

Jules Bonin-Ducharme

Provincial Math Facilitator for the CFORP,
Supporting the Ontario Ministry of
Education's math initiatives.
@jboninducharme



Le centre d'innovation pédagogique

Who thinks, learns!

The top 5 strategies used to engage high school students in critical thinking.

bit.do/julesbdNCTM2017

 @jboninducharme

Think, pair, share

What makes a rich math task rich?



How to develop rich math tasks that foster critical thinking?

Ontario mathematical Processes

Problem
Solving

Reflecting

Reasoning and
Proving

Connecting

Representing

Tools and
computational
strategies

Communicating

<http://www.edugains.ca/newsite/math/mathprocesses.html>

Standards for Mathematical Practice

Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.

6 prompts to help create questions and tasks that invite thinking

Critique the
piece

Judge the
better or best

Rework the
piece

Decode the
puzzle

Design to
specs

Perform to
specs

Celebrate mistakes!

In this class, mistakes are:

Expected
Inspected
Respected

Source: Jo Boaler

You might know this one!

$$a = b$$

$$a^2 = ab$$

$$a^2 - b^2 = ab - b^2$$

$$(a + b)(a - b) = b(a - b)$$

$$a + b = b$$

$$2b = b$$

$$2 = 1$$

Wait! What???

$$(n+1)^2 = n^2 + 2n + 1$$

$$(n+1)^2 - (2n+1) = n^2$$

$$(n+1)^2 - (2n+1) - n(2n+1) = n^2 - n(2n+1)$$

$$(n+1)^2 - (n+1)(2n+1) = n^2 - n(2n+1)$$

$$(n+1)^2 - (n+1)(2n+1) + \frac{(2n+1)^2}{4} = n^2 - n(2n+1) + \frac{(2n+1)^2}{4}$$

$$(n+1)^2 - 2(n+1)\left(\frac{2n+1}{2}\right) + \left(\frac{2n+1}{2}\right)^2 = n^2 - 2n\left(\frac{2n+1}{2}\right) + \left(\frac{2n+1}{2}\right)^2$$

$$\left((n+1) - \frac{2n+1}{2}\right)^2 = \left(n - \frac{2n+1}{2}\right)^2$$

$$(n+1) - \frac{2n+1}{2} = n - \frac{2n+1}{2}$$

$$n+1 = n$$

$$x^2 - 5x + 6 = 2$$

$$(x - 2)(x - 3) = 2$$

$$x - 2 = 2 \quad \text{or} \quad x - 3 = 2$$

$$x = 4 \quad \text{or} \quad x = 5$$

This is an «awesome » mistake!
How would you respond to this student?

Starting with mistakes!

$2^5 = 10$	$100^{1/2} = 50$
$(-2)^3 = 8$	$7^{-2} = -49$
$-6^2 = 36$	$(y^3)^5 = y^8$
$37^0 = 0$	$\frac{x^5}{x^9} = x^4$

source: [Andrew Stadel](#)

Visualize, Verbalize, Verify

Pay attention to Spatial Reasoning



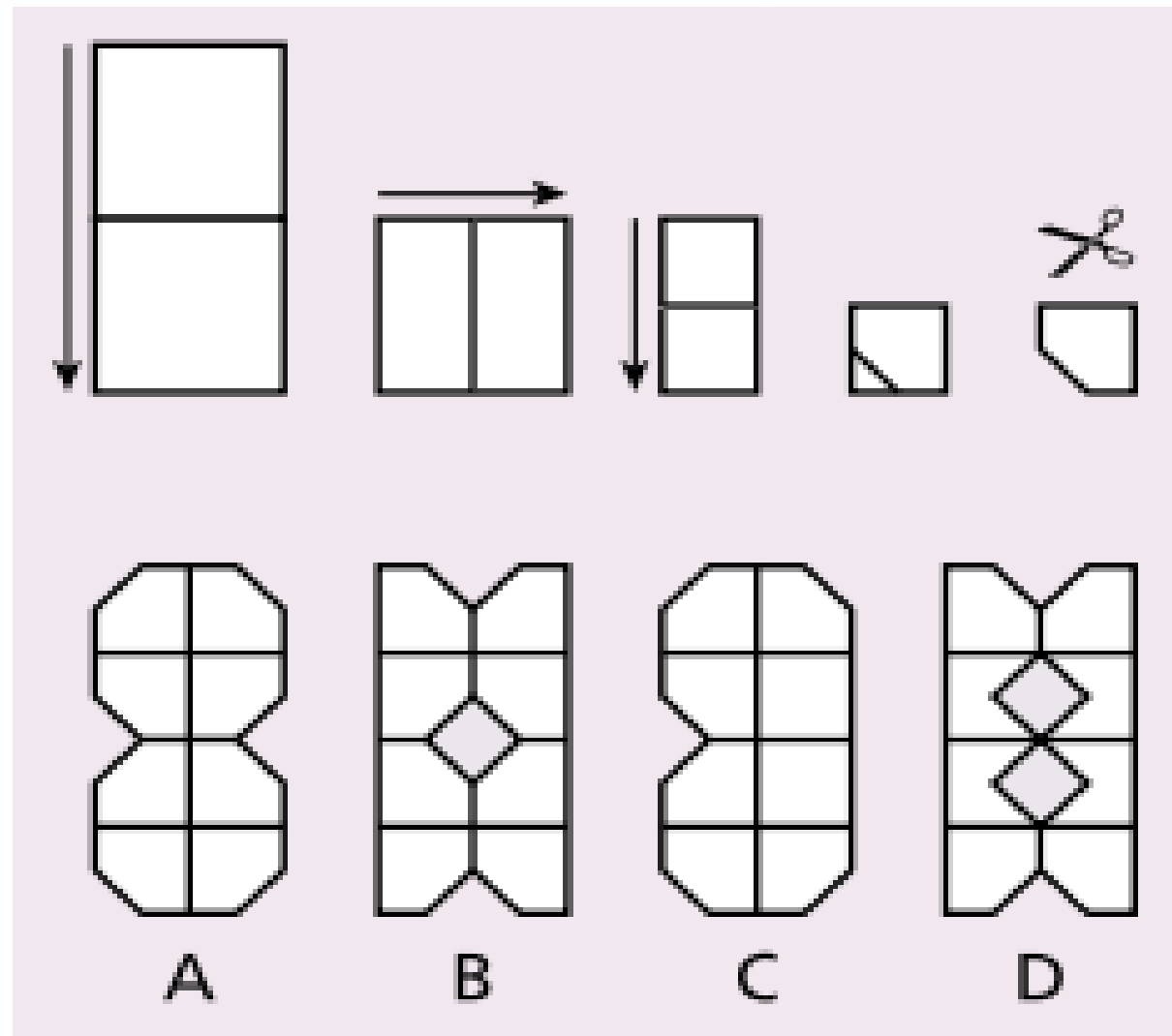
Visualize



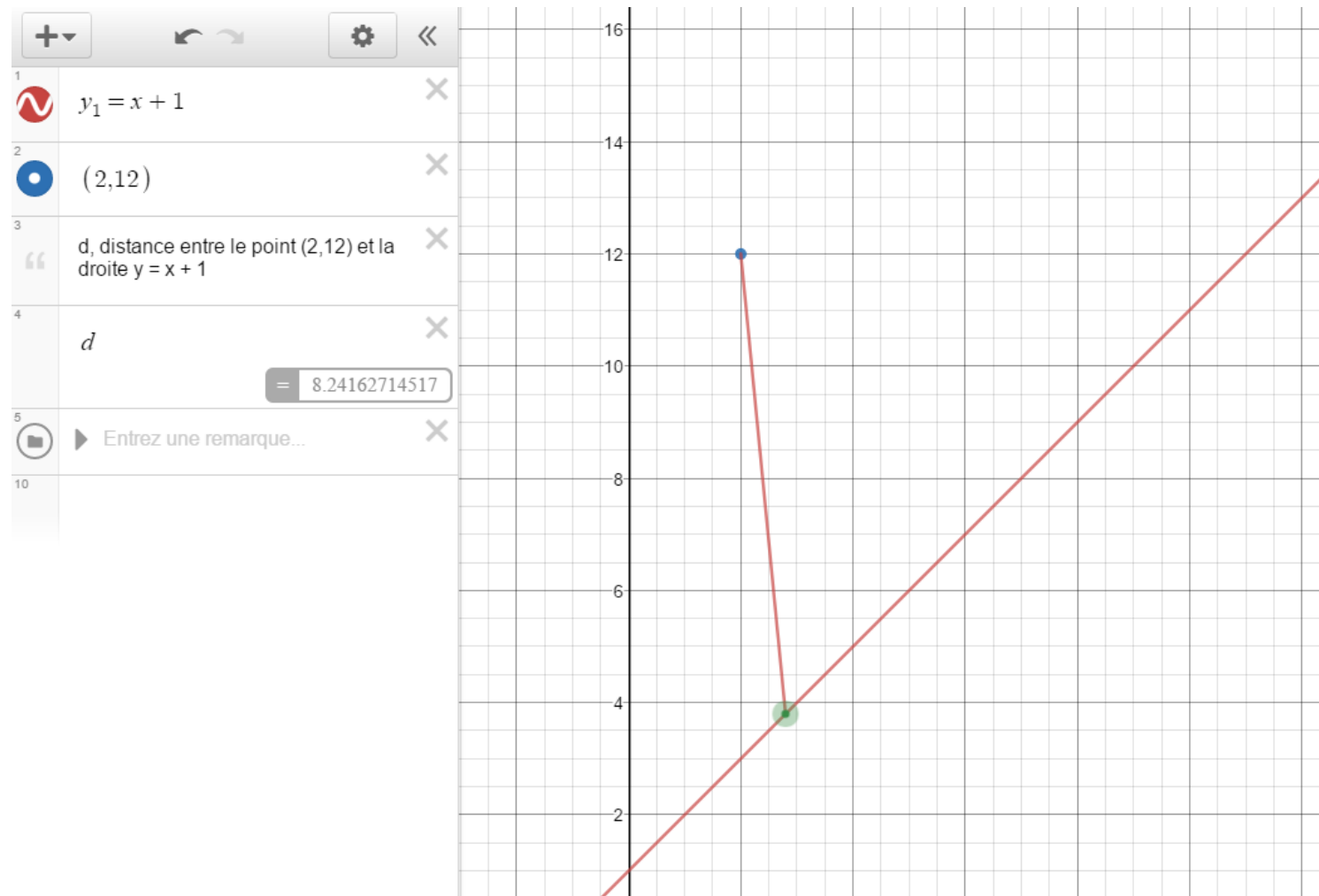
Verbalize



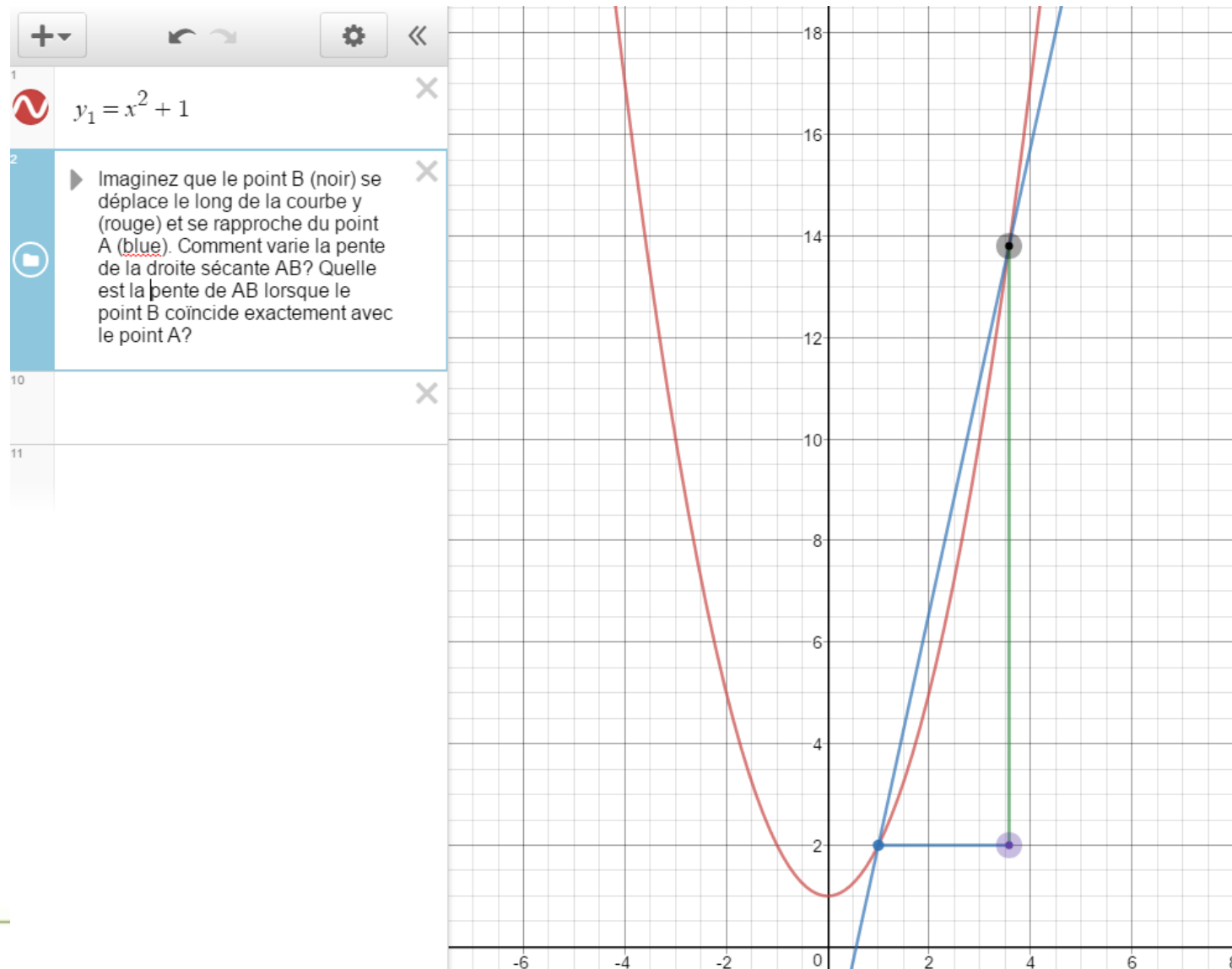
Verify



Distance between a point and a line



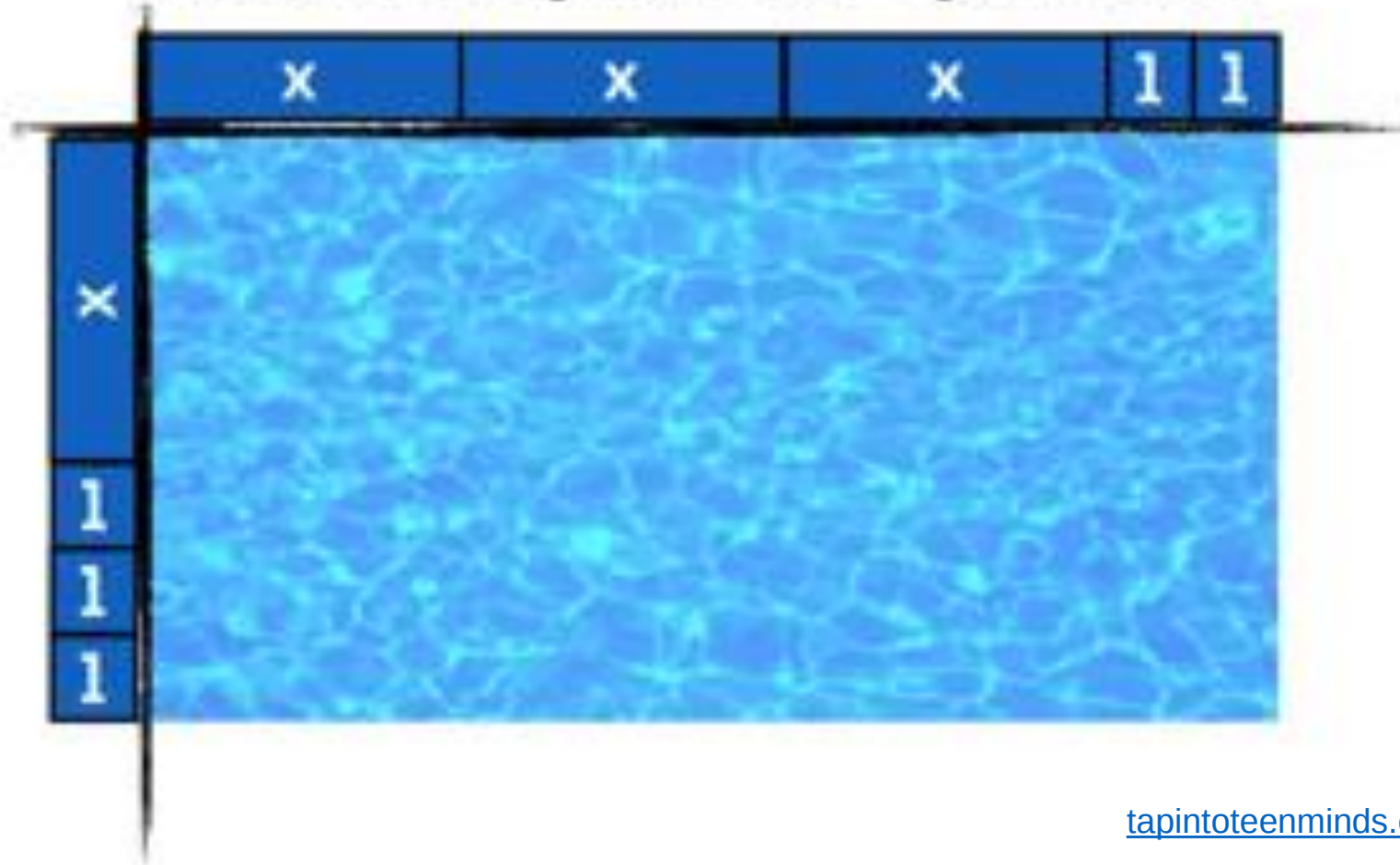
The secant of a parabola



Steal models!



What's My Area Equation?

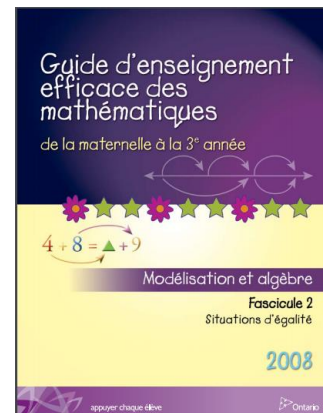


tapintoteenminds.com/progression-of-multiplication/

Dynamic double number line

How can we determine if this equality is true?

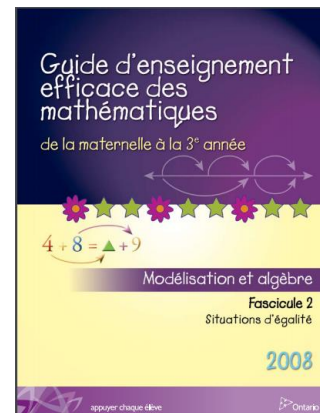
$$115 + 35 = 50 + 50 + 50$$



Dynamic double number line

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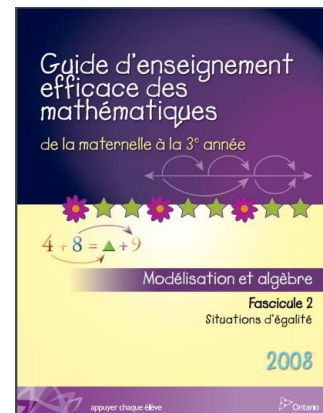
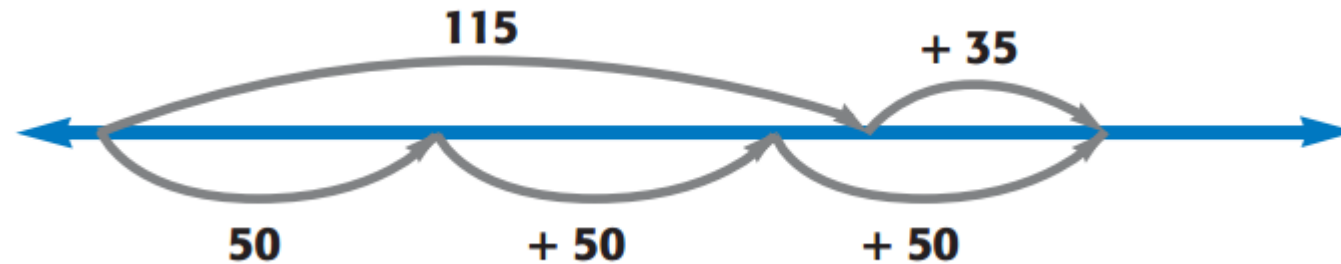
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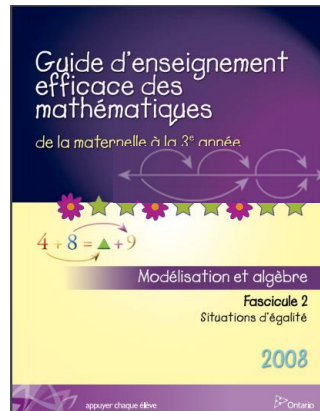
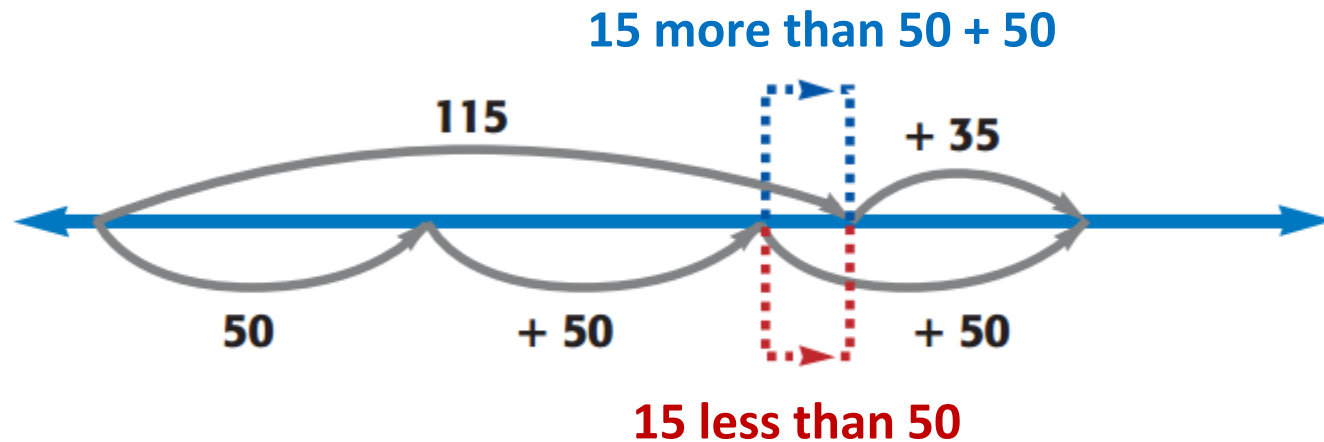
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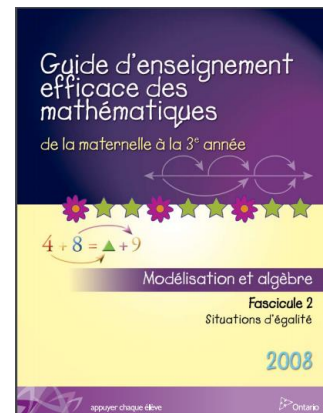
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Dynamic double number line

How can we determine if this equality is true?

$$115 + 35 = 50 + 50 + 50$$



Oh No! The Numbers Fell Off Our Number Line



<http://tmerc.ca/m4yc/peterborough.html>

Clothesline math!

$$2x+7$$

$$16$$

source: [Andrew Stadel – clothesline math activities](#)

Clothesline math!

$$2x+7$$

0

16

source: [Andrew Stadel – clothesline math activities](#)

Clothesline math!

$0x$

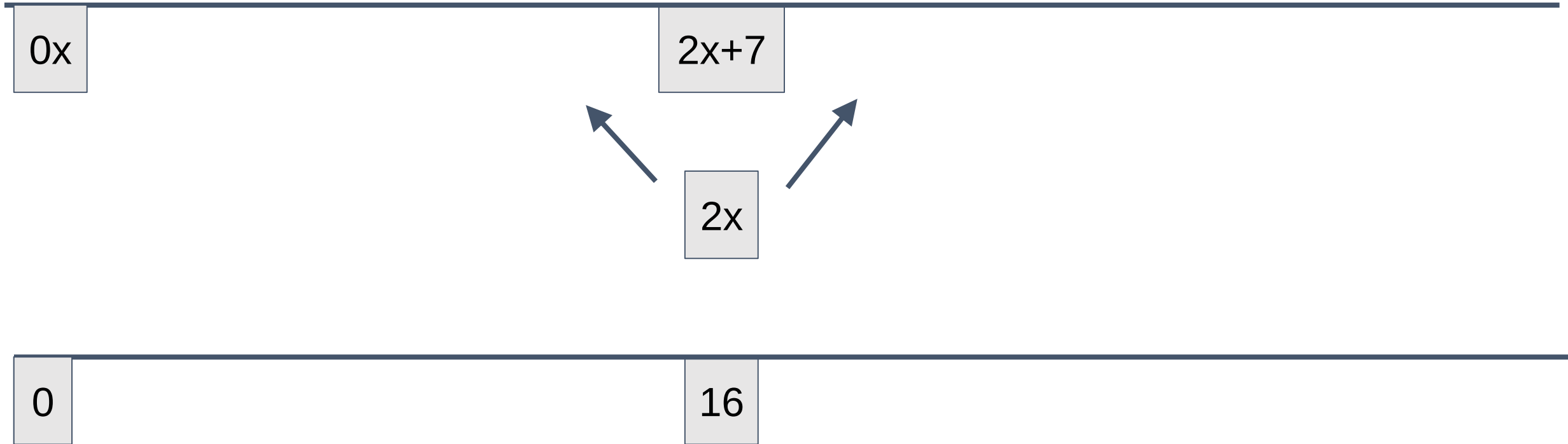
$2x+7$

0

16

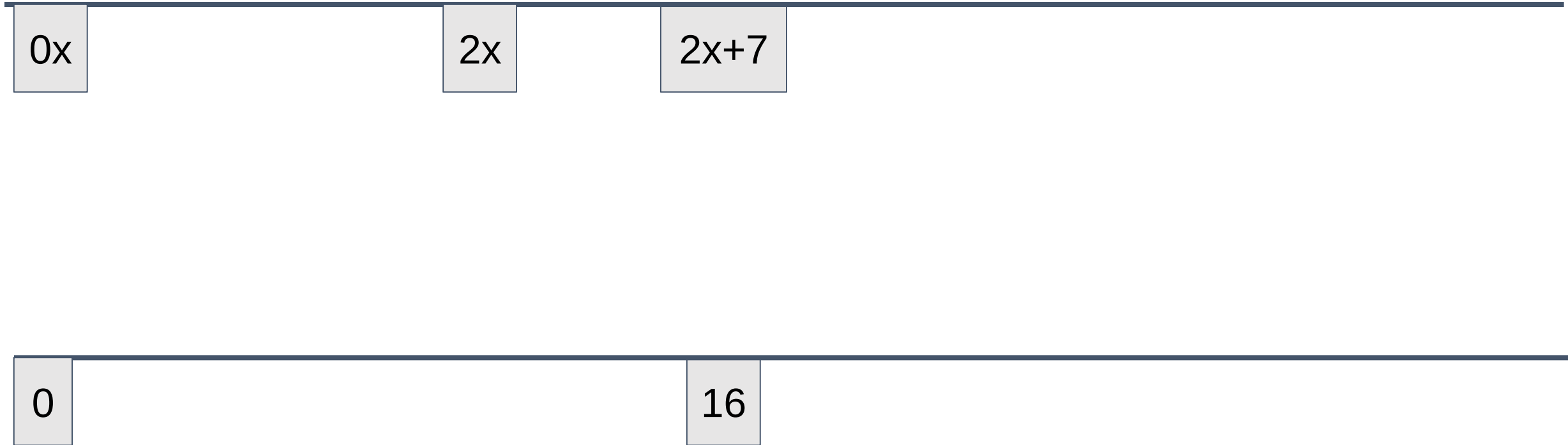
source: [Andrew Stadel – clothesline math activities](#)

Clothesline math!



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Clothesline math!

$0x$

$2x$

$2x+7$

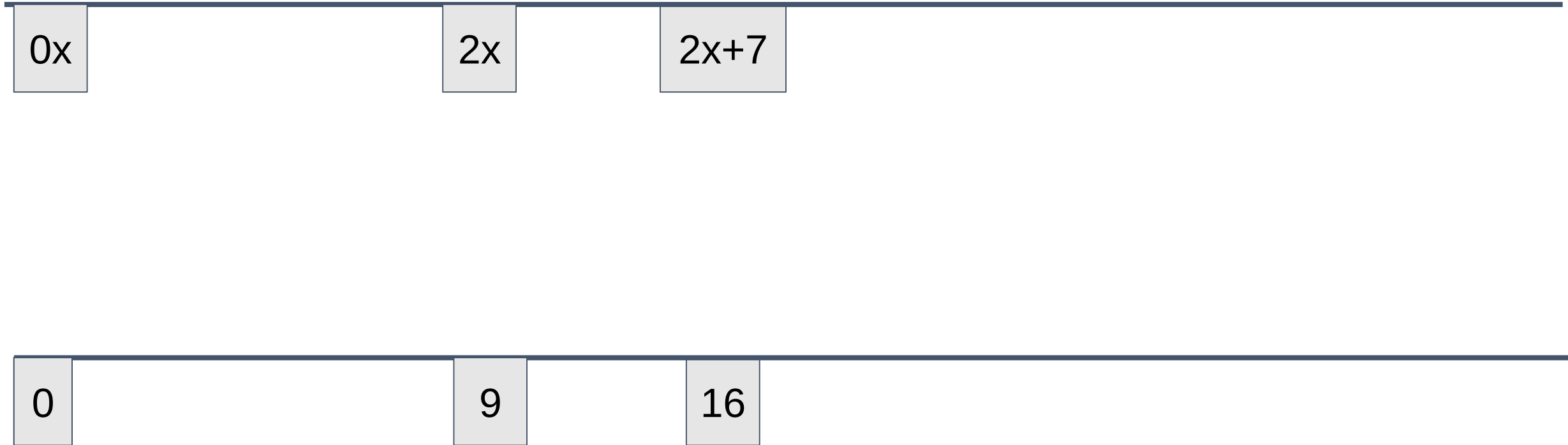
0

?

16

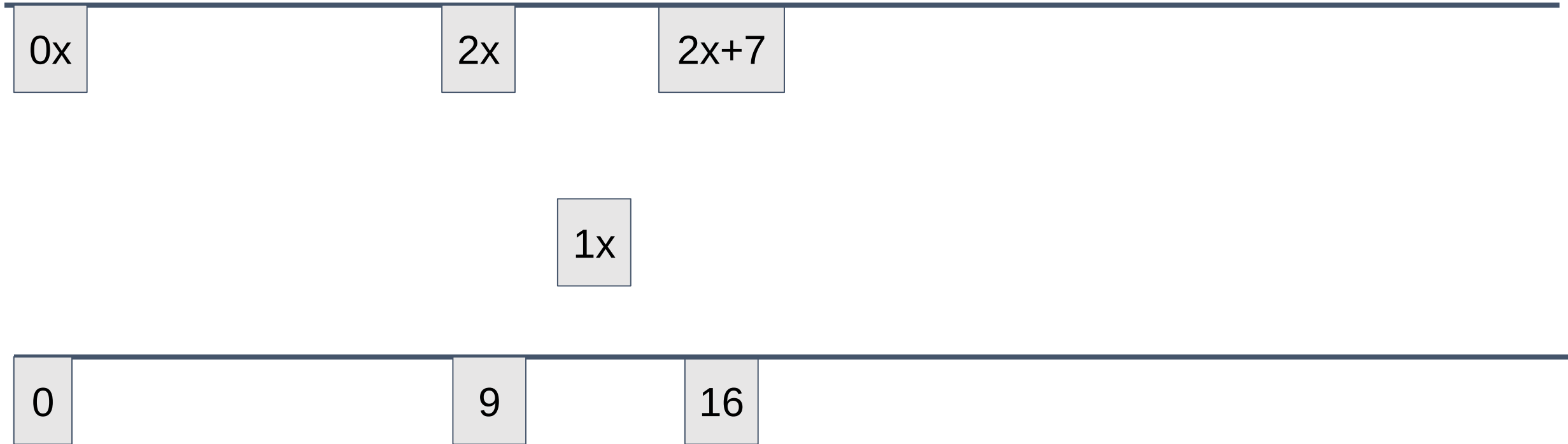
source: [Andrew Stadel – clothesline math activities](#)

Clothesline math!



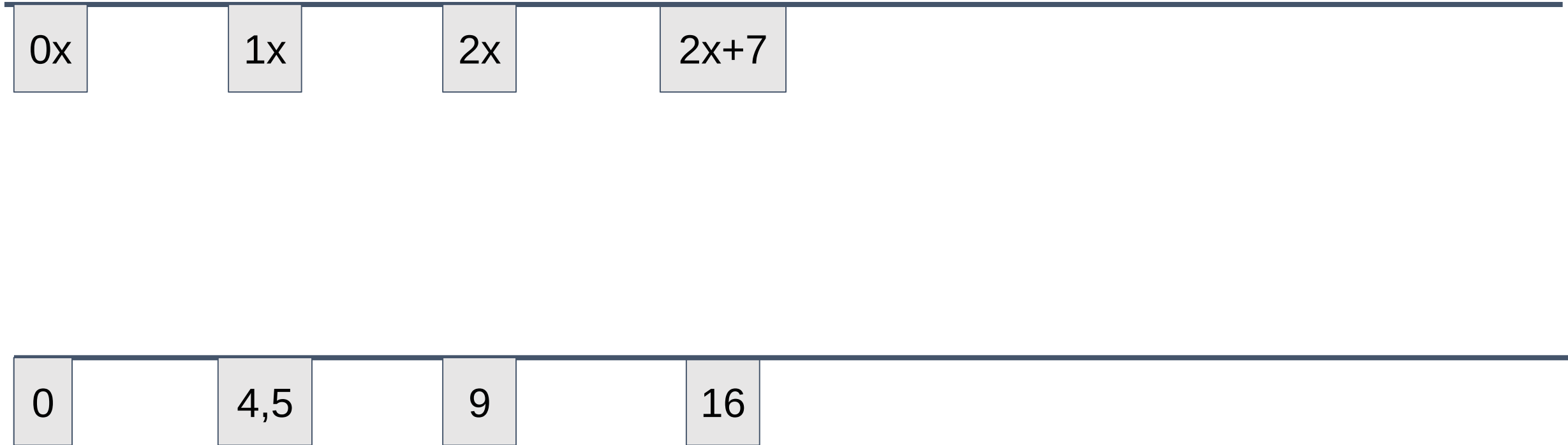
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Clothesline math!



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Clothesline math!



source: [Andrew Stadel – clothesline math activities](#)

Clothesline math!

$$7x - 11 = 5x + 3$$

source: [Andrew Stadel – clothesline math activities](#)

Clothesline math!

source: [Andrew Stadel – clothesline math activities](#)

$0x$

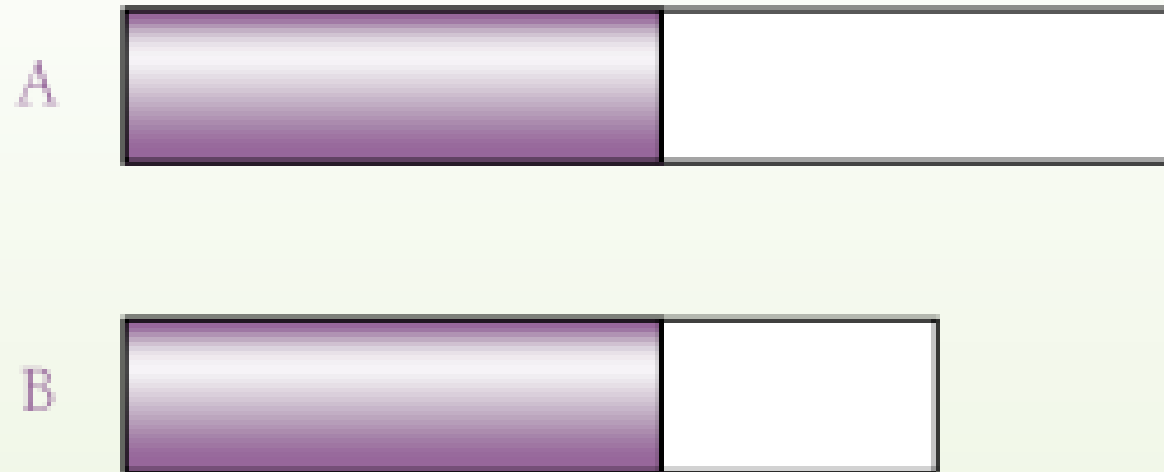
0

$7x-11$

$5x+3$

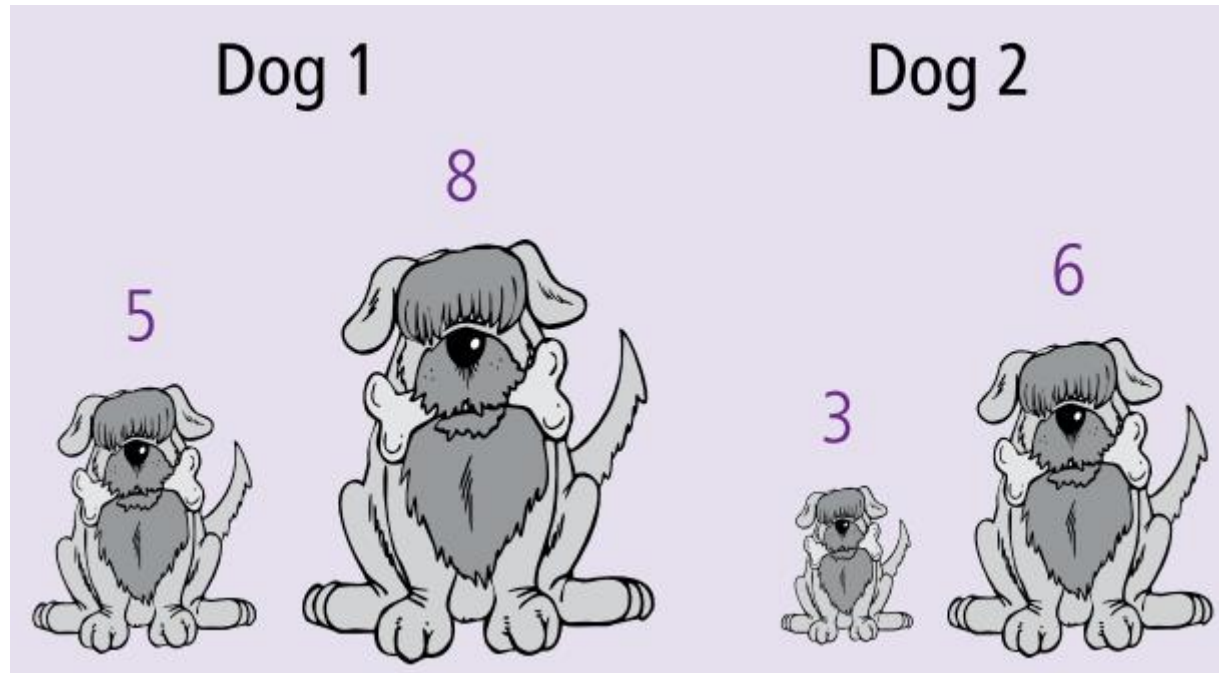
Ask ambiguous questions!

Which shape is more purple?



Adapted from Marian Small (2008, p. 254)

Which dog grew more?



The solution to a fairly simple equation is 3.5 more than the solution to a complicated equation.

What might the equations be?

How many circles are there in the 113th figure?

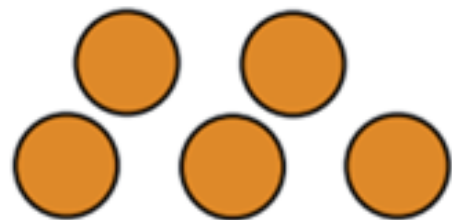


Figure 1

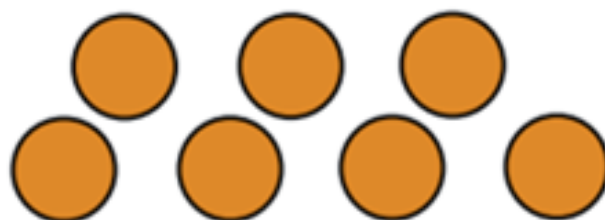


Figure 2

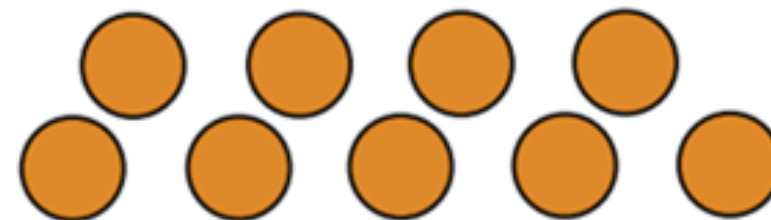


Figure 3

How does this pattern grow?

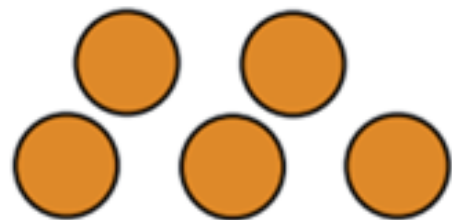


Figure 1

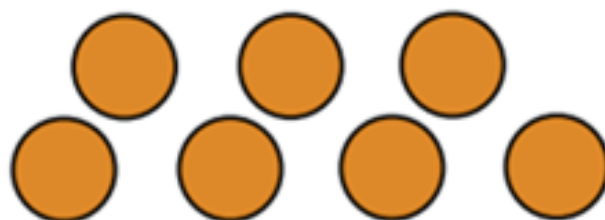


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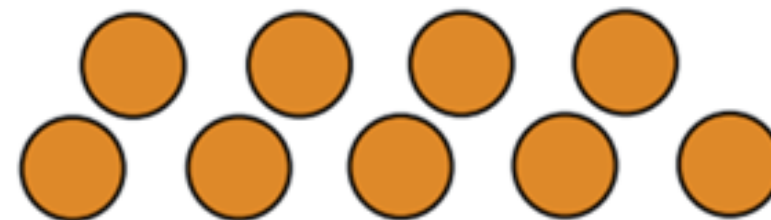


Figure 3

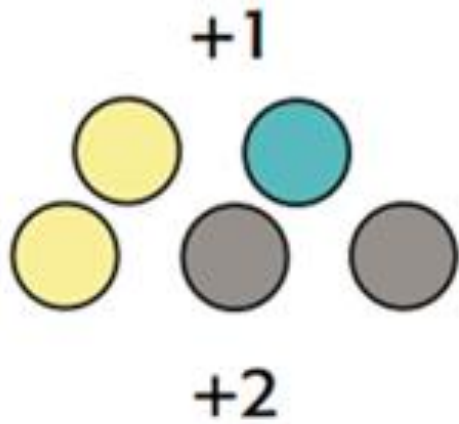


Figure 1

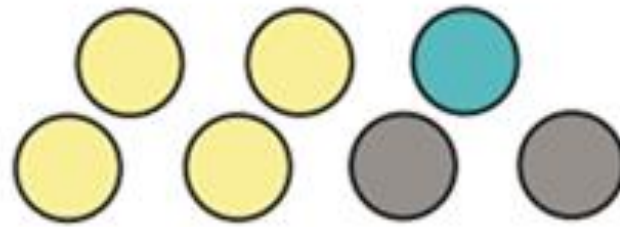


Figure 2

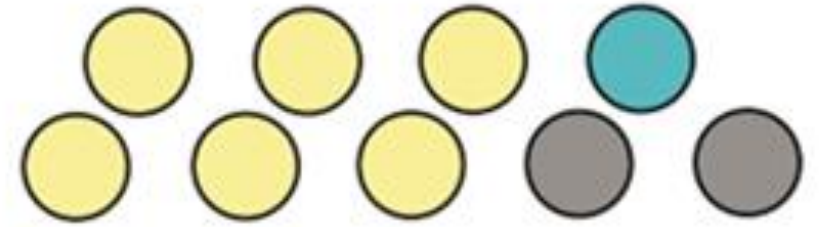


Figure 3

$$(n + 1) + (n + 2)$$

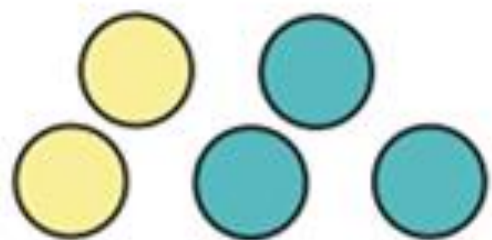


Figure 1

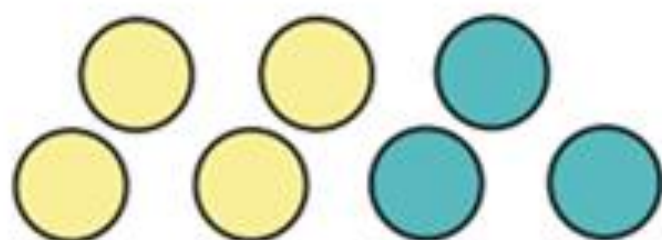


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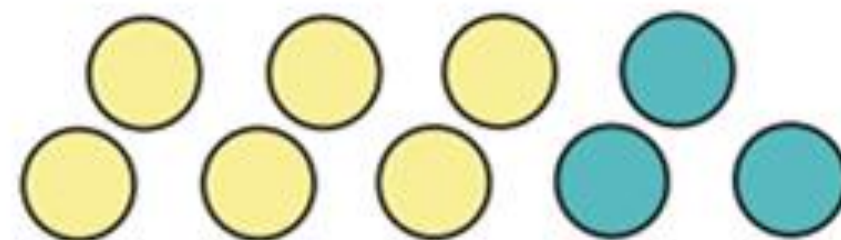


Figure 3

$$n + n + 3$$

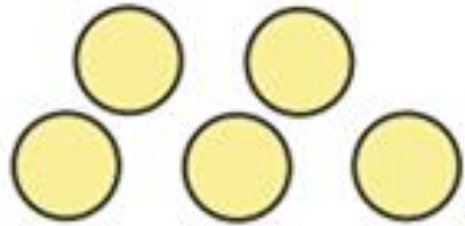


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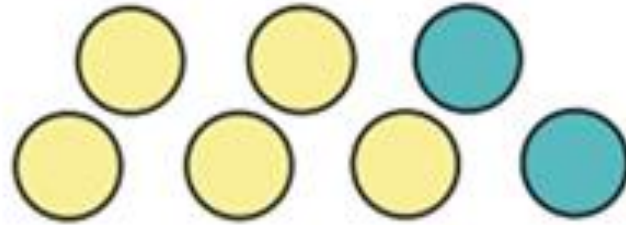


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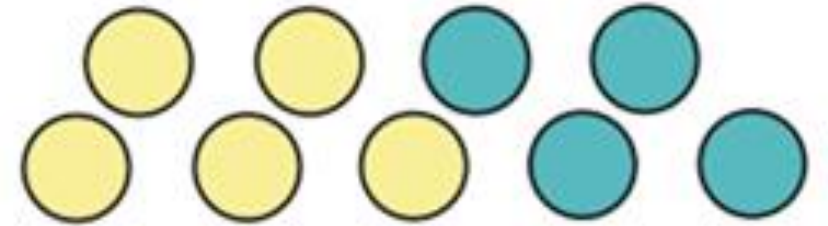
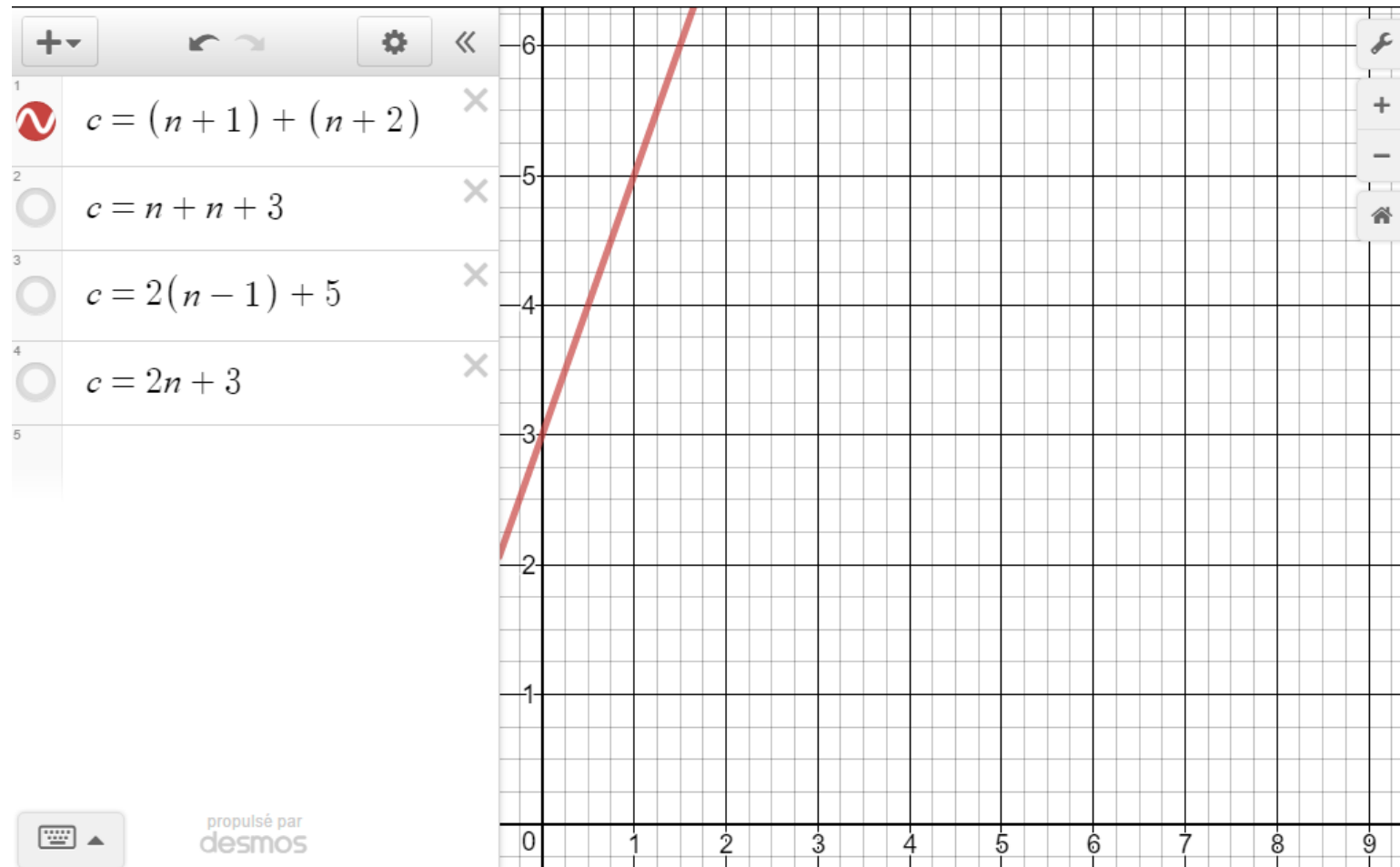


Figure 3

$$5 + 2(n - 1)$$

Graphic representation



Visual Patterns

1-20 21-40 41-60 61-80 81-100 101-120 121-140 141-160 161-180 181-200

201-220 TEACHERS GALLERY CONTACT

Click on a pattern to see a larger image and the answer to step 43. What is the equation?

Math fights!

Sam: It is not possible to multiply two fractions and get a product with a smaller denominator than the ones you started with.

Kevin: Yes, you can.

With whom do you agree?

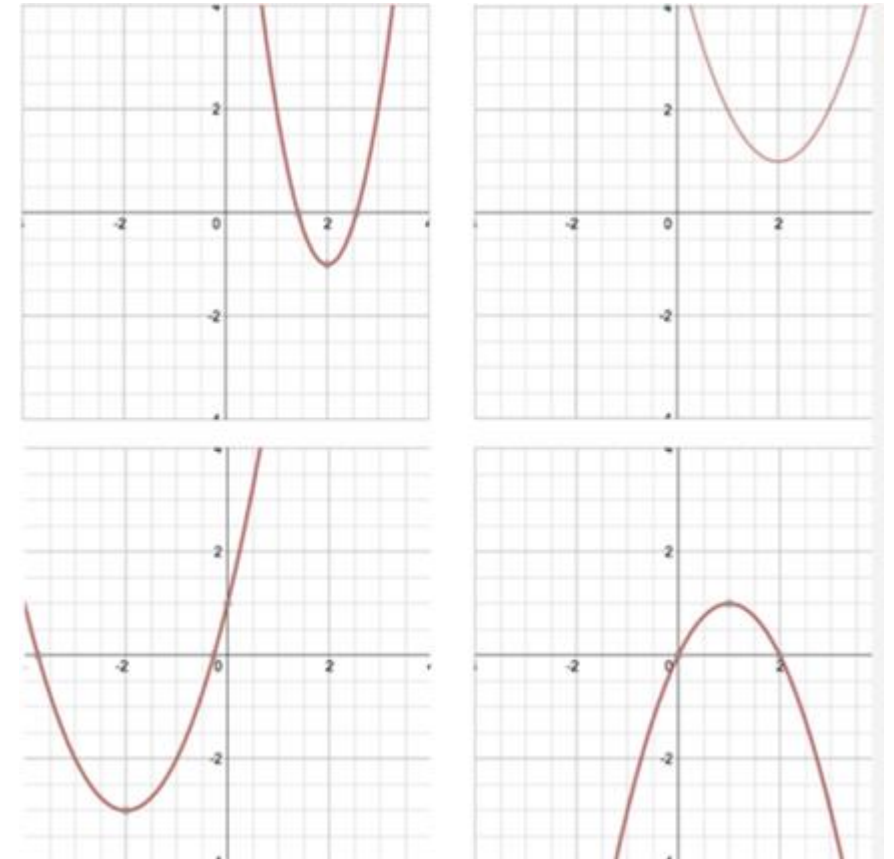
Julie: Every function of the form $y = a \sin(x - h) + b$ can be renamed as a function of the form $y = c \cos(x - k) + d$.

Xavier: Some can, but not every.

With whom do you agree?

Which one doesn't belong?

Which graph doesn't belong?



source: wodb.ca

Developing arguments

Is the circumference of the bottle smaller, equal or higher than the height of the bottle?



source: [Andrew Stadel](#)



Tweet the strategy that you find the most interesting and tell me why. #thinkingmath

1. Celebrate mistakes
2. Visualize, Verbalize, Verify
3. Steal models
4. Ask ambiguous questions
5. Math fights

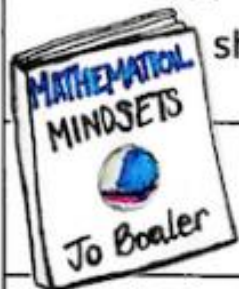
Recommendations for Task/Lesson Design

Open the task to encourage multiple methods, pathways and representations.

Pose a problem before teaching the method.

Design a task that allows all learners to contribute to the learning and have room for extension.

Make opportunities for students to authentically share their thinking with peers.



Add a visual component.

Add the requirement to convince and reason, be skeptical.

Powerful Questions to develop a deep level of understanding

How do you see that idea?

Why does that answer make sense?

Why does that method work?

How is that method connected to others?

How can that idea be represented in different ways?

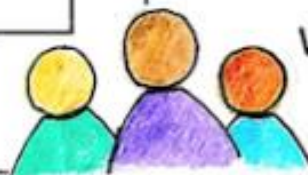
Can you prove it?

Can you prove it visually?

Can you justify your thinking?

Can you predict what would happen if....?

Did you make any interesting mistakes?





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