The following pages contain student and teacher resources for a lesson that you can use with your students. More information can be found at www.mathandteaching.org.

CATEGORICAL DATA

Summary (Ready)	Goals (Set)
We will use two-way tables to display the frequencies and relative frequencies of categorical data. We will examine patterns of association in bivariate categorical data. We will draw conclusions about possible associations.	• Construct two-way frequency and relative frequency tables using bivariate data. • Interpret two-way frequency tables. • Conduct a survey.

Warmup (Go)

Numerical data is data consisting of numbers.

Below are some data about Ms. Robertson's 8th grade math class.

- Number of boys……..…. 12
- Number of girls…… 18
- Number of students who ate breakfast this morning……. 20
- Number of students who passed the math test today..…. 25

Use the data to answer the questions below.

1. How many students are in the class? n = _____

2. What percent of the students in the class are boys? _____

   What percent of the students in the class are girls? _____

   Why must the sum of these percents equal 100%?

3. What percent of the students ate breakfast this morning? _____

   What percent of the students passed the math test today? _____

   Why is it possible that the sum of these percents doesn't equal 100%?

WHAT IS CATEGORICAL DATA?

Categorical data is data sorted into categories, such as colors, ranges of measurements, or other attributes of the data. Generally there are only finitely many categories.

1. For each topic below, list categories for organizing collected data. Then create a possible survey question that you could use to collect data for each category.

Topic	Possible Categories	Possible Survey Question(s)
Example: Hair color	Black, brown, blonde, red, other	What color is your hair? OR Do you have dark hair?
Music		
Art		
Fruit		
Ecosystems		

2. If you were reading a table that had categorical data labeled “swimming,” “football,” and “basketball,” what might be the larger topic that is being explored?

3. Choose a topic that you could collect data about and list possible categories related to that topic.

   Topic: _____

   Categories:

CATEGORICAL QUESTIONS

Categorical survey questions are used to collect categorical data. These questions usually have word responses. For example, "Do you own at least one dog?" is a categorical survey question because the answer is either "yes" or "no."

Numerical survey questions are used to collect numerical data. Numerical data sometimes comes from counting. It can sometimes come from measurements. These questions usually have number responses. For example, "How many dogs do you own?" is a numerical survey question because the answer is a number like zero or five.

State whether each question in the table below is a categorical or numerical survey question.

Question	Categorical (C) or Numerical (N)?
1. Are you male or female?	
2. What is your favorite color?	
3. How many dress shirts do you own?	
4. Are you an only child?	
5. How many siblings do you have?	
6. What were the class scores on the test?	
7. What types of birds did we observe today?	
8. What are the average family incomes of different cities in my state?	

9. State whether the question is categorical or numerical. Then rewrite the question in the other form.

   "How many video games do you own?" Type: _____

   Rewritten in other form:

10. Create a categorical question and then rewrite it as a numerical question.

BIVARIATE CATEGORICAL DATA

Bivariate data is data that has two variables based on the same population.

The results of an 8th grade survey about favorite color are below.

	Pink	Red	Blue	Purple	TOTAL
Boys	2	10	10	3	
Girls	8	2	3	7	
TOTAL					

1. Complete the table by finding totals for the rows and the columns.
2. How many students total were surveyed? n = _____
3. How many girls were surveyed? _____
4. What percent of girls preferred pink? _____
5. How many boys were surveyed? _____
6. What percent of boys preferred pink? _____
7. How many students preferred pink? _____
8. What percent of the students who preferred pink are girls? _____
9. What percent of the students who preferred pink are boys? _____
10. Compare the **questions** from problems 4 and 8. How are they different?

11. Compare your **answers** from problems 4 and 8. Why are they different?

12. The table above is called a two-way table. Explain what you think a two-way table is in your own words.

CHORES AND CURFEWS

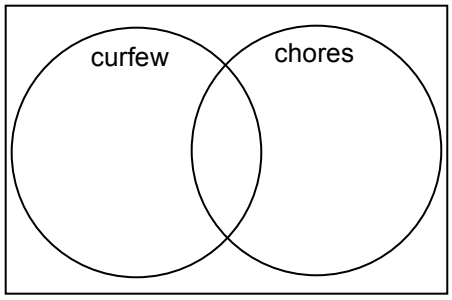
Ten different 8th graders were asked the following questions:

- Do you have a curfew?
- Are you assigned chores at home?

Data was collected on their responses and recorded in the table below.

	Students A through J									
	A	B	C	D	E	F	G	H	I	J
Curfew	Yes	No	No	Yes	Yes	No	Yes	No	No	Yes
Chores	Yes	No	Yes	Yes	No	No	Yes	Yes	No	Yes

1. Use the sections in the Venn diagram to record the number of students in each category. Be sure to include the number of students with neither chores nor curfew outside of the circles.



2. How many students were surveyed? _____
   This is called the number of observations (*n*) or sample size for the survey.

3. How many students surveyed had chores? _____

4. How many students surveyed had a curfew? _____

5. What percent of students had both chores and a curfew? _____

6. What percent of students had neither chores nor a curfew? _____

7. What percent of students who had chores also had a curfew? _____

8. What percent of students who had a curfew also had chores? _____

9. Madhav thinks that most students have both a curfew and chores. Does the data support Madhav's claim? Explain.

TWO-WAY FREQUENCY TABLES

A frequency table is a table that lists items and the number of times they occur in a data set.

A two-way frequency table is useful for displaying bivariate categorical data.

1. Use the data on the previous page to complete the table.

	Students with **Curfew**	Students with **No Curfew**	**TOTAL**
Students with **Chores**			
Students with **No Chores**			
TOTAL			

Based on this table:

2. Circle the number that indicates the sample size for the whole survey.

   What percent of all students had chores? _____

3. Draw a square around the number that indicates the total number of students who had chores.

   What percent of students who had chores also had a curfew? _____

4. Draw a triangle around the number that indicates the total number of students who did not have a curfew.

   What percent of the students who did not have a curfew also did not have chores?

   _____

5. Draw a parallelogram around the number that indicates the total number of students who had a curfew.

   What percent of students who had a curfew also had chores? _____

6. Raji thinks that most students who had chores were more likely to have a curfew. Does the data support Raji's claim? Explain.

RELATIVE FREQUENCY TABLES

A frequency table with raw data can be used to create a relative frequency table that contains percents. The strength of any association or relationship between two variables can be easier to describe using percents.

1. Here is one way to create a relative frequency table about curfews and chores. Use data from the previous page to complete the table.

	Table 1: Curfew and Chores		
	Curfew	**No Curfew**	**TOTAL**
Chores (*n* = _____)			100%
No Chores (*n* = 4)	$\frac{1}{4}$ = 25%		100%

2. Here is another way to create a relative frequency table about curfews and chores. Use data from the previous page to complete this table.

	Table 2: Curfew and Chores	
	Curfew (*n* = 5)	**No Curfew** (*n* = _____)
Chores		
No Chores	$\frac{1}{5}$ = 20%	
TOTAL	100%	100%

3. Circle the percent of students with both curfews and chores in each table. Why is this percent different in the two tables?

4. Raji thought that students who had chores were more likely to have a curfew as well. Do these relative frequency tables support her claim? ___ Explain.

5. Barbara states, "Of the students who were surveyed, students without curfews are less likely to have chores compared to those with a curfew."

   Draw squares around the percents that show the association between not having a curfew and not having chores. Then determine if Barbara's reasoning is correct.

CATEGORICAL DATA

These are some notes and suggestions for how the lesson might be taught in your classroom. It is offered as a suggested pathway, not necessarily as a script. More information can be found at www.mathandteaching.org.

Summary

Students use two-way tables to display the frequencies and relative frequencies of categorical data. Students examine patterns of association in bivariate categorical data. Students draw conclusions about possible associations.

Goals

- Construct two-way frequency and relative frequency tables using bivariate data.
- Interpret two-way frequency tables.
- Conduct a survey.

PREVIEW / WARMUP

Whole Class

Student Page 0
Categorical Data

Lesson 10.1 contains challenging and subtle concepts, and may require more direct instruction and guidance than other lessons in this program.

- Introduce the goals and standards for the lesson. Discuss important vocabulary as relevant. Students complete the warmup activity.

  Why must the percentages of boys and girls in the class total 100%? Because all (100%) of the students are either boys or girls.

  Why doesn't the sum of the percentage of students who ate breakfast and the percentage students who passed the math test have to total 100%? Because it is possible for some students to have breakfast and also pass the math test. Students can be counted in both categories.

- At the conclusion of the warmup exercise, make it explicit to students that the prior knowledge they used will be extended to new knowledge in this packet.

INTRODUCE 1 / EXPLORE 1

Whole Class/ Partners

Student Page 1 What is Categorical Data?

Student Page 2 Categorical Questions?

- Explain the meaning of categorical data, and use hair color as an example.

  If we wanted to survey people about music preferences, what are some categories we could use to organize our data? Answers will vary, but some categories could include rock, rap, country, etc.

  What are some survey questions we might ask? Answers will vary, but some questions might be: What is your favorite music? What do you like to listen to on the radio? Where do you listen to music the most?

- Students complete the rest of the table with a partner. Then ask students to share responses with the rest of the class.

  Is the question "Do you like art?" a categorical question? Yes. ***What would the categories be?*** "Yes." "No." Some of the categorical questions later in the lesson have "Yes/No" responses.

- Students answer the open-ended questions below the table before sharing responses.

  > Demographics (statistics relating to populations of people) could be an interesting topic for discussion to promote critical thinking for all students. ***"How are people organized into categories?"*** may be a worthwhile question to ask your students depending on their maturity and social awareness. Such a question invites a discussion about the challenges and consequences of organizing people by race, gender, age, socioeconomic status, sexual orientation, etc. Students encounter demographic data in future math and non-math classes. Use your discretion and assess the culture of your classroom before entering into such a discussion. See TN5 for further discussion on this topic.

- Students distinguish between categorical and numerical survey questions.

  [Problem 9] Is the question categorical or numerical? Numerical is the obvious answer, but one could think of the answer in categories (0, 1, 2, 3, 4, etc…. or 0-2, 3-5, 6-8, etc.).

  How would you write the question as a categorical question? Possibilities include, "Do you own any video games?" or "What kind of video games do you own?" (e.g sports, puzzles, role playng games etc.)

- [Problem 10] Have students share their examples of categorical and numerical questions, and critique the clarity of the questions.

INTRODUCE 1 / EXPLORE 1 (Continued)

Whole Class/ Individuals/Partners

Student Page 3 Bivariate Categorical Data

- Review the meaning of categorical and bivariate data. Ask some clarifying questions about the table. Check to see that students can total the rows and columns correctly.

  What two topics are measured and displayed in this table? Gender in the two rows and favorite color in the four columns.

  What categories are used to describe gender? "Boy" and "girl."

  What categories are used to describe the favorite color? "Pink," "red," "blue," and "purple."

  How many students were surveyed in total? Both the "total" column and row add to 45. Remind students that we use "*n*" to represent the total population in a survey.

- Students finish the rest of the page individually or collaboratively. Support as needed.

- Students may find it difficult to identify the correct denominator when finding percents. The following prompts and problems are meant to help students become aware of how they need to read carefully to determine which number to use as a denominator.

  How are questions from problems 4 and 8 different? In problem 4, students find the percent of girls who prefer pink. In problem 8, students find the percent of "pink favorites" who are girls.

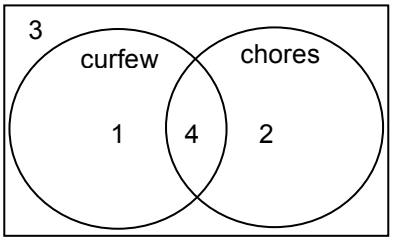
  How are the answers from problems 4 and 8 different? In problem 4, students find the percent out of the total girls (8/10). In problem 8, they find percent out of the total students who like pink (2/10)

- Ask similar questions to help students compare problems 5 and 9.

- Students share their understanding of two-way frequency tables.

Student Page 4 Chores and Curfews

- Introduce the survey questions and help students complete the Venn diagram using the data in the table. This data is used on the following page



  How many students were surveyed? 10. ***How may had chores?*** 6. ***How many had curfew?*** 5. ***What percent of students had both chores and a curfew?*** 4/10 = 40%.

  What percent of students had neither chores nor a curfew? 3/10 = 30%.

  What percent of students who had chores also had a curfew? 4/6 (about 67%).

  What percent of students who had curfew also had chores? 4/5 = 80%.

  Does the data support Madhav's claim that most students have both a curfew and chores? No, only 40% of students have both curfew and chores. However, some students might suggest (and correctly so) that there appears to be an association between the two variables and that students with a curfew are more likely to also have chores and vice versa.

INTRODUCE 2 / EXPLORE 2

Whole Class/ Partners

Student Page 5 Two-Way Frequency Tables

- Introduce two-way frequency tables as a tool for displaying bivariate categorical data. Help students fill in the table using the Venn diagram on the previous page. As students do problems, be sure they understand that each number in the "total" columns and rows can be used to represent the sample size for different subgroups. Some questions are repeats from the previous page for emphasis.

	Students with **Curfew**	Students with **No Curfew**	**TOTAL**
Students with **Chores**	4	2	6
Students with **No Chores**	1	3	4
TOTAL	5	5	10

What is the sample size of the whole survey? 10.

See TN7 for more teaching strategies to help students.

What is the total number of students who had chores? 6.

What percent of all students had chores? 6/10 = 60%
What percent of student who had chores also had a curfew? 4/6 is about 67%.

What is the total number of students who did not have a curfew? 5. ***What percent of the students who did not have a curfew also did not have chores?*** 3/5 = 60%.

What is the total number of students who had curfew? 5. ***What percent of students who had curfew also had chores?*** 4/5 = 80%.

- Students analyze Raji's claim and assess its validity using evidence in the table.

[Problem 6] Does the data support Raji's claim that most students who had chores had a curfew? Yes. ***Explain.*** 67% (4 out of 6) of students who had chores had curfews.

How is Raji's claim different from Madhav's on the previous page? Madhav uses all the students in his denominator. Raji is analyzing using subsets of the sample size in her denominators.

Why might there be a relationship between students having chores and having a curfew? What might it say about parenting behavior? Answers will vary.

Do you think this relationship might exist if we conducted this survey in class? Answers will vary.

INTRODUCE 2 / EXPLORE 2 (Continued)

Whole Class/ Partners

Student Page 6 Relative Frequency Tables

- Explain that it can be easier to see if inferences like Raji's are accurate by creating relative frequency tables. Students calculate the percentages for each cell in the relative frequency tables. Identifying the correct denominator (*n*) to use for each row (in Table 1) and each column (Table 2) is critical. The numerator can be found in the appropriate field in the frequency table.

Table 1: Curfew and Chores			
	Curfew	**No Curfew**	**TOTAL**
Chores (*n* = 6)	4/6 = 67%	2/6 = 33%	100%
No Chores (*n* = 4)	1/4 = 25%	3/4 = 75%	100%

Table 2: Curfew and Chores		
	Curfew (*n* = 5)	**No Curfew** (*n* = 5)
Chores	4/5 = 80%	2/5 = 40%
No Chores	1/5 = 20%	3/5 = 60%
TOTAL	100%	100%

Circle the percent that compares curfews and chores in each table. Why are the two percents that compare curfews and chores different in each table? They have different sample sizes. In Table 1, the sample size is 6 (the total students with chores). In Table 2, the sample size is 5 (the total students with curfews).

- Students explain why both Raji's and Barbara's inferences are correct and cite percents from the table as evidence. ***Does Barbara's argument support or refute Raji's argument?*** Answers will vary. But together, their arguments seem to support the idea that there is a relationship between these categorical variables.

SUMMARIZE

Whole Class

- Formatively assess student understanding of key vocabulary. Students can create a word bank on a classroom wall that lists definitions and examples of categorical and numerical data, categorical data questions, bivariate data, frequency tables, and relative frequency tables. Use examples from the lesson to assist in the discussion.

  What is bivariate data? Data that has two variables. ***What is categorical data?*** Data sorted into categories. ***What do you think bivariate categorical data means?*** Bivariate categorical consists of two attributes that are collected from each member of the sample population such as hair color and eye color of students at a school.

  How are relative frequency tables a useful tool for analyzing bivariate data? They help us see possible associations between variables.

CLOSURE

Whole Class

- Review the goals, standards, and vocabulary for the lesson.