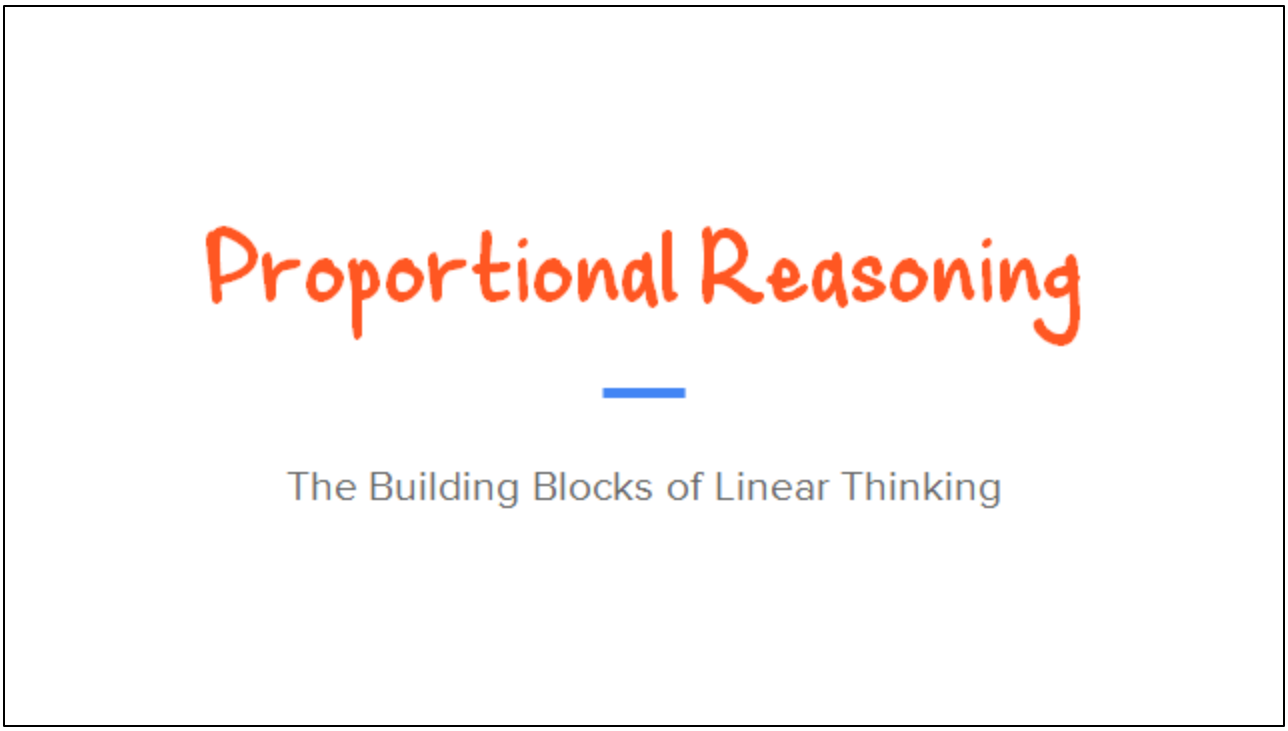
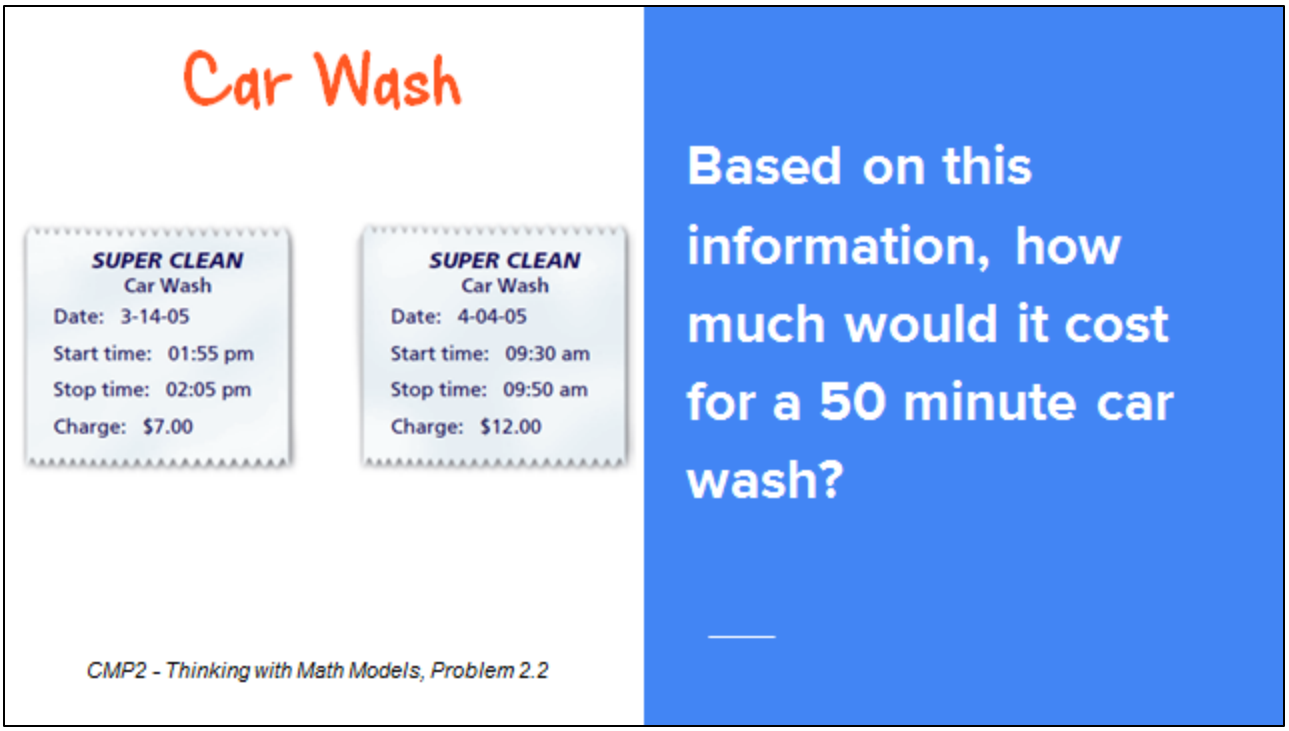


This is called a “workshop” which means we’re learning together.
As a result of our conversations, some of us may begin to think differently about the connection between proportionality and linearity or rethink what/how we’ve been teaching, others may solidify their thinking about instruction, while others may develop some language for how to coach or work with teammates around instruction.



What was your approach? Justify your thinking. Can you represent the information in a different way?

What assumptions are we making? – That it's linear (so we'd need to establish that important information for students).

Some possible common student errors:
Susie got $31 - adding the costs for 10 min + 20 min + 20 min (but that includes the y-int three times)
Sally got $35 - multiplying the cost for 10 min by 5 (but that includes the y-int five times)
What do Susie and Sally not understand? What do they understand?

- Sally has some understanding of proportional reasoning, however she's using the rate y/x rather than the rate change in y/change in x.

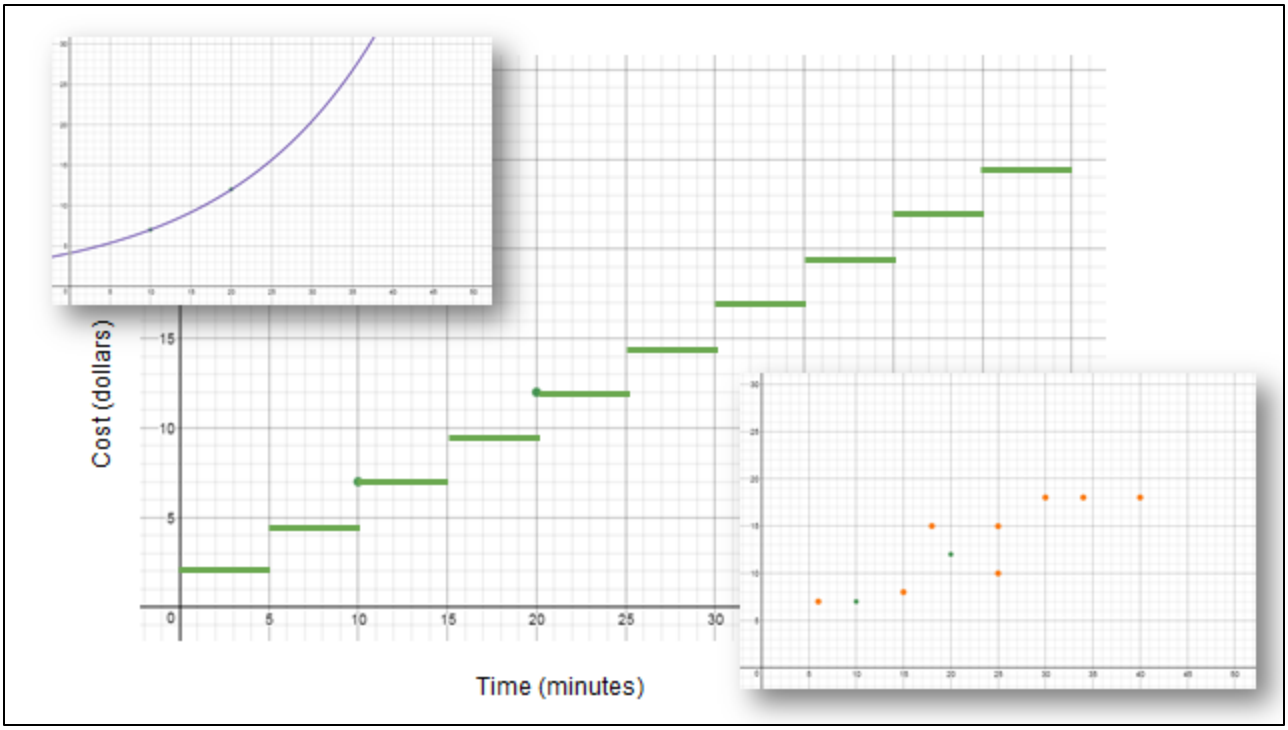
How do we build on Sally's reasoning but help her understand the difference?

What happens if we plot (10,7) and (20,12) on a coordinate graph (Desmos.com), then add students' answers?
What might we want them to notice?
What questions would we ask?

Reflections for us (the teacher):

- Why is linearity so comfortable? - the predictability created by the steadiness of the rate of change
- If we know the relationship is linear, how does that shape how we approach the problem? - we start looking for that unit rate
- Was this a proportional linear relationship? How do you know?
- But, does this situation involve proportionality?

Think about the proportional reasoning we needed to find the cost for 50 minutes!



You ASSUMED it's linear, but what if his cost/time relationship was…

- Step function puts 50 minute cost at $24.50
- Exponential function puts 50 min cost at about $60
- Or maybe at Crazy Carl's Car Wash, Carl just looks you over and randomly chooses a price!

Who are you, anyways?

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Session Learning Goals:

Increase awareness of how early explorations of proportionality contribute to students' understanding of linearity

Equip teachers to build on students' prior experiences and knowledge to deepen connections between proportionality and linearity

We'll look at several common-ish tasks and discuss how to amplify them to connect proportional reasoning and linear thinking

BUILD

Building conceptual and procedural understanding

NCTM Strand Description:

When students possess conceptual understanding of mathematical concepts, they recognize connectedness amount mathematical ideas, are ***fluent in multiple forms of representations****, and can communicate and justify the strategies they employ in problem-solving situations. Sessions in this strand will highlight mathematical ideas that* ***enable students to choose flexibly among methods and strategies****.*

Setting the GPS

(TEKS) 8.4 The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope.

(CCSS) 8.EE.B Understand the connections between proportional relationships, lines, and linear equations.

https://www.smore.com/13wnu

The title of our session is Prop Reasoning: The building blocks of linear thinking. We know this is where they're headed, and even beyond as they enter Algebra 1.

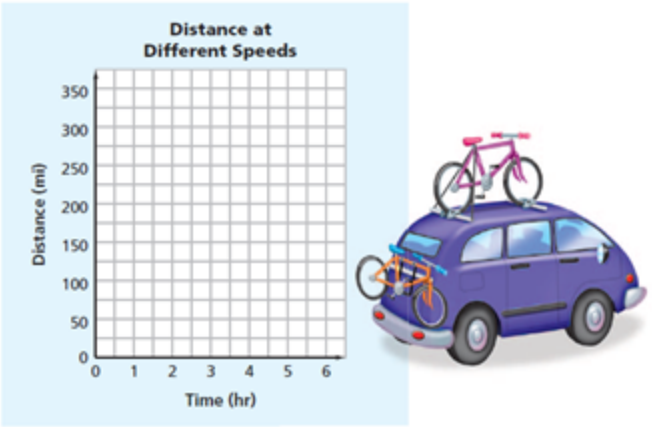
What are the foundational concepts related to proportionality that are the building blocks for this new learning?
On your chart paper - jot down concepts that take students to this point in their learning (limiting ourselves to 5th, 6th, 7th, and 8th). Create a visual representation, like a concept map or a scaffold or…

If you'd like some help to jog your memory, we have a link to the standards on our Smore.

Lap #1

The Ocean Bike Tour Company uses a table and a graph to estimate their travel time for different average speeds on their drive back to Atlantic City.

Time (hrs)	Distance for Speed of 50 mph	Distance for Speed of 55 mph	Distance for Speed of 60 mph
0			
1			
2			
3			
6			



CMP2 - Variables and Patterns, Problem 3.1

This task focuses on learning that usually happens a year or two (at least) before the car wash task.
This could be the beginnings of a 5th grade task, or perhaps 6th

- TEKS 5.8C has students graphing from input/output table, or TEKS 5.4C - given a rule extend the pattern and graph
- CCSS Math 5.OA.B.3 - Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

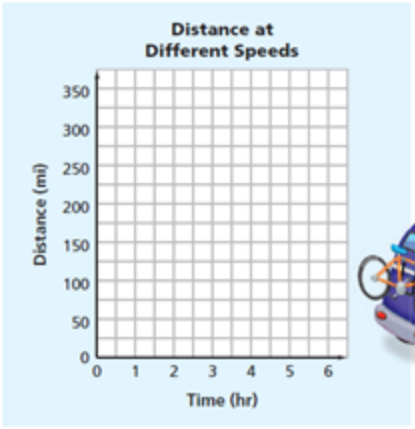
Lap #1

For each of the three average speeds:

1. Look for patterns relating distance and time in the table and graph. Write a rule in words for calculating the distance traveled in any given time.

2. Write an equation for your rule, using letters to represent the variables.

3. Describe how the pattern of change shows up in the table, graph, and equation.



CMP2 - Variables and Patterns, Problem 3.1

Here are some of the explorations for this task from Connected Math.

Where is proportionality in this?
What questions might you ask your students to bring out the proportionality?
Did you look back at the table to plot every point? Did anyone use a pattern to find the next point?
What are some other points on this line? How do you know without going back to the table or equation?
Were all three lines equally easy to graph? Why not? (easier when the rate of change is a more visible "rise and run")
Why are these straight lines? Why do they diverge? What does the rule we wrote MEAN? (For every (t,d) value on the graph, the miles is 50 times the hours)
What's similar about the three columns in the table? What's different?

As we build these concepts in earlier grades, are we doing it in a way that would support the understandings students need later when working with linear relationships? (emphasizes proportionality, but stretches to linearity as well) When you consider the focus of the learning for this task, what conversations with students would support the later learning they will be exploring in the car wash task?

NOTE: We aren't asking earlier grades to pre-teach slope!!! Even though we're right there with our graphs, tables, equations… we're not asking you to teach 8th grade math to 6th graders. We're still staying in our lane (so to speak) with a focus on the

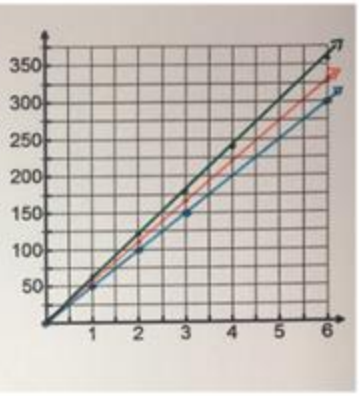
6th grade (or whatever) standard. **We are layering it with the process standards (mathematical practices) with it to promote deeper understanding.**

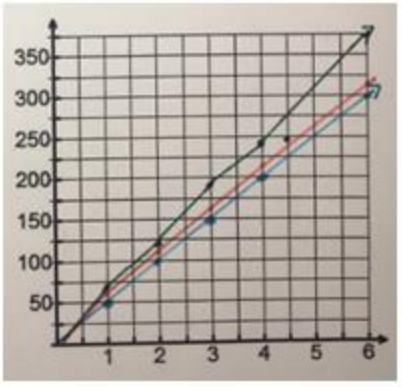
REFLECTION: Some may have seen this as a expressions/equations/relationships task. We're trying to explore ways to connect that to proportionality.
Summarize on the back of your task sheet. Jot down some questions that would stretch the learning in this task. Could even make a T chart (questions → future content connection)
(Example: Why do the lines diverge? → later: The slope of the line affects the steepness of the graph)

TEKS 6.5A, 6.6ABC
CCSS math 6.RP.A.3A, 6.EE.C.9

Lap #1

Time (hrs)	Distance for Speed of 50 mph	Distance for Speed of 55 mph	Distance for Speed of 60 mph
0	0	0	0
1	50	55	60
2	100	110	120
3	150	165	180
6	300	330	360
t	50t	55t	60t





Assume the picture on the left is what we want to see from student work.
If the right represents what you see from a student, you might be ELATED! They started at (0,0). They connected the coordinates. Their coordinate points are really close to where they should be. CELEBRATION, right? Yes, but we cannot stop there.
The blue and orange lines are drawn fairly parallel. Why is that a big deal?
The green line is a bit jagged. How would we help students see that - if plotted perfectly (which can be hard) - it should be a straight line? Why is that a big deal?

Lap #2

Nanette has taken a lucrative job with BMW. She is working on the fuel gauge programming and has to do many conversions for the US and international models. The BMW-3 series has a fuel tank capacity of 16 gallons. What is this capacity in liters for the international model?

Let's look at what might be a 7th grade task.
(TEKS 7.4E and CCSS 6.RP.A.3D, 7.RP.A.3)

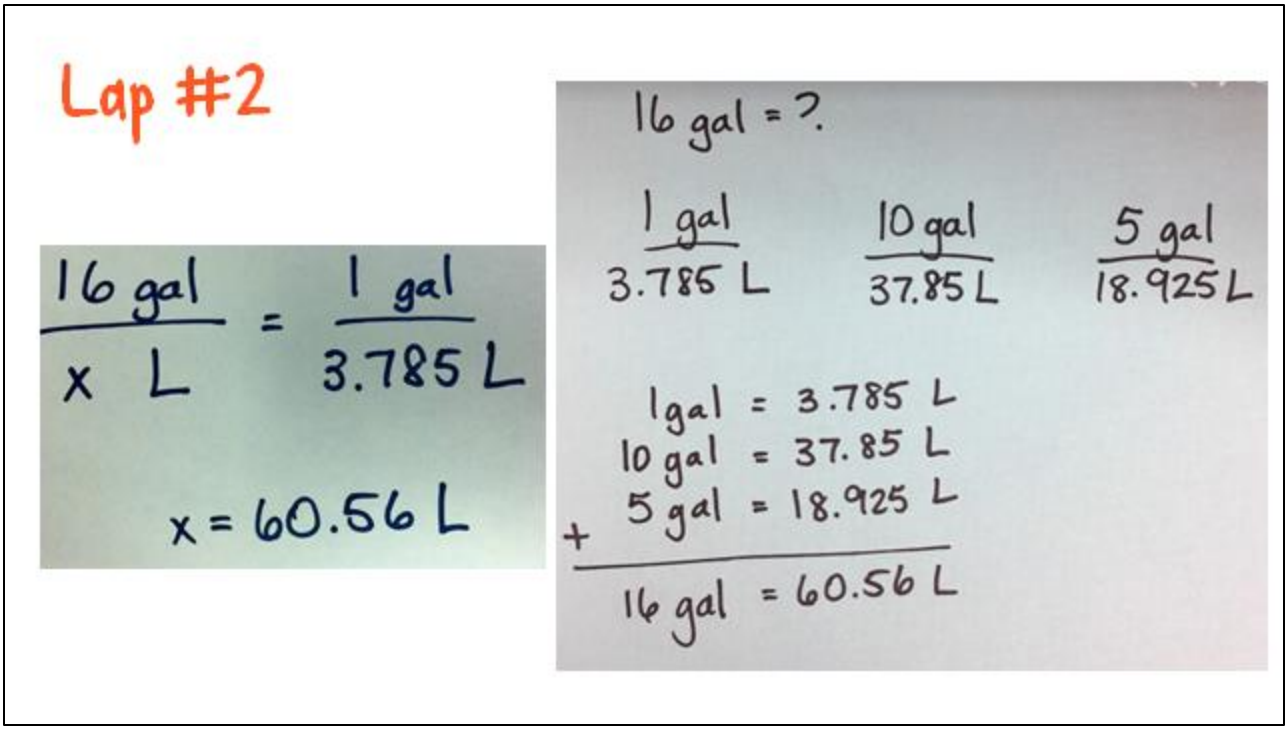
On your own- think about how you would solve this problem. How would your students solve this? (possibly a straightforward proportion)
(approx 60.6, 1 gal = 3.785 L)

But, how else could we represent this relationship? (table, graph, etc.)
How might we amplify it to explore the linearity of it?

- Suppose a student scaled up/down to solve 1 gal = 3.785 L, then 10 gal = 37.85, etc to get to 16 gallons. (how do we know we could add 10 gallons to 1 gallon to 5 gallons?)
- We could represent each of those ratios in a table and graph (Desmos)

What questions could you ask (staying in your lane) that would stretch to later understanding of linearity?
What if we included the process column in our table? What connections could we then make to a multiplicative relationships?

REFLECTION: The last "lap" (task) was something we look at as "expressions, equations, relationships" and we considered ways to amplify the proportionality in it. For "lap" 2 we might categorize this task as a ratio/proportionality problem. We've discussed ways to amplify it, making connections to linearity. Jot down some notes on the back of your task sheet.



Examples of possible student work.

- The work on the left allows us to see the scaling between these two ratios. Each value is getting 16 times larger.
- The work on the right gives us an "in" for exploring this relationship more deeply.

Why is it that you can add those three ratios and get a fourth, equivalent ratio, but we could not in the car wash receipt problem? How do we help Sally see the difference?

The Process Column

gallons		liters
1		3.785
2	2(3.785)	7.57
10	10(3.785)	37.85
16	16(3.785)	60.56
g	g(3.785)	

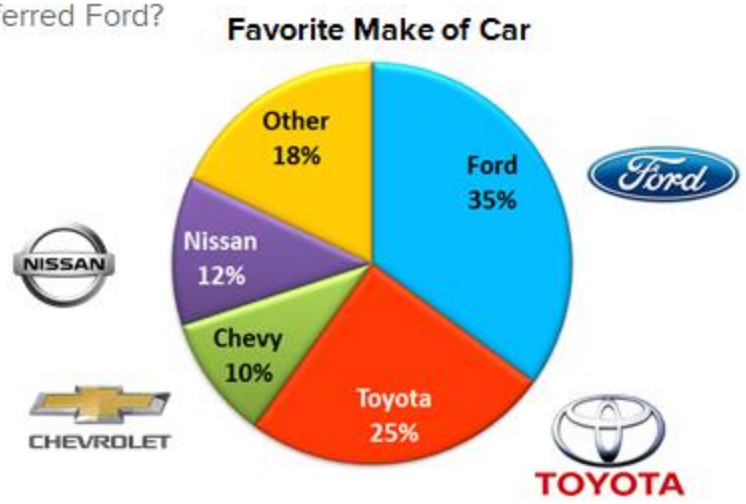
We would of course want students to see how proportions could be used to convert across measurement systems. We want to honor the proportionality in the original problem. But if we then explore using a table we can amplify it to make connections to linearity.

The process column helps students connect numeric patterns with general rules and algebraic representations.

We can then (after students create their own tables with a process column) go to Desmos.com and see what our corresponding values look like in a graph (before asking for the general rule)

Lap #3

Johnny surveyed 360 people asking them their favorite make of car. How many people preferred Ford?



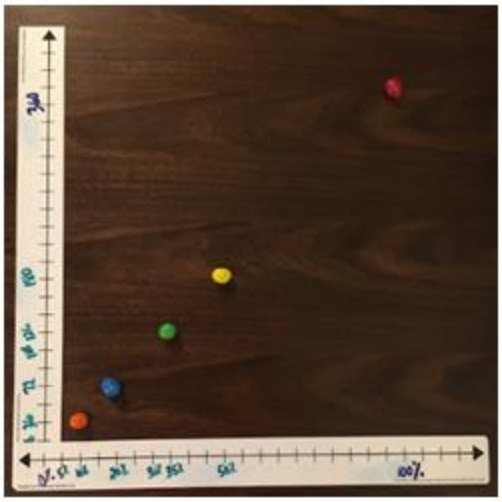
How would <u>you</u> solve this problem? (maybe 0.35 x 360 or set up a percent proportion…= 126 people)

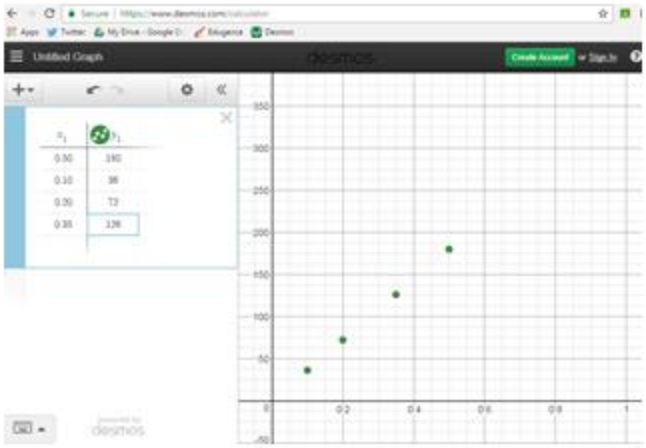
How could we turn this into a building block for linear thinking?
Let's make a table (percent, number of people), but let's add a process column.
What would it look like if we graphed (percent, number of people)?
Before we make that leap, let's represent this information using a double number line. (0 to 100% and 0 to 360)

<u>REFLECTION</u>: Some might categorize this as a computational problem, but it's really about proportionality. The double number line could help us deepen that proportionality connection, which can help with later understanding of linearity. Jot down some notes on the back of your task sheet.

Lap #3 - Double Number Lines

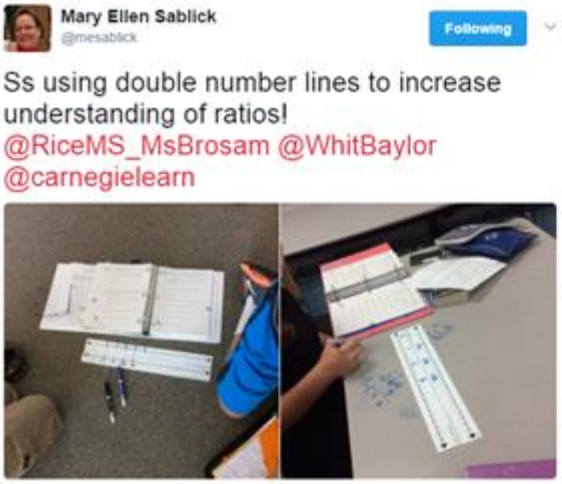
Lap #3 - Double Number Lines

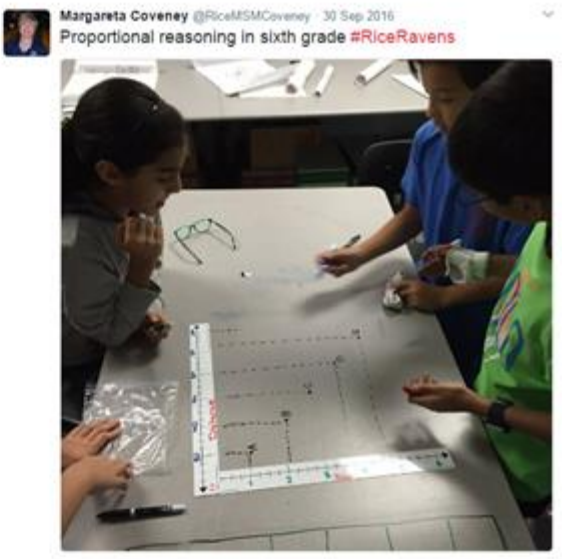




Rotate one of the number lines 90 degrees and have students plot COORDINATING values (coordinates).
Could also use Desmos.com to plot corresponding values from the double number line – put (percentage, portion of people) into a table and examine what the graph looks like.
What would you use for the x-values? The decimal form of the percentage, or the amount out of 100? Why? (importance is in the conversation about what each axis represents)
What does the slope represent in this graph? (not that we would necessarily explore that with 7th graders)

Double Number Lines





Some 6th grade Plano ISD teachers showing how they use double number lines to help students explore equivalent ratios and proportionality.

Lap #4

Jimmy was selling cars. His first month car sales earned $1500 in profit and he made $450 in commission. His second month his sales earned $3200 in profit and he made $960 in commission. His third month his sales earned $4000 in profit. How much did he make in commission the third month?

How would you solve this problem? Probably use a proportion? (ANSWER = 1200)

How could we turn this into a building block for linear thinking?

What would this situation look like in a table? What conversations could we have with students around a process column?

What might be a benefit of representing this situation graphically? What would the slope represent?
How could this help students verify and make sense of their answer?

How would it help us identify the percent commission? (slope = 0.3, which means he gets 30% of the profits)

NCTM Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics: Grades 6-8

Essential Understanding 1. Reasoning with ratios involves attending to and coordinating two quantities.

Essential Understanding 8. A rate is a set of infinitely many equivalent ratios.

Essential Understanding 10. Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.

Chapter 1 gives us 10 essential understandings related to proportional reasoning:
EU 1 - We can use multiple representations to further explore these two coordinating quantities.
EU 8 - This includes that rate we call slope. We must help students develop an understanding that a slope can be transformed into an infinite number of equivalent ratios, which helps identify more data in our linear relationship.
EU 10 - So we must help students explore proportional relationships more deeply.

What might thwart this type of thinking?

In response to the Essential Understandings presented by NCTM, what might thwart this type of thinking for students?

Nix the tricks – Earlier grades need to stop using “butterfly” (cross multiplication)

Constant of Proportionality vs Slope

What is the difference?

$$k = \frac{y}{x} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

BONUS: 8th grade, this is for YOU!
What's the difference - IS there a difference?

Let's just consider their "formulas".
They are both ratios. They both represent the rate of change between x and y.
BUT are you teaching that these are synonymous? What's the danger in that?
(proportionality exists in every linear relationship as the rate of change between x and y. It just so happens that this rate of change is equivalent to y/x when we have a proportional linear relationship)

Examples of misconceptions:
If there is a slope --> then it is proportional, or linear=proportional
Are coefficient and slope synonymous? (NO- Later students will put linear equations in other forms, like standard form.)

Session Learning Goals (revisited):

Increase awareness of how early explorations of proportionality contribute to students' understanding of linearity

Equip teachers to build on students' prior experiences and knowledge to deepen connections between proportionality and linearity

Reflection - As a result of our explorations...

What might you stop doing?

What might you start doing?

What might you do differently?

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