



Division: Let's Help
Students Make
Sense of It



Hello!

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What Makes Division So Difficult
for Students?

Turn and Talk with a neighbor



Children as young as kindergarten age can understand and solve division word problems.

Baroody, 1998

Do you agree?

Why??

- Context → it supports all learners
- Informal Strategies → use what they already know
- Routine → they **do** division already



BIG IDEA

The properties of multiplication and addition provide the mathematical foundation for understanding computational procedures for multiplication and division, including mental computation and estimation strategies, invented algorithms, and standard algorithms.



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SMPS??

NCTM, 2011

1 of the 8 Effective Mathematics Teaching Practices



Implement tasks
that promote
reasoning and
problem-solving

So, Consider the Following

Money

Intro: Context? Better than blocks

Follow-up: Why This Way? Did
Anyone Have any Leftovers?
Really?

Extension: Incorporate coins.
Which ones?

Partitive or Quotative? What?

Let's Be Smarties!

Intro: How many people could you give 3 "Smarties"??

What did you do with your leftovers?

How Were The Actions in
These Two Tasks Different?
Talk w/ a Neighbor?

Partitive & Quotitive

Fair Sharing or Divvy Up

The number of groups or shares to be made is known, but the number of objects in (or size of) each group or share is unknown.

Example: There are 12 apples on the counter. If you are sharing the apples equally among 3 bags, how many apples will go in each bag?

Measurement

The number of objects in (or size of) each group or share is known, but the number of groups or shares is unknown.

Example: There are 12 apples on the counter. If you put 3 apples in each bag, how many bags will you fill?

Division Quilts

TCM, Sept. 2015

You Need:

- Grid Paper
- Number Card (or Die)
- Crayons, Markers, or ???



Your Task: Partition Your Quilt

The Grid Paper is your
blank quilt

The Number Card is your
divisor

Identify as many of those
sets (divisors) you can by
using different colors or
designs to visualize each
unique set



Partitive or Quotitive??

Another Effective Math Teaching Practice



Build
Procedural
Fluency from
Conceptual
Understanding

$$3 \overline{) 39}$$

$$5 \overline{) 92}$$

$$2 \overline{) 87}$$

$$3 \overline{) 68}$$

My Students Can't Do Division!!!



Ryan, Tysen and Leo found a dog. After returning her safely, they got a \$39 reward to share fairly. How much money will each person get?

Jennika had \$87 when she went to the fair. If all the rides were \$2 each, how many rides could Jennika go on?

My Students **CAN** Do Division!!!



Your class has raised \$92 so far for your field trip. If the trip costs \$5 per students, how many kids would be able to go on the trip if it were today? How much more money is needed so another student could go on the trip?

$$3 \overline{) 39}$$

$$5 \overline{) 92}$$

$$2 \overline{) 87}$$

$$3 \overline{) 68}$$



Link the work of
high cognitive
demand to the
procedural practice

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$$3 \overline{) 39}$$

$$5 \overline{) 92}$$

$2 \overline{) 87}$	
$\begin{array}{r} 6 \\ 81 \\ \hline 10 \\ 71 \end{array}$	3 rides
	5 rides

$$3 \overline{) 68}$$

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Partitive or Quotitive?

Your mother promised you a birthday party for you and some of your friends at the ***Hard Rock Cafe*** in San Antonio. She said she will spend up to \$792. If each person costs \$33, how many friends could you invite to your party?

What are your first thoughts?

Which of These is More Familiar?

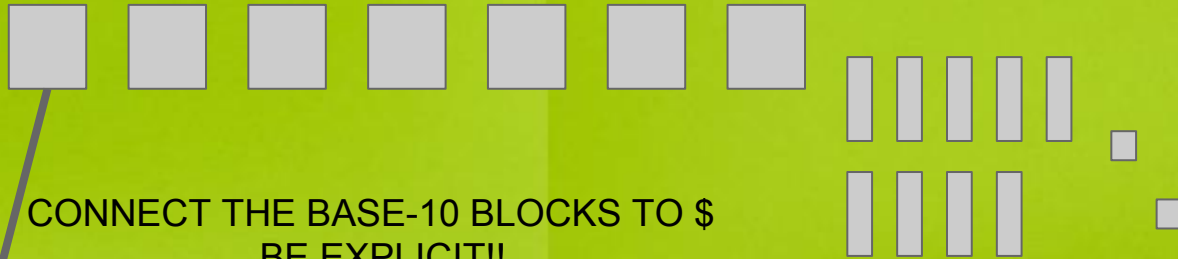
$$33 \overline{) 792}$$

\$33 per person $\overline{) \$700 + \$90 + \$2 \text{ total}}$



Take Nothing For Granted

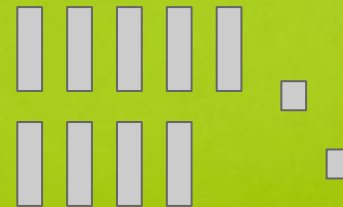
33



3 → 33's

Which of These is More Familiar?

33



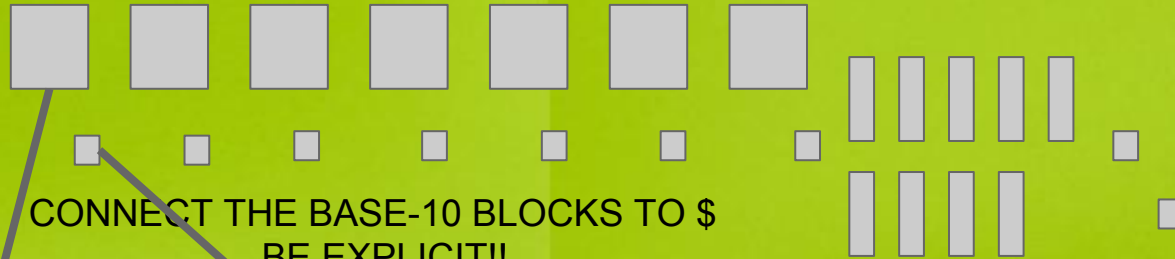
CONNECT THE BASE-10 BLOCKS TO \$
BE EXPLICIT!!



3 → 33's → 7 times

Which of These is More Familiar?

33



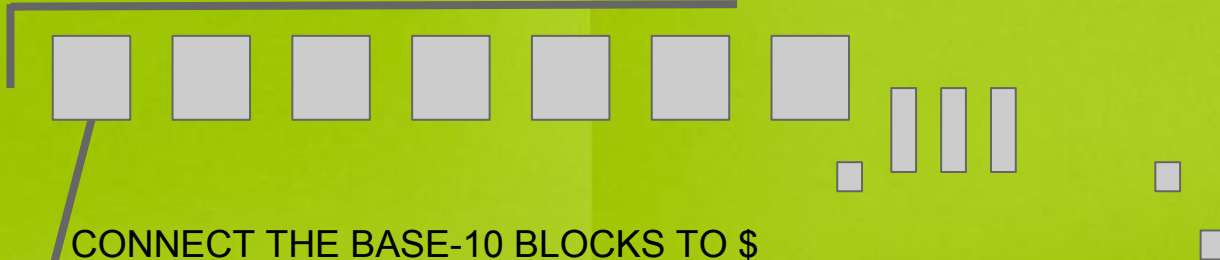
3 → 33's → 7 times



left → 7 times

With Some Final Re-Arranging

33



3 → 33's → 7 times



left → 7 times

Another Effective Math Teaching Practice



Use and
connect
mathematical
representations

BUILD TOWARDS ALGORITHMIC UNDERSTANDING

Basic Division Models

Partial Quotients

Distributive Property

Area Model

$$\begin{array}{r} 52 \text{ R } 10 \\ 45 \overline{) 2350} \\ \underline{- 900} 20 \times 45 \quad (20) \\ 1450 20 \times 45 \quad (20) \\ \underline{- 900} 10 \times 45 \quad (10) \\ 550 2 \times 45 \quad (2) \\ \underline{- 450} \\ 100 \\ \underline{- 90} \\ 10 \end{array}$$

$$\begin{array}{c} \swarrow 30 + 60 \searrow \\ 90 \div 6 \\ (30 \div 6) + (60 \div 6) \\ 5 + 10 = 15 \end{array}$$

$$656 \div 2 = 328$$

300	20	8
656	56	16
$\underline{- 600}$	$\underline{- 40}$	$\underline{- 16}$
56	16	0

$300 + 20 + 8 = 328$

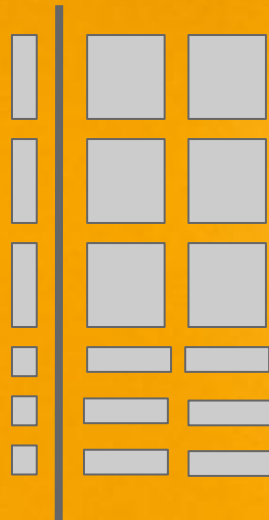
Where do the base-10 blocks fit??

Area Model??

33 guzinta 792 or ...

\$33 dollars/ person with the number of people unknown

\$33 or



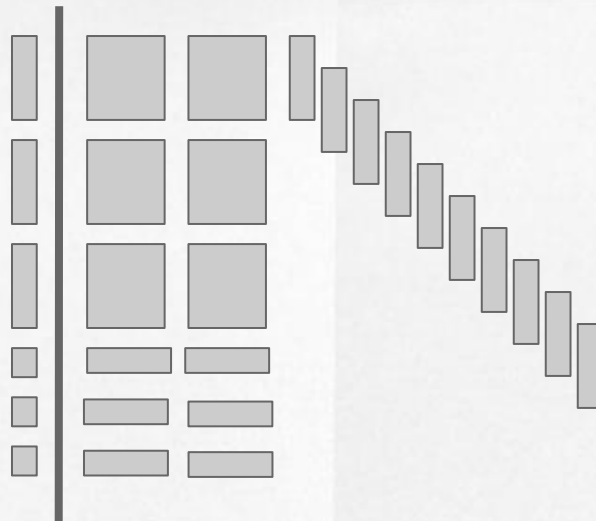
132 left . . . decompose the remaining 100

Area Model??

33 guzinta 792 or ...

\$33 dollars/ person with the number of people unknown

\$33 or

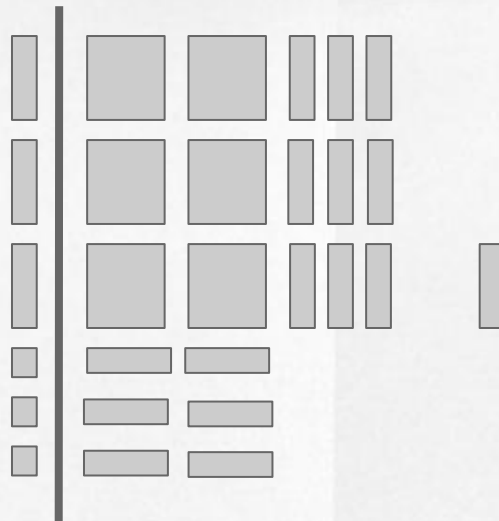


Area Model??

33 guzinta 792 or ...

33 groups of unknown size ...

33 or



Rearrange ... and
the other 32

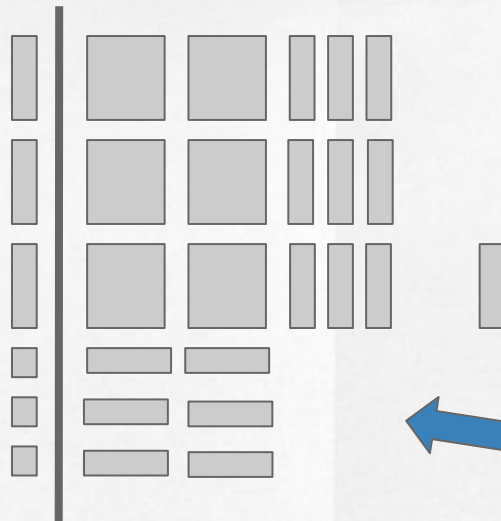


Area Model??

33 guzinta 792 or ...

33 groups of unknown size ...

33 or



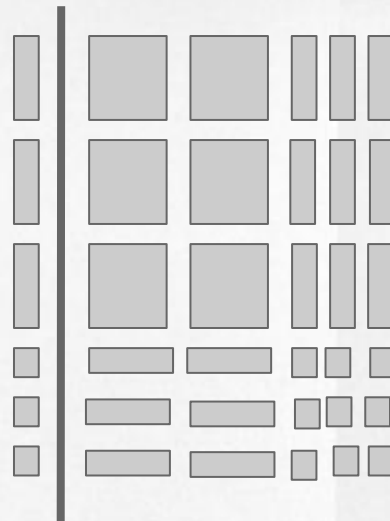
Need 9, decompose the
10

Area Model??

33 guzinta 792 or ...

33 groups of unknown size ...

33 or



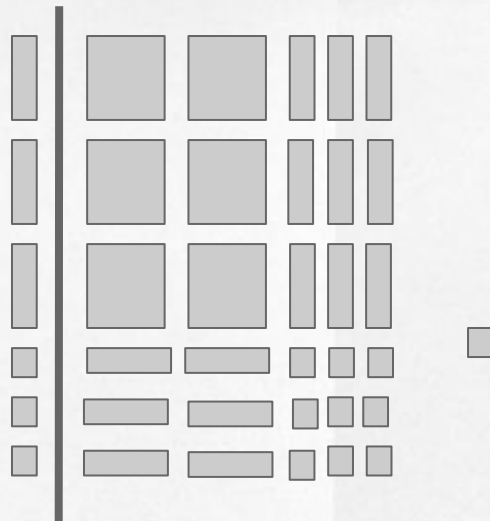
Need 9, decompose the
10

Area Model??

33 guzinta 792 or ...

33 groups of unknown size ...

33 or



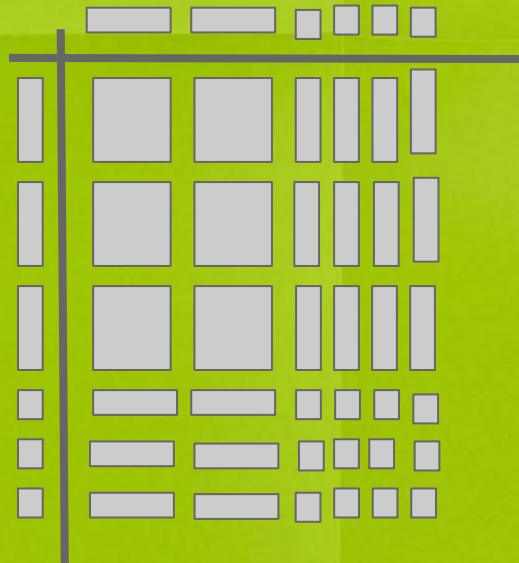
33 left → another column

Area Model??

33 guzinta 792 or ...

33 groups of unknown size ...

33 or



33 rows, with 24 in
each row

Another Mathematics Teaching Practices



Support
productive
struggle in
learning
mathematics

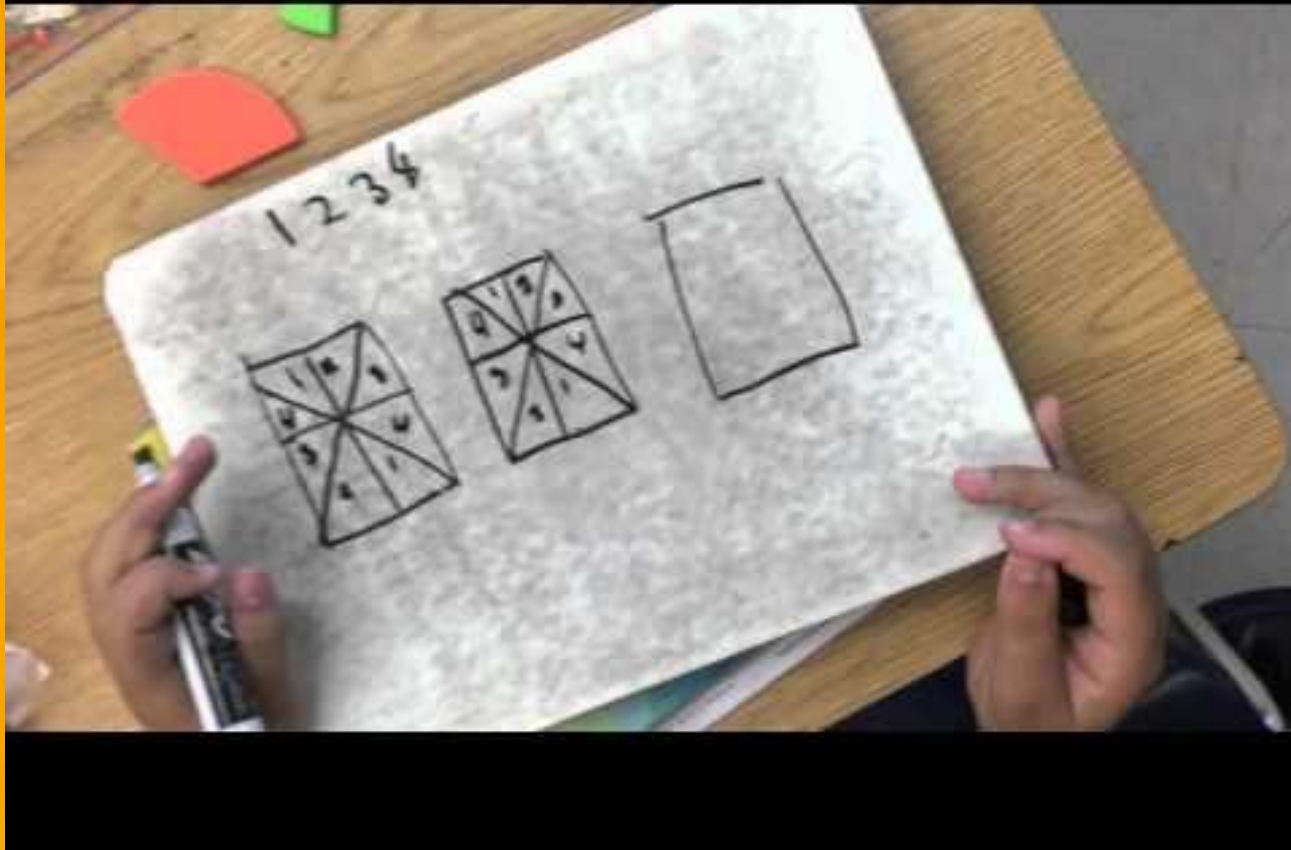
Get More Complicated???

You and 3 friends come home from school.
Your wonderful mother has made:



If you share them fairly, how much sandwich
does each person get?

What Might A Student Do??



Another Mathematics Teaching Practices



Elicit and use
evidence of
student
thinking

Another Context?

How many



are in



Another??



If you were going to divvy the Hershey Bar among yourself and 5 friends, how many would each person get?



$$12/12 \div 6 = ?? \quad 12/12/6$$

Or 1 whole bar/6 people or $\frac{1}{6}$ of the whole

What if you had $\frac{1}{2}$ of a Hershey bar and 4 people? How many of would each person get?



How Many Does This Make??



Establish
mathematical
goals to to
focus learning

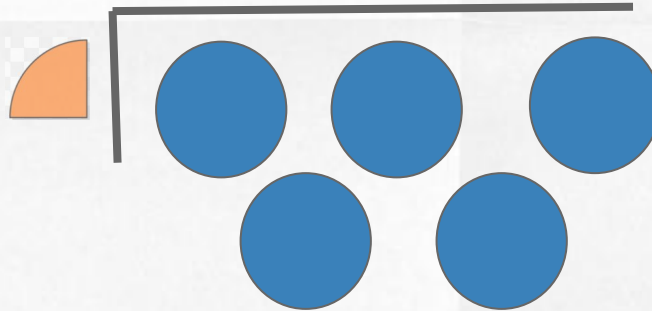
What would these look like
algorithmically?

$$\frac{12}{12} \div 6$$

$$\frac{6}{12} \div 4$$

Another Avenue For Sense-Making

$$5 \div \frac{1}{4}$$



Same as our “How many



in 5 whole dollars?”

Here is Why We Reason Why?

$\frac{3}{4} \div \frac{1}{8}$: “How many $\frac{1}{8}$ -sized (of the original pie) pieces of pie can you serve from $\frac{3}{4}$ of a pie?”

Algorithmically

$$\frac{3}{4} \div \frac{1}{8} \rightarrow \frac{3}{4} \times \frac{8}{1} = \frac{24}{4}$$

Why does
this work??

$$\frac{3}{4} \div \frac{1}{8} \rightarrow \frac{3}{4}$$

$$\frac{1}{8}$$

Algorithmically

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$$\frac{3}{4} \div \frac{1}{8} \rightarrow \frac{3}{4}$$

$$\frac{1}{8} \times \frac{8}{1} =$$

Inverse Property for Fraction Multiplication

Algorithmically

$$\frac{3}{4} \div \frac{1}{8} \longrightarrow \frac{3}{4} \times \frac{8}{1} = \frac{24}{4}$$

Why does
this work??

$$\begin{array}{r} \frac{3}{4} \div \frac{1}{8} \longrightarrow \frac{3}{4} \times \frac{8}{1} = \\ \hline \frac{1}{8} \times \frac{8}{1} = \end{array}$$

Algorithmically

$$\frac{3}{4} \div \frac{1}{8} \rightarrow \frac{3}{4} \times \frac{8}{1} = \frac{24}{4}$$

Why does
this work??

$$\begin{array}{rcl} \frac{3}{4} \div \frac{1}{8} & \rightarrow & \frac{3}{4} \times \frac{8}{1} = \frac{24}{4} \\ & & \frac{1}{8} \times \frac{8}{1} = 1 \\ \hline & & \frac{24}{4} = 6 \end{array}$$

THE Algorithmic LIE We Tell Kids

$$\frac{3}{4} \div \frac{1}{8} \longrightarrow \frac{3}{\frac{1}{2}} \longrightarrow 6$$

Why does
this work??

Shift THE Algorithmic LIE We Tell Kids
Why Not

$$\frac{3}{4} = \frac{1}{8} \times \frac{?}{??}$$

Use what they
do know about
multiplication
and division:
think relationally.



BIG IDEA

The properties of multiplication and addition provide the mathematical foundation for understanding computational procedures for multiplication and division, including mental computation and estimation strategies, invented algorithms, and standard algorithms.

Algorithm?

$\frac{1}{4} \div \frac{3}{4} \rightarrow \frac{1}{4} \times \frac{4}{3} \dots$ it's makes sense??



To close, some humor:

Teacher: Now class, when I ask you this question, I want you all to answer at once. How much is six plus four?

Class: At once!!!

Our students will do whatever we ask so be sure you ask questions that will move your students to achieve your mathematical goals.