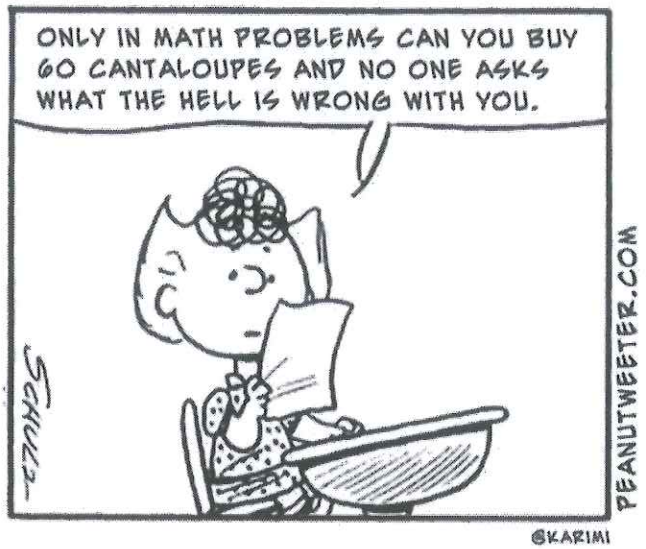
Wait…..
We Didn't Do An Example Like THAT!



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#NCTMAnnual
April 2017

5 Keys to Creating Better Learning Opportunities

(1) Withhold information
(2) Don't answer every question they ask
(3) Increase the difficulty - raise the bar
(4) Make it realistic - make it matter!
(5) Make learning active (Desmos, QR Code Scavenger Hunts, etc)

Examples for the Classroom

Class Example - "Stay Composed"

Let $h(x) = f(g(x))$. Fill in the table below so that the values of each expression can be found. No, you all don't have to have the same answer.

x	$f(x)$	$g(x)$	$h(x)$
-3			
-1			
4			
7			

Once you have what you think you need, write the answer somewhere your partner can't see. Then give this paper to your partner. Ask them to use your table to evaluate each problem below.

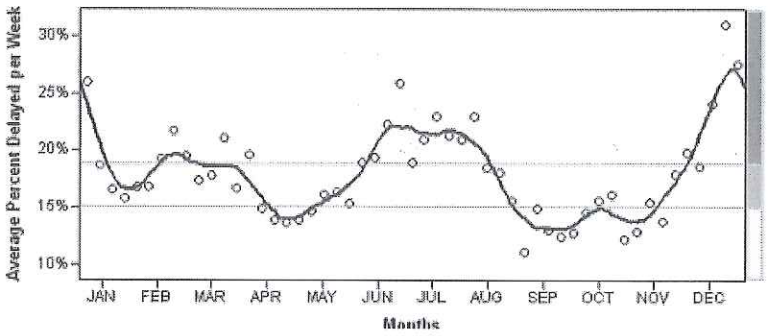
a. $f(g(-1)) =$ b. $h(f(7)) =$ c. $g(h(4)) =$

Class Example - What is Domain? Let's be Realistic!

If you usually give this.....	Try this.....
$f(a)=\frac{4}{a-7}$	$f(a)=\frac{4}{\sqrt{a-7}}$
$f(x)=\frac{1}{x+3}$	$f(x)=\frac{\ln x}{x+3}$
$y=e^x$	$y=\frac{e^x}{x^2-1}$
$y=\log(x+5)$	$y=\frac{e^x}{\log(x+5)}$

Or how about this.....

The graph below shows the percentage of flights delayed each month for the previous year. Indicate the domain and range for the data shown.



Class Example – Next Level SOH CAH TOA

Planes Above

Student Instructions

- Go to www.wolframalpha.com
- Type in "planes above" in the search bar
- You will receive a list of the planes traveling in your area (based on IP address)
- Suppose you were on the coding project for this feature. Verify the slant distances using trig ratios!
- Bonus: Refresh after 5 minutes and calculate the speed of the airplane!

Performance Assessment Tasks

You Wanna a Piece of This?

Flat Tax

The federal income tax laws are very complicated. There have been various proposals to simplify the system. Many of these proposals are often referred to as a flat tax. To most people, a flat tax implies a single tax rate. This is usually not the case in most of the proposals. For example, a tax proposal endorsed by 1996 presidential candidate Steve Forbes proposed that a person would be taxed at a rate of 17% only on the income they earned above $13,300. The first $13,300 would not be taxed at all.

1) Why is Steve Forbes flat tax a piecewise function?
2) Give an equation for Steve Forbes' flat tax where income is the input (x) and the amount of the federal income tax is the output (y). What is the domain for your function?
3) How much would a person earning $20,000 per year pay in federal income tax? What percent of that person's income is used to pay federal income tax?
4) How much would a person earning $40,000 per year pay in federal income tax? What percent of that person's income is used to pay federal income tax?
5) You should see from your answers to questions 3 and 4 that the percent of income paid in federal income tax is not the same for everyone since you are only taxed on what you make above $13,300. Give a formula for the function where the input is the income of an individual and the output is the percent of that income that goes towards the federal income tax. What is the domain and the range of this function?

NOT a "Piece"ful Winter

During the post-Christmas blizzard, snow fell at a remarkable rate and accumulated very rapidly. The final volume total was a staggering 330 cubic feet! The rate, $f(t)$, in cubic feet per hour, at which you and your friends remove snow from the school parking lot at time t hours after midnight is modeled by

$$f(t)=\begin{cases}0, & 0\le t<8\\ 112, & 8\le t<9\\ 98, & 9\le t\le 11\end{cases}$$

1) Explain the significance of each piece of this equation. Discuss the meaning of each number. Consider why the rate was lower for the last piece of the equation.
2) Let $r(t)$ represent the total amount of snow, in cubic feet, that you and your friends have ***removed*** from the parking lot at time t hours after midnight. Construct $r(t)$ as a piecewise-defined function with domain $0\le t\le 11$.
3) Did you and your friends remove all of the snow from the parking lot by 11 am? Justify your response. If not, how much snow remained at 11 am?

Ben Franklin's Will

The Franklin Technical Institute of Boston owes its existence to a provision in a codicil to Benjamin Franklin's will. In part the codicil reads:

I wish to be useful even after my Death, if possible, in forming and advancing other young men that may be serviceable to their Country in both Boston and Philadelphia. To this end, I devote Two thousand Pounds Sterling, which I give, one thousand thereof to the Inhabitants of the Town of Boston in Massachusetts, and the other thousand to the inhabitants of the City of Philadelphia, in Trust and for the Uses, Interests and Purposes hereinafter mentioned and declared.

Franklin's plan was to lend money to young apprentices at 5% interest with the provision that each borrower should pay each year along.

...with the yearly interest, one tenth part of the Principal, which sums of Principal and Interest shall be again let to fresh Borrowers...If this plan is executed and succeeds as projected without interruption for one hundred Years, the Sum will then be one hundred and thirty-one thousand Pounds of which I would have the Managers of the Donation to the Inhabitants of the Town of Boston, then lay out at their discretion one hundred thousand Pounds in Public Works...The remaining thirty-one thousand Pounds, I would have continued to be let out on Interest in the manner above directed for another hundred Years...At the end of this second term if no unfortunate accident has prevented the operation the sum will be Four Millions and Sixty-one Thousand Pounds.

It was not always possible to find as many borrowers as Franklin had planned, but the managers of the trust did the best they could. At the end of 100 years from the receipt of the Franklin gift, in January 1894, the fund had grown from 1000 pounds to almost 90,000 pounds. In 100 years the original capital had multiplied about 90 times instead of the 131 times Franklin had imagined.

(a) What annual rate of interest, compounded continuously for 100 years, would have multiplied Benjamin Franklin's original capital by 90?

(b) In Benjamin Franklin's estimate that the original 1000 pounds would grow to 131,000 in 100 years, he was using an annual rate of 5% and compounding once each year. What rate of interest per year when compounded continuously for 100 years would multiply the original amount by 131?

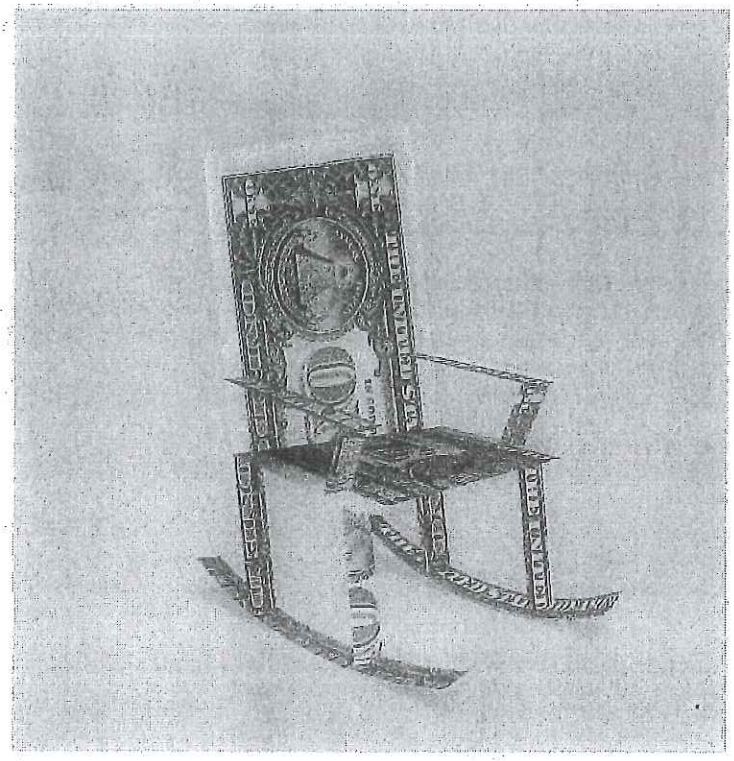
(c) Assuming that Franklin was correct in his use of an interest rate of 5%, how much would his initial gift of 1000 pounds in 1794 be worth today (in pounds)?

(d) Look up the current conversion rate from pounds to dollars. Calculate the amount found in (c) to dollars.

THE BUILDING BLOCKS FOR YOUR RETIREMENT

Saving for your future isn't quick. But with these tips, it can be easy

BY KATHY KRISTOF



ROCKING CHAIR LIFESTYLE "The goal is to get up in the morning and work on whatever project you want," says J. Money. "If you save enough, you can do that."

WHEN THE BLOGGER AND personal finance expert known as J. Money started saving for the future, retirement was the furthest thing from his mind. He was in his late 20s, working for a tech startup, and thinking about buying a house and getting married. Now, roughly a decade later, he runs his own company and extols the life-altering virtues of saving prodigiously and starting young.

Money's Washington, D.C.–based business, which publishes his ad-supported blogs Budgets Are Sexy and Rockstar Finance, earns $60,000 to $160,000 annually, depending on how hard he chooses to work, he says. Money, who uses a pen name to protect his financial privacy, started publicly tracking his net worth on his first blog back in 2008. At the time, he had just $56,000 in retirement savings. Today, he has nearly 10 times that amount, and he is on track to become almost a millionaire by his 45th birthday. That's probably not enough for a full retirement, of course, but it's a more than ample cushion to have by that age—and he's already reaping the benefits of his planning ahead.

The savings Money built up in his 20s gave him the wherewithal to start his own business, and already have offered him a measure of financial freedom. He and his wife have two young boys; and now, when there's a lot going on with his young family, he can take time off without wondering whether the break will destroy his long-term plans.

"It's about having the freedom to wake up and do what you want to do and not think about finances," says Money. "I want to work on projects I'm passionate about without worrying about whether they're going to be profitable."

As he's found over the past 10 years of saving for his own goals and advising others how to do so, smart retirement planning boils down to a few simple truths.

TIME IS ON YOUR SIDE. USE IT

The earlier you start socking away money, the more time you give compounding to work for you. That's simply the effect of earning returns on your returns. For instance, if you began with $100,000 and earned 10 percent interest, you'd start the following year with (1) —which, at 10 percent, would add another (2) to your savings the following year before you've made your own contribution. With a small account

ILLUSTRATION BRIAN CRONIN

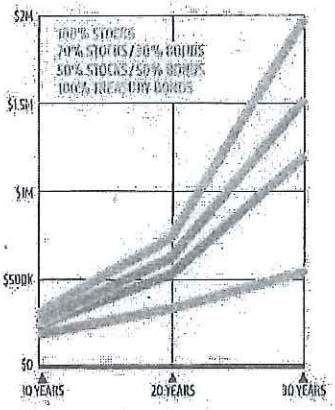
and a short time frame, compounding's impact feels marginal. But as your balance builds over decades, it does yeoman's work.

Consider Money, now 37 years old. He contributes an average of $20,000 per year, or up to 25 percent of his income: $15,000 goes into a tax-deferred retirement plan for self-employed people, known as a SEP-IRA, and an additional $5,000 of after-tax money goes into a Roth IRA. His current balance is about $500,000, thanks in part to more than $300,000 in investment returns.

And compounding is just starting to gain momentum for Money. If he stopped contributing to savings now—an unlikely possibility—he would still nearly (3) his current nest egg by his 45th birthday, assuming his investments earn about 8 percent on average. (A portfolio that's half stocks and half bonds averaged an 8.3 percent annual return during the 90-year period ending in 2015, according to Morningstar, which shares an owner with *Inc.*) If that average holds, by the time he hits age 65, Money's current investments could be worth more than (4) million. In other words, a $200,000 investment could earn you more than (4) million in returns, thanks to compounding and lots of time. If Money had started saving a decade later, he would likely accumulate (5) as much by age 65, assuming he earned the same 8.3 percent average annual return and invested the same amount. The price of procrastination: $2.5 million.

THE IMPACT OF AGGRESSIVE INVESTING (9)

Stocks are far more volatile than bonds. But if you have decades until you need your money, the rewards of investing aggressively can outweigh the risks. How $100,000 would grow based on historical returns, according to the 2016 SBBI Yearbook:



WHAT YOU'LL GET FOR WHAT YOU SAVE

(7) years is the amount of time it takes to double the value of your nest egg, assuming about an 8 percent average annual return and no added contributions.

1.27%
The annual fee you'll pay for the average actively managed mutual fund.

18.2%
The amount of the average stock investor's return that would have been lost, over the past 10 years, to that average fee.

$344,730
The amount you would lose over 30 years to that fee, assuming you started with $100,000 and averaged an 8 percent return before investment management fees were deducted.

$1,632
The amount of additional monthly retirement income that you could generate for 30 years from high $344,730. (Assumes 4 percent rate of return while in retirement.)

Sources: Morningstar, Dinkytown.net calculator

TAKE RISKS WHEN YOU'RE YOUNG

Money has earned considerably more than 8 percent on his investments and is likely to continue doing so, because he invests almost all of his retirement assets in stocks. If he were older, he'd be in more danger of losing money that couldn't be recovered. But at his age, the risk is likely to pay off (see chart, above).

The investment world measures risk by "volatility," or the amount a security's value swings up and down. By that standard, stocks are about three times more volatile than government bonds, but have historically earned nearly twice the average annual return (10 percent versus 5.6 percent, roughly). In their worst years, big-company stocks lost 43.3 percent of their value, while bonds lost 14.9 percent; a portfolio made up of 50 percent stocks and 50 percent bonds suffered a 24.7 percent loss in its worst year. However, over long stretches, stocks look far less risky. If you look at the rolling 10-year periods tracked by Morningstar, only four have produced losses. During the remaining 76 periods, stocks earned money, usually considerably more than bonds.

Those higher average returns can make a vast difference. For example, with a diversified portfolio, Money would expect to accumulate that (4) million by the time he reaches 65; but if he keeps the bulk of his wealth in stocks, the average 10 percent in market returns could boost his portfolio to (6) million.

DON'T PAY HIGH FEES FOR FANCY ACCOUNTS

Every dollar you pay to a fund manager is a dollar that can't compound in your account. The average actively managed equity mutual fund charges about 1.27 percent of assets each year, to pay managers to buy and sell stocks on your behalf, according to Morningstar. But many index funds charge just a fraction as much, because they don't hire high-priced investment managers to pick stocks. Instead, those funds buy and hold every stock that makes up a set index.

Money buys index funds, specifically Vanguard's Total Stock Market Index Fund (VTSAX), which charges a paltry 0.05 percent of assets each year. That allows him to pay about (7) less per year in investment management fees, a cost differential that will grow with his balance. "I hate to think how much I lost to fees before I switched to index funds," he says.

IT'S NOT ABOUT A ROCKING CHAIR

Yes, saving for retirement is your goal—but if you follow these steps, you'll also get a more immediate payoff in peace of mind. The bulk of Money's assets is invested in tax-favored retirement plans that he's not likely to tap for decades, but he's already benefiting from the happiness that financial security can offer. "When I first started, I thought money was for buying a lot of stuff. I wanted things," he says. "Over the years, I realized that I didn't care about the money. I cared about the lifestyle. Now the goal is to get up in the morning and work on whatever project you want. If you save enough, you can do that."

—KATHY KRISTOF, *author of* Investing 101, *is an award-winning financial writer and journalist.*

Who Cares?

It's only 2.5 Million Dollars Anyway

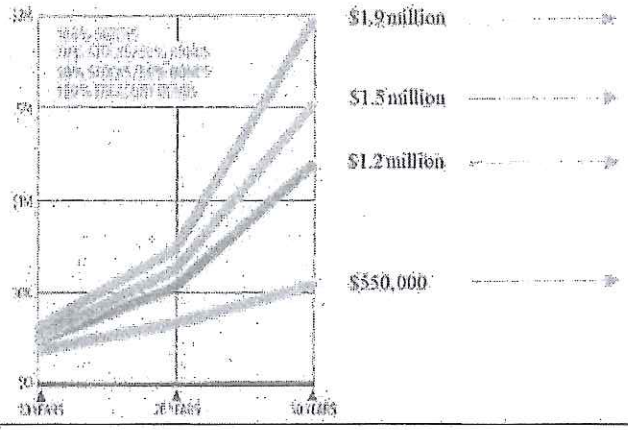
Read the attached article very carefully. I think you'll find it highly informative. However, you'll also notice that some of the key figures used to back up the writers' claim are missing. That's where you come in. Now that you are such a ***Financial Phenom***, I need you to fact check this article and see if the claims are indeed correct, or if she's just trying to sell a story.

Directions: Simply read along and when you come to a numbered blank, go to work trying to find the number that belongs there! I've provided space below for your work and answers.

Assumption: We are using continuous compound interest ONLY!

Good luck!

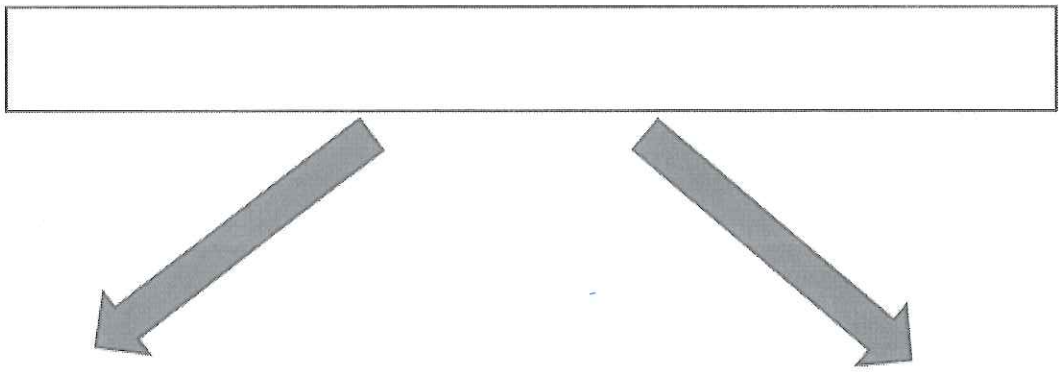
#	Work and Setup	Answer
1		
2		
3		
4		
5	*Hint: Assumes $200,000 invested at age 37 instead of presumably 27 (late 20s mentioned in the article)*	

6		
7	*Hint: Assume the account balance is $500,000*	
8		
9	This chart (from the article) showed how a hypothetical investment of $100,000 may grow. Use the predicted values of the accounts after 30 years to find the interest rates required for each account to achieve its final value. 	***Interest Rates***

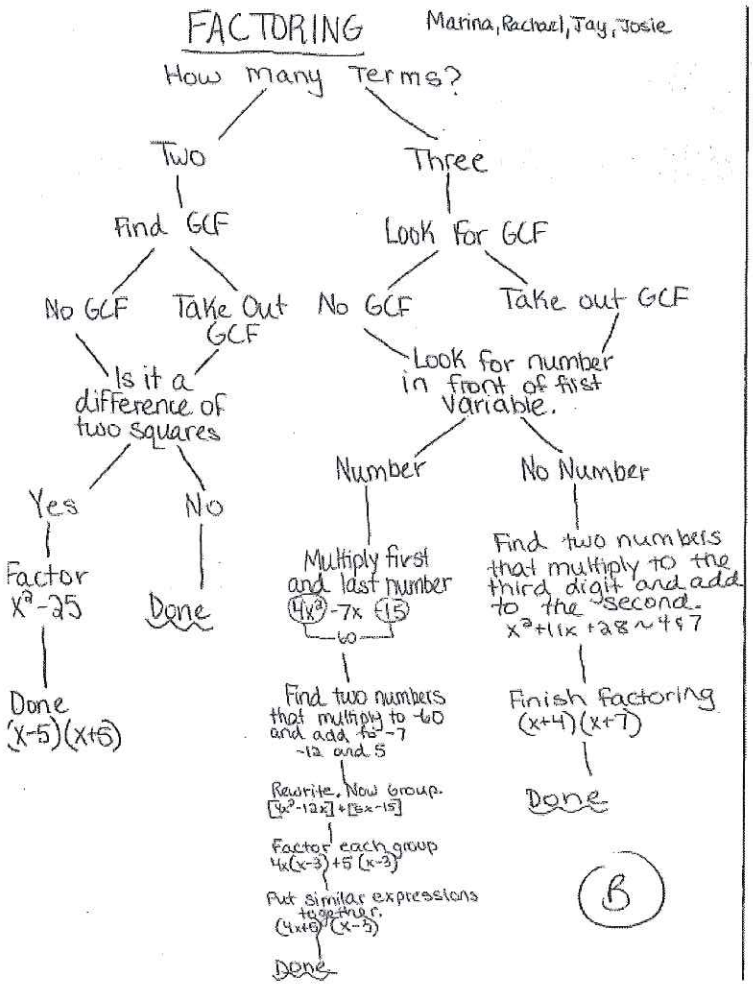
Active Learning Opportunities

Log and Exponential Equation FLOWCHART – YOUR DESIGN!

Why not try to do it on your tablet?



Factoring FLOWCHART



Creating a QR Code Scavenger Hunt

(1) Design the Hunt
- Will students be traveling all around the school? Inside? Outside?
- Consider time constraints if students will be walking from clue to clue.
- Will students be in teams or by themselves?
- If math, will a calculator be involved?
- Do students all have a smartphone?

(2) Design the Assessment

(3) Make a rules sheet and discuss the entire process BEFORE the day of the hunt

(4) Create your QR Codes and an Answer Sheet for students

(5) TEST EACH CODE!!!

(6) Post wherever, and test the codes one more time!