

National Council of Teachers of Mathematics – April 2017

Session #103 - Using Manipulatives to Deepen Understanding of CCSS Fraction Progressions

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- Lesson Title:** Meaning of Fractions using Number Lines, Geoboards, and Counting Bears
- Topic:** Introduction to Fractions
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- Audience:** Pre-service and In-service teachers of elementary mathematics

Learning Goals: We created the following learning goals to align with the Common Core State Standards about Numbers and Fractions (most are pulled from grade three with one grade two standard). We used a progressions document on fractions in grades 3-5 to create these learning goals along with our own experiences teaching mathematics content and methods courses for prospective elementary school teachers.

https://commoncoretools.files.wordpress.com/2011/08/ccss_progression_nf_35_2011_08_12.pdf

1. Meaning of fractions

a. Same sized pieces (Partitioning and shape vs. value):

Understand the fraction $1/b$ as the quantity formed by 1 part when the whole is partitioned into b equal parts (3.NF.A.1). Recognize that equal shares of identical wholes need not have the same shape (2.G.A.3).

b. Iterating unit fractions:

Understand the fraction a/b as the quantity formed by a parts of size $1/b$. In other words, a indicates the number of copies of the unit fraction $1/b$ (3.NF.A.1).

2. Different representations of fractions

Understand three diagram models for representing rational numbers: linear, area, and discrete. In other words, the whole can be a line segment, a shape (circle, rectangle, etc.), and a collection of objects, or any one finite entity susceptible to subdivision and measurement (3.NF.A.2).

3. Specifying the whole

Understand that the value of a fraction is dependent on the whole. In other words, the size of the pieces depends on the referent whole. For example, the fraction $\frac{1}{2}$ can be placed between 0 and 1 on a number line, and is different than finding $\frac{1}{2}$ of a number ($\frac{1}{2}$ of 2, for instance).

Background

This lesson was written with several goals in mind. First, we were interested in facilitating a collaboration between mathematics methods and content instructors. This would necessitate collaboration across departments and colleges. Second, we were interested in learning more about co-teaching strategies. We selected *lesson study* as a vehicle to reach these goals, which is a collaborative process involving goal setting, planning, teaching and observing, and debriefing on a research lesson.

This lesson plan is the product of our lesson study on an introductory fractions lesson. This cycle of lesson study involved two rotating instructors co-teaching a version of this lesson two times during the Spring of 2016 and four times during the Fall of 2016. After the teaching of the lesson, instructors alongside observing colleagues from three different departments (elementary and bilingual education, mathematics, and secondary education) debriefed for one to two hours on changes they would make to the lesson if they were to teach it again. The lesson plan is the fourth draft of our introductory lesson on understanding the meaning of fractions. It captures the accumulation of all changes made to our initial lesson plan.

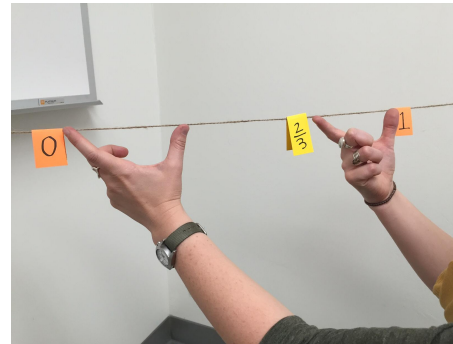
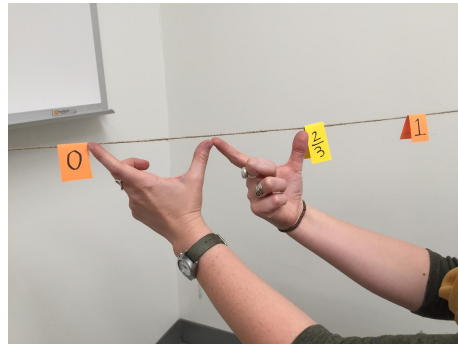
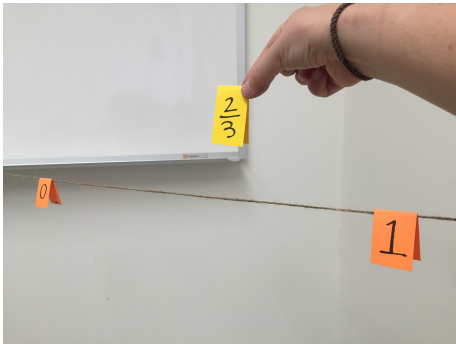
Materials

Launch:

- nametag stickers (one per student),
- photocopies of half-sheet “Warm Up” prompt (one per student)

Explore Station 1 – number line:

- approximately 3-5 long pieces of string tied or taped in one corner of the room (one per subgroup),
- printed & cut sets of fraction tents (one set per subgroup; make the tents based on the “Station 1: Number Line” handout),
- photocopies of “Station 1: Number Line” handout (one per student)



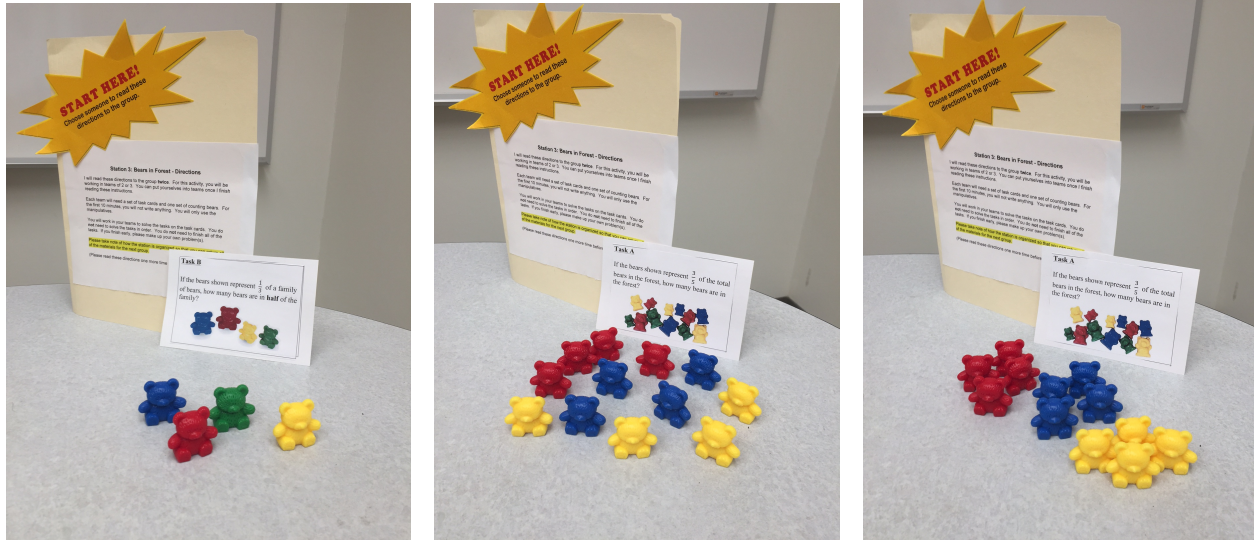
Explore Station 2:

- geoboards (approximately $\frac{1}{3}$ as many as the number of students in the class),
- sets of rubber bands (approximately $\frac{1}{3}$ as many sets as the number of students in the class),
- photocopies of “Station 2: Geoboards” handout (one per student),
- several pre-constructed geoboards to facilitate discussion (see “Ready-made boards” image in the table below)



Explore Station 3:

- one printed copy of the directions (consider taping to a folder and standing up at the station),
- approximately 3-5 sets of task cards (one per subgroup, see “Bears in Forest - Tasks” handout),
- several sets of counting bears (one per subgroup),
- photocopies of “Station 3: Bears in Forest - Solution Sheet” (approximately 2 times the number of students in the class)



Discussion:

- sets of role cards (one per subgroup, see “Discussion - Group Roles” handout),
- printed copies of group discussion prompts (one per subgroup, see “Discussion Prompt” handout)

Launch

Timing: 5 minutes

Co-teaching Model: Team teaching / One teach, One Assist

Description: Instructor A (note that Instructor A refers to the instructor of record for the course) will introduce the lesson, briefly introduce the co-teaching project, and introduce Instructor B (note that Instructor B refers to the second co-instructor, who is not the instructor of record for the course). Any other observers will briefly introduce themselves (name and role/department).

Instructor A will have written the following question on the board as a warm up. Students will begin to answer the question on the half-sheet of paper as they enter the room:

“If you were asked to **explain what a fraction is** to your younger cousin or sibling, what would you say? Jot down any ideas/words/pictures/contexts that come to mind.”

Give students 1-2 minutes into the period to finish. The students should retain the papers for revisiting in the closing.

Transition to Stations: Instructor A will tell students that they will be rotating to three different stations. “Please make and wear a nametag sticker, place belongings along the side of room (clear off the tables), and bring a pencil.” Assign students to each station and explain that they will be rotating clock-wise.

Instructor A will say that this lesson was designed intentionally to target particular Common Core standards on fractions.

Explore

Timing: 45 minutes (15 minutes per station)

Co-teaching Model: Station teaching

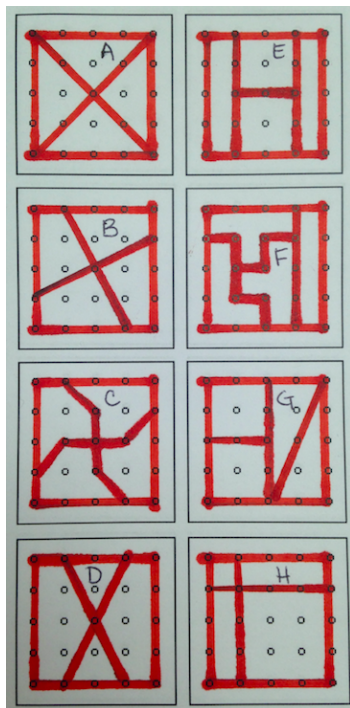
Description: During the explore part of the lesson, groups of approximately 10-12 students will rotate through each of three stations. They will remain at each station for 15 minutes. Throughout the rotation, students will be exposed to different representations of fractions (linear, area, and discrete) and different key concepts of fractions to address the learning goals.

Instructor Does	Students Do	Tips for Instructors
<u>Station 1: Number Line Station</u> Leader: Instructor A “How can this piece of string be used with fractions?” - Instructor models the placement of the first two numbers on the number line (see the first task on the handout) and explains that the students are to precisely place the third fraction based on the locations of the first two. Do not solve the first task for the students. State the importance of placing the fractions in the order on the	<u>Station 1: Number Line Station</u> - Students reflect on where they've seen number lines before in real life. - Students work in partners or triads placing fractions on a number line. - Students play with placing unit fractions and copies of unit fractions on number line, specifying where the whole is. - Students do this for ten minutes. - In last five minutes, students reflect by connecting actions with physical number line to symbolic	<u>Station 1: Number Line Station</u> - Before the lesson, tape the number line (or tie it to available objects) so that students don't have to hold it and miss out on placing the fractions. - Write the name of the fraction on both sides of the fraction tent so that it is easy to see. - Make fraction tents that are long so that they can stay securely on the line. - Students may question whether they can place the third number

handout. - Clearly instruct students to CLEAR the number line after each prompt and start fresh - Students form groups of triads or fours. - Instructor gives one sheet per group. - Instructor will allow groups to work at their own pace and rotate to probe with particular questions as necessary. “Does it matter where we place the two fractions, e.g. 0 and 1?” “Can you show me as precisely as possible where my whole is if this is where (unit fraction) is?” “How do we decide where numbers go?” (brainstorm where they see ordered numbers: temperatures, elevators, reading a book left to right, rulers) “So convention is that the smaller numbers are on the left and larger numbers are on the right).” - Instructor measures unit fractions and iterates it to see if placements are as accurate and precise as possible. - Instructor relates to math practice #6 “attention to precision”. - Instructor will rephrase or revoice what students say to try to use mathematical vocabulary related to learning goals (e.g. unit fraction, iterating, partitioning, equal parts,	representations of number line and fractions on a sheet of paper. Students will turn in their handouts, even if unfinished, before rotating to the next station.	outside of the first two. - Students may place $\frac{1}{2}$ (when given 0 and 2 on a number line) at the location of 1. This happens when students take $\frac{1}{2}$ of 2, instead of $\frac{1}{2}$ of 1. This is the result of a referent unit error. - Students may place $\frac{3}{4}$ (when given 0 and $\frac{1}{4}$) at $\frac{3}{4}$ of $\frac{1}{4}$, which is $\frac{3}{16}$. This happens when assuming a referent whole equivalent to $\frac{1}{4}$ of the fraction $\frac{3}{4}$, instead of assuming a referent whole equivalent to 1. - Students may place $\frac{5}{8}$ to the left of $\frac{3}{4}$ rather than to the right, implying that $\frac{5}{8}$ is less than $\frac{3}{4}$. Teacher can encourage them to think about how much less than 1 whole each fraction is. - Students may order all numbers first on the line and disregard the sequence of tasks. - Teacher should encourage students to “Clear their number line” after each task. Students might ask if they can use other fractions other than the three given in each task, or if they can use blank spots to serve as place holders. This is fine. - Teacher should make sure that students place the fractions in the sequence given. For instance, if the prompt says, “here is 0, here is
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...) • Instructor may challenge students to place fractions without the same denominator on number line. • Instructor can explain the grade levels where students learn content. • Instructor will do this for ten minutes (see handout for particular problems). • Last five minutes of station devoted to recording explorations on a sheet. Stress that students do not need to finish the entire sheet, nor do they need to answer the problems in order. • Students place semi-completed worksheets in envelope.		5/3, show 1,” make sure that the student places 0 and 5/3 first, and then shows 1. Some students placed 0 and 1 first, and then placed 5/3. This lowers the cognitive demand of the task; order was intentional. • Remind students to place their solution sheets in the envelopes and that they need not write solutions to all tasks. (Note: We found that if the students rotated with their handouts, they felt pressure to finish the handout from the previous station rather than fully engaging in the next station).
<u>Station 2: Geoboards Station</u> Leader: Instructor B • Instructor begins by allowing students to freely explore with geoboards and rubber bands. • Instructor will have several ready-made geoboards (see image) to highlight key concepts. Ideally, the instructor will draw from student-produced boards; the ready-made boards are prepared as a back-up. • “How can these rubber bands and geoboards be used with fractions?” • Instructor directs students to break	<u>Station 2: Geoboards Station</u> • Students experience playing with rubber bands on geoboards. • Students then try to find one way to break 16 squares on geoboard into four pieces. • Students discuss whether four pieces need to be equal or not. • Students discuss the name of each of those four equal pieces to be “fourths” or “one-fourth”. • Students find another way to partition geoboard into four equal parts. • Students construct arguments as to how they know their fourths are	<u>Station 2: Geoboards Station</u> • A main goal of this station is to highlight the difference between “four pieces” and “fourths.” When instructed to break the geoboard into four pieces, any of the geoboards depicted in the left column are valid. However, when instructed to break the geoboard into fourths, the four pieces must be equal (must have equal areas) such as geoboards A, B, C, E, F, and G. Geoboards D and H represent four pieces but do not represent fourths. • Be on the lookout for students who

the geoboard into four parts. Listens to see whether or not students break them into equal parts. • If the parts are equal, get students to refer to them as “fourths”, confirming that one equal sized piece of four equal sized pieces is called “fourths” or “one-fourth”. If student says “equal fourths”, instructor will press on this redundancy to clarify that a fourth is a fourth when it is one of four equal valued pieces. Be clear about why the phrase “equal fourths” is redundant. • Instructor continues to facilitate conversations of students as they explore ways to show fourths. Ideally, the instructor uses student-produced boards to facilitate the discussion. Ready-made boards are available, if needed (see image). • Last five minutes of station devoted to recording explorations on a sheet (there may or may not be time for this). • Students place semi-completed worksheets in envelope. <i>Image: Possible ready-made boards to facilitate discussion (Note: some represent fourths A, B, C, E, F, G and some do not D, H)</i>	equally sized and of size $\frac{1}{4}$. • In last five minutes, students reflect by connecting actions with geoboards to symbolic representation of geoboards on paper.	say “equal fourths.” It is important to identify “equal fourths” as redundant because to say “fourths” is to say “four equal pieces.” Therefore, to say “equal fourths” is redundant and unnecessary. This may foster a misconception so it is important to address it. • Many students will relentlessly want to count the number of interior pegs as an indicator of whether the pieces are equal. Turn to geoboard E pictured in the left column as a counterexample to this conjecture. Reinforce that the geoboard represents an area model so the pieces are equal if they have equal areas (and the number of interior pegs are not a measure of area). • Think about using the word congruent to tease apart some of the distinctions between “equal in appearance” to “equal in value” • Note: During some iterations of the lesson, there was plenty of time for the handout. During other iterations we did not get to it at all. The handout can always be assigned for homework. • Optional: For those students interested in understanding how the perimeter of a polygon relates to the number of pegs in the interior of the polygon, check out
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Station 3: Counting Bears Station

Leader: Independent station

- This is an independent station, so the instructor must have the instructions, task cards, and counting bears set up before the students arrive to the station.
- See handout for particular problems.
- Students will work without the solution sheets for the first 10 minutes. When there are 5 minutes remaining, an instructor

Station 3: Counting Bears Station

- One student will read the posted instructions to the entire group.
- The group will split into pairs or threes.
- Sub-groups will follow the instructions found on task cards.
- Students will use counting bears as a discrete model to solve several tasks that relate a quantity of bears to a fraction, and then finding another quantity with that information

Pick's Theorem:

<http://illuminations.nctm.org/Unit.aspx?id=6504>

- Remind students to place their solution sheets in the envelopes and that they need not find nine different ways to represent fourths (Note: We found that if the students rotated with their handouts, they felt pressure to finish the handout from the previous station rather than fully engaging in the next station).

Station 3: Counting Bears Station

- It is important to give the students 10 minutes to solve the tasks *without paper*. In the first teaching of this lesson, students were given paper right from the start. We found that the students felt a need to solve each task symbolically; some groups never touched the manipulatives. In the second teaching of the lesson, we did not provide paper for ten minutes and each and every group touched the

must visit this station and provide the solution sheets. Students place semi-completed worksheets in envelope.	(e.g. if the following amount of bears represents $\frac{3}{4}$ of all the bears in the forest, show the total number of bears or show $\frac{1}{4}$). - Students will work on the tasks for 10 minutes without solution sheets (no paper). - When 5 minutes remain, the students will be given solution sheets and asked to work out their solutions on paper. Students will place their solutions sheets in the envelopes.	manipulatives. We found that this encouraged more conversation and better conceptual explanations. When five minutes remain, solution sheets are provided and the students are given the option to write out their solutions to some/all of the tasks. Remind students to place their solution sheets in the envelopes and that they need not write solutions to all four tasks, nor do they need to solve the tasks in order (Note: We found that if the students rotated with their handouts, they felt pressure to finish the handout from the previous station rather than fully engaging in the next station).
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Discussion

Timing: 25 minutes

Co-teaching Model: One teach (Instructor A), one assist (Instructor B)

Description: The students will be seated in groups of 3-4. Instructor A will provide instructions for facilitating the group discussion. The students will discuss the prompt in their groups with each group member taking an assigned role. Following the group discussion, Instructor A will lead a whole-class share out while Instructor B takes notes of key points on the board or a poster.

Instructor Does	Students Do
- Instructor A will explain that each group member will play a role in the small-group discussion. - Both instructors will rotate the room, listening to group	- The students will each take a role card and will read over his/her role. - Students in different roles will facilitate different aspects

discussions. Instructors will have their own copy of the handout for the purpose of taking notes on selecting and sequencing groups' solutions. • While rotating the room, the instructors will search for the targeted student responses listed below. These groups will be called upon during the whole-class share aloud. • Instructor A will call the attention of the whole class and lead a whole-class share aloud • During this share aloud, Instructor B will take notes of key points by typing into a projected table (see below). • Instructor A will conclude the class by summarizing key points	of the discussion ○ The <i>scribe</i> will take notes for the group ○ The <i>spokesperson</i> will speak for the group during the share aloud ○ The <i>facilitator</i> will keep the time and read the question prompts ○ The <i>manager</i> will be certain that everyone is following their roles and that everyone is given equal opportunity to speak ○ (Note that for a group of 3, one person can serve the role of both <i>facilitator</i> and <i>manager</i>) • Students will discuss the learning goals in their small groups • After the small group discussion, students (specifically the <i>spokesperson</i> for each group) will participate in a whole class share-aloud.
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Table for whole-class share aloud

Learning Goal	Number Line	Geoboards	Counting Bears
1a. Meaning: same-sized pieces			
1b. Meaning: iterating unit fractions			

2. Different representations			
3. Specifying the whole			

Exit Ticket

Now revisit the warm-up question. Answer with a different color pen or on a new sheet of paper:

“If you were asked to **explain what a fraction is** to your younger cousin or sibling, what would you say? Jot down any ideas/words/pictures/contexts that come to mind.”

Handouts (see Handouts document)