The Problem of Word Problems

SUPPORTING STUDENT ACCESS

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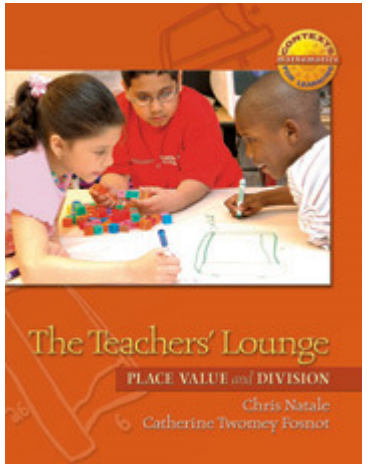
Problems in Context, Problems with Context

Yesterday, I was in the teachers' lounge and there are two vending machines – one that sells only bottled water and one that sells different flavors of juice. The person who services the machines was there filling the water machine. I asked the man how many bottles the machine could hold and he told me 156! He had a cart with bottles and the bottles came in six-packs. I wonder, how many six-packs will the machine hold?



(Natale & Fosnot, 2007)

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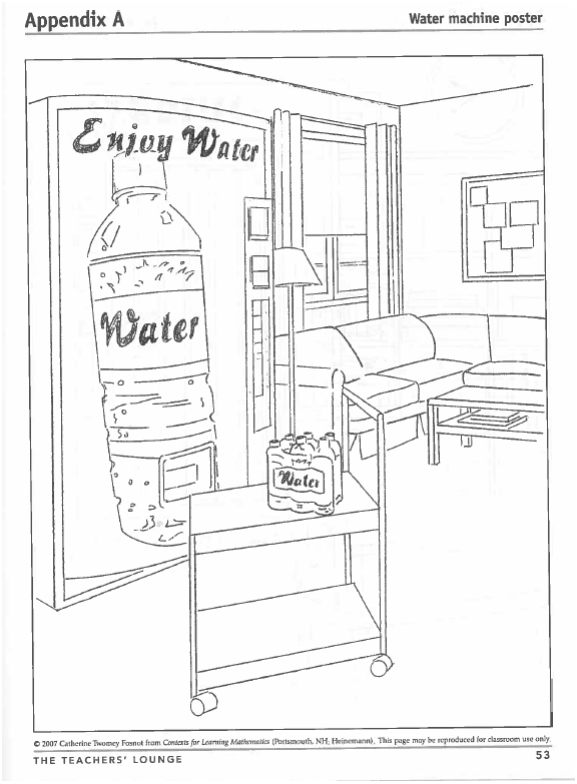
Appendix A

Water machine poster

Enjoy Water

Water

Water

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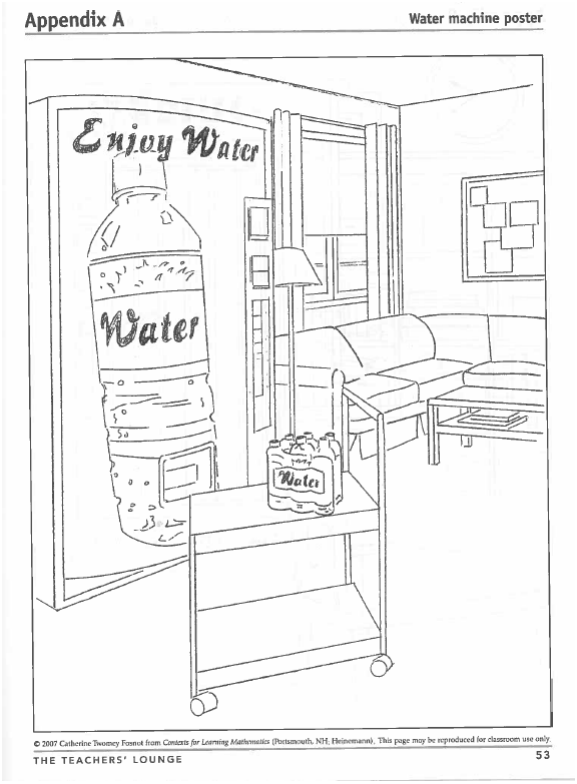
WHAT STRUGGLES MIGHT STUDENTS HAVE IN GETTING ACCESS TO THIS TASK?

HOW MIGHT YOU SUPPORT THEM?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Barriers to Access

Students experience a variety of barriers when trying to enter mathematical stories:

- Reading: Decoding, Fluency, Comprehension, and Inference
- Cultural Knowledge and Varied Personal Experiences
- High-frequency and Academic Vocabulary
- English Language Proficiency

Two Types of Need

1. Stuck Outside: Need to Gain Entry

   Students may not understand enough of the story to comprehend the problem and find an entry point. These students are likely not working because of their confusion.

2. Tangled Up Inside: Working Under Misunderstanding

   Student may believe they understand the problem and begin work, while misunderstanding key elements of the story or question. The work students are doing will not lead to a solution and it therefore not productive.

Setting the Instructional Stage

Instructional Vision: Use tasks to provoke productive struggle toward big conceptual ideas. Students develop their own strategies and work to refine them over the unit of study.

Unit Structure: Modeled on Contexts for Learning (Fosnot, 2007, 2010), big conceptual ideas are explored through multi-day contexts.

Lesson Structure:

1. Launch: Introduction of the day's task on the carpet
2. Worktime: Students work in self-selected partnerships and teacher confers with students at work
3. Discussion: Students discuss findings and key conceptual questions

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When do we support student access?

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Stanford | Graduate School of Education

What is Conferring?

CONFERRING: TEACHER-STUDENT INTERACTIONS WHILE STUDENTS ACTIVELY GRAPPLE WITH PROBLEMS WHICH ELICIT AND NUDGE STUDENT THINKING

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Elicit Student Thinking

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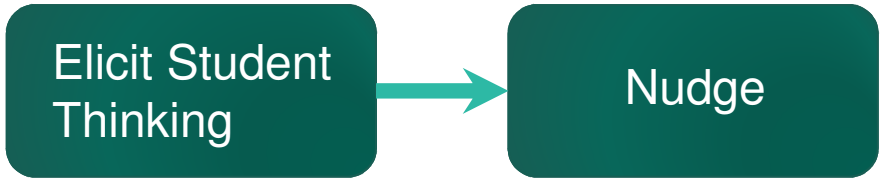
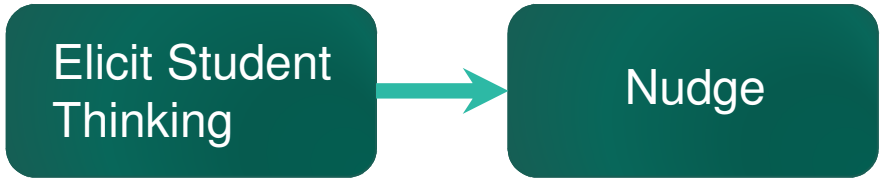


What is Conferring?

Conferring: Teacher-students interactions while students actively grapple with problems which elicit and nudge student thinking



Nudges that support access are about:

- Understanding the story
- Identifying the mathematical ideas in the story
- Using the story to develop a model that is:
  - Connected to the story
  - Accurate
  - Clear to students
  - Efficient for students

Six Moves to Support Access

From preliminary analysis of video recordings of 15 lessons, we have identified the following six types of moves to support student access to problems in context:

1. Retelling
2. Visualizing
3. Expanding
4. Acting Out
5. Modeling
6. Connecting the Representation to the Story

Retelling

What it is: Prompting students to retell the story aloud

Use it to:

- Assess comprehension and identify where understanding broke down
- Support understanding of vocabulary and the story sequence
- Support identifying the mathematics in the story

Sample Moves:

- "What's happening in the story?"
- "What happened next?"
- "Let's go back to the story (or text)."
- "What does the story say about that?"
- "Tell me the story again."
- Refer students to a chart or story as an anchor text.

Visualizing

What it is: Prompting students to make a mental image of the story or particular elements of the story, such as the setting or key objects

Use it to:

- Support understanding of vocabulary
- Bridge (and leverage) varied students experiences
- Develop a more detailed understanding of the story and mathematically important parts

Sample Moves

- "What does it look like?"
- "Can you picture what is happening? Tell me about it."
- "Have you ever seen a ___? What does it look like?"

Expanding

WHAT IT IS: CO-CONSTRUCTING WITH STUDENTS ADDITIONAL STORYTELLING DETAILS THAT MAKE THE CONTEXT CLEARER OR MORE VIVID

USE IT TO:

- SUPPORT STUDENTS IN VISUALIZING AND COMPREHENDING THE STORY
- MOTIVATE THE QUESTION BEING ASKED

SAMPLE MOVES

- "So, imagine…"
- "What would ___ do (to accomplish the action in the story)?"
- *These moves are highly contextual.*

Acting Out

What it is: Performing the story physically

Use it to:

- Make the action – typically the source of the mathematics – concrete
- Moves students toward modeling

Sample Moves

- "Let's do it. Who's going to be ___?"
- "How can we act it out?"
- "I'll be ___. Tell me what to do."

Modeling

What it is: Prompting students to represent the story with objects or pictures

Use it to:

- Represent and make visible the mathematics of the story
- Support students in physically manipulating and reasoning about the mathematics
- Move toward developing a strategy for solving the problem

Sample Moves

- "How could we do it with some objects?"
- "What could we use to represent the ____?"
- "How could we show what's happening?"

Connecting the Representation to the Story

WHAT IT IS: CLARIFYING THE MEANING OF STUDENTS' REPRESENTATIONS

USE IT TO:

- SUPPORT STUDENTS IN REASONING ABOUT THE WORK THEY HAVE ALREADY BEGUN
- RECONNECT STUDENTS TO WHAT ACTION THEY HAVE MODELED AND WHAT THEY STILL NEED TO
- IDENTIFY AND ADDRESS MISUNDERSTANDINGS ABOUT THE STORY'S MEANING
- SUPPORT PRECISION

SAMPLE MOVES

- "What does this number represent in the story?"
- "What does this (object, circle, tally, dot, etc.) represent in the story?"
- "Where are the _____ (items or people from the story)?"

Orchestrating the Moves

SETTING THE SCENE:

A pair of students, Joanna and Jessinia, were working on the vending machine problem, trying to figure out how many six-packs of water would fit in a machine that held 156 bottles.

They had an answer of 35 on their papers but struggled to explain it.

They had also been working on a problem about the juice vending machine which had six flavors arranged in columns inside the machine.

Orchestrating the Moves

COACH: **CAN YOU TELL ME WHERE THAT CAME FROM?**

JESSINIA: YOU CAN SHOW THAT HERE [POINTING TO DRAWING OF VENDING MACHINE] BECAUSE IT'S ALL BLOCKED. INSTEAD OF USING THE WATER, I USED THE JUICE, AND TO ADD SIX OF THOSE, AND I COUNTED IT.

COACH: SO I HAVE A QUESTION. **IS THE JUICE MACHINE SET UP THE SAME WAY AS THE WATER MACHINE?**

JESSINIA: YEAH.

COACH: THEY'RE THE SAME ON THE INSIDE?

JOANNA: YEAH, 'CAUSE, IT SAYS RIGHT THERE [POINTING TO CLASS CHART WITH PROBLEM] THAT THEY BOTH HAVE 156 BOTTLES.

COACH: THEY BOTH HAVE 156 ON THE INSIDE.

JOANNA: YEAH.

COACH: **BUT I'M WONDERING IF THE INSIDE OF THE MACHINE, WITH THE WATER, IS IT LIKE THIS WITH THE COLUMNS.**

JOANNA AND JESSINIA: NO.

JOANNA: I THINK IT'S DIVIDED UP IN ROWS.

Connecting Representation to the Story

Visualizing

Orchestrating the Moves

COACH: **IT COULD BE. WE JUST DON'T KNOW, DO WE? IT COULD BE BY ROWS, IT COULD BE BY COLUMNS, IT COULD BE BY SOMETHING ELSE. WE CAN'T SEE IT, CAN WE? SO DO YOU THINK THIS IS GOING TO HELP YOU – THAT THE JUICE MACHINE IS GOING TO HELP YOU TO THINK ABOUT THE WATER MACHINE?**

JESSINIA: NO

JOANNA: NO, NOT REALLY.

Visualizing

COACH: MAYBE NOT? OKAY. CAN WE SET THIS ONE ASIDE, BECAUSE MAYBE THIS IS A SEPARATE PROBLEM AND WE'RE GOING TO FOCUS ON THE WATER. SO I NOTICE THAT YOU DID A COUPLE OF THINGS HERE. SO ONE THINGS IS THAT YOU KNOW THAT 156 IS 100, 50, AND 6. YOU KNOW THAT, RIGHT? AND YOU CONFIRMED THAT BY ADDING THEM TOGETHER, AND YOU SAW THAT IT IS TRUE, RIGHT HERE [POINTING TO STUDENT WORK]. BUT NOW I'M WONDERING, YOU TOLD ME THAT YOU'RE TRYING TO FIGURE OUT HOW MANY SIX-PACKS, HOW MANY GROUPS OF 6, WILL FIT –

JOANNA: - IN THERE –

Modeling

COACH: - IN THERE. **HOW COULD YOU SHOW THAT SOMEHOW?**

JOANNA: I TRIED TO SHOW JESSINIA A STRATEGY, BUT I DON'T KNOW IF IT WILL TAKE LONG.

Orchestrating the Moves

COACH: **WAS THAT WHAT THIS WAS HERE [POINTING TO STUDENT WORK]. (PAUSE) WHY DON'T YOU TELL ME ABOUT THIS.**

JOANNA: I WAS GOING TO PUT 6…6 CIRCLES AND THEN…

COACH: SO I SEE A CIRCLE, AND INSIDE I SEE 6 DOTS.

JOANNA: YEAH.

COACH: SO WHAT DOES THIS REPRESENT? CAN YOU TELL ME WHAT THAT MEANS?

JESSINIA: SIX.

JOANNA: SIX. I WAS GOING TO PUT SIX CIRCLES AND SIX DOTS IN EACH, AND SEE HOW MUCH WATER BOTTLES WE NE

COACH: - SO, FOR YOU, REPRESENT, AGAIN?

JOANNA: THE SIX, THE SI

COACH: THE SIX. DOES I

JOANNA: YEAH.

COACH: IT DID. OKAY, SO

JOANNA: BECAUSE I THAT, UM, WHEN WE WERE IN THIRD GRADE, WE HAD ANSWERS, WE HAD TO DIVIDE THEM.

COACH: OKAY.

JOANNA: SO WHAT I KNEW ABOUT THIS WAS, UM, WHENEVER YOU PUT IT, LIKE, DOTS, LIKE 6 DOTS, WHATEVER WAS THE PROBLEM, YOU PUT SIX DOTS. I PUT SIX DOTS AND SIX CIRCLES AND IT GAVE YOU AN ANSWER, YOU PUT IT THERE. YOU PUT IT UP HERE, AND IT GIVES YOU AN ANSWER. IT EQUALS 150, 150, 150…BUT WE DON'T KNOW…

Connecting Representation to the Story

What moves do you see in the rest of this transcript?

Orchestrating the Moves

COACH: **WAS THAT WHAT THIS WAS HERE [POINTING TO STUDENT WORK]. (PAUSE) WHY DON'T YOU TELL ME ABOUT THIS.**

JOANNA: I WAS GOING TO PUT 6…6 CIRCLES AND THEN…

COACH: SO I SEE A CIRCLE, AND INSIDE I SEE 6 DOTS.

JOANNA: YEAH.

COACH: **SO WHAT DOES THIS REPRESENT? CAN YOU TELL ME WHAT THAT MEANS?**

JESSINIA: SIX.

JOANNA: SIX. I WAS GOING TO PUT SIX CIRCLES AND SIX DOTS IN EACH, AND SEE HOW MUCH WATER BOTTLES WE NEEDED. AND, SO THEN –

COACH: - **SO, FOR YOU, WHEN YOU WERE DRAWING THIS, WHAT DID THIS REPRESENT, AGAIN?**

JOANNA: THE SIX, THE SIX…

COACH: THE SIX. DOES IT REPRESENT THE 6 FROM THE 156?

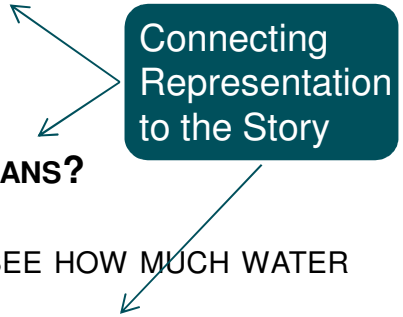
JOANNA: YEAH.

COACH: IT DID. OKAY, SO WHY DID YOU WANT TO REPRESENT THAT SIX.

JOANNA: BECAUSE I THAT, UM, WHEN WE WERE IN THIRD GRADE, WE HAD ANSWERS, WE HAD TO DIVIDE THEM.

COACH: OKAY.

JOANNA: SO WHAT I KNEW ABOUT THIS WAS, UM, WHENEVER YOU PUT IT, LIKE, DOTS, LIKE 6 DOTS, WHATEVER WAS THE PROBLEM, YOU PUT SIX DOTS. I PUT SIX DOTS AND SIX CIRCLES AND IT GAVE YOU AN ANSWER, YOU PUT IT THERE. YOU PUT IT UP HERE, AND IT GIVES YOU AN ANSWER. IT EQUALS 150, 150, 150…BUT WE DON'T KNOW…



Orchestrating the Moves

COACH: **SO, LET'S GO BACK TO THE STORY.** I'M GOING TO TURN THIS [PAPER] OVER BECAUSE I KNOW WHAT THIS REPRESENTS NOW AND I KNOW WHAT YOU DID HERE. **BUT I WANT TO START US BACK AT THE BEGINNING JUST FOR A MOMENT TO SEE IF WE CAN FIND A WAY TO MOVING ON THE PROBLEM. ALRIGHT? ALRIGHT. SO, IF WE GO BACK TO THE BEGINNING, CAN YOU TELL ME WHAT JESSICA IS DOING? WITH THE WATER MACHINE – WHAT'S HAPPENING?**

JESSINIA: SHE'S –

JOANNA - SHE'S TRYING TO, UM, SHE KNOWS HOW TO OPEN THE MACHINE, BUT HER PROBLEM, THE PROBLEM THAT SHE DOESN'T KNOW HOW TO DO IS HOW MUCH WATER BOTTLES DOES SHE NEED TO PUT IN THE VENDING MACHINE.

COACH: SHE DOESN'T KNOW HOW MANY WATER BOTTLES TO PUT IN THE VENDING MACHINE. **AND IS IT JUST HOW MANY WATER BOTTLES THAT SHE DOESN'T KNOW?**

JOANNA: IT'S PACKS.

COACH: IT'S PACKS SHE DOESN'T KNOW, RIGHT? **SO I WANT YOU TO IMAGINE JESSICA HAS THIS CART [POINTS TO PICTURE] AND SHE'S AT HER TRUCK AND SHE'S TRYING TO TAKE THE PACKS OFF –**

JOANNA: - I THINK SHE'S GOING TO TAKE 4 PACKS…

COACH: SHE'LL START WITH 4 PACKS? (PAUSE)

JOANNA: 6, 12,…

Retelling

Expanding

Orchestrating the Moves

COACH: AND YOU'RE GOING TO SEE WHERE THAT GETS YOU? **OKAY, SO, HOW COULD YOU SHOW, WITH SOME KIND OF DRAWING, THE FOUR PACKS THAT YOU'RE GOING TO TAKE?** IS THERE SOME – SO MAYBE WE NEED A FRESH PLACE TO DRAW. [JESSINIA HOPS UP] YOU'RE GOING TO GET SOMETHING?

JESSINA: YEAH.

COACH: JESSINIA'S GOING TO GO GET SOMETHING. LET'S WAIT A SECOND FOR HER. (LONG PAUSE) [JESSINIA RETURNS WITH PAPER.] OKAY, SO YOU WANT TO START WITH 4 PACKS, AND JUST SEE HOW THAT IS? **SO CAN YOU GUYS THINK, HOW COULD WE SHOW THE FOUR PACKS HERE?**

Modeling

JOANNA: I THINK I'LL USE THE STRATEGY, I THINK I'LL USE THE STRATEGY THAT I DID BEFORE. LIKE, HERE I WANT TO PUT FOUR OF THESE [POINTING TO CIRCLES].

COACH: OKAY, AND **WHAT IS EACH CIRCLE GOING TO REPRESENT NOW?**

JOANNA: THEY'RE GOING TO REPRESENT, UH, THE WATER BOTTLES AND…

COACH: **IS EACH CIRCLE A WATER BOTTLE?**

JOANNA: NO, THEY'RE SIX-PACKS.

Connecting Representation to the Story

COACH: **THEY'RE SIX-PACKS. OKAY, SO THE CIRCLE IS THE SIX-PACK AND INSIDE THE CIRCLE –**

JOANNA: - THERE'RE SIX BOTTLE, WATER BOTTLES.

COACH: OKAY, SO WHY DON'T YOU GO AHEAD AND DRAW THAT.

The Value of this Approach

- Maintains student authority over the mathematics
- Keeps the math and context integrated
- Allows the teacher to serve as a resource for equitable access to contexts
- Maintains the cognitive demand of the task
- Portable across grades, content, and contexts

Think about Your Own Context

Think a moment in which students were clearly confused about a problem in context.

- What was happening?
- What was the story context?
- What could you now imagine saying to support access?

Which of the types of moves we looked at today have the most potential to support your students?

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Orchestrating the Moves

SETTING THE SCENE:

A group of four students were trying to solve a problem involving a submarine sandwich that had been cut into five equal pieces for five friends to share. Before eating one friend, Lawrence, gets up and has to run off to soccer practice. He passes his piece to Sam and walks out. How much of the sub sandwich does Sam get?

The students in this group were stuck outside the problem. The teacher asked what they were trying to figure out and one student, George, said "The numerator."

Orchestrating the Moves

TEACHER: WE'RE TRYING TO FIGURE OUT THE NUMERATOR FOR…SAM'S AMOUNT OF THE SUB?

ANGELA: YEAH.

TEACHER: WHAT ARE YOU THINKING?

ANGELA: 2/5

TEACHER: WHAT WAS THAT, ANGELA?

ANGELA: 2/5

TEACHER: OKAY, WHY?

DESTINY: SO, I'M THINKING, THAT IT MIGHT BE MORE, BECAUSE IT MIGHT BE LIKE FIVE PIECES CUT IN HALF.

TEACHER: SAY THAT AGAIN.

DESTINY: I THINK IT'S LIKE CUTTING FIVE PIECES IN HALF.

TEACHER: **WHY ARE YOU CUTTING FIVE PIECES IN HALF? WHAT DOES THAT MEAN?**

DESTINY: I THINK IT'S LIKE FOUR…

TEACHER: FOUR WHAT?

DESTINY: FOUR PIECES, CUT…

.

Orchestrating the Moves

TEACHER: **OKAY, SO, THERE'S FIVE PIECES [ARRANGING PIECES OF THE SANDWICH ON THE TABLE], YES?**

DESTINY: MHMM.

TEACHER: **AND LAWRENCE LEAVES. WHAT HAPPENS TO THE SUB?**

ANGELA: IT'S, UH,…

GEORGE: IT GETS ALL MIXED UP. 'CAUSE IT'S NOT FAIR THAT SAM GET TWO PIECES.

TEACHER: OKAY, DID I SAY ANYTHING ABOUT, **IS THERE ANYTHING IN THE STORY ABOUT ANJELICA, SARA, OR CAMERON BEING UPSET?**

GEORGE: NO.

TEACHER: SO, IS THIS A PROBLEM ABOUT FAIRNESS?

GEORGE: NO.

TEACHER: OKAY.

ANGELA: I DISAGREE WITH THAT, BECAUSE, UH –

TEACHER: WHAT DO YOU DISAGREE WITH?

ANGELA: BECAUSE IT DOESN'T JUST, JUST, LIKE, JUST BECAUSE LAWRENCE LEAVES IT…LIKE, THIS IS LIKE, SAM'S PIECE AND LAWRENCE GAVE THIS ONE TO HIM, AND SO IT'S STILL LIKE FIVE PIECES.

TEACHER: **SO, TELL ME IF I'M WRONG. SO, WHAT YOU'RE SAYING ANGELA IS LIKE, LET'S PRETEND YOU'RE ANGELICA [TO DESTINY PASSING HER A PIECE OF SUB], YOU'RE SAM [TO ANGELA], YOU'RE CAMERON [TO SANDRA], YOU'RE SARA [TO GEORGE], AND I'M LAWRENCE. OKAY, YEAH? OKAY, THEN I'M LIKE, "OH, I'VE GOT TO GO TO SOCCER PRACTICE! SEE YOU GUYS LATER!" [STAND UP AND PUSHES HER PIECE OF SUB TO SAM. LEAVES THE TABLE]** OKAY, SO HOW DOES THAT CHANGE THE SITUATION?

GEORGE: IT CHANGE THE SITUATION BECAUSE SAM GETS 2/5 OF THE SUB