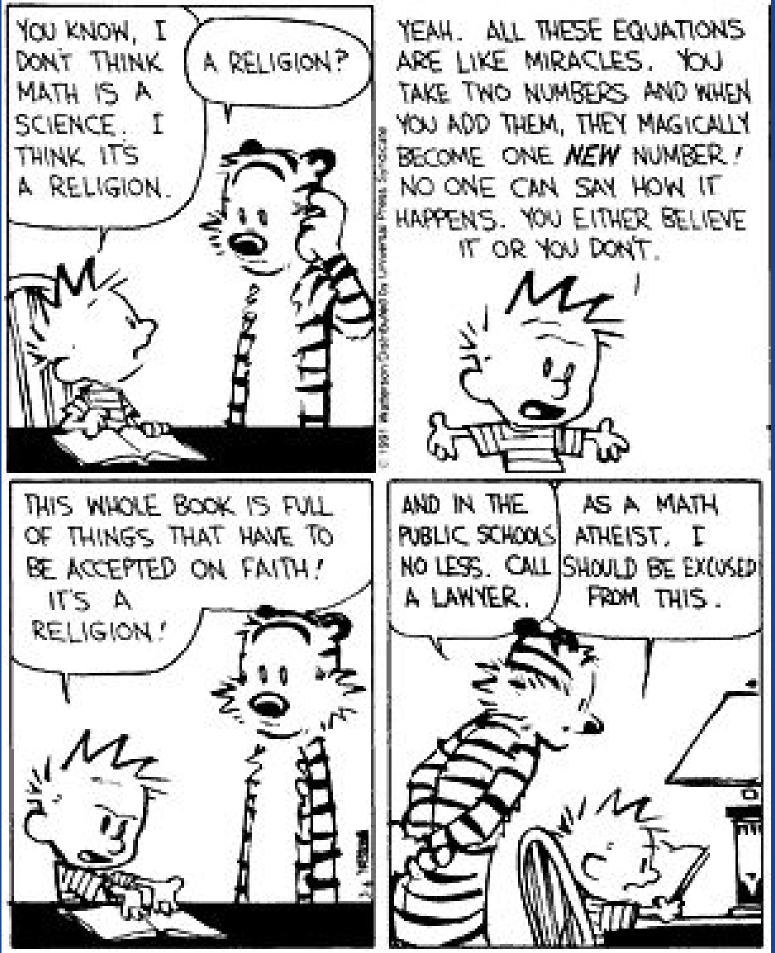
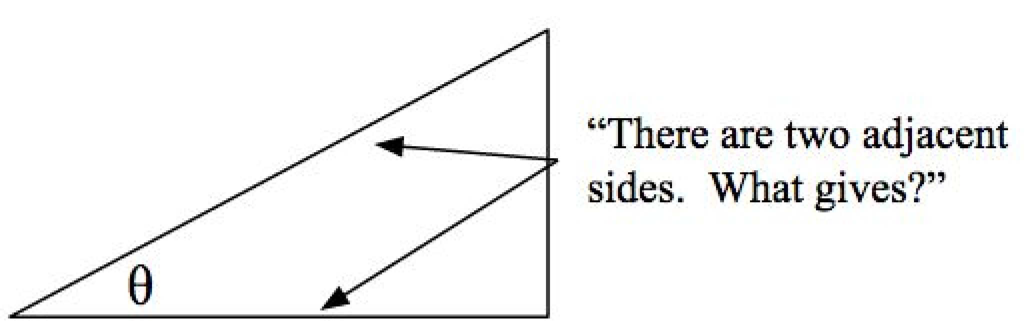
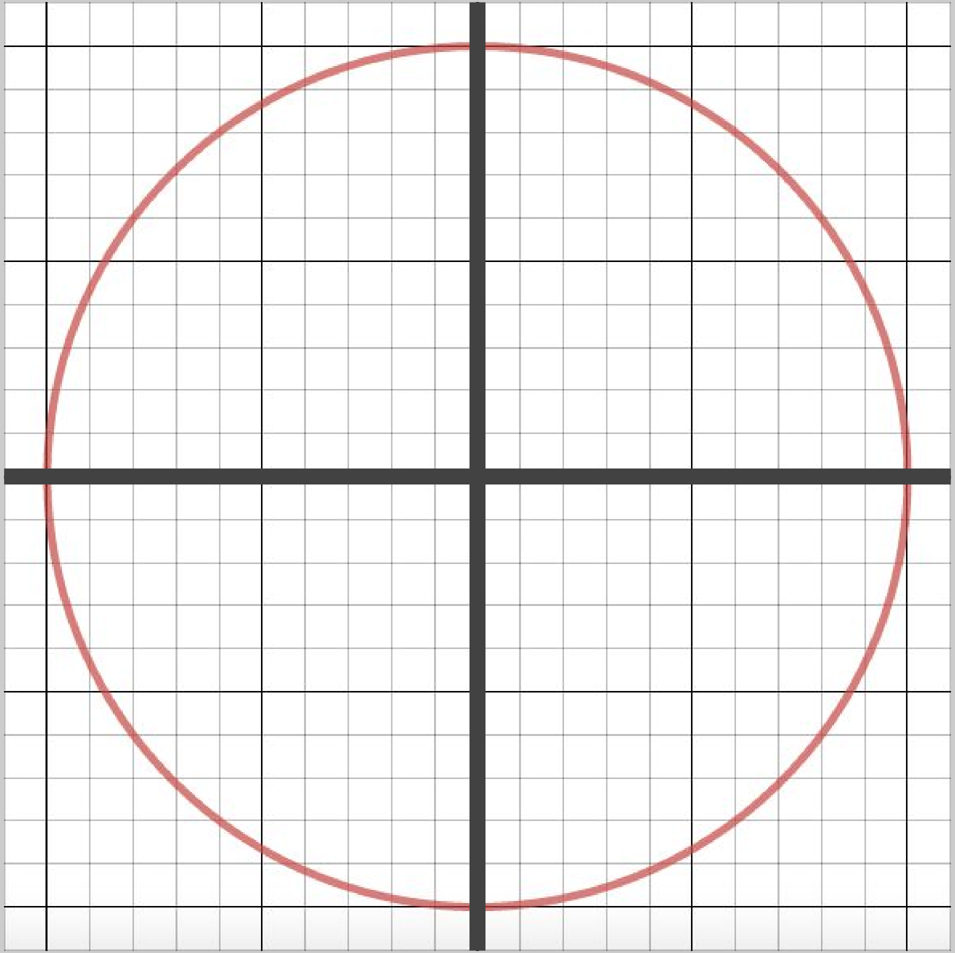

YOU KNOW, I DON'T THINK MATH IS A SCIENCE. I THINK IT'S A RELIGION.
A RELIGION?
YEAH. ALL THESE EQUATIONS ARE LIKE MIRACLES. YOU TAKE TWO NUMBERS AND WHEN YOU ADD THEM, THEY MAGICALLY BECOME ONE NEW NUMBER! NO ONE CAN SAY HOW IT HAPPENS. YOU EITHER BELIEVE IT OR YOU DON'T.
THIS WHOLE BOOK IS FULL OF THINGS THAT HAVE TO BE ACCEPTED ON FAITH! IT'S A RELIGION!
AND IN THE PUBLIC SCHOOLS NO LESS. CALL A LAWYER.
AS A MATH ATHEIST, I SHOULD BE EXCUSED FROM THIS.

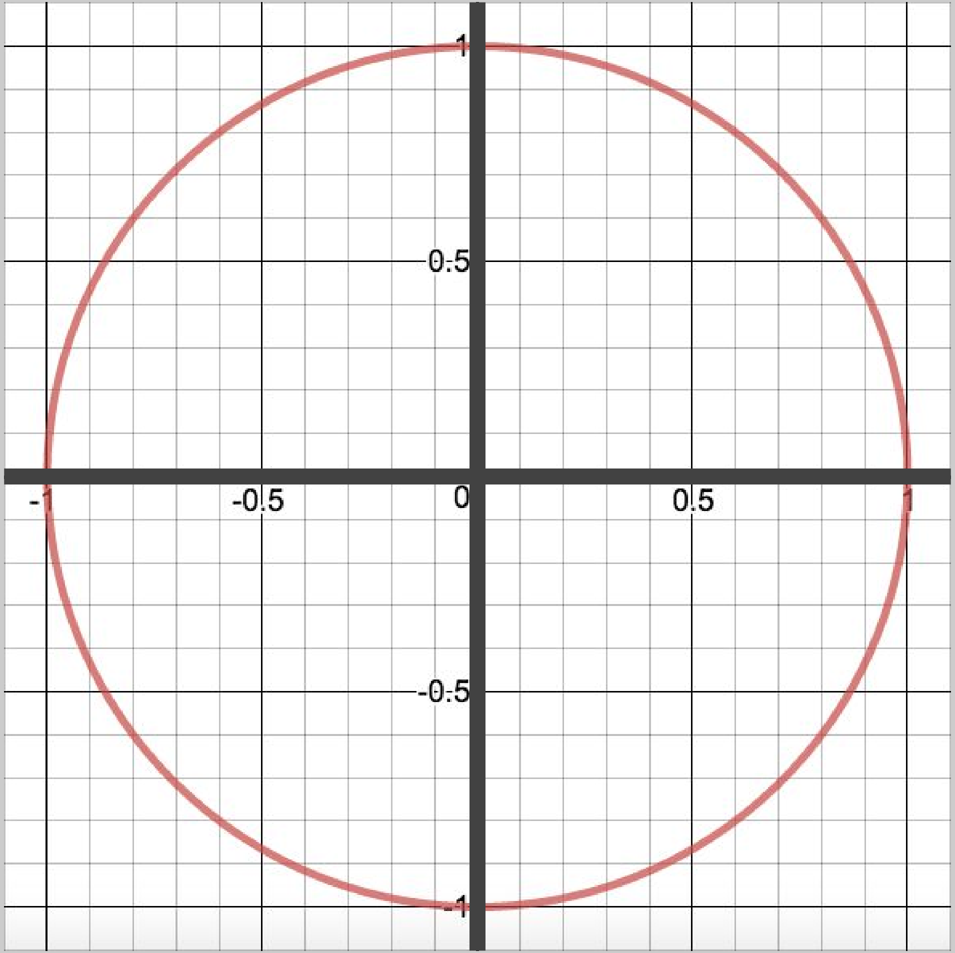
Students' struggles with trigonometry:

- Students' understanding of ratio can be loose
- 

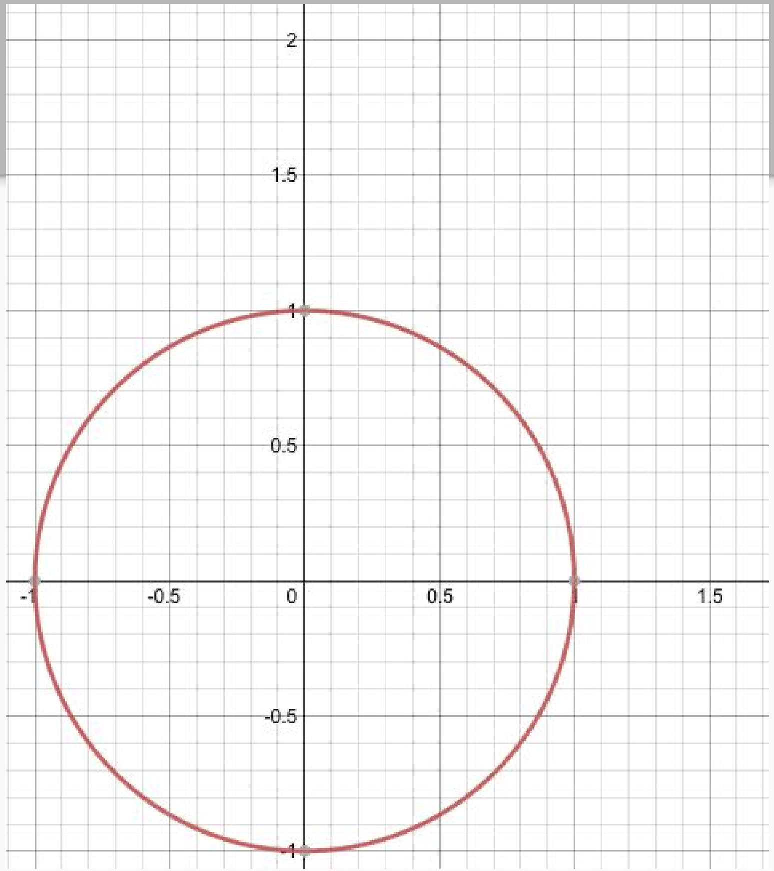
- Like square roots, students cannot evaluate trig functions

Welcome, class!



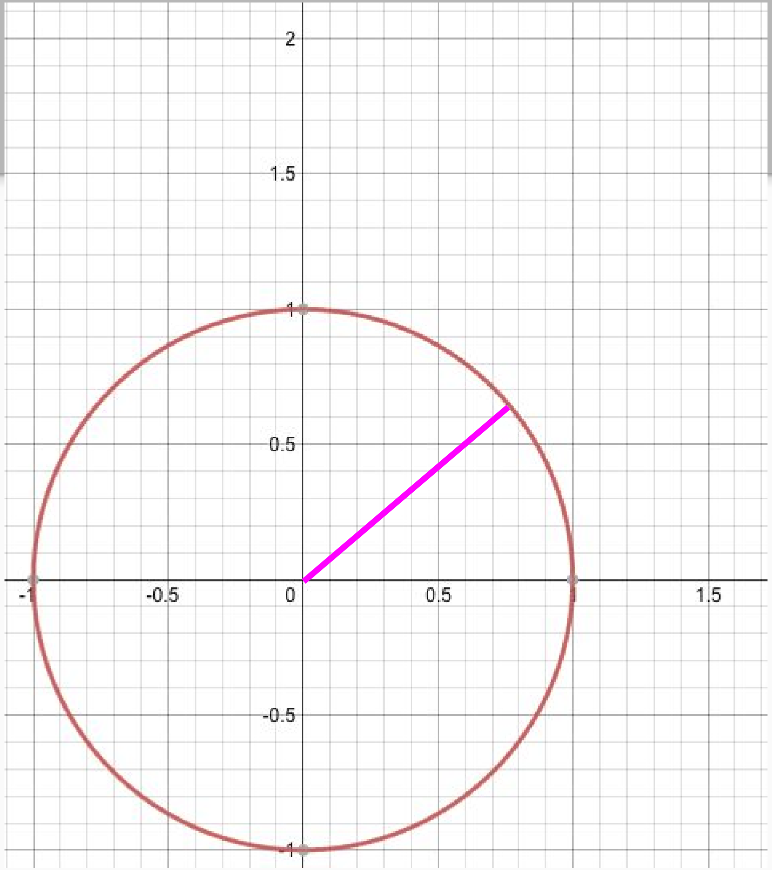

1
0.5
-1
-0.5
0
0.5
1
-0.5
-1

<u>Here's your sine</u> (and your cosine)



- Draw a ray at a 40º angle counterclockwise from the x-axis originating from the origin.
- Give the coordinates of where the ray intersects the circle. Call this point P.
- Draw a perpendicular segment from point P to the x-axis.

Here’s your sine (and your cosine)



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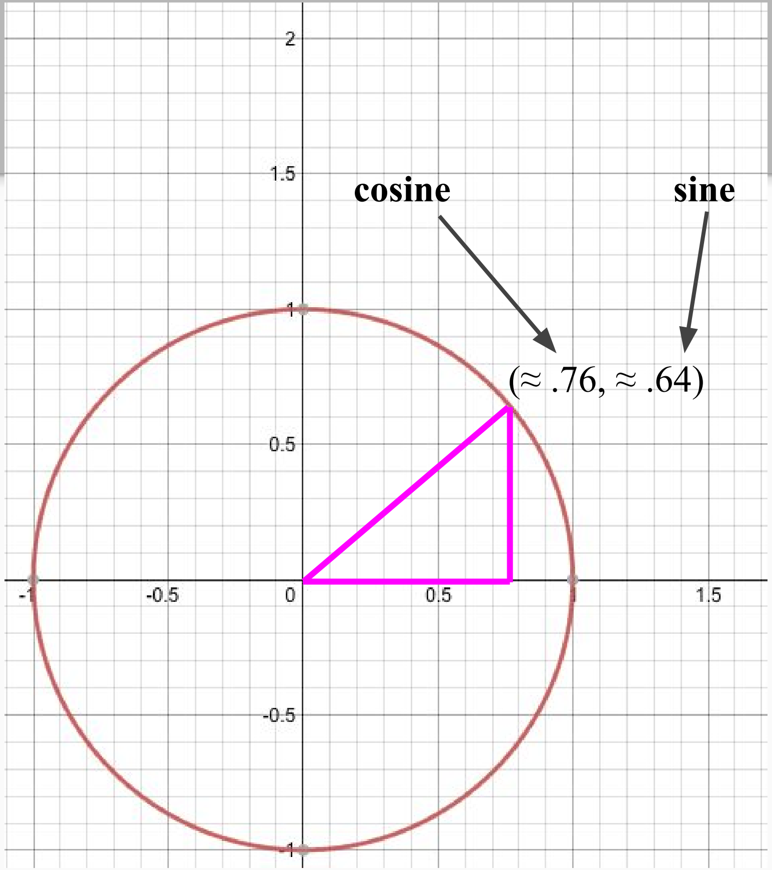
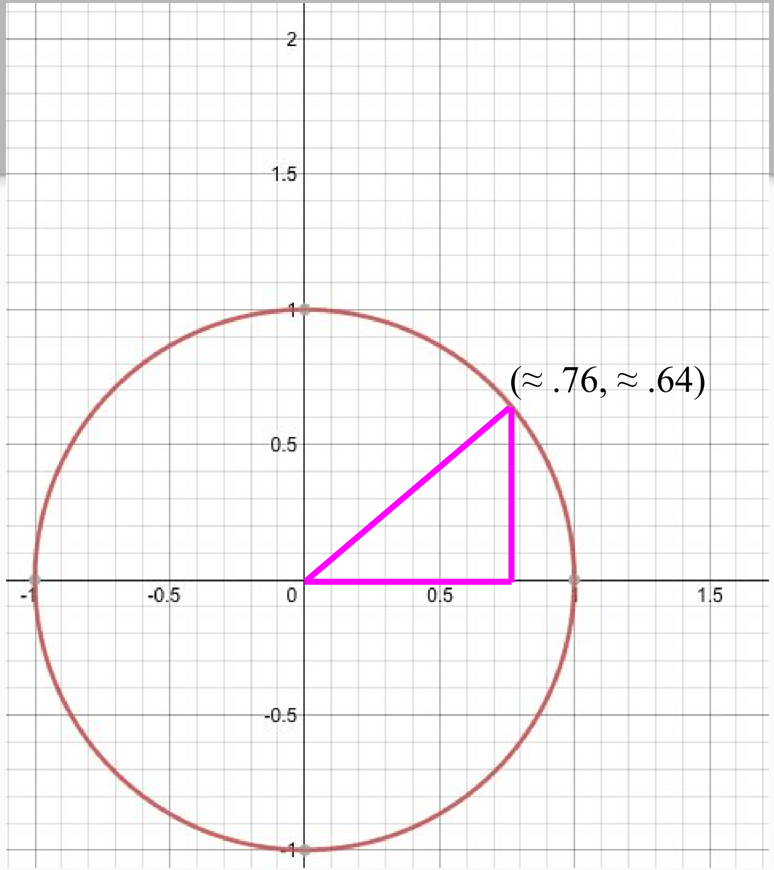


- Draw a ray at a 40º angle counterclockwise from the x-axis originating from the origin.
- Give the coordinates of where the ray intersects the circle. Call this point P.
- Draw a perpendicular segment from point P to the x-axis.
  - **Sine is the y-value**
  - **Cosine is the x-value**

Here’s your sine (and your cosine)

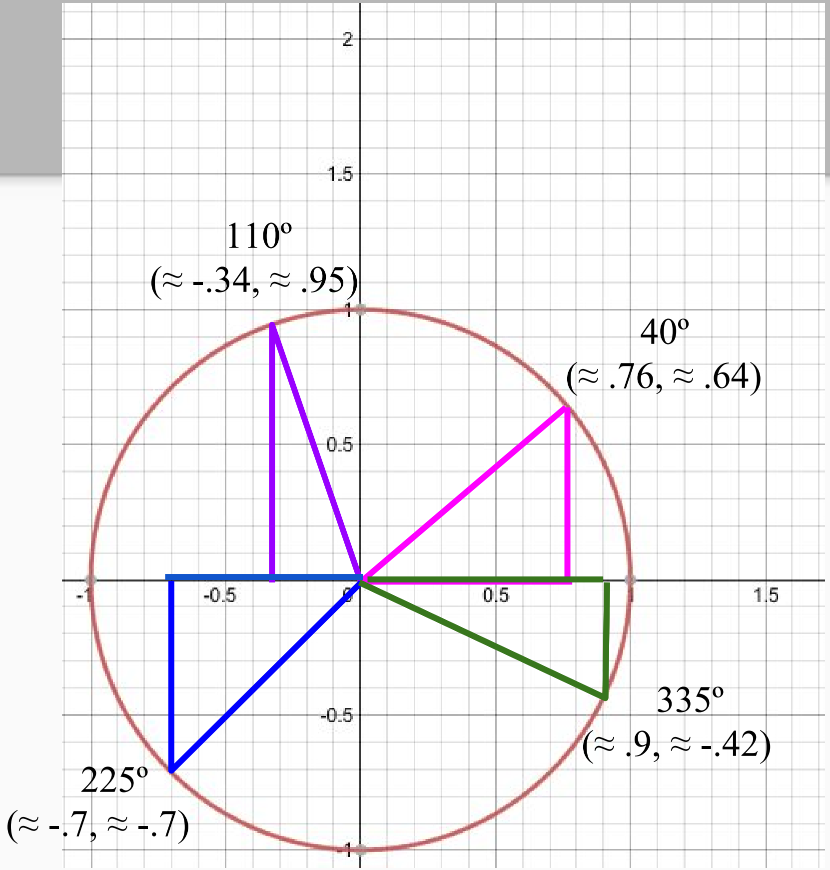


Get a feel for what is happening...

Complete the same procedure for the following angles:

- 110º
- 225º
- 335º

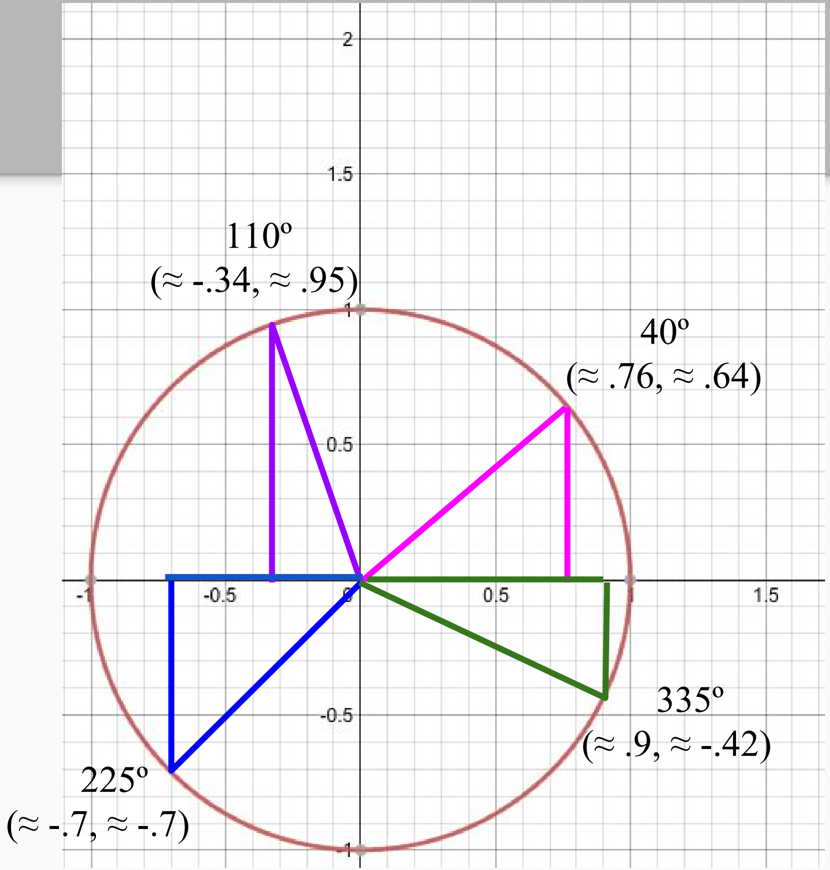
Here's your sine (and your cosine)



Complete the same procedure for the following angles:

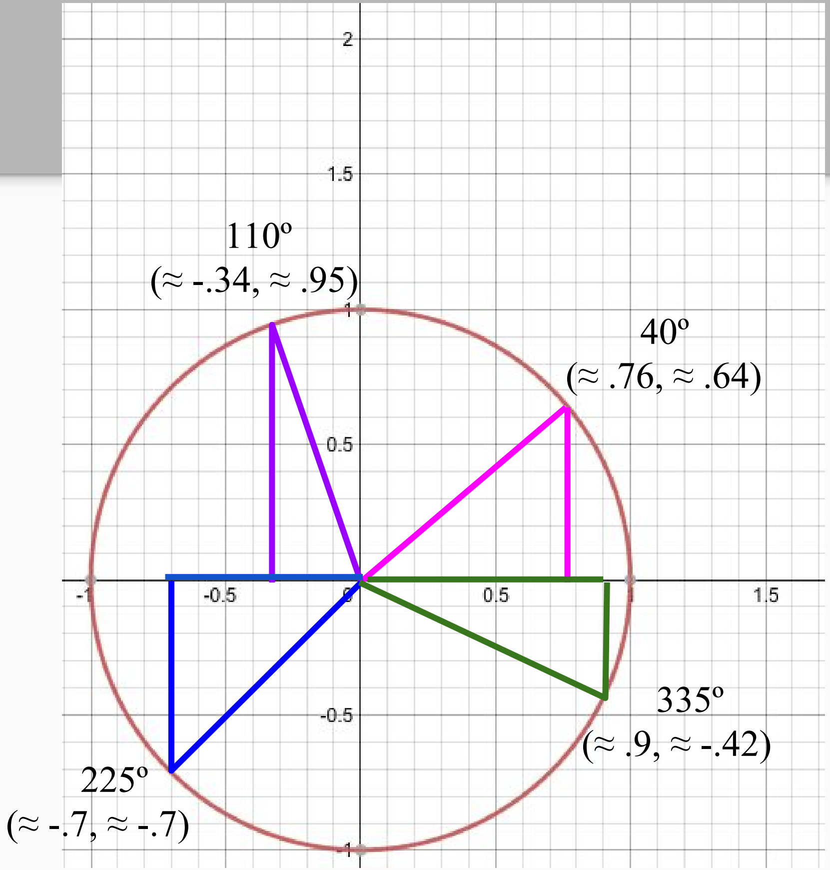
- 110º
- 225º
- 335º

Here's your sine (and your cosine)



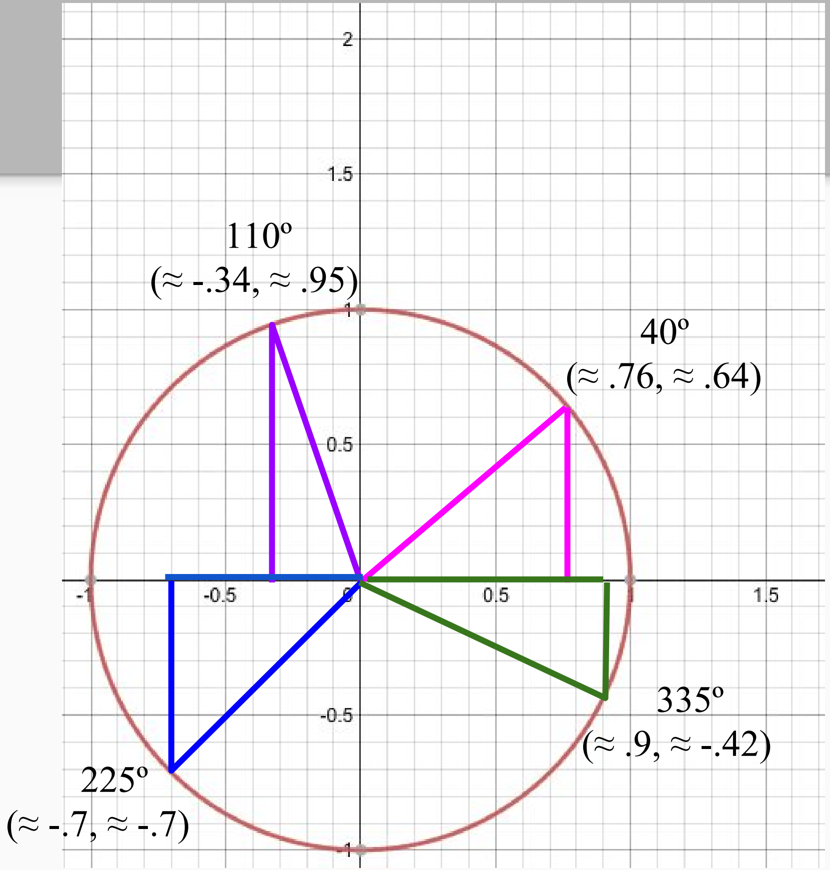
- What do you notice about the sine and cosine values?

Here's your sine (and your cosine)



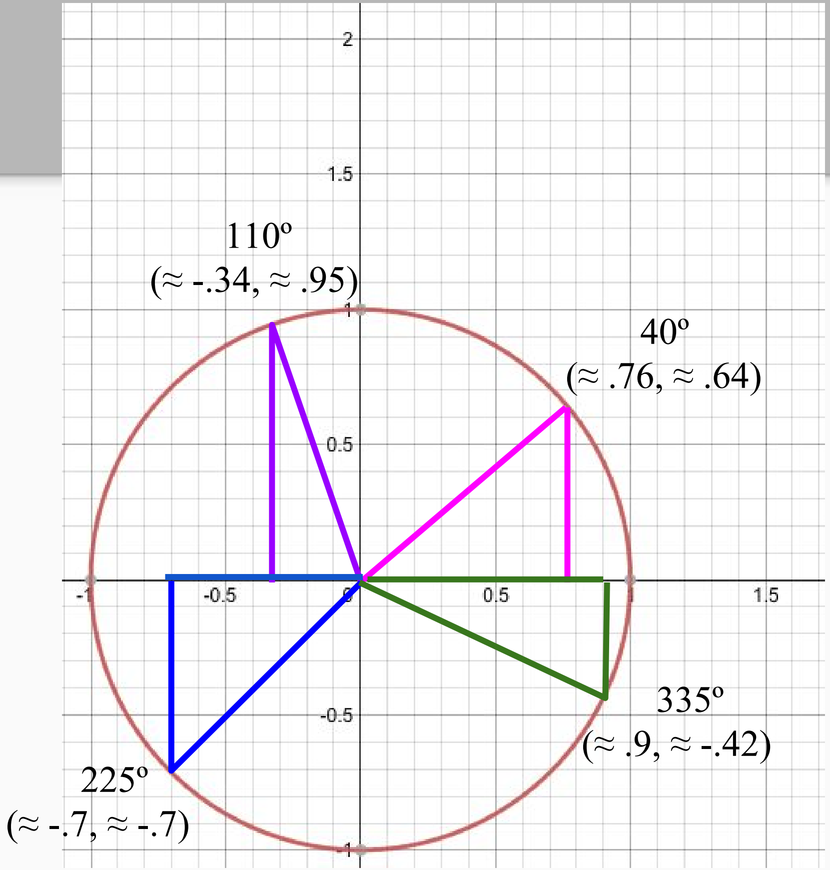
- What do you notice about the sine and cosine values?
- Is this possible: sin x = 2?

Here's your sine (and your cosine)



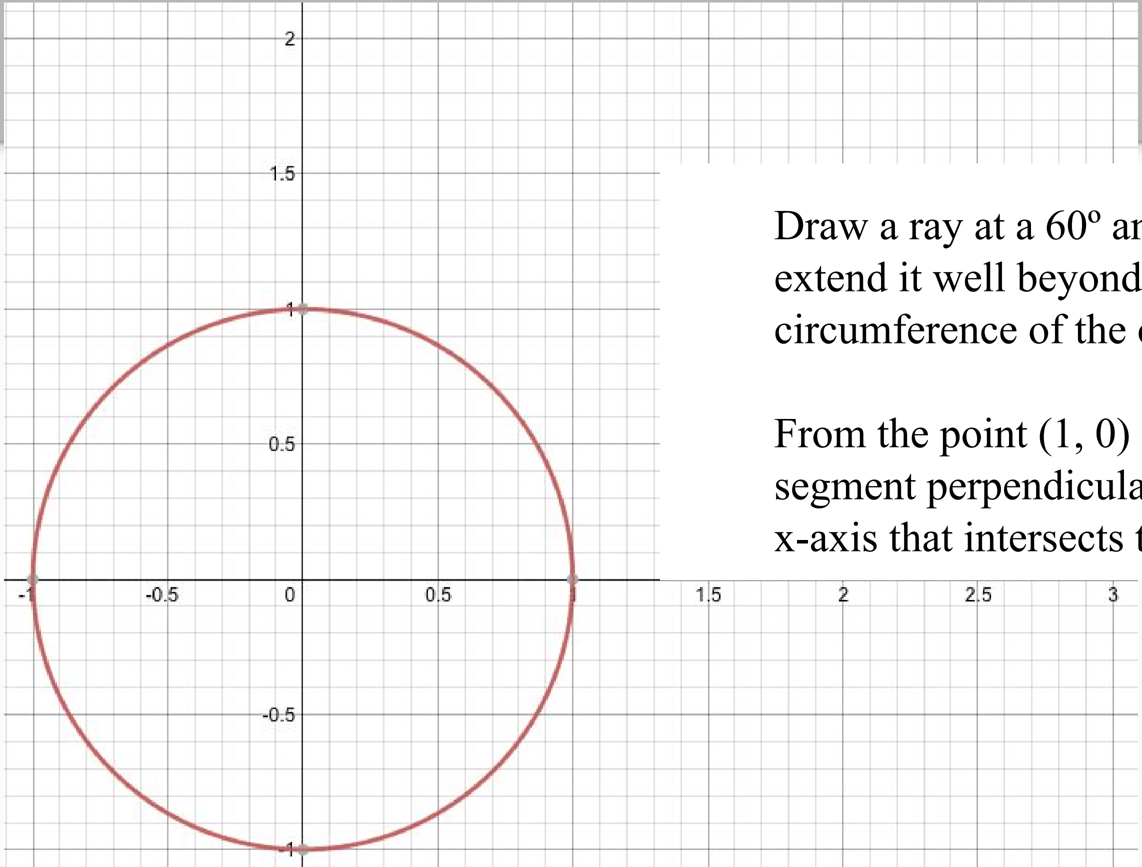
- What do you notice about the sine and cosine values?
- Is this possible: sin x = 2?
- **Which is bigger: sin 25º or sin 10º?**

Here’s your sine (and your cosine)



- What do you notice about the sine and cosine values?
- Is this possible: sin x = 2?
- Which is bigger: sin 25º or sin 10º?
- **Which is bigger: sin 25º or cos 25º?**

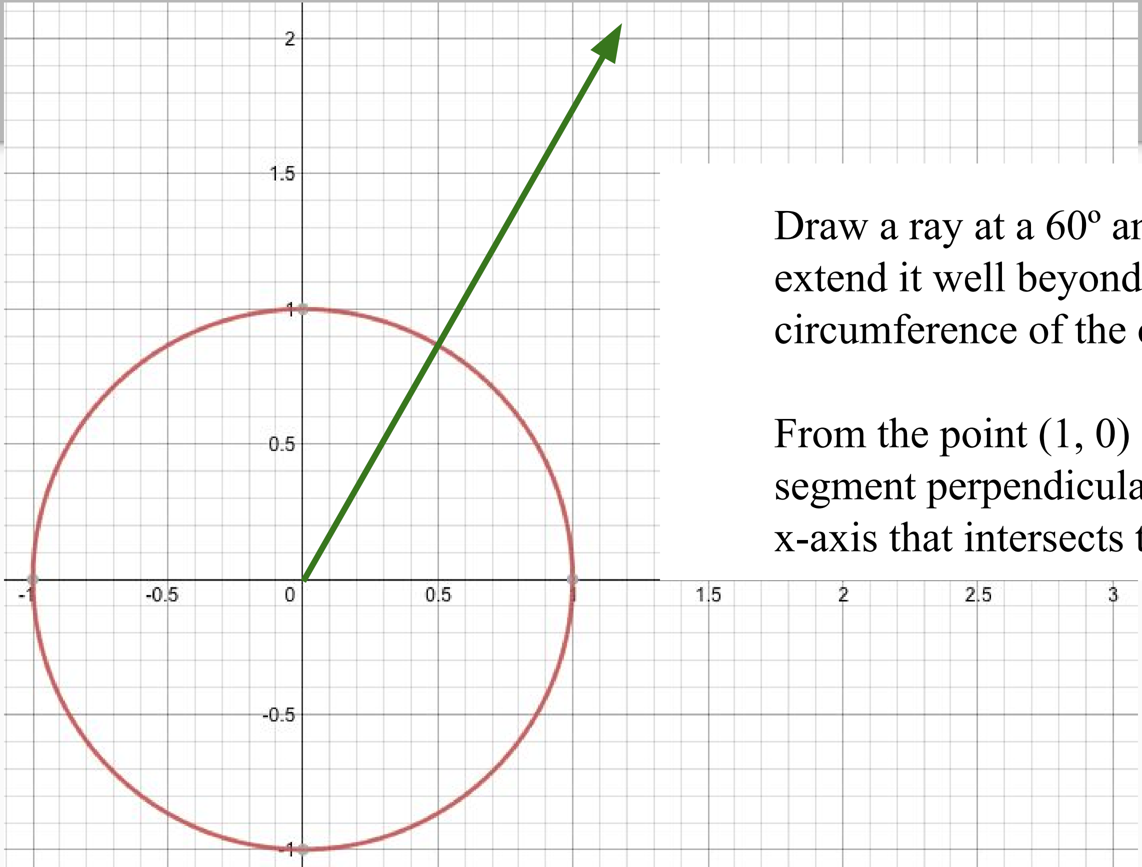
Let’s get **tan**...



Draw a ray at a 60º angle and extend it well beyond the circumference of the circle.

From the point (1, 0) draw a segment perpendicular to the x-axis that intersects the ray.

Let’s get **tan**...



Draw a ray at a 60º angle and extend it well beyond the circumference of the circle.

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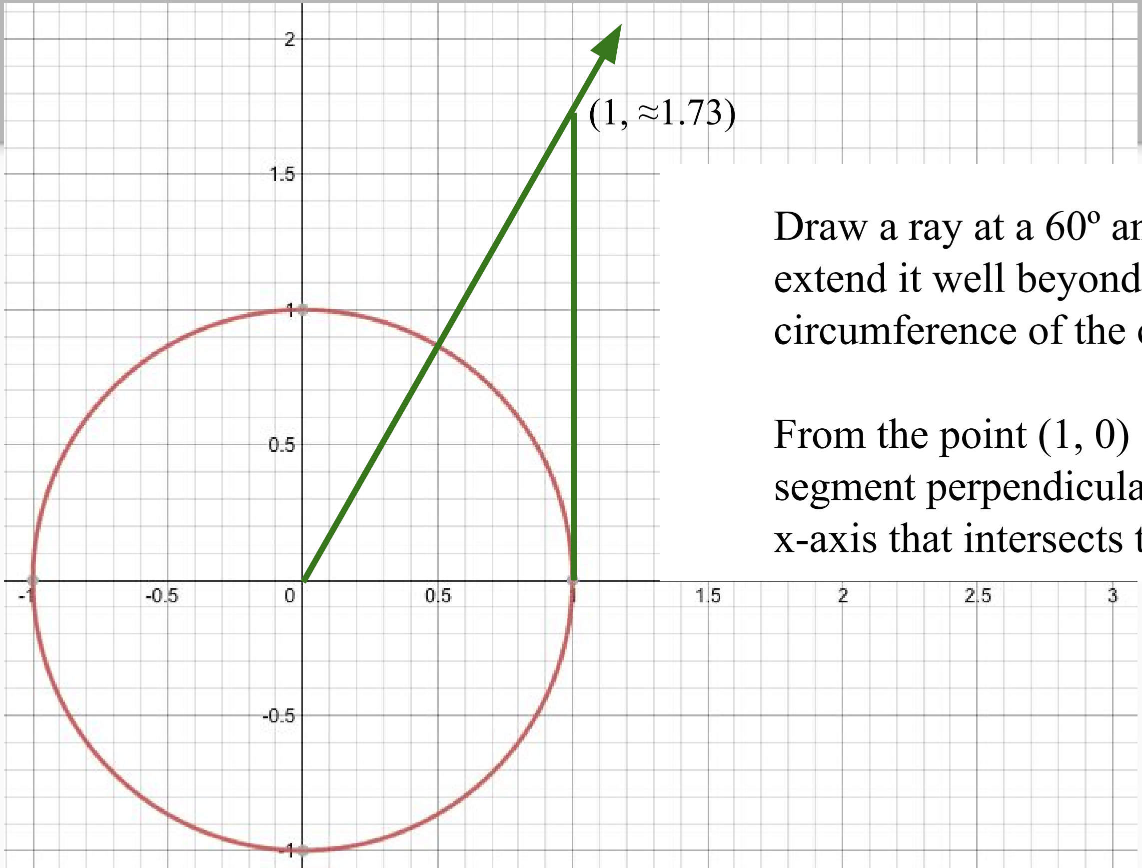


Draw a ray at a 60º angle and extend it well beyond the circumference of the circle.

From the point (1, 0) draw a segment perpendicular to the x-axis that intersects the ray.

To Desmos to explore:

Tangent

Circle dilation!

So how can we use this?

<u>An example</u> (Adapted from *Geometry Connections, 2008* from CPM)
While traveling around the beautiful city of San Francisco, Juanisha climbed several steep streets. One of the steepest, Filbert Street, has a slope angle of 31.5º according to her guidebook.

Once Juanisha finished walking up 100 feet on Filbert street, she wanted to know how high she had climbed. Draw a picture to model this situation and determine how high she climbed.

"Any time students are doing more on task work than the teacher, students are learning."

- Every Teacher Ever

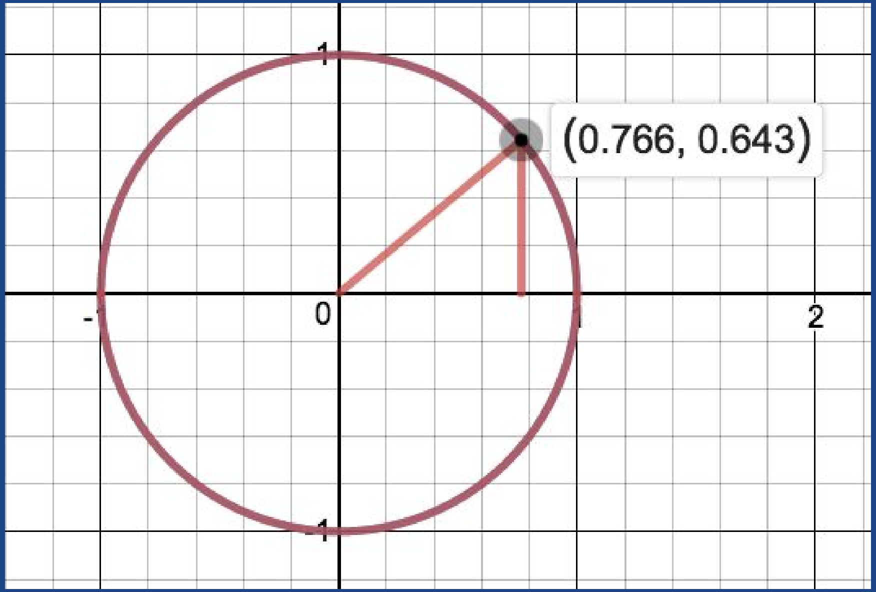
So how can we use this?

Another example (Adapted from *Geometry Connections, 2008* from CPM)

What is the horizontal distance Juanisha traveled? (“As the crow flies”)

To the solution!

$r{\bullet}\sin\theta$ = opposite $r{\bullet}\cos\theta$ = adjacent

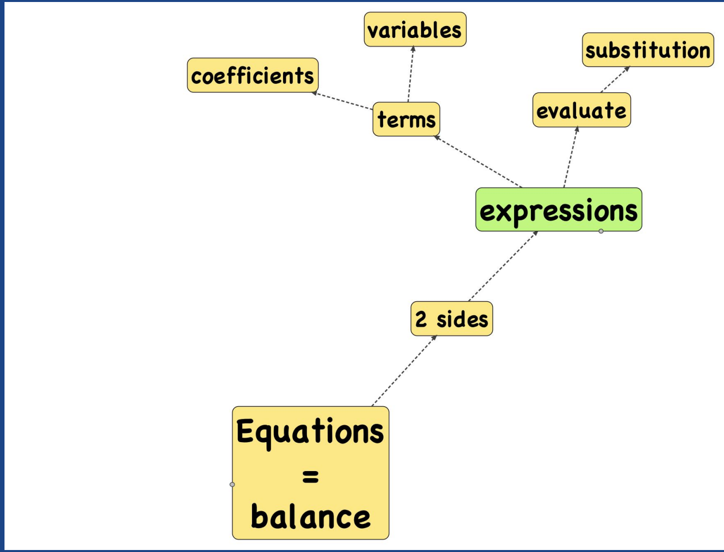


Why does this help students OF ANY LEVEL?

- Connects to what is already understood
- Estimates

Why does this help students OF ANY LEVEL?

Concept Maps and Christmas lights



What does the research say?

<u>PROCEPT</u> - combination of processes and concepts

Ex) $2 + 3 = 5$
$2 + 1 + 1 + 1 = 5$

Ex)
$4^2 = 4 \times 4 = 16$

What does the research say?

PROCEPT - combination of processes and concepts

Trigonometric functions can be understood as procepts when combining the concepts of sine and cosine with rotations. (Weber, 2005)

sin a $\neq$ 2 because…

$sin\ 90^o = 1$ *because…* Desmos - https://goo.gl/4gimT7

What does the research say?

1. Students shown how to execute a procedure to accomplish a specific trig task (i.e. evaluate sine of a given angle)
2. Students applied procedure in a number of cases, about five or six. Instructor circled the room to make sure students were obtaining reasonable results.
3. Students were asked questions that required reasoning.
    a. Without going through the calculations, which is bigger sin 23 or sin 37?
    b. Is sin 145 positive and why?

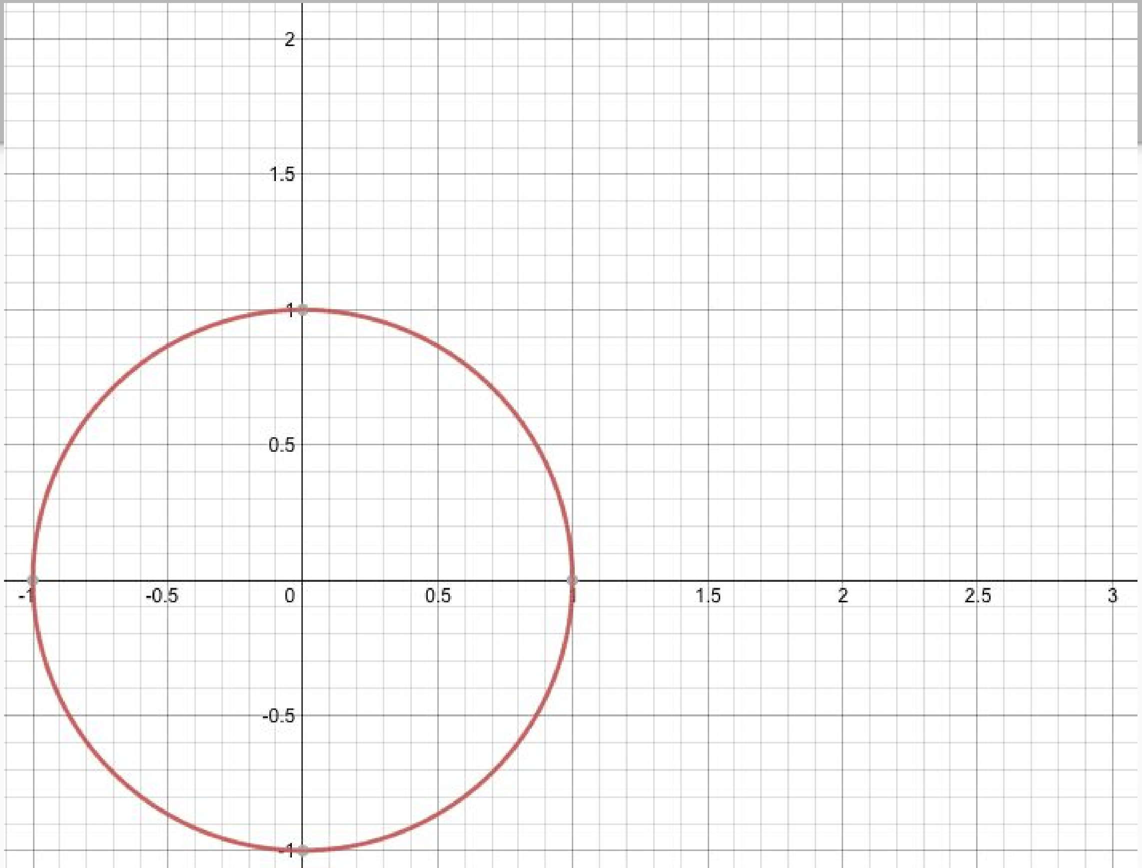
Use the unit circle and protractor to answer these questions.

Google Form - https://goo.gl/htWpyx

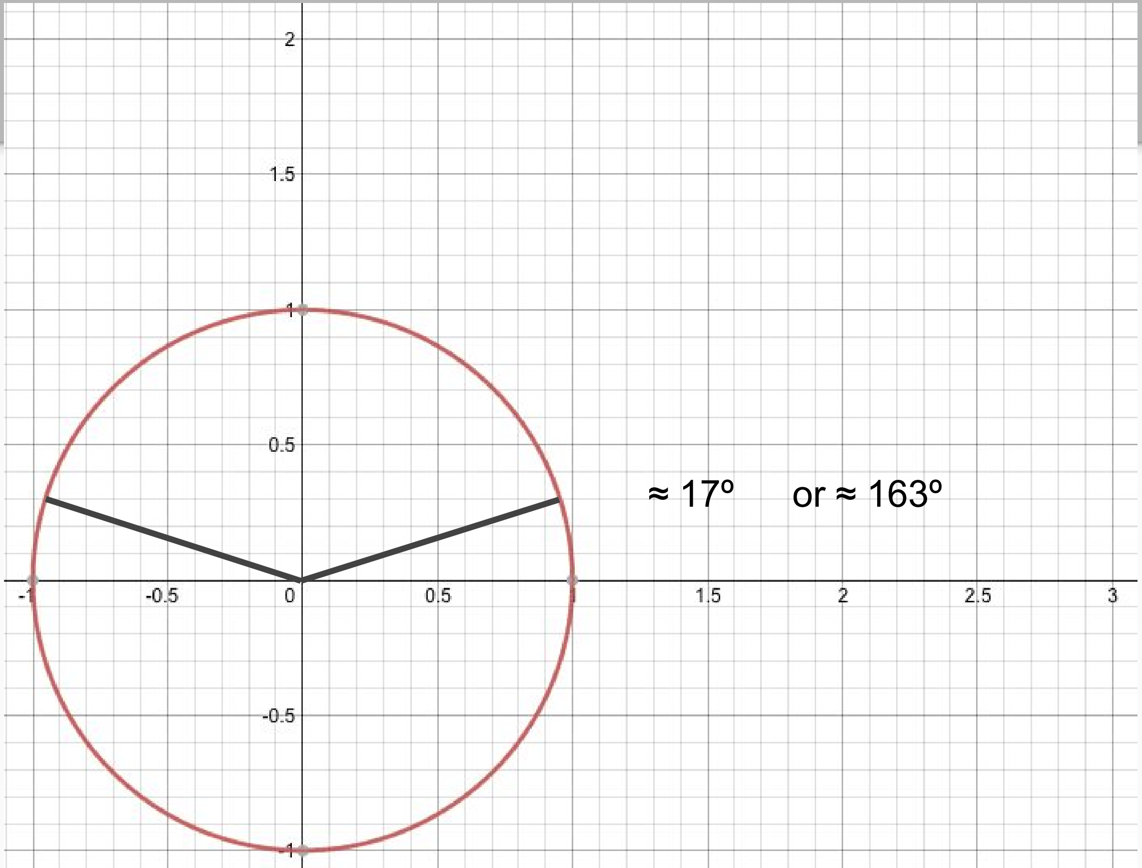
a. What is the range of sin x?
b. Why is sin 270º = -1?
c. In what quadrants are cos x POSITIVE?
d. Which is bigger cos 30º or sin 30º? Why?
e. Which is bigger cos 30º or sin 60º? Why?

Inverse operations:

How we can find θ such that $\sin \theta = 0.3$?



Inverse operations:
Describe how we can find θ such that sin θ = 0.3?

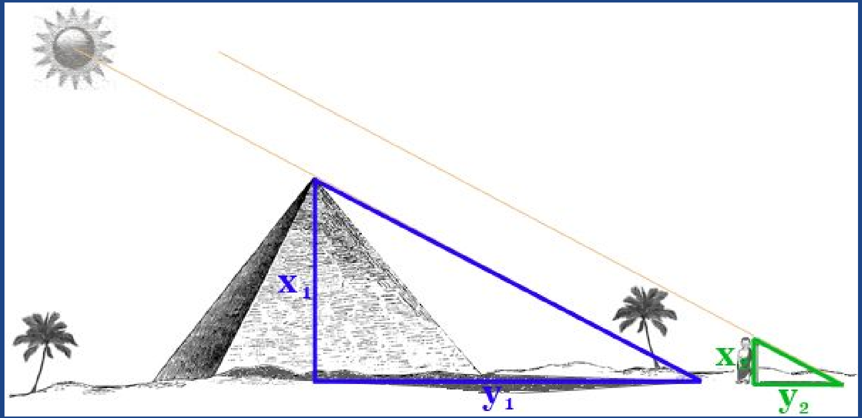


Let’s look at history...

<u>**Thales of Miletus (624 - 546 BCE)**</u>

Determined heights of pyramids using shadows

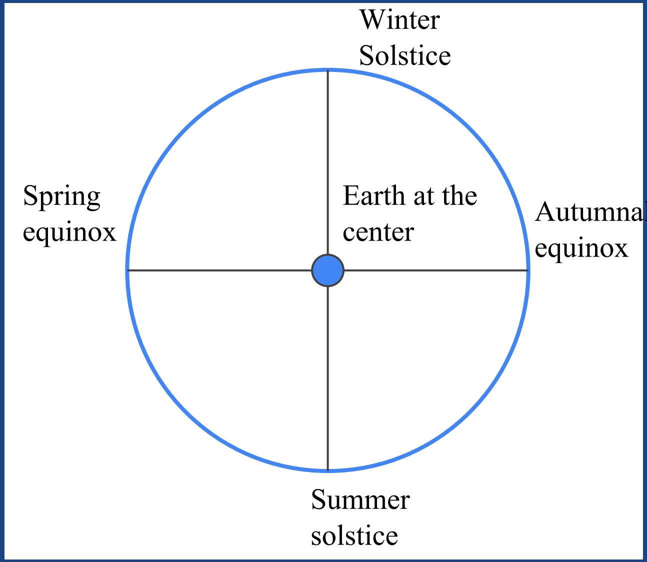
Not necessarily trigonometry, but similarity and ratios



Let’s look at history...

Hipparchus of Nicaea (190 - 120 BCE)

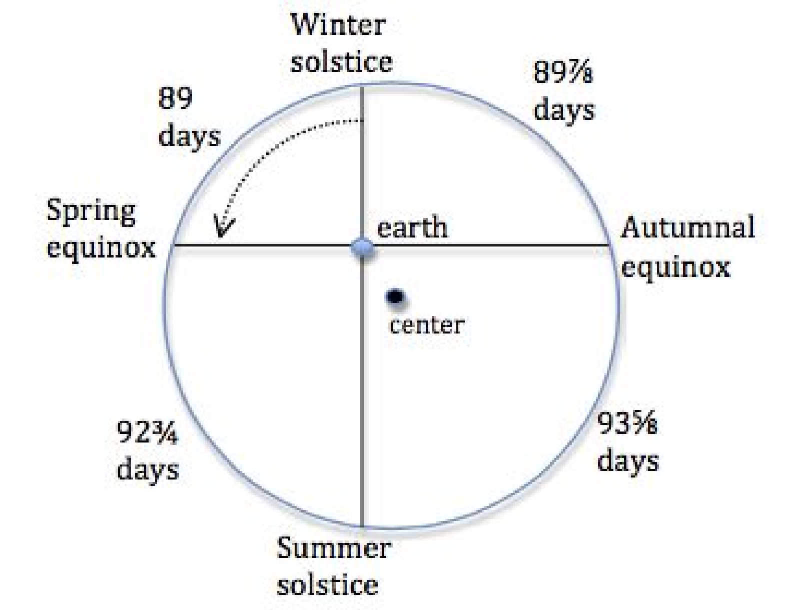
- First to use a table of approximate arcs and chords
- **If the Earth was at the center and the orbits all circular, how long would we EXPECT the seasons to be?**



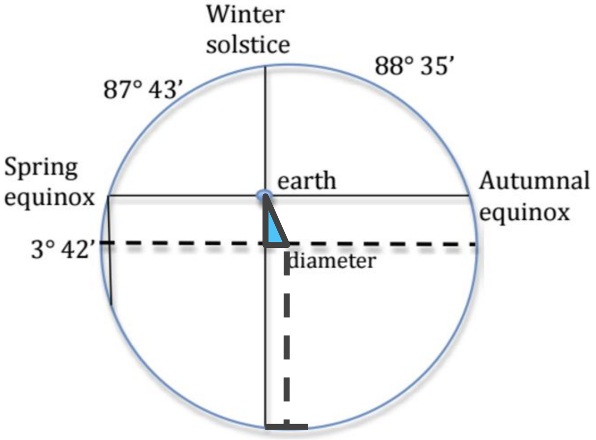
Let’s look at history...

<u>Hipparchus of Nicaea (190 - 120 BCE)</u>

- This is what Hipparchus observed.



Let’s look at history...



Convert days into arc lengths

$$\frac{89}{365.25} \cdot 360 \approx 87 + \frac{43}{60}$$

winter $87^\circ\ 43'$,

spring $91^\circ\ 25'$,

summer $92^\circ\ 17'$,

fall $88^\circ\ 35'$.

Let's look at history...

Convert days into arc lengths

$$\frac{89}{365.25} \cdot 360 \approx 87 + \frac{43}{60}$$

winter $87^\circ\ 43'$,

spring $91^\circ\ 25'$,

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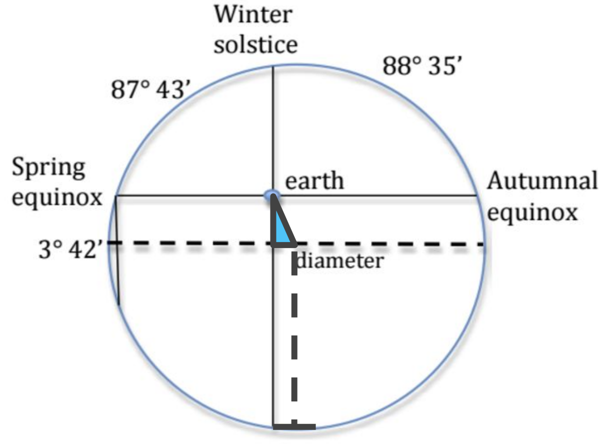
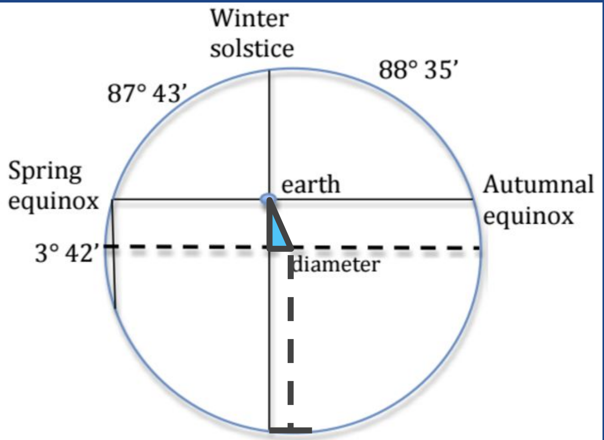
fall $88^\circ\ 35'$.

Vertical Displacement

87º 43' + 88º 35' = 176º 18'



Let’s look at history...



Convert days into arc lengths

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winter 87° 43′,

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summer 92° 17′,

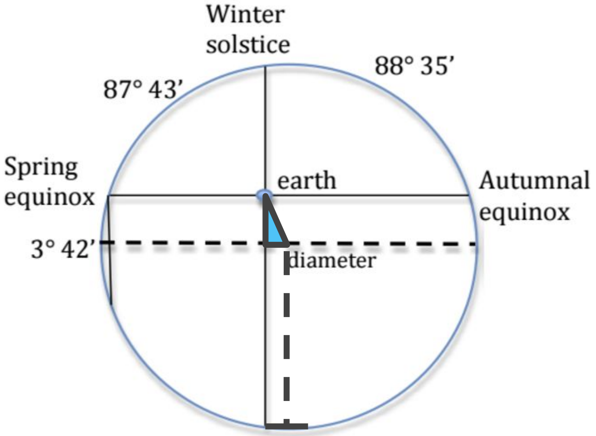
fall 88° 35′.

Vertical Displacement

87º 43’ + 88º 35’ = 176º 18’

180º - 176º 18’ = 3º 42’

Let’s look at history...



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87º 43’ + 88º 35’ = 176º 18’

180º - 176º 18’ = 3º 42’

crd (3º 42’) = 2sin(1º 51’) = 2sin1.85º = .065 au

Let's look at history...

Convert days into arc lengths

$$\frac{89}{365.25} \cdot 360 \approx 87 + \frac{43}{60}$$

winter $87^\circ\ 43'$,

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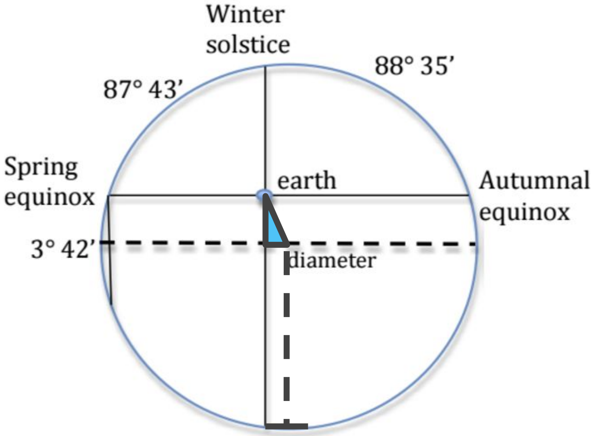


Vertical Displacement

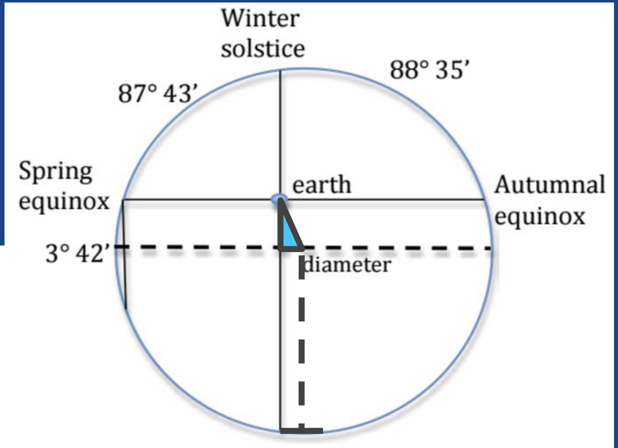
87º 43' + 88º 35' = 176º 18'

180º - 176º 18' = 3º 42'

crd (3º 42') = 2sin(1º 51') = 2sin1.85º = .065 au

.065 / 2 = ***.0325 au***

Let's look at history...



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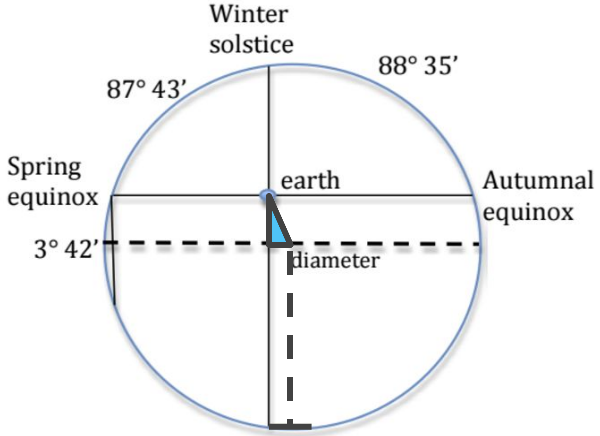
Horizontal Displacement

87º 43' + 91º 25' = 179º 08'

Let's look at history...



<u>Convert days into arc lengths</u>

$$\frac{89}{365.25} \cdot 360 \approx 87 + \frac{43}{60}$$

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<u>Vertical Displacement</u>

87º 43' + 88º 35' = 176º 18'

180º - 176º 18' = 3º 42'

crd (3º 42') = 2sin(1º 51') = 2sin1.85º = .065 au

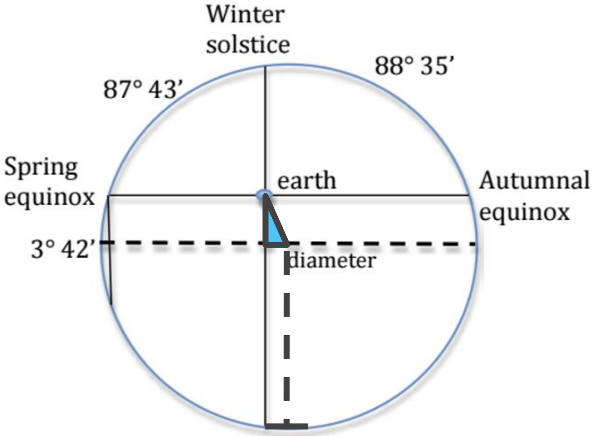
.065 / 2 = ***.0325 au***

<u>Horizontal Displacement</u>

87º 43' + 91º 25' = 179º 08'

180º - 179º 08' = 52'

Let's look at history...



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.065 / 2 = ***.0325 au***

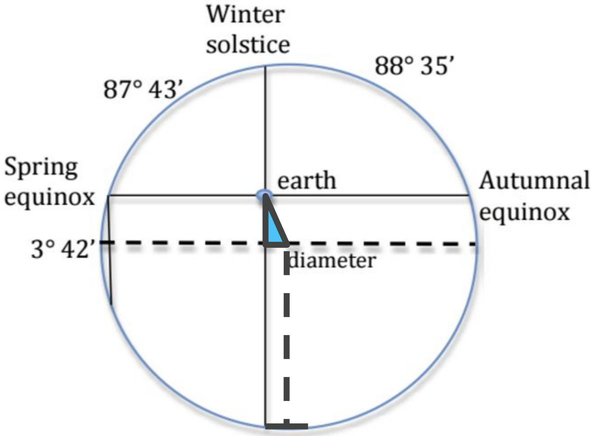
Horizontal Displacement

87º 43' + 91º 25' = 179º 08'

180º - 179º 08' = 52'

crd (52') = 2sin(26') = 2sin .433º = .015 au

Let's look at history...



Convert days into arc lengths

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Vertical Displacement

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crd (3º 42') = 2sin(1º 51') = 2sin1.85º = .065 au

.065 / 2 = ***.0325 au***

Horizontal Displacement

87º 43' + 91º 25' = 179º 08'

180º - 179º 08' = 52'

crd (52') = 2sin(26') = 2sin .433º = .015 au

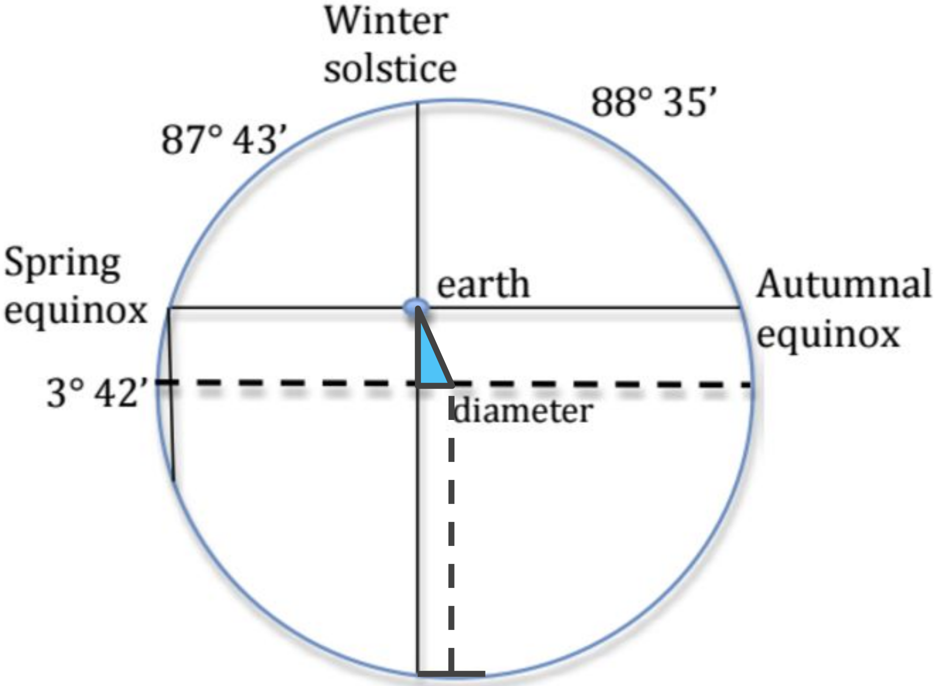
.015 / 2 = ***.0075 au***

Let's look at history...

<u>Vertical Displacement</u>
.0325 au

<u>Horizontal Displacement</u>
.0075 au

<u>Distance to the "center of the universe"</u>:



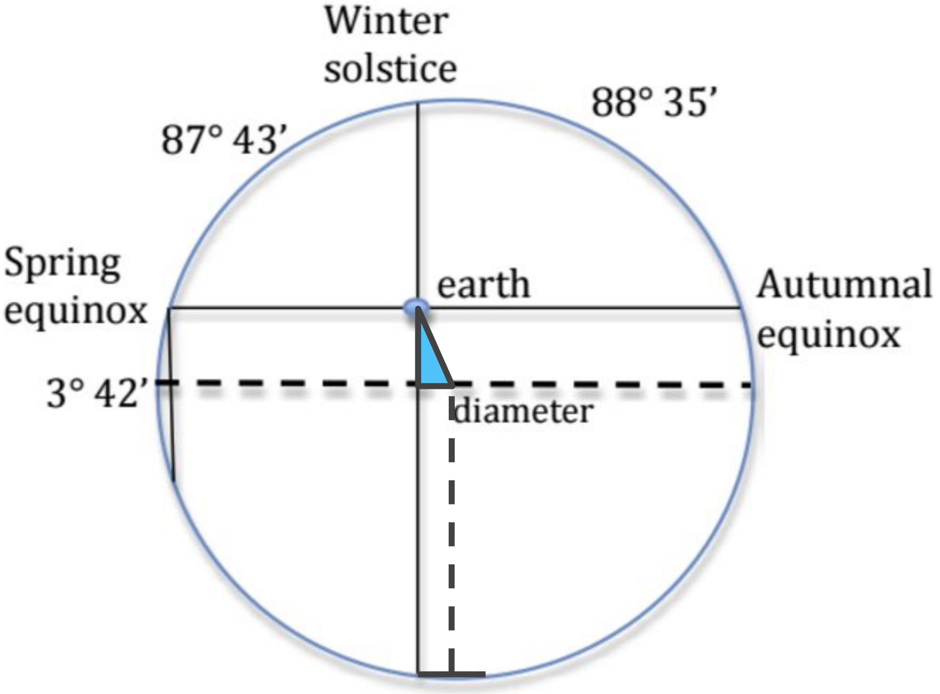
Let’s look at history...

Vertical Displacement
.0325 au

Horizontal Displacement
.0075 au

Distance to the “center of the universe”:

$\sqrt{(.0075^2+.0325^2)}$ = ***.0334 au***



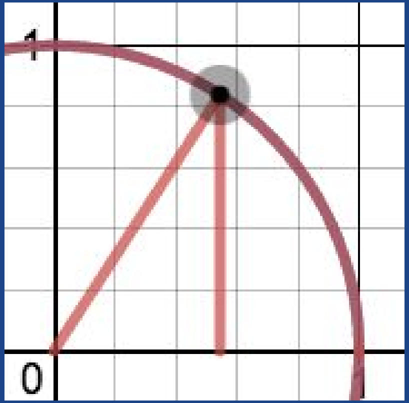
Let’s look at history...

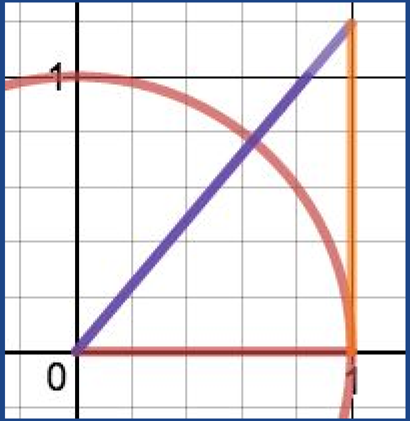
Textbooks changed over time

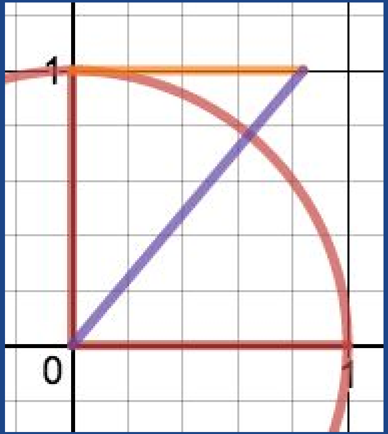
What are other reasons teachers should consider using the unit circle in Geometry class? Link

Pythagorean Trig Identities. Now with Visuals!

$\sin^2 x + \cos^2 x = 1$ $\qquad 1 + \tan^2 x = \sec^2 x$ $\qquad 1 + \cot^2 x = \csc^2 x$







Cool connections (probably not with high schoolers, though)

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y^2)]}}$$

$$r = \frac{X \cdot Y}{||X|| \cdot ||Y||} = cos\theta$$

Center the data first

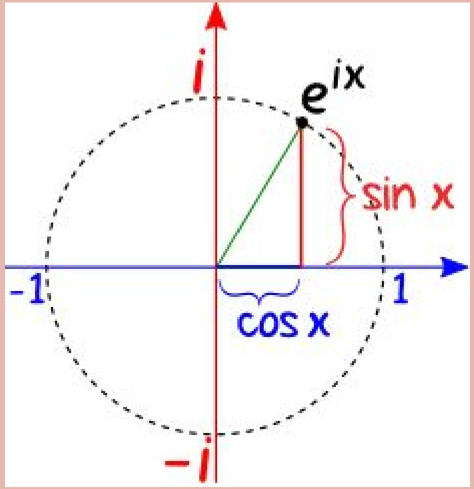
Cool connections (probably not with high schoolers, though)

$$e^{i\pi} + 1 = 0$$

Cool connections

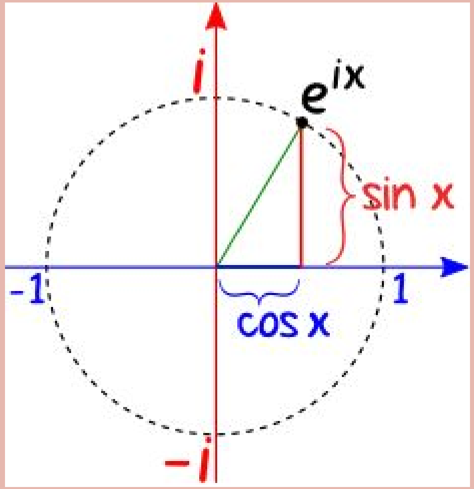
$$e^{i\pi} + 1 = 0$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$



Cool connections

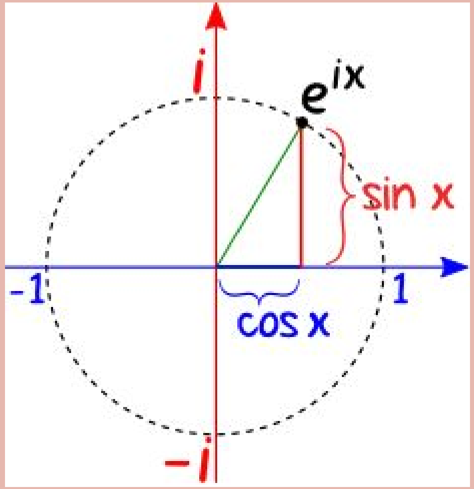
$$e^{i\pi} + 1 = 0$$



$$e^{i\theta} = \cos\theta + i\sin\theta \;\rightarrow\; e^{i\pi} = \cos\pi + i\sin\pi$$

Cool connections

$$e^{i\pi} + 1 = 0$$



$$e^{i\theta} = \cos\theta + i\sin\theta \;\rightarrow\; e^{i\pi} = \cos\pi + i\sin\pi = -1 + 0$$

Anecdotes

Students

Professors and undergrads

Making Sense of Trigonometry in Geometry Class through the Unit Circle

A case for introducing trig functions with the unit circle in Geometry class...

Jon Southam
Math Teacher - Sonoma Valley High School
Trellis Fellow

Thank you!

Jon Southam
Email - jsoutham@sonomaschools.org
Twitter - @jonsoutham
Sonoma Valley High School
Trellis Fellow

Links:

This presentation - https://goo.gl/Vaufjj
Unit Circle 5 Lesson Unit - https://goo.gl/waJMmk
Desmos Unit Circle - https://goo.gl/4gimT7
Desmos Sec, Csc, and Cot - https://goo.gl/fjEKDj
Unit circle to be written on - https://goo.gl/su2Ekf

Sources:

Bressoud, D. M. (2010). Returning to the beginnings of trigonometry—the circle—has implications for how we teach it. *Mathematics Teacher*, 104(2), 107-112.

Weber, K. (2005). Students' Understanding of Trigonometric Functions. *Mathematics Education Research Journal*, 17(3), 91-112.

Weber, K. (2008). Teaching trigonometric functions: Lessons learned from research. *Mathematics teacher*, 102(2), 144-150.