Skydiving

Extreme skydivers try to set records for very high altitude jumps. These jumps are typically made not from airplanes, but from hot-air balloons. The record for extreme jumping is 39 kilometers (about 24.2 miles) set by Felix Baumgautner during which he set a record for fastest free-fall velocity of 1357.64 km/hr (843.6 mi/hr). In this lab, you will determine how fast such a skydiver is going at his fastest and how long it takes him to reach the ground.

Expectations: You may work with one other student from class to complete this lab. No other resources are allowed.

This is a lab you will do in Microsoft Excel, a Google Spreadsheet, or Geogebra. Be sure to keep your spreadsheets for Parts 1-4 on separate tabs. You will turn in your file as part of your project. See the end of this handout for more information about what to turn in.

Part One: Ignoring air resistance.

In this first part, we will ignore air resistance. In such a case, the acceleration that a skydiver experiences is always equal to $-9.8\frac{m}{s^2}$. (We'll use meters and seconds for all distance and time measurements.)

Since acceleration is the derivative of velocity, we may use Euler's method to compute the skydiver's velocity based on his previous velocity and acceleration. Similarly, since velocity is the derivative of altitude, we may also use Euler's method to compute the skydiver's altitude based on his previous altitude and velocity.

Open a blank Excel spreadsheet and create four columns labeled:

Time	Altitude	Velocity	Acceleration

You may wish to go to the "view" tab and use the "freeze panes" function to "freeze" the first row, containing these labels. That way when the numbers go off the bottom of the page, you'll still be able to read the column names.

Now we need to give our system of differential equations some initial values. Rather than have the skydiver jump from 39 kilometers, let's have him jump from just 4 kilometers to start with:

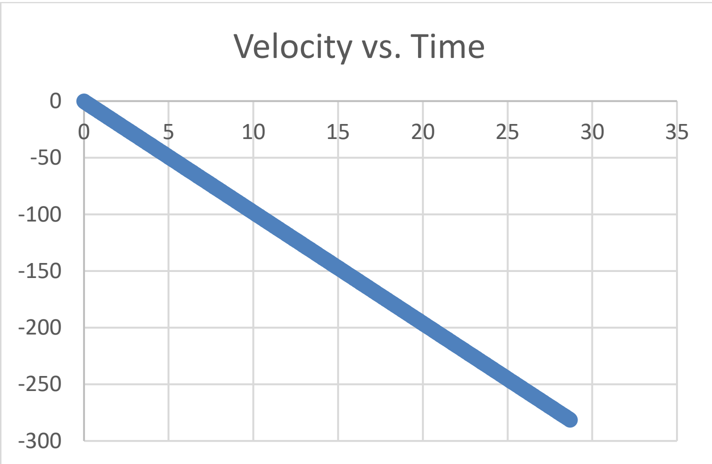
Time	**Altitude**	**Velocity**	**Acceleration**
0	4000	0	-9.8

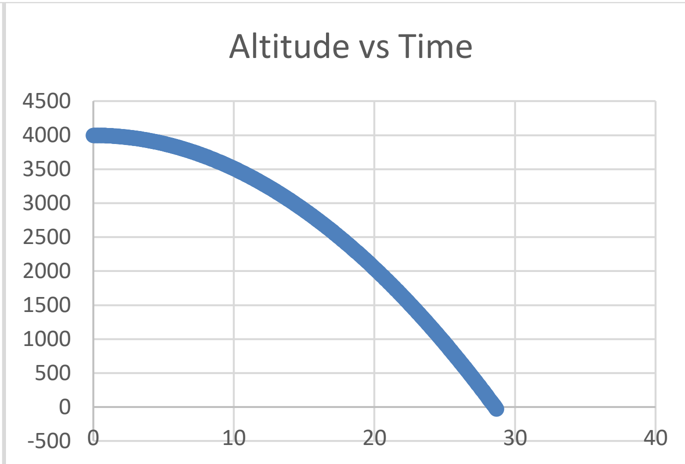
Now comes the part where we use Euler's method—we'll use $\Delta t = 0.05$ second. Determine how to compute altitude, velocity, and acceleration at different values of t. Once you type your formulas in, you should see the following values:

	A	B	C	D
1	**Time**	**Altitude**	**Velocity**	**Acceleration**
2	0	4000	0	-9.8
3	0.05	4000	-0.49	-9.8
4	0.1	3999.9755	-0.98	-9.8
5	0.15	3999.9265	-1.47	-9.8
6	0.2	3999.853	-1.96	-9.8

Now you can select the bottom four cells simultaneously, grab the tiny black square located at the lower right of the rightmost square, and drag it downward. The formulas in the cells will copy into the next row(s), but always referring to the immediately preceding row, not to the topmost row.

Keep dragging downwards until the skydiver hits the ground. *How fast is he going when he does?* (You can convert meters per second to miles per hour by multiplying by 2.23.)
Plot velocity versus time, by selecting just those two columns (use the control key to select two columns at once), then click on the "Insert" tab, and choose "Scatter" from among the "Charts." (You probably want a line-connected scatterplot rather than points—the latter would make your plot appear very thick.)





Once you have verified that your Euler's method is working correctly, go back and generalize your model as much as you can by replacing constants, such as starting altitude, starting velocity, starting acceleration, etc. with parameters set up in the top row of your spreadsheet.

Part Two: Air resistance

Air resistance is roughly proportional to the square of velocity, and it operates in the opposite direction—in this case, that means it *adds* to the constant $-9.8\frac{m}{s^2}$:

$$\text{Acceleration} = -9.8 + kv^2,$$

where v is the skydiver's velocity, and k is a constant of proportionality. The constant k depends upon several things:

$$k = \frac{C\rho S}{2m}$$

where:

- C is the "coefficient of air resistance" (equal to 0.57);
- ρ is air density measured in kilograms per cubic meter;
- S is the surface area of the skydiver (that is, his surface area perpendicular to his direction of motion), measured in square meters;
- and m is his mass, measured in kilograms.

For our skydiver, we have the following values:

$C = 0.57$, $\rho = 1.3$, $m = 75$, and $S = 0.7$ (parachute closed) / $S = 25$ (parachute opened)

We need to modify our Excel document to take into account the air resistance, and we'll have the skydiver open his parachute when he is one kilometer above the ground. Here are the columns we'll need.

	A	B	C	E	F
1	Time	Altitude	Velocity	k	Acceleration
2	0	4000	0	???	???
3					

Determine how to use this new information to first determine the value of k, and then how to determine acceleration using this value of k to take air resistance into consideration.

Part Three: Parachute

It's about time that we incorporate a parachute into our model for our skydiver so they don't hit the ground at such a high velocity. When opening the parachute, the skydiver (almost) instantly changes their surface area. Skydivers either wait until they reach a certain altitude before opening their parachute, or they wait for a certain number of seconds to pass after jumping. Modify your model so that the skydiver opens their parachute when 1000 meters above the ground.

You may find the following Excel command useful for this:

```
IF[ altitude > threshold, Value if TRUE, Value if FALSE]
```

i.e. `IF[B2 > 1000, … , … ]`

This should allow you to write one expression that can return two values, depending on if the current altitude is above or below 1000 meters.

1. **1.** Make plots of acceleration vs. time, velocity vs. time, and altitude vs. time. Explain how these graphs relate to the jump your skydiver is making.

2. Modify your spreadsheet to help you answer the following questions.

a) A skydiver who weighs 84 kg uses a parachute with a surface area of 20 square meters. If he jumps from a height of 500 meters and pulls the ripcord after 10 seconds,

i. how fast is he going when he lands?

ii. how far from the ground is the skydiver when the chute opens?

iii. how long does the dive last?

iv. what is the maximum velocity of the skydiver on this dive?

b) By considering the differential equation, explain why if you double the weight of the skydiver and double the area of the chute, the jump takes the same amount of time and you land with the same velocity.

Part Four: High Altitude Jumps

Now we're ready to have our skydiver jump from 40 kilometers. Felix Baumgautner pulls his chute at an altitude of approximately 2610 meters. There's another change that must be made as well: air pressure (ρ) is no longer constant. It is about 1.3 kilograms per cubic meter at sea level, but it decreases by about 0.01% for every meter higher you go. Modify your Excel document to allow for these changes in altitude and air pressure.

3. Make plots of acceleration vs. time, velocity vs. time, and altitude vs. time. Explain how these graphs relate to the jump your skydiver is making.

4. How fast is the skydiver falling when he's falling fastest? How long does it take him to reach the ground?

What to turn in (one copy per group):

1) You must upload your Excel file containing your spreadsheet for each part on a separate tab. Plots from questions 1 and 3 should be found at the top of the appropriate tab.

2) A Word document with explanations of the graphs in questions 1 and 3, and solutions for the problems asked in questions 2 and 4.

Videos and an article on high altitude jumps:

https://www.youtube.com/watch?v=raiFrxbHxV0

http://www.nytimes.com/2014/10/25/science/alan-eustace-jumps-from-stratosphere-breaking-felix-baumgartners-world-record.html?smid=nytcore-ipad-share&smprod=nytcore-ipad&_r=0