Fluency, Discourse, and Standards for Mathematical Practice

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Session 639

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What Does It Mean to Be Fluent in Math?

- Math fact fluency
- Computational fluency
- Numerical fluency
- Procedural fluency

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2

Definitions of Fluency

- **Math fact fluency** is the ability to recall the answers to basic math facts automatically and without hesitation.
- **Computational fluency** involves a combination of being accurate, efficient, and flexible when working with basic facts (addition, subtraction, multiplication, and division).

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Definitions of Fluency

- **Numerical Fluency** is the ability to compose and decompose numbers flexibly, efficiently, and accurately within meaningful contexts.
- **Procedural fluency** is the ability to: *apply procedures accurately, efficiently, and flexibly;* transfer procedures to different problems and contexts; *build or modify procedures from other procedures;* recognize when one strategy or procedure is more appropriate than another .

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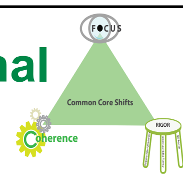
A Few Thoughts About Fluency

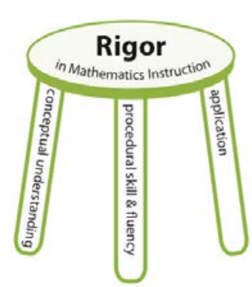
- Fluency is about understanding, strategies, and efficiency—not merely speed
- Conceptual understanding and strategies are the foundations on which fluency is built
- Helping students develop fluency requires purposeful instructional moves.

5

Common Core Instructional Shifts: Rigor



- The Common Core requires a balance (*requiring equal intensity in time, activities, and resources in each*) of:
  - Conceptual Understanding
  - Procedural Skill and Fluency
  - Application/Problem Solving



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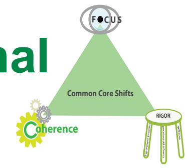

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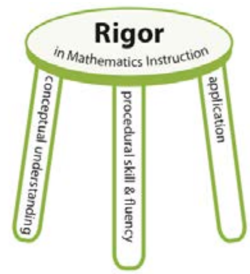

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Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. *Build **procedural fluency** from conceptual understanding.*
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

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Build Procedural Fluency from Conceptual Understanding

- Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding ***so that students, over time, become skillful in using procedures flexibly*** *as they solve contextual and mathematical problems.*

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Build Procedural Fluency from Conceptual Understanding

- Procedural Fluency should:
  - Build on a foundation of conceptual understanding;
  - Result in generalized methods for solving problems; and
  - Enable students to flexibly choose among methods to solve contextual and mathematical problems.

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10

Build Procedural Fluency from Conceptual Understanding

- Students must be able to do much more than carry out mathematical procedures. They must know:
  - Which procedure is most appropriate and most productive in a given situation
  - What a procedure accomplishes
  - What kinds of results to expect
- Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.

(Martin, 2009, p. 165)

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11

What Does Fluency Look Like?

- What approaches to these three problems would you consider to be showing more or less potential evidence of fluency?

A. 1002 – 998 =

B. 57 x 4 =

C. 159 ÷ 13 =

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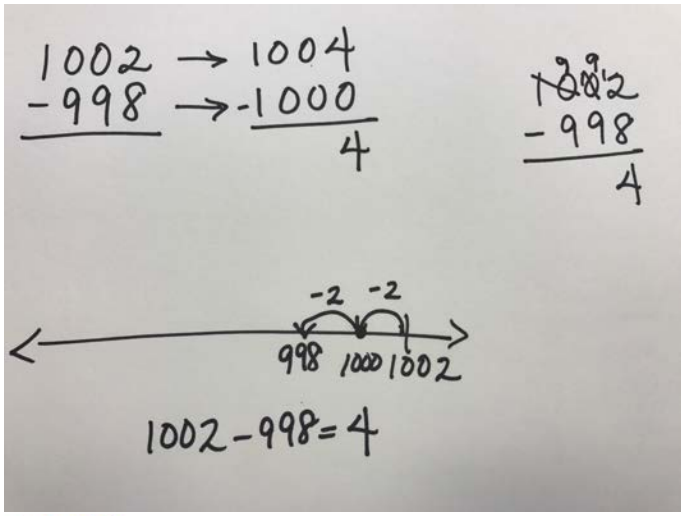
1002 – 998
More or Less Fluent?



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57 x 4
More or Less Fluent?

David's solution

$$\begin{array}{r} {}^{+2}57 \\ \times\ 4 \\ \hline 288 \end{array}$$

I multiplied 7 and 4 and got 28. I put down the 8 and carried the 2. Then I added the 2 and the 5 and got 7 and multiplied it by 4 and got 28. I put down the 28 and got 288.

Anna's solution

4×57
$4 \times 50 = 200$
$4 \times 7 = 28$
$200 + 28 = 228$

I did it in parts. First I multiplied 4 x 50 and got 200. Then I multiplied 4 and 7 and got 28. Then I just added those two parts together to get the answer.

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National Council of Teachers of Mathematics (2014). *Principles to Actions: Ensuring mathematical success for all.* NCTM: Reston, VA.

14

159 ÷ 13
More or Less Fluent?

- Cara thought about the problem as, ”How many 13s are in 159?"
  - She knew that ten 13s is 130, leaving her 29 remaining in the dividend, and that she could take two 13s out of that 29, giving her a quotient of 12 with a remainder of 3.
- Armand counted by 13s until he reached 52. Then he added some 52s to get as close to 159 as possible (52 + 52 + 52 = 156).
  - He knew there were four 13s in each of those 52s, so he had twelve 13s, with a remainder of 3.

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Teacher and Student Actions

Build procedural fluency from conceptual understanding
Teacher and student actions

What are *teachers* doing?	What are *students* doing?
Providing students with opportunities to use their own reasoning strategies and methods for solving problems.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using.
Asking students to discuss and explain why the procedures that they are using work to solve particular problems.	Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.
Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Determining whether specific approaches generalize to a broad class of problems.

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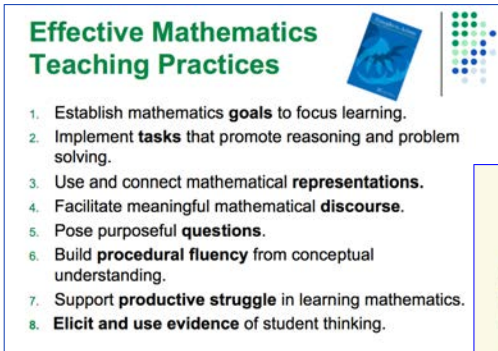
ETPs and SMPs to Support Students' Procedural Fluency


Effective Mathematics Teaching Practices

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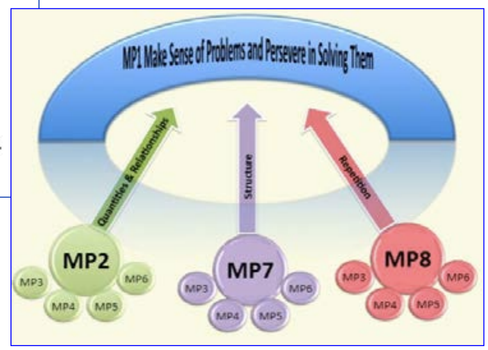

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17

Meaningful Mathematics Discourse

- We know that to truly develop mathematical thinking, students need to be active participants–they should be *both* **doing math** *and* **talking about math.**

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18

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Discourse to Support Procedural Fluency

- "…students must have ample opportunities to take part in a variety of rich, structured conversations—as part of a whole class, in small groups, and with a partner…
- …Being productive members of these conversations requires that students **contribute** accurate, relevant information; **respond to and develop** what others have said; **make comparisons and contrasts**; and **analyze and synthesize** a multitude of ideas in various domains."

—Common Core State Standards Initiative (ELA Standards)

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20

Meaningful Mathematics Discourse

- Mathematical discourse gives students opportunities to:
  - Share ideas and clarify understandings
  - Construct convincing arguments
  - Develop a language for expressing mathematical ideas
  - Learn to see things from a variety of perspectives

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21

Teacher and Student Actions

Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

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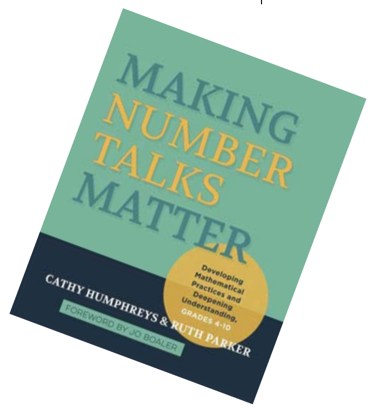
National Council of Teachers of Mathematics (2014). *Principles to Actions: Ensuring mathematical success for all.* NCTM: Reston, VA.

22

Number Talks

- Number talks are about students making sense of their own mathematical ideas.
- The minute ***we*** start to explain, we take little bits of their ideas—and their autonomy as thinkers—away.



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23

Number Talks and Discourse

- Number Talks support students to:
  - Think and reason quantitatively...not just focus on how to compute but knowing and flexibly using different properties of operations
  - Make connections and look for relationships while "doing mathematics."
  - Share strategies for their mathematical processes to clarify their thinking, using correct mathematical language both orally and in writing.
  - Recognize math concepts go beyond memorized procedures.

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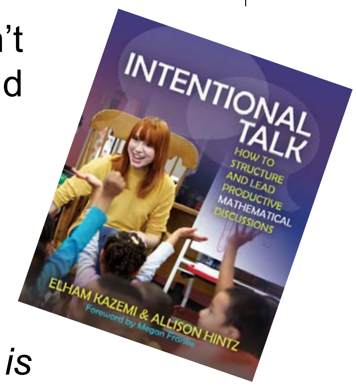
Intentional Talk

- Mathematics discussions aren't just about show-and-tell: "stand up, sit down, clap, clap, clap."
- Knowing what to do with students' ideas and teaching children how to meaningfully participate in discussions can be a lot more daunting…*but it is doable*.



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Kazemi & Hintz, *Intentional Talk*, 2014 25

Principles for Classroom Discussions

1. Discussions should achieve a mathematical goal, and different types of goals require planning and leading discussions differently.
2. Students need to know what and how to share so their ideas are heard and are useful to others.
3. Teachers need to orient students to one another and the mathematical ideas so that every member of the class is involved in achieving the mathematical goal.
4. Teachers must communicate that all children are sense makers and that their ideas are valued.

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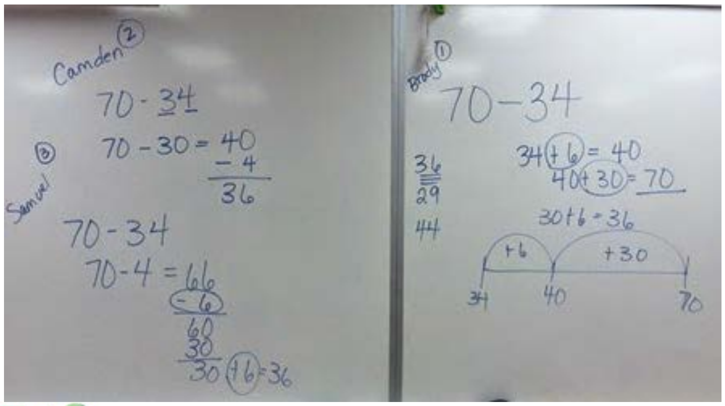
Kazemi & Hintz, *Intentional Talk*, 2014 26

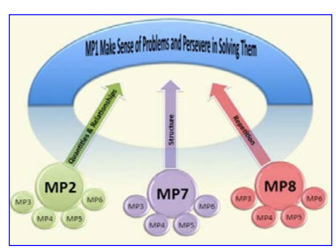
Open Strategy Sharing

- After posing a problem for students, the teacher asks them to share various strategies and records their approaches as they talk.



Targeted Discussion Structures & Their Goals

1. Compare and Connect
2. Why? Let's Justify
3. What's Best and Why?
4. Define and Clarify
5. Troubleshoot and Revise



Kazemi & Hintz, *Intentional Talk*, 2014

Targeted Discussion Structures & Their Goals

- Compare and Connect
  - To compare similarities and differences among strategies
- What's Best and Why?
  - To determine a best (most efficient) solution strategy in a particular circumstance

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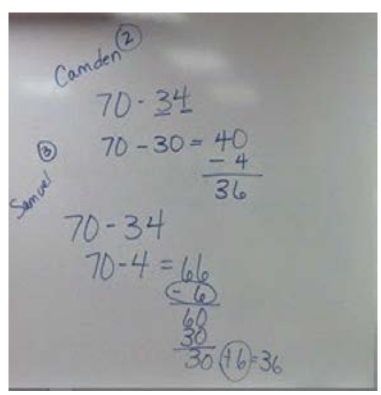
Kazemi & Hintz, *Intentional Talk*, 2014 29

Compare and Connect

- Decide which strategies you want students to compare and connect
- Identify connections you think are important for student to notice
- Anticipate what students may notice…and how you might respond to support their ideas
- Make sure you are clear on the mathematical idea you want to target in the compare/connect discussion.



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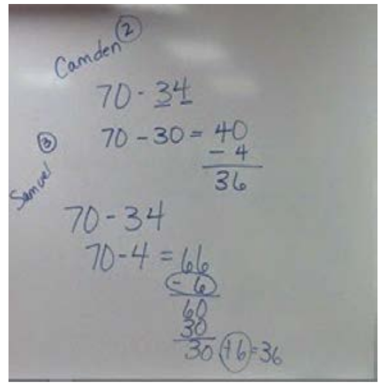
30

What's Best and Why?

- The teacher structures the discussion in one of two ways:
  - Shows a particular strategy and then asks students to generate an effective use of that strategy
  - Shows a few different ways to solve a problem and asks students to figure out which one is the most efficient strategy for that problem.



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31

When to Use These Targeted Discussion Approaches?

- The problem can be solved in more than one way and you expect that your students will use a variety of ways
- *You want to support your students in making sense of different strategies*
- You want to prompt students along to slightly more sophisticated strategies
- *You want to compare the use of different mathematical tools or representations to solve the problem.*

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32

ETPs and SMPs to Support Students' Procedural Fluency

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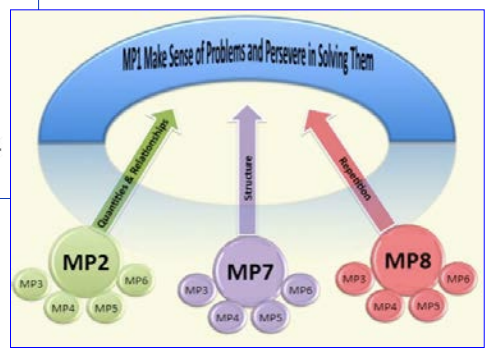

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33

Implications for Practice

- What opportunities/routines for students to build procedural fluency from conceptual understanding are you now thinking of incorporating into your practice (your classroom or your work with teachers)?
- What behaviors/approaches/thinking will you be looking for from students to give evidence of their fluency?

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Wrapping It Up

- Feel free to contact me: ccarrol@wested.org
- Please take the time to me give feedback on my session, it helps my planning for future sessions and it helps the conference committee determine your needs and wants.

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35