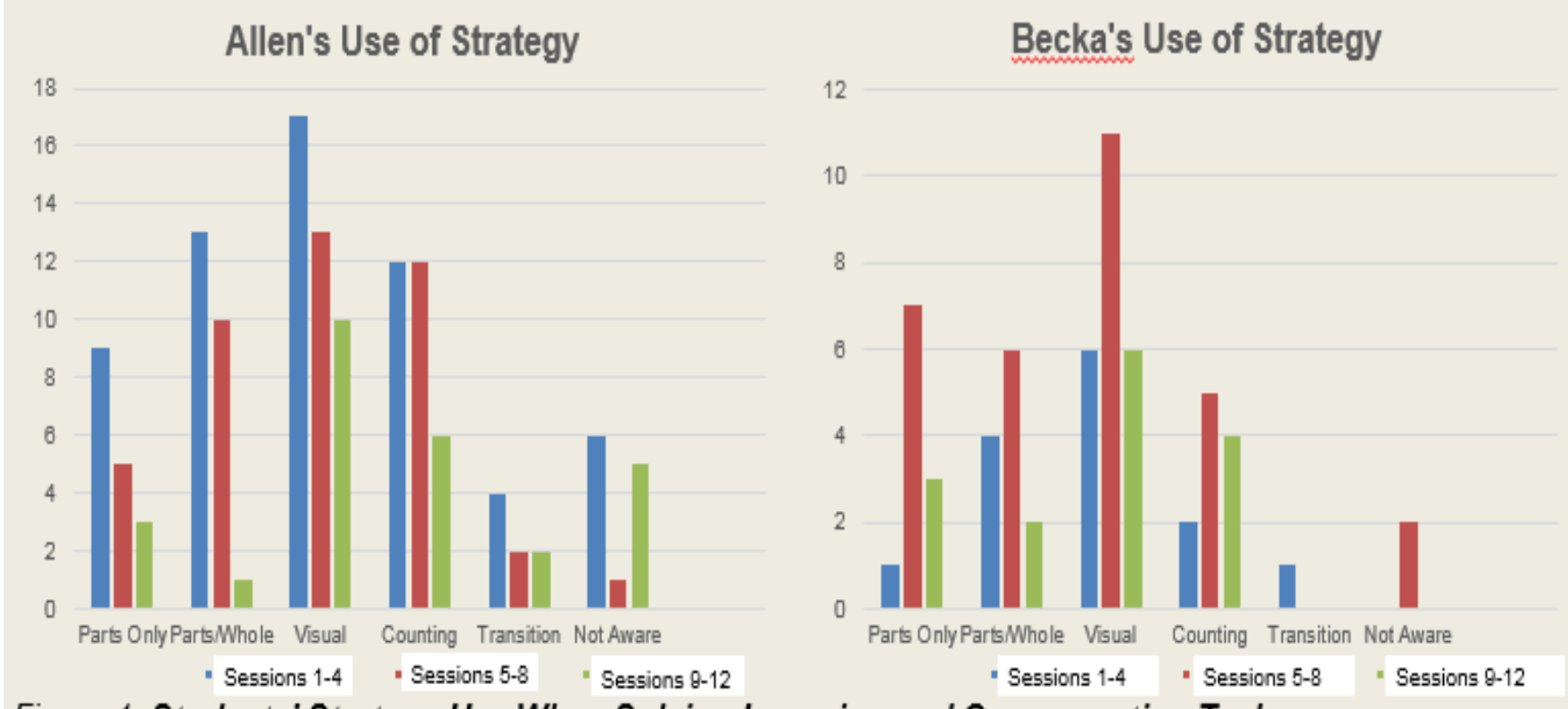


Figure 1. ***Students' Strategy Use When Solving Inversion and Compensation Tasks***

Next Steps …

- Turn to a neighbor – what will you use from this workshop?
- What could still be helpful?
- Record your feedback on the post-its and place it in the "Parking Lot" before leaving.

Thank You!

If we teach today's students as we taught yesterday's, we rob them of tomorrow.

-John Dewey

Beth L. MacDonald
beth.macdonald@usu.edu
Kristy Litster
chemile@hotmail.com
M. Jill Ashby
jill.ashby@aggiemail.usu.edu

UtahState University