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Exploring Yellowstone National Park with Mathematical Modeling

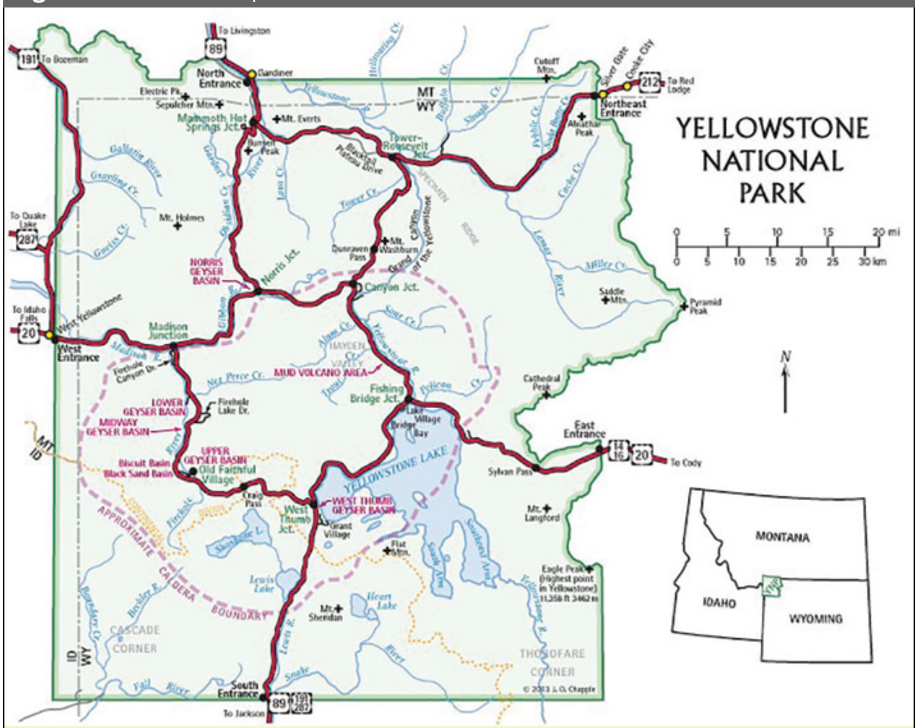
Using maps as part of a measurement activity engaged fifth graders as they studied the area of a national park.

With the 2016 national park centennial and such initiatives as Find Your Park, students and teachers alike are encouraged to explore state and national parks as their outdoor classrooms. From terrain change to average daily temperature, many aspects of the outdoors can be explored through mathematics. Because of our location, several fifth-grade classrooms across our district have the opportunity to visit Yellowstone National Park. Exploring Yellowstone is both relevant and motivating for our students because most have already visited or learned about some features of the park. For these reasons, as teachers, we thought that Yellowstone could act as an ideal context in which to investigate measurement topics and engage in the process of mathematical modeling.

Mathematical modeling, a practice standard in the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010), is a process by which students develop and use mathematics as a tool to make sense of the world around them (Lesh and Doerr 2003). Students investigate a real-world situation by asking mathematical questions; along the way, they need to decide how to use appropriate tools and mathematical content knowledge. After devising a solution, students interpret their model to determine if it makes sense and if it can be modified.

The practice of mathematical modeling is of growing importance in mathematics education because it bolsters mathematical proficiencies beyond traditional content knowledge. Modeling tasks are distinct from many other mathematical activities because of their openness. Instead of being presented with a specific mathematical problem, students are presented with a situation and allowed to problem pose to determine what mathematical questions might be appropriate to ask. In this sense, modeling provides a purpose to mathematics and empowers students to view everyday situations through a mathematical lens. Modeling tasks are also open in terms of solution strategies. Students are able to choose how they would like to solve the problem and demonstrate proficiency and creativity by using and connecting known mathematical knowledge (Greer, Verschaffel, and Mukhopadhyay 2007; Lehrer and Schauble 2007). Finally, solutions to modeling tasks are also open. Students must discuss and determine if their solution best meets the needs of the situation at hand, if their solutions can be revised, and if there are multiple applicable solutions. Modeling tasks challenge students to use mathematics as a tool to ask questions about and make sense of a messy, real-world problem.

Fig. 1 The authors adapted the Yellowstone Park task from Beckmann (2010).



Your task today is to find the area of Yellowstone National Park using any of the following tools (please be ready to present your solution at the end of class):

- Modeling dough
- Graph paper (different sizes: 1 inch, 1/2 inch, 1/4 inch, and 1/8 inch)
- 1 inch tiles
- String

1. Describe how you found the area of the park and the tool you used. What did you do first, second, third, and so on? Why did you proceed in this manner?

2. Could you envision using another tool to find the area of the park? How might you use that particular tool?

DEVELOPING THE MODELING TASK AND ANTICIPATING STUDENTS' RESPONSES

By late elementary school and early middle school, students have developed a wealth of knowledge about length, perimeter, and area and are just beginning to investigate area of irregular shapes, decomposing area into smaller regions, and scale. Posing a modeling task related to measurement topics offers an opportunity to take time to connect ideas and delve into a real-world problem to foster greater mathematical understanding and flexibility. When posing a modeling task, determining what students might find relevant and interesting is important because this will encourage engagement and

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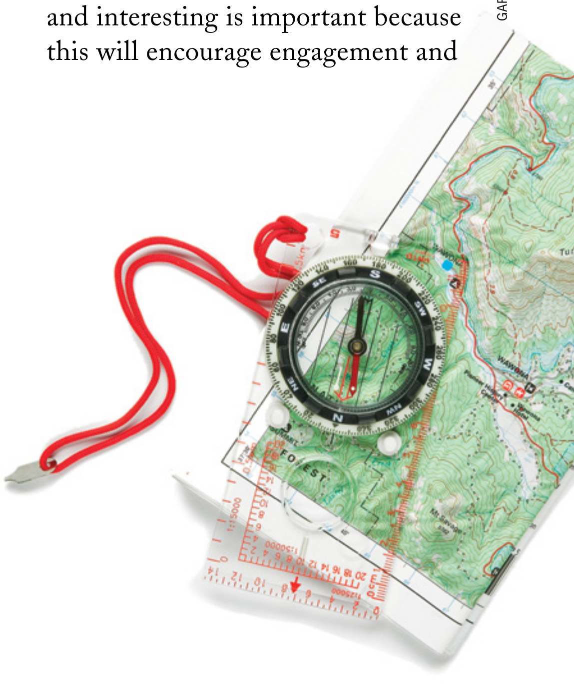
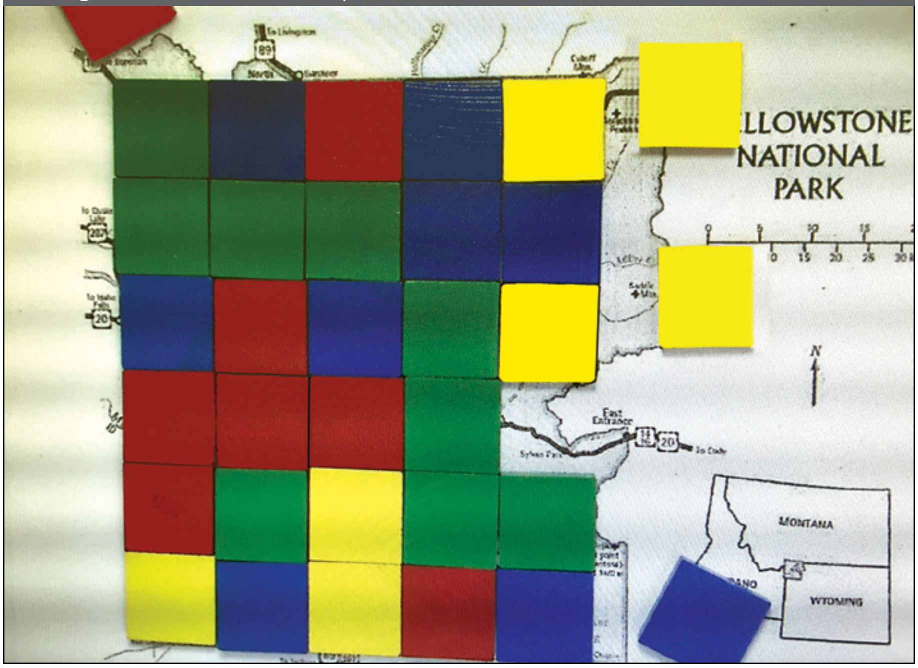


understanding (Carlson et al. 2016).

Our students were planning to visit Yellowstone National Park and needed to learn to make sense of park maps, so we thought this park context would be an ideal way to hold their interests while investigating geometry and scale. We decided that our initial question would be this: "How big is Yellowstone National Park?" Even though we specifically focused on this park, this task is easily adaptable to investigating other regions, such as cities, states, or countries.

When posing a large, open-ended question, anticipating possible student responses is important, specifically the mathematics that they bring to the situation, how they might go about the task, questions that may arise, and discussions that could develop (Carlson et al. 2016; Smith and Stein 2001). We decided to focus specifically on finding the area of Yellowstone National Park and illuminating ideas surrounding composition and decomposition of irregular figures, precision, and scale. We thought students might need to discuss and agree on how they might measure area as well as how they would use knowledge of map scale, units, and area formulas to aide them. We also discussed that we could make this task more demanding by considering comparisons between multiple maps and units. We decided that if students were successful with determining models to measure Yellowstone, then we would branch into other questions and ask, "Which is bigger, Yellowstone National Park or Yosemite National Park?"

Fig. 2 When students who used the Square Tiles strategy compared their initial area measurements with those who used other strategies, they realized an error in their thinking about the area of each square.



ENACTING THE LESSON

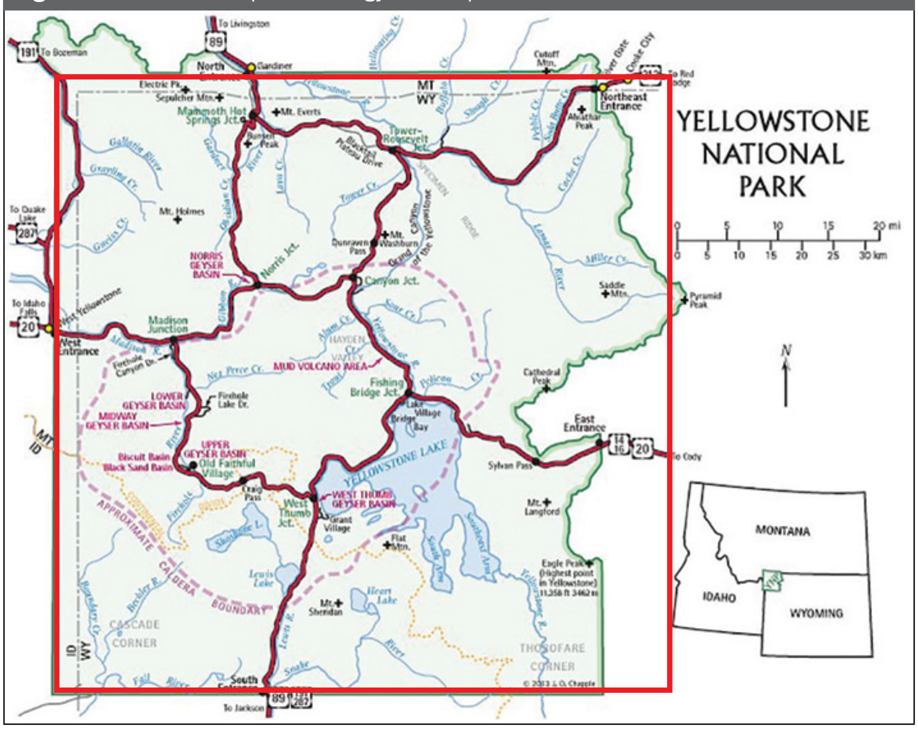
We began the lesson by posing the following question: "How big is Yellowstone National Park?" Part of the modeling process is determining, as a mathematical community of learners, the mathematical question and how to define ambiguous words, such as "big." Several students suggested that the class could measure height, length, width, or area to determine how big something is. The class decided to investigate one attribute at a time, so that students' models would be comparable. Students decided that their mathematical question would be "What is the area of Yellowstone National Park?"

To gauge initial ideas about space and size, we asked students to estimate the area before beginning the modeling task. Students responded that it was difficult to estimate something they had little experience with. With answers like 1 acre or 5000 square miles, they felt they were guessing at random. One student drew on experiences and stated, "I know it has to be almost 100 miles long because my parents and I drove across the park, and it took a few hours." Going into the task, students did not have a clear sense of how big the region on the map might be.

Posing a modeling task related to measurement topics offers an opportunity to take time to connect ideas and delve into a real-world problem.

Fig. 3 One student outlined the perimeter of the park.

Fig. 4 The Area of a Square strategy was implemented.



STUDENTS' MODELS FOR FINDING AREA

After discussion, we posed a task (see **fig. 1**) adapted from Beckmann (2010, p. CA-288). We hoped each tool would elicit different types of thinking about the problem, and we encouraged students to choose a tool that made sense to them. At the end of class, we asked each group to present their model following consultations with other groups to consider possible limitations. Below we describe several student models that emerged and the ways in which students' described these models.

Square Tiles

Several groups asked for square tiles so that they could decompose the park into square units. They used the scale of the map and noticed that the tile's side length was 10 miles. After positioning the tiles, a student counted 29 full tiles and then multiplied 29 × 10, resulting in 290 square miles for the area (see **fig. 2**). We saw this as an initial opportunity to discuss geometry in relation to map scale and asked students to use another tool and reflect on their response. When implementing a secondary strategy, groups realized that something was off in their thinking because measurements for the park differed greatly. We asked them to consider the scale of the map as well as their unit. One student stated, "Each square has two sides, which would be 10 miles × 10 miles, or 100 square miles." Students realized that instead of 290 square miles, it should be 2900 square miles and that even though a 1 inch × 1 inch square also represents 1 square inch, a 10 mile × 10 mile square does not represent 10 square miles.

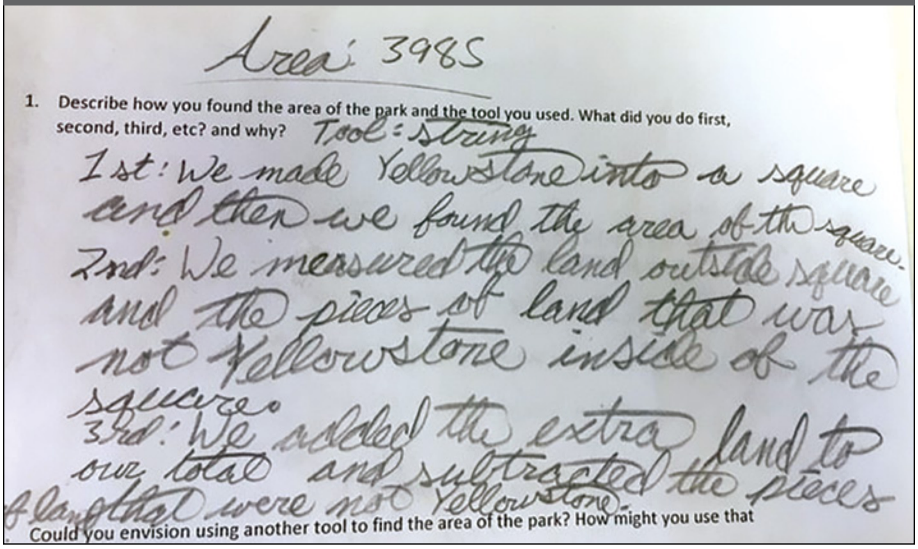
As students discussed and implemented this strategy, they commented that they were unsure of what to do with the extra space. Part of the park is rectilinear and can be tiled fairly easily, but portions of the park jut out (see **fig. 1**). Students decided that the model was quick, but it did not account for all of the park. In response, some students ignored extra parts or estimated fractional parts of tiles.

Strings or Lengths

Looking at the scale of the map, many students were drawn to the idea of using increments to measure something about the park. Using playdough or string, students outlined the perimeter of the park, marking increments of 20 miles, based on the map scale (see **fig. 3**), to find the perimeter. After consulting with a teacher, the students wondered what this could tell them about the area of the park.

Students knew how to find the area of a rectangle or square and visualized Yellowstone Park as a square with some irregular pieces removed. Groups went about this strategy in two different ways. One group boxed in the park and adjusted the square until, visually, it appeared that the amount of land inside the square was the same as the amount of land outside the square. They discussed that the pieces outside the square would fill in the empty space and make a whole square, so they could then multiply the length and the width. The second group boxed in the park (see **fig. 4**) using a square. Then, using the graph paper unit as a measuring tool, they subtracted portions inside the square that were not Yellowstone and added portions of the park that were outside the square. Using this method, they estimated 3985 square miles. See **figure 5** for one student's explanation.

Fig. 5 Students described their thinking about the Area of a Square strategy.


Area: 3985

1. Describe how you found the area of the park and the tool you used. What did you do first, second, third, etc? and why? Tool: string

1st: We made Yellowstone into a square and then we found the area of the square.
2nd: We measured the land outside square and the pieces of land that was not Yellowstone inside of the square.
3rd: We added the extra land to our total and subtracted the pieces of land that were not Yellowstone.

Could you envision using another tool to find the area of the park? How might you use that

Fig. 6 Area was modeled quite clearly by this student's use of the Modeling Dough strategy.

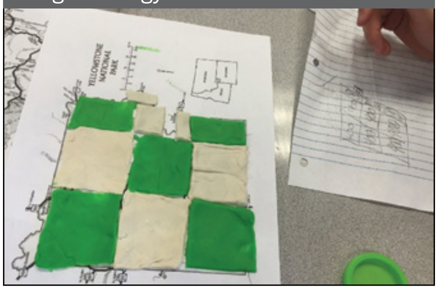
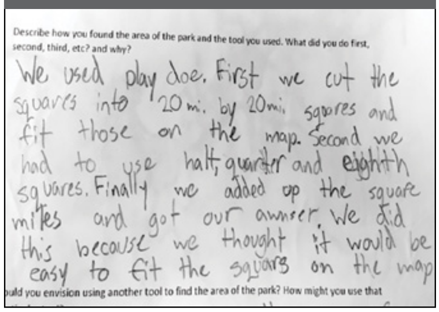


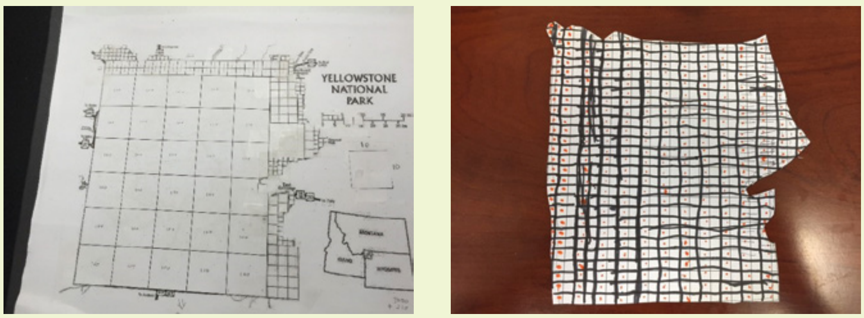
Fig. 7 Then, this strategy's use was explained.


Describe how you found the area of the park and the tool you used. What did you do first, second, third, etc? and why?

We used play doe. First we cut the squares into 20 mi. by 20 mi. squres and fit those on the map. Second we had to use half, quarter and eighth squares. Finally we added up the square miles and got our awneer. We did this because we thought it would be easy to fit the squars on the map

...uld you envision using another tool to find the area of the park? How might you use that

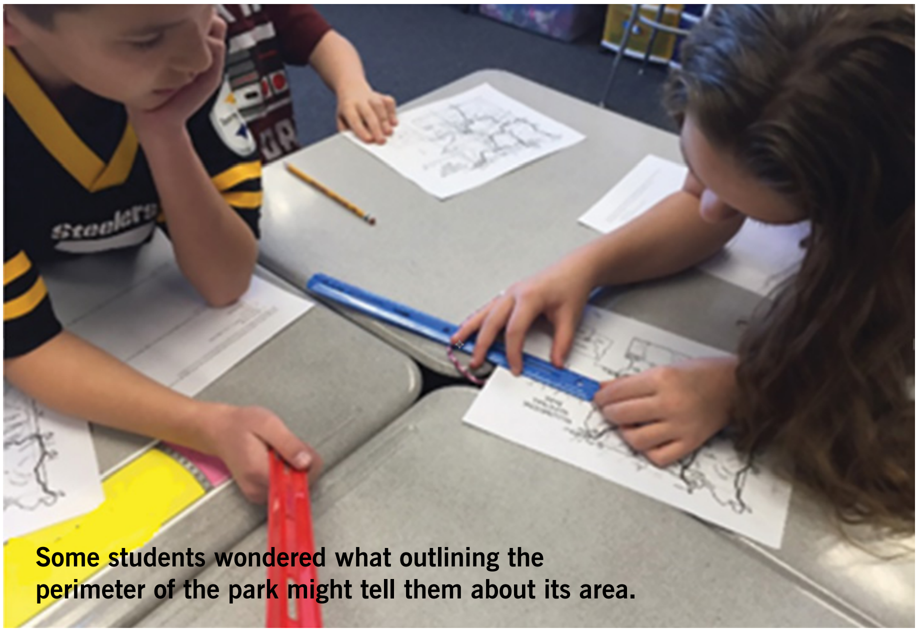
Fig. 8 These student work examples illustrated how graph paper came into play.



(a) (b)

Modeling Dough

Many students were drawn to the modeling dough and used it, in conjunction with the map scale, to create units. One group (see **fig. 6–7**) created square units and used side lengths to measure the map scale (20 miles × 20 miles, or 400 square miles). They found that six of these units could fit within the park. Next, they created half-size units (10 miles × 20 miles, or 200 square miles) and found that two of these units fit, followed by two units of one-fourth size (10 miles × 10 miles, or 100 square miles) and four units of one-eighth size (10 × 5, or 50 square miles). The students totaled their units to estimate the area of the park to be 3200 square miles.



Some students wondered what outlining the perimeter of the park might tell them about its area.

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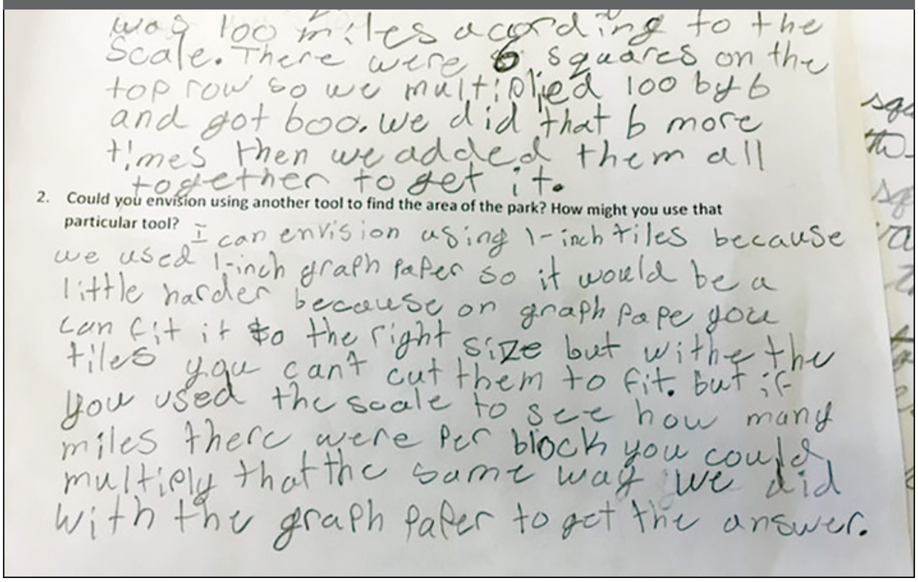
Fig. 9 Benefits and limitations are reflected on.



was 100 miles according to the Scale. There were 6 squares on the top row so we multiplied 100 by 6 and got 600. We did that 6 more times then we added them all together to get it.

2. Could you envision using another tool to find the area of the park? How might you use that particular tool? I can envision using 1-inch tiles because we used 1-inch graph paper so it would be a little harder because on graph pape you can fit it to the right size but with the tiles you can't cut them to fit. But if you used the scale to see how many miles there were per block you could multiply that the same way we did with the graph paper to get the answer.

Graph Paper

A few students cut the park out of the map and traced it onto graph paper or cut out graph paper and glued it on to the park map (see **fig. 8a**). They found that the graph paper helped them be more precise. Some students used one-eighth-inch-square graph paper because it fit the best; others began with one-inch-square graph paper and gradually worked down to smaller units, as needed. For example, the group whose work is represented in **figure 8b** began by cutting out one-inch squares and found that they could use 31 one-inch squares. Next, they moved to one-fourth-inch and one-eighth-inch squares, as needed, and discovered it would take an additional 82 one-fourth-inch squares and 68 one-eighth-inch squares. Students saw that 16 one-fourth-inch squares made a square inch and 64 one-eighth-inch squares made a square inch, and they used this information to reason that 82 one-fourth-inch squares were about 5 square inches and 68 one-eighth-inch squares were about a square inch. This amounted to about 36 square inches, or 3600 square miles. One of the key findings for the group was relating and comparing units to one another to determine how many of each type would fit into a square inch.

Lesson Discussion

At the end of the lesson, we wanted students to explain their thinking and reasoning about area as well as consider limitations of their model. When reflecting on the task, students discussed that their models showed how they envisioned the park. Some saw the park as one large shape and used the map scale to help them create lengths to approximate the area of a square. Others imagined the park as separate pieces and used the map scale to help them decompose the park and create individual units to measure.

After revealing the actual area of the park, which is approximately 3458 square miles, students commented that all their models seemed to provide reasonable estimates. We then asked students to consider why differences existed between models if they were all measuring the same park. One student stated, "Some of the models are more precise." This sparked a conversation on what it means to be precise. Students discussed that the way in which the park was decomposed into smaller shapes mattered. Tools like the square tiles were less precise because they could not cover the whole park or be broken apart (see **fig. 9**). Estimating the number of partial larger tiles that should be used was difficult. When students used smaller square units, like the one-eighth-inch graph paper or units made of dough, they could measure and account for more of the park accurately.

Although the dough and graph paper methods were more precise,

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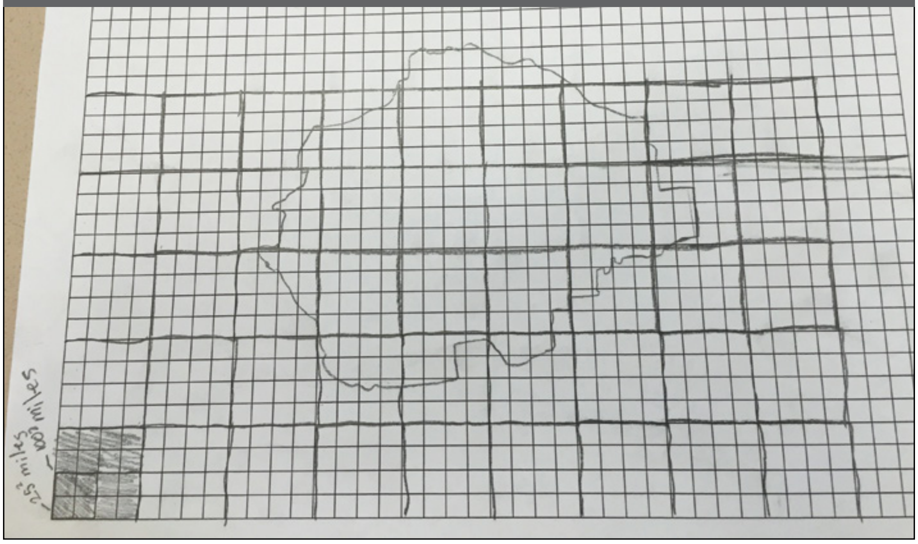
students also commented that they were more time-consuming. Because Yellowstone National Park is almost a rectilinear shape, using square tiles provided a quick and reasonable estimate. Later in the lesson, when students moved on to investigate other parks, like Yosemite National Park, they realized that using only tiles to approximate the area was difficult, so they incorporated graph paper to help accurately account for irregularity (see **fig. 10**).

Fig. 10 A student measured Yosemite National Park using graph paper.



A GATEWAY TO DEEPER MATHEMATICAL KNOWLEDGE

The Yellowstone Park task highlights important pedagogical features of mathematical modeling. First, mathematical modeling acts as a gateway mathematical practice in that it allows access to many other mathematical practices. When measuring Yellowstone, students were able to make sense of a real-world problem mathematically and choose appropriate tools to help them measure (i.e., graph paper, dough, tiles). Students connected knowledge about units, lengths, widths, and area measurement to form different models. They constructed viable arguments for generating the area and critiqued other models. By considering, comparing, and contrasting models, students were able to consider both precision and time and how they might modify their measuring techniques in the future to find the area of irregular shapes. All these practices emerged organically and with relative ease when investigating a task of importance to students.

The modeling task also fostered deeper mathematical content knowledge by allowing students to use and connect mathematical ideas surrounding measurement topics in

We then asked students to consider why differences existed between models if they were all measuring the same park.

meaningful ways. From the teachers' perspectives, the task illuminated how students envisioned area and helped to scaffold ideas about scale and composite shapes. It also acted as a way to draw several mathematical ideas together.

Last, the modeling task allowed for creativity and choice as well as questions for future discussion. Students commented that they enjoyed being given the freedom to choose how they wanted to solve the task. The task also sparked their curiosity about the area of other regions in the country and how they compared with one another. Students asked, "How big is the town compared to Yellowstone?" and "How does Google Maps® estimate area?" In closing, the Yellowstone task highlights that modeling can be used to help students engage in both mathematical content and practices and become creative, flexible thinkers.

Let's Chat about Modeling Yellowstone Park

On Wednesday, April 19, 2017, at 9:00 p.m. EST, we will expand on "Exploring Yellowstone National Park with Mathmematical Modeling" (pp. 462–70), by Megan H. Wickstrom, Ruth Carr, and Dacia Lackey. Join us at #MTMSchat.

We will also Storify the conversation for those who cannot join us live. Our monthly chats fall on the third Wednesday of the month.

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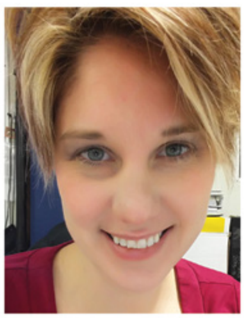

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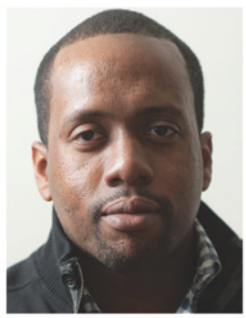

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