Introduce Function Concepts and Linear Functions Geometrically(!)

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Scott Steketee **21st Century Partnership for STEM Education** **stek@21pstem.org**
Daniel Scher **KCP Technologies** **dscher@kcptech.com**

The CCSSM expects students to understand transformations as functions. Algebra students exploit this standard using Web Sketchpad: They vary the variables of geometric transformations; experience domain, range, and rate of change; and connect their learning to the graph of *y=mx+b.*

Richard Feynman famously said *What I cannot create, I do not understand.* The focus of today's session is on activities that enable students to create their own functions, with variables that students actually vary, with relative rate of change that they observe, analyze, and control, functions whose behavior creates patterns and pictures, all with the goal of developing students' deep and broad understanding of function concepts.

Big Idea: Definition of Function

The first group of activities helps students to define *function* and to distinguish one family from another based on the variables' relative rate of change.

Big Idea: Transformations Are Functions.

These activities also introduce four transformation families as functions, with tasks that involve function notation and restricted domains.

Big Idea: Variables Really Vary.

In all these activities, students drag the independent variable and observe the effect on the dependent variable, creating a strong connection between variation and physical movement.

Big Idea: Functions in Geometry and Algebra

The second group of activities begins by restricting the domain of geometric to a number line.

Students discover that dilation corresponds to multiplication and that translation corresponds to addition. Students compose these functions on a number line and then on a dynagraph, and play the dynagraph game to match their function to a mystery function.

Big Idea: The Slope of a Graph Shows the Relative Rate of Change of the Variables.

Students compose dilation and translation, rotate the dependent variable's axis, and trace the point determined by the independent and dependent variables. The trace is a straight line, giving rise to the name *linear function.*

Benefits:

In the course of these activities, students:

- Drag independent variables,
- Construct rules and dependent variables,
- Observe and describe relative rate of change,
- Restrict the domain and observe the range,
- Use function notation meaningfully,
- Connect transformations to algebra,
- Use composition to create linear functions,
- Build dynagraphs to show rate of change, and
- Construct Cartesian graphs based on motion.

Materials:

Presentation web page:
geometricfunctions.org/links/nctm2017/

Investigate Transformations as Functions:
geometricfunctions.org/links/intro-functions/

Connect Geometry and Algebra via Functions:
geometricfunctions.org/links/geom-and-algebra

Our blog: sineofthetimes.org

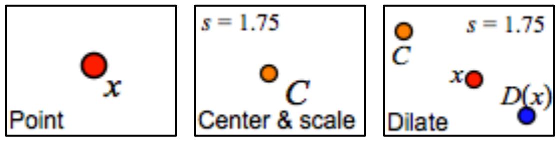
Please email us with your questions and suggestions!

Dilate Family 1 Names:

In this activity you will dilate a point and compare the motion of the point to the motion of its dilated image.

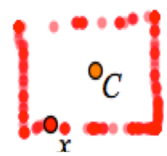
DILATE A POINT

Begin by dilating a point and describing how the variables behave.

1. In your browser open geometricfunctions.org/links/dilate-family/. Go to **page 2.**
2. Use the first three tools 

to create a dilate function. When you use the Dilate tool, be sure to match point x to point x.
3. Drag independent variable x on the screen and observe the behavior of $D_{C,s}(x)$.
4. Tap the scale factor s and change its value to 0.50.

Q1 Turn on tracing and drag x in to make a rectangle. Draw a picture of the result. Be sure to mark x, C, $D_{C,s}(x)$ in your picture.



Q2 As you drag x, how does $D_{C,s}(x)$ behave? Fill in the blanks below.

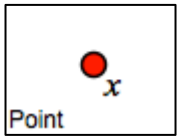
s = 0.50	**Drag *x* left**	**Drag *x* up**
Which way does $D_{C,s}(x)$ move?		
Which variable moves faster?		

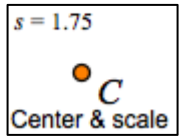
Q3 Drag x to try to find fixed points of the dilate function. (Remember, a *fixed point* is a place where x and $D_{C,s}(x)$ come together at the same time.)

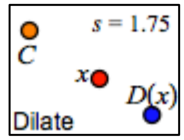
What did you find out?

Dilate Family 2 Names:

USE DIFFERENT SCALE FACTORS

5. On **page 3** use 





to create a dilate function.

6. Tap the scale factor $s = \boxed{1.75}$ and make its value 2.00.

Q4 Drag x left. Which way does $D_{C,s}(x)$ go, and how fast? Drag x up. Which way does $D_{C,s}(x)$ go, and how fast?

s = 2.00	**Drag *x* left**	**Drag *x* up**
Which way does $D_{C,s}(x)$ move?		
Which variable moves faster, x or $D_{C,s}(x)$?		
Which variable makes a longer trace, x or $D_{C,s}(x)$?		

Q5 Is there a connection between the speed of the variables and the lengths of their traces? If so, describe it.

Q6 Change the value of s to –1.00. What happens now when you drag x?

s = –1.00	**Drag *x* left**	**Drag *x* up**
Which way does $D_{C,s}(x)$ move?		
Which variable moves faster?		
Which makes a bigger design?		

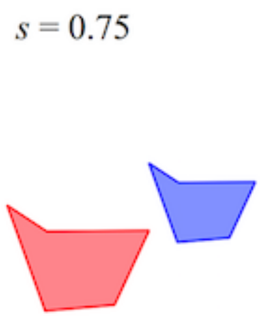
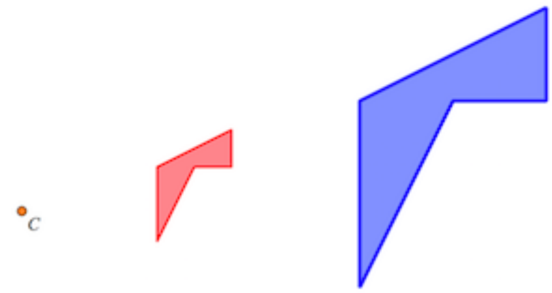
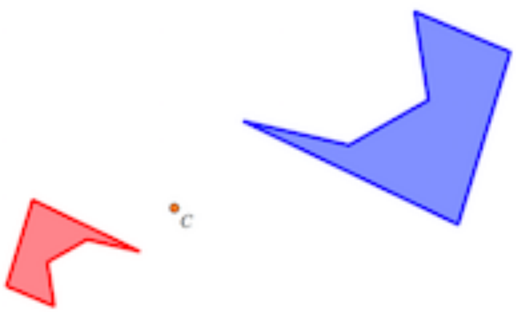
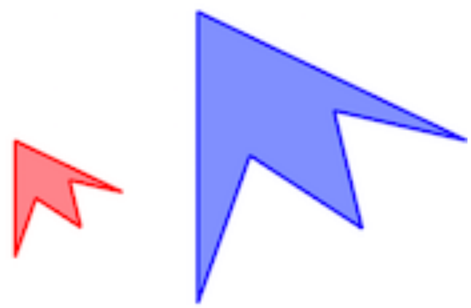
Q7 What do you think would happen if you make s = 0.00? Test your guess.

Prediction:	Actual result:

Dilate Family 3 Names:

CHALLENGES

7. On **page 4** construct a dilate function, restricting x to the red polygon and using the scale factor shown. Then adjust the center point so the dependent variable traces out the perimeter of the blue polygon. Mark the location of your center point in the space below.
8. **Pages 5, 6, and 7** are similar. Follow the directions on each page and mark your center points and scales below below.

Page 4	Page 5
	
Page 6	Page 7
	

DILATION GAMES

9. On **page 8**, you can see independent variable *x,* center point *C,* and scale factor *s*. But the dependent variable $D_{C,s}(x)$ is hidden. Drag the yellow circle to show where you think $D_{C,s}(x)$ is hiding, and press *Check* to see if you were right. Then a new problem appears for you to try. Practice this game until you get it right every time.
10. Play the Dilation Games. (The link is below the sketch.) In each game your job is to find a different missing element. Keep track of your scores as you get better.

MORE CHALLENGES

11. **Pages 9, 10, and 11** contain more puzzles that you can use to extend your understanding of the dilate function.

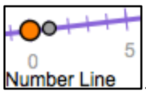
Create a Dynagraph Name:

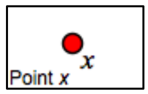
In this activity you will compose a function on a number line, as you have already done. But this time you will display the dependent variable on its own number line to make it easier to compare the motion of the two variables. We will call the two number lines the *input axis* and the *output axis.*

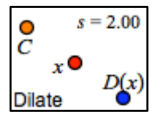
DILATE

Begin by constructing the input axis.

1. Open http://geometricfunctions.org and navigate to Activities | Cartesian Connection | Create a Dynagraph. Go to page 2 of the sketch.

2. Construct a 

, construct 

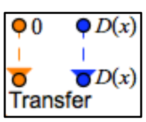
on the line, and 

point *x*. Adjust the number line to make it horizontal.

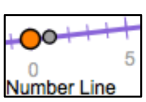
Q1 Each line in this table describes the relative motion of x and $D(x)$. Find a scale factor that creates the given motion, and write it down. Check your result by varying *x.*

Relative speed	Relative direction	*Scale factor s*
$D(x)$ goes slower than x.	$D(x)$ goes the opposite direction as x.	***s* =**
$D(x)$ goes faster than x.	$D(x)$ goes the same direction as x.	***s* =**
$D(x)$ goes the same speed as x.	$D(x)$ goes the opposite direction as x.	***s* =**
$D(x)$ goes slower than x.	$D(x)$ goes the same direction as x.	***s* =**

CREATE THE OUTPUT AXIS

3. Use the 

tool. Attach glowing point *0* to point *0* of the input axis, and attach glowing $D(x)$ to $D(x)$ on the input axis.

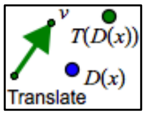
4. Use the 

tool. Attach the new number line to the lower orange point and make it horizontal. This new number line will be the output axis.

5. You should be able to drag the origin of the output axis up or down, but not sideways. Also, the lower $D(x)$ point should always stay on the output axis.

TRANSLATE

Use the output axis to translate $D(x)$.

6. Use the 

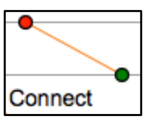
tool to translate $D(x)$ (on the output axis) by vector v. Be sure to attach point v to the axis. The result, $T(D(x))$, is "the *translation* of the *dilation* of x." Drag point v back and forth to observe the result of the translation.

INVESTIGATE

Q2 For each row of the table, edit s to the value shown, and drag points v and x so they have the indicated values. Then record the values for $D(x)$ and $T(D(x))$.

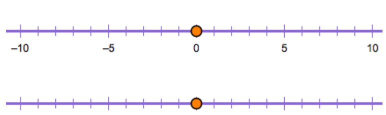
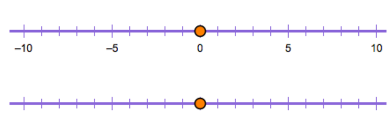
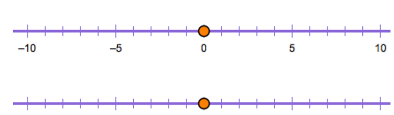
s	v	x	$D(x)$	$T(D(x))$
2.50	–3.00	–2.00		
–2.00	1.00	–3.00		
0.50	2.50	2.00		

7. Press the Hide Transfer and Hide Dilation buttons.

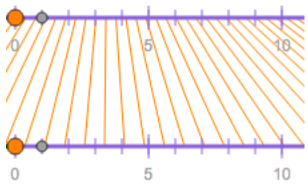
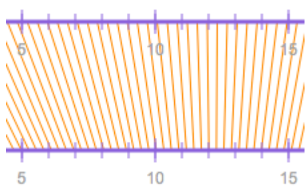
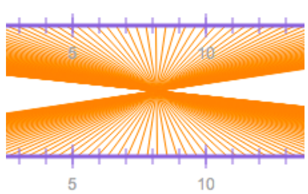
8. Use the 

tool to connect x with $T(D(x))$, and turn tracing on. Experiment using different scale factors to see what the traces of the connecting line look like.

Q3 For ***a*** below, set the dilation scale factor (s) and the translation vector (v) as shown. Then vary x and draw in some of the connecting lines that are traced. Then do **b** and **c.**

a	***b***	***c***
$s = 2.0$	$s = -3.0$	$s = 0.5$
$v = -3.0$	$v = 5.0$	$v = 2.0$
		

Q4 For the traces on the left, circle the correct words on the right, and tell how you know.

	$T(D(x))$ is moving **| faster | slower | at the same speed |** as *x.* $T(D(x))$ is moving in the **| same | opposite |** direction as *x.* I know this because:
	$T(D(x))$ is *moving* **| faster | slower | at the same speed |** as *x.* $T(D(x))$ is moving in the **| same | opposite |** direction as *x.* I know this because:
	$T(D(x))$ is *moving* **| faster | slower | at the same speed |** as *x.* $T(D(x))$ is moving in the **| same | opposite |** direction as *x.* I know this because:

SOLVE A MYSTERY

9. On page 3 create another dynagraph just as you did on page 2.

10. Use the Animate tool to create an Animate x button, and use the ?? Mystery tool to create a mystery function, labeled ??(x).

11. For your first mystery function, start with level 1. To solve the mystery, adjust s and v to make $T(D(x))$ line up exactly with the mystery function.

You'll know you have it right when you can press Animate x and see $T(D(x))$ and ??(x) lined up exactly all the way across the screen. Once you have solved three or four mystery functions answer the question below.

Q5 If a friend asks for your help in solving mystery functions, what hints and suggestions would you give her?

PLAY THE MYSTERY GAME

12. Tap the link beneath the sketch to "Play the Dynagraph Game." Start on Level 1, but with practice you should be able to master Level 3 or, even better, Level 4.