Integrating Algebraic Thinking In Elementary Math: The Power of a Routine

Melissa Canham

@Melissa_Canham

Glenda Martinez

@GCMartinez23

Common Components of CGI / CCSS-M Classrooms

- Problem solving is the focus of instruction; teachers pose a variety of problems.
- Many problem-solving strategies are used to solve problems. Children decide how they should solve each problem.
- Children communicate to their teachers and peers how they solved the problems.
- Teachers understand children’s problem-solving strategies and use that knowledge to plan instruction.

What is Algebraic Thinking?

Algebraic thinking involves the construction and representation of patterns and regularities, deliberate generalization, and most important, active exploration and conjecture. (Kaput, NCTM, 1993).

"Children can learn arithmetic in a way that provides a basis for learning algebra."

Thinking Mathematically,
Carpenter, Franke, & Levi

What does Algebraic Thinking look like in Elementary CCSS-M?

- "Using objects or drawings"
- "Use strategies such as…"
- "Strategies based on place value, properties of operations, and the relationship between…"
- "Flexible thinking"

Relational Thinking

Relational Thinking

- Students are able to express a number in terms of other numbers and operations on those numbers
  - Takes into account the fundamental properties of operations and equality
- The equal sign is viewed as expressing a relation between numbers

The Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is a Routine?

- 5 – 15 minutes during the opening of math time
- Provide students with meaningful ongoing practice with:
  - Number Sense
  - Place Value
  - Computational Fluency
  - Properties of Operations
  - Fractions
  - Standards for Mathematical Practice
  - Listening to others' strategies

Routines

- Allows teachers to fill in the number sense holes that students are coming in with
- Helps to introduce new strategies in a way that gets students thinking
- Forces students (especially older students) to think outside the standard algorithm
- Helps to address the fluency standard

Goals for Making Properties of Arithmetic Explicit

- All students have access to basic mathematical properties
- Students understand why the computation procedures they use work the way they do
- Students apply their procedures flexibly in a variety of contexts
- Students recognize the connections between arithmetic and algebra and can use their understanding of arithmetic as a foundation for learning algebra with understanding

- Thinking Mathematically, Carpenter et. al

True/False Number Sentences

CGI Approach to Problem Solving

When problems are sequenced in ways that are sensitive to children's developing understanding, children can solve both word problems and equations without explicit instruction.

Extending Children's Mathematics: Fractions and Decimals Susan B. Empson & Linda Levi (2012)

True/ False Number Sentences

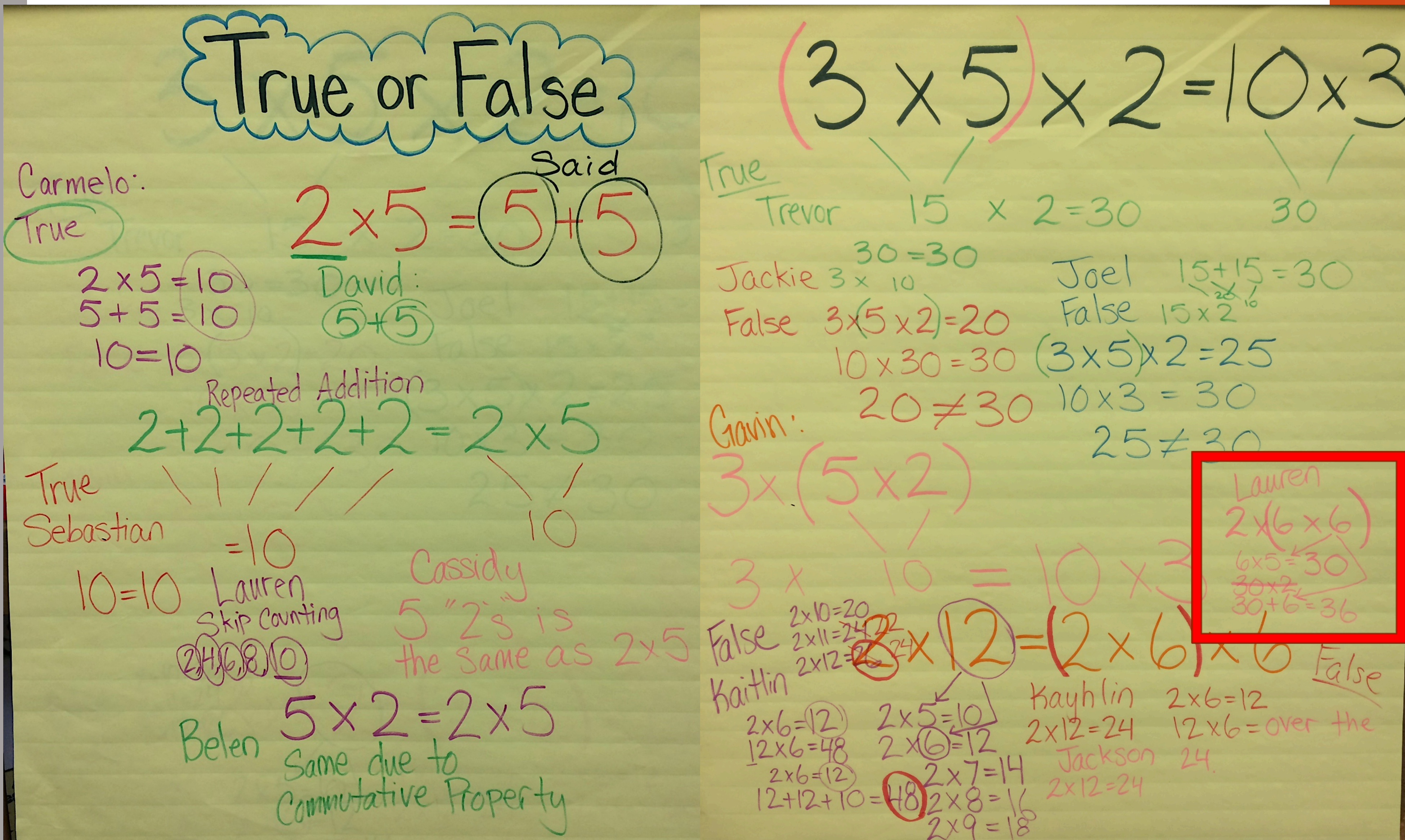
$2 \times 5 = 5 + 5$

$2 + 2 + 2 + 2 + 2 = 2 \times 5$

$5 \times 2 = 2 \times 5$

$3 \times 5 \times 2 = 10 \times 3$

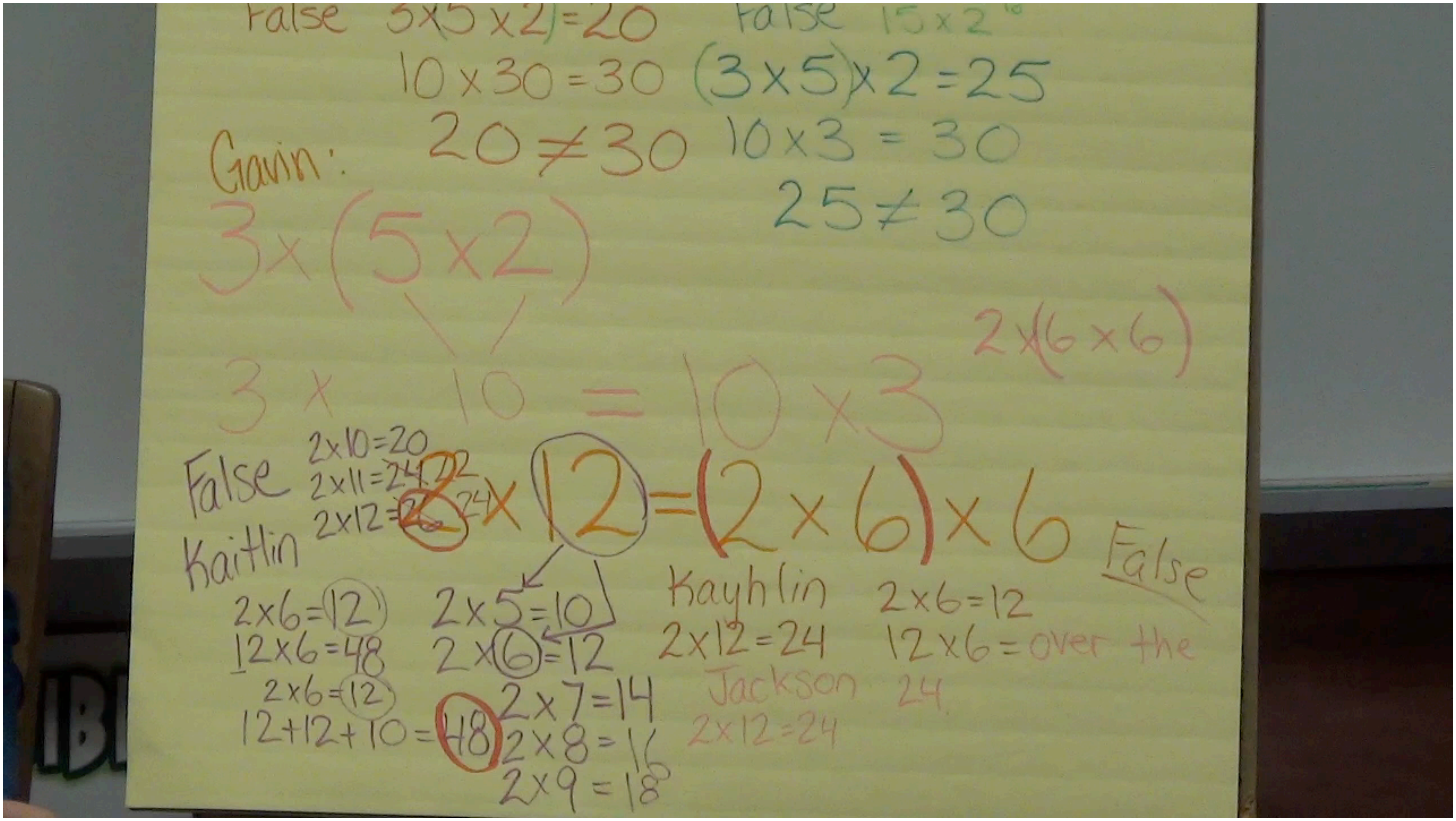
$2 \times 12 = 2 \times 6 \times 6$


True or False
Carmelo:
True
2x5=5+5
Said
2x5=10
5+5=10
10=10
David:
5+5
Repeated Addition
2+2+2+2+2=2x5
True
Sebastian
=10
10
10=10
Lauren
Skip Counting
2,4,6,8,10
Cassidy
5 "2's" is the same as 2x5
Belen
5x2=2x5
Same due to Commutative Property
(3x5)x2=10x3
True
Trevor
15 x 2=30
30
30=30
Jackie 3x 10
False 3x(5x2)=20
10x30=30
20≠30
Joel
False
15+15=30
15x2
(3x5)x2=25
10x3=30
25≠30
Gavin:
3x(5x2)
3 x 10 = 10 x 3
Lauren
2x(6x6)
6x5=30
30x2
30+6=36
2x12=(2x6)x6
False
Kaitlin
2x10=20
2x11=22
2x12=24
2x6=12
12x6=48
2x6=12
12+12+10=48
2x5=10
2x6=12
2x7=14
2x8=16
2x9=18
False
Kayhlin
2x12=24
2x6=12
12x6=over the 24.
Jackson
2x12=24

"The primary goal in giving students these number sentences is not to teach students efficient ways to solve algebra equations; it is to engage them in thinking flexibly about number operations."

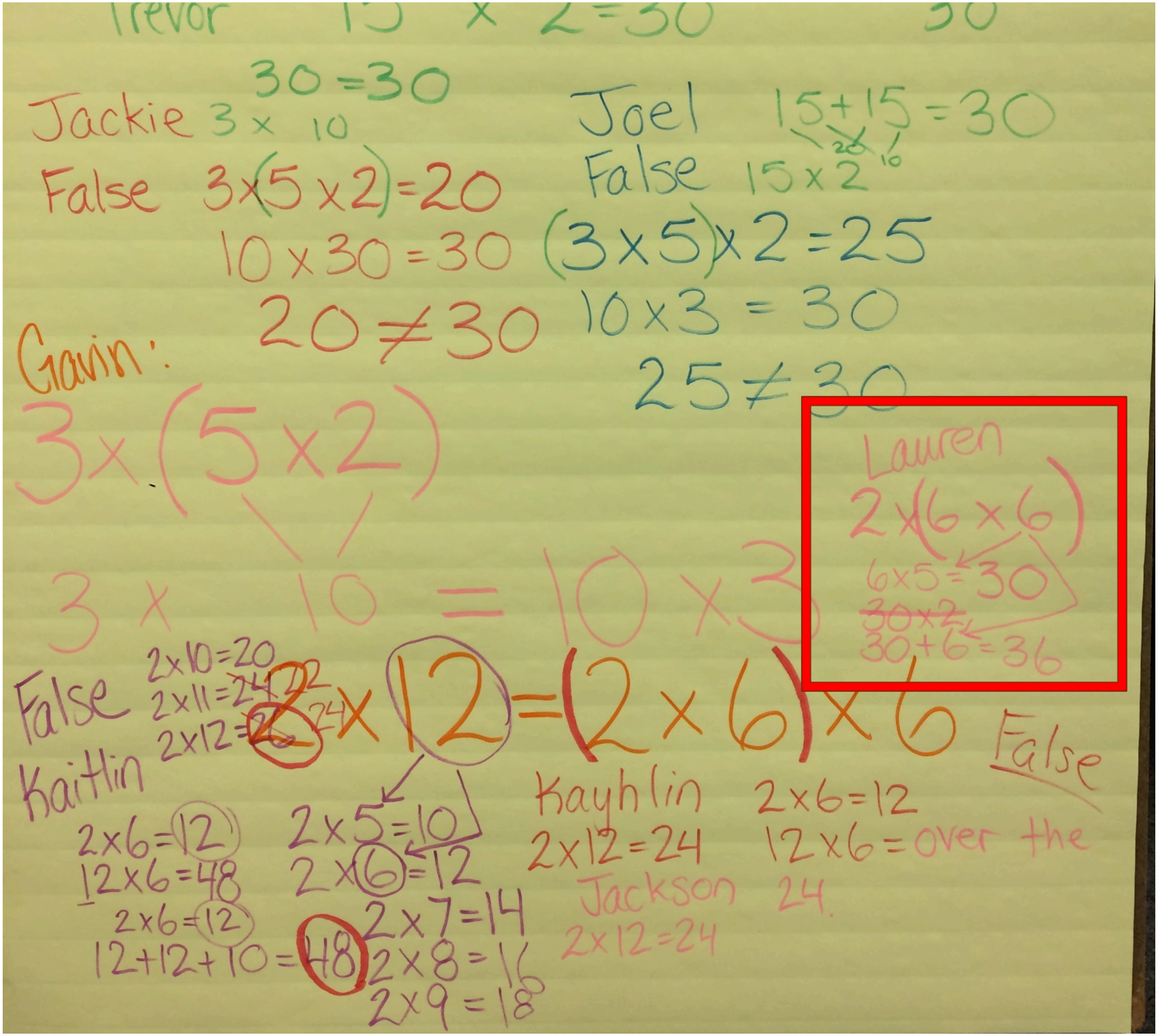
- Thinking Mathematically, Carpenter et. al, pg. 73

True/False Number Sentences



What Did You Notice?

- What algebraic thinking was evident?
- How did the teacher respond?
- If these were your students, where would you go next with these ideas?


Trevor 15 x 2 = 30 30
30 = 30
Jackie 3 x 10
False 3 x (5 x 2) = 20
10 x 30 = 30
20 ≠ 30
Joel 15 + 15 = 30
False 15 x 2
(3 x 5) x 2 = 25
10 x 3 = 30
25 ≠ 30
Gavin:
3 x (5 x 2)
3 x 10 = 10 x 3
Lauren
2 x (6 x 6)
6 x 5 = 30
30 x 2
30 + 6 = 36
2 x 12 = (2 x 6) x 6
False
2x10=20
2x11=22
2x12=24
Kaitlin
2x6=12
12x6=48
2x6=12
12+12+10=48
2x5=10
2x6=12
2x7=14
2x8=16
2x9=18
Kayhlin
2x12=24
2x6=12
12x6=over the 24.
Jackson
2x12=24

"We are not proposing that it is always necessary to introduce equations or use names of properties with children. What is important is that children meaningfully engage with these properties and that they begin to explicitly recognize how the mathematics they are using depends on them."

- Children's Mathematics, Carpenter et. al, pg.179

Open Expressions

For each expression below, choose two different values that would make calculations "easy". Then simplify the expressions.

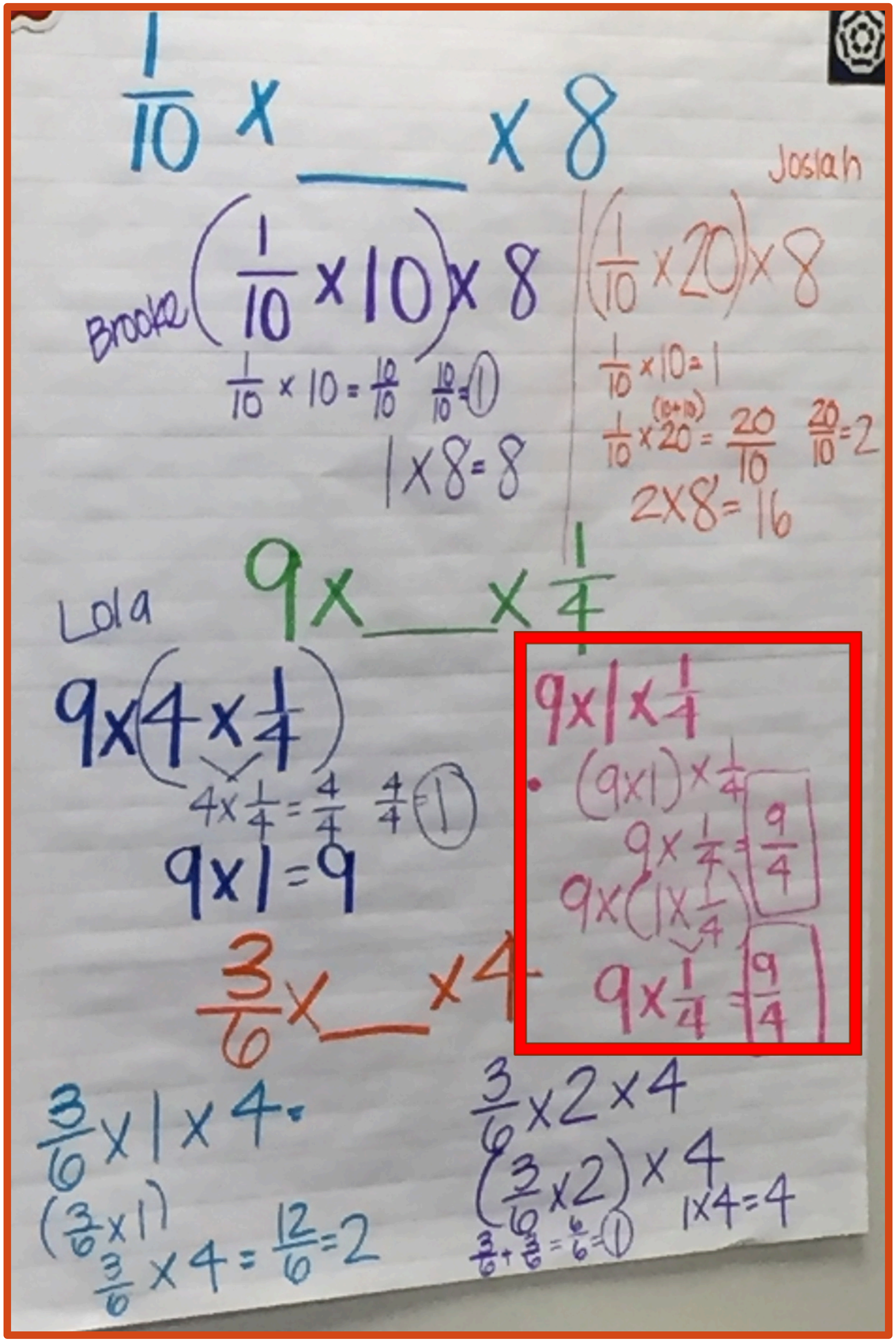
$$\frac{1}{10} \times \underline{\qquad\qquad} \times 8$$

$$9 \times \underline{\qquad\qquad} \times \frac{1}{4}$$

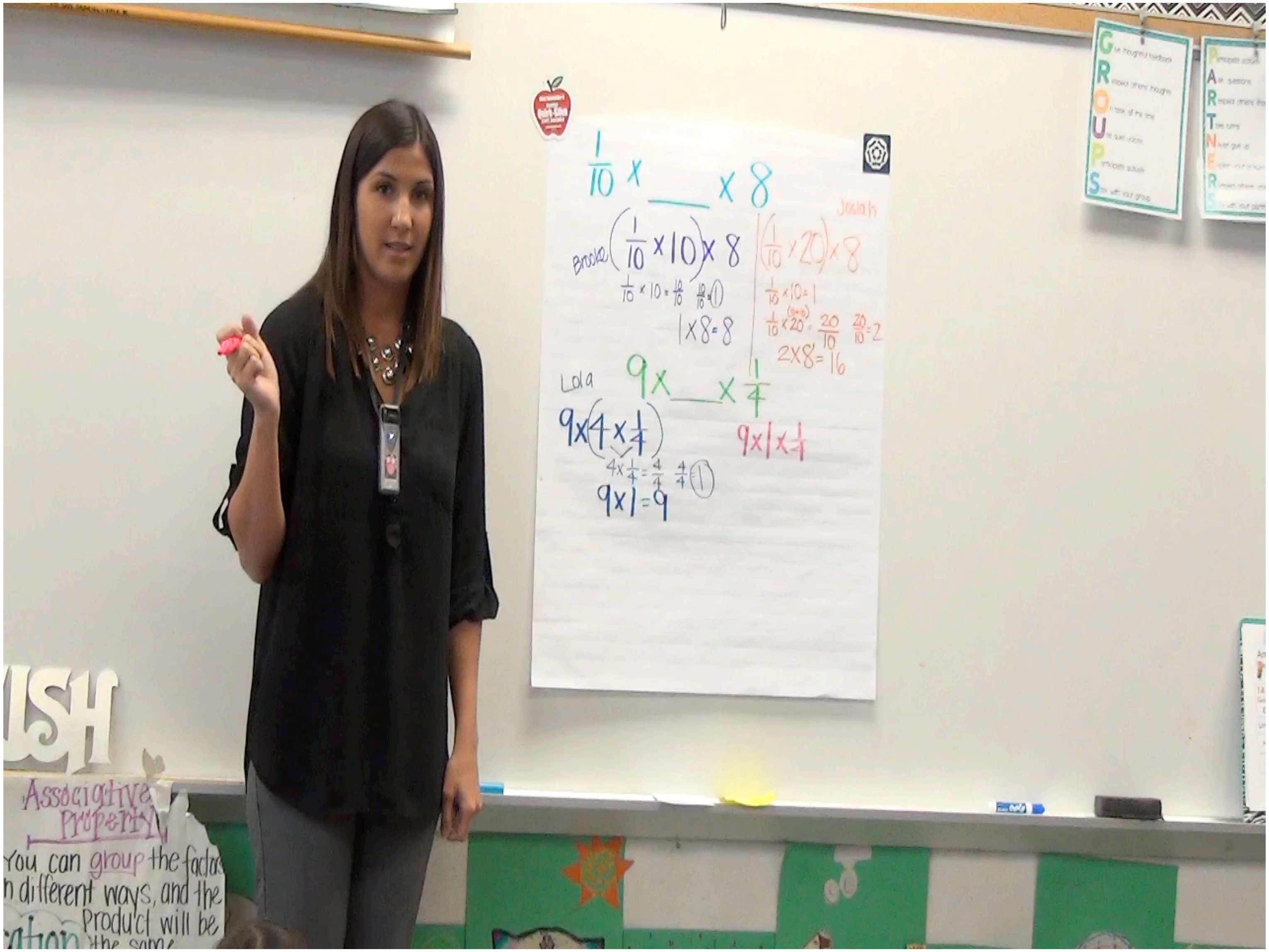
$$\frac{3}{6} \times \underline{\qquad\qquad} \times 4$$

“Learning about whole numbers should provide a foundation for learning about fractions and decimals and it should involve a seamless transition to learning algebra.”

- Children’s Mathematics, Carpenter et. al, pg. xx


1/10 x ___ x 8
Brooke (1/10 x 10) x 8
1/10 x 10 = 10/10 10/10 = 1
1 x 8 = 8
Josiah
(1/10 x 20) x 8
1/10 x 10 = 1
1/10 x 20 (10+10) = 20/10 20/10 = 2
2 x 8 = 16
9 x ___ x 1/4
Lola
9 x (4 x 1/4)
4 x 1/4 = 4/4 4/4 = 1
9 x 1 = 9
9 x 1 x 1/4
(9 x 1) x 1/4
9 x 1/4 = 9/4
9 x (1 x 1/4)
9 x 1/4 = 9/4
3/6 x ___ x 4
3/6 x 1 x 4
(3/6 x 1)
3/6 x 4 = 12/6 = 2
3/6 x 2 x 4
(3/6 x 2) x 4
3/6 + 3/6 = 6/6 = 1
1 x 4 = 4

Open Expressions



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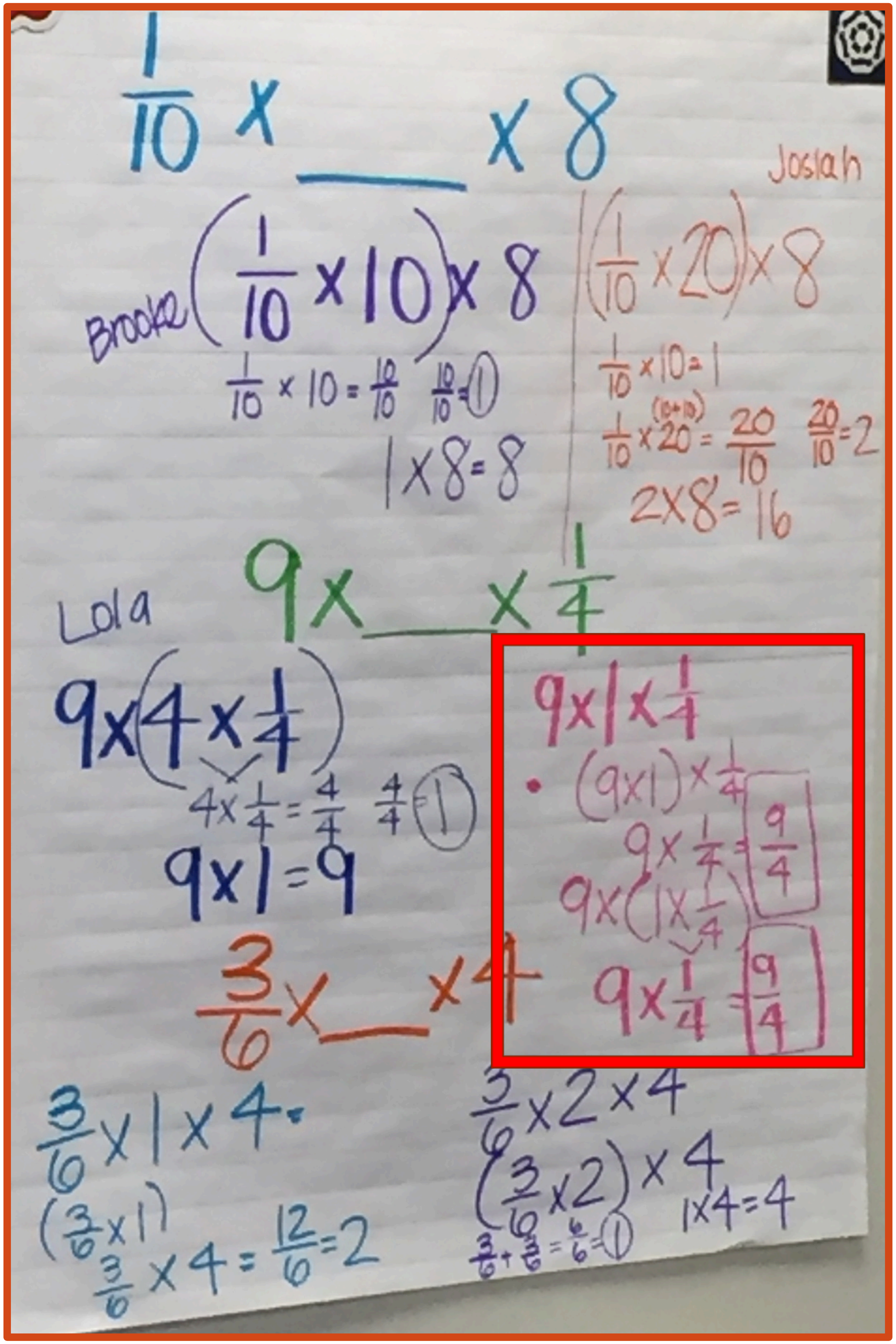
1/10 x ___ x 8
Josiah
Brooke
(1/10 x 10) x 8
1/10 x 10 = 10/10   10/10 = 1
1 x 8 = 8
(1/10 x 20) x 8
1/10 x 10 = 1
(10+10)
1/10 x 20 = 20/10   20/10 = 2
2 x 8 = 16
9 x ___ x 1/4
Lola
9 x (4 x 1/4)
4 x 1/4 = 4/4   4/4 = 1
9 x 1 = 9
9 x 1 x 1/4
(9 x 1) x 1/4
9 x 1/4 = 9/4
9 x (1 x 1/4)
9 x 1/4 = 9/4
3/6 x ___ x 4
3/6 x 1 x 4
(3/6 x 1)
3/6 x 4 = 12/6 = 2
3/6 x 2 x 4
(3/6 x 2) x 4
3/6 + 3/6 = 6/6 = 1
1 x 4 = 4

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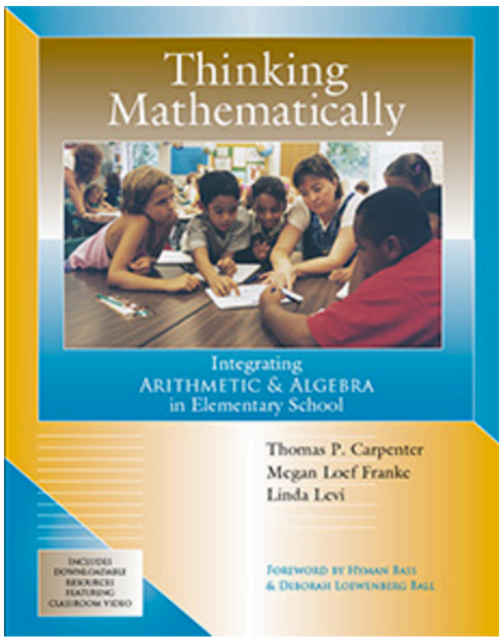
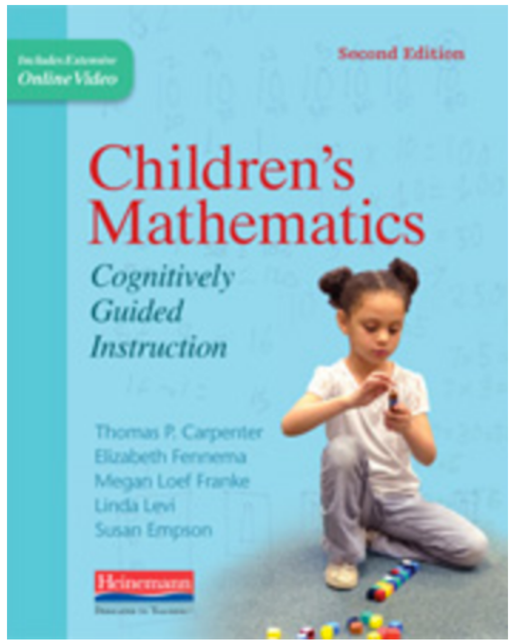
Resources


Thinking Mathematically
Integrating
Arithmetic & Algebra
in Elementary School
Thomas P. Carpenter
Megan Loef Franke
Linda Levi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Children's Mathematics
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Thomas P. Carpenter
Elizabeth Fennema
Megan Loef Franke
Linda Levi
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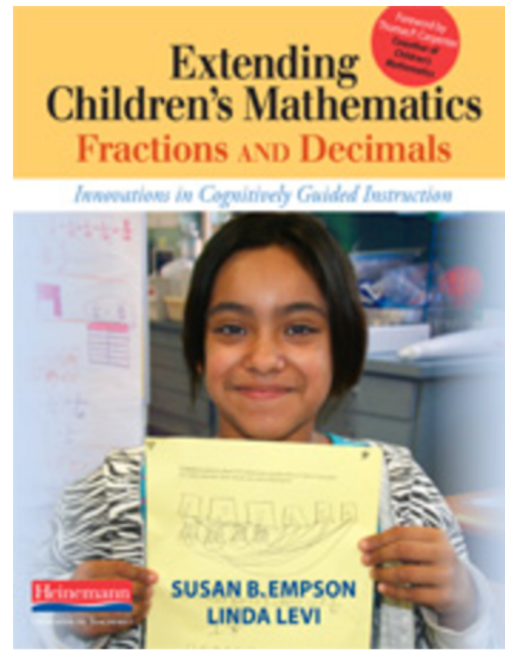
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Innovations in Cognitively Guided Instruction
Susan B. Empson
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Thank You!

Melissa Canham: mcanham@dusd.net

@Melissa_Canham

Glenda Martinez: gmartinez@dusd.net

@GCMartinez23

DUSD CGI Website: www.dusd.net/cgi

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