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Principles to Action with Tech

Scott Farrar
@farrarscott

NCTM 2017

Goals:

1. Understand what tech can/should support
2. Have teachers leverage that support

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student.desmos.com

695S7

Desmos Classroom Activities x Guest

Secure <https://student.desmos.com>

desmos

Welcome!



Enter your class code:

Sign in to come back to your work later:

 or [Sign in with Desmos](#)

Your experience?

Think of a modern tech tool you've
used or observed in a classroom,

what made it 'good'? or not?

talk with your neighbor

Share!

Think of a tech example...

Think of a modern tech tool you've used or observed in a classroom.

What made the tech good for learning? Or, not good for learning?

Submit to Class

Farrar: Principles to Action with x

Secure <https://student.desmos.com/activitybuilder/student/58dedfbc9cccc90613e1d928>

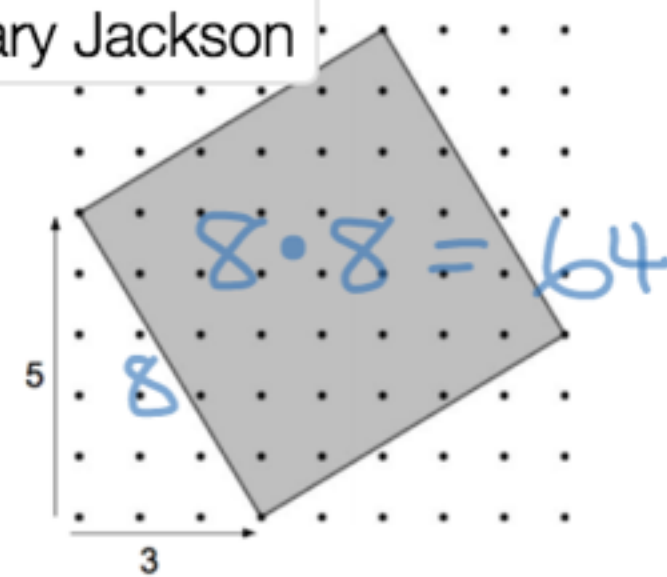
Guest 2 of 5 Guest Create Account or Sign In

Find the area of the tilted square. Show your reasoning.

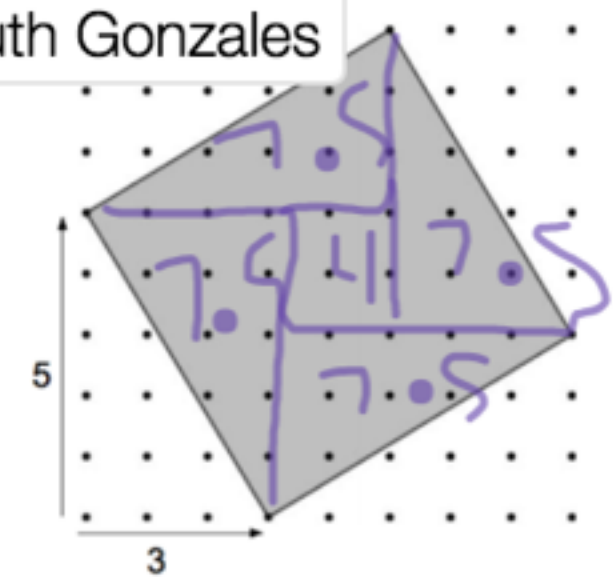
The diagram shows a tilted square on a grid of dots. The square is shaded gray. Its vertices are located at grid intersections. A vertical line segment with arrows at both ends is drawn to the left of the square, spanning the same vertical distance as the square's height, and is labeled with the number 5. A horizontal line segment with arrows at both ends is drawn below the square, spanning the same horizontal distance as the square's base, and is labeled with the number 3. The grid is 10 units wide and 10 units high.

Aggregation

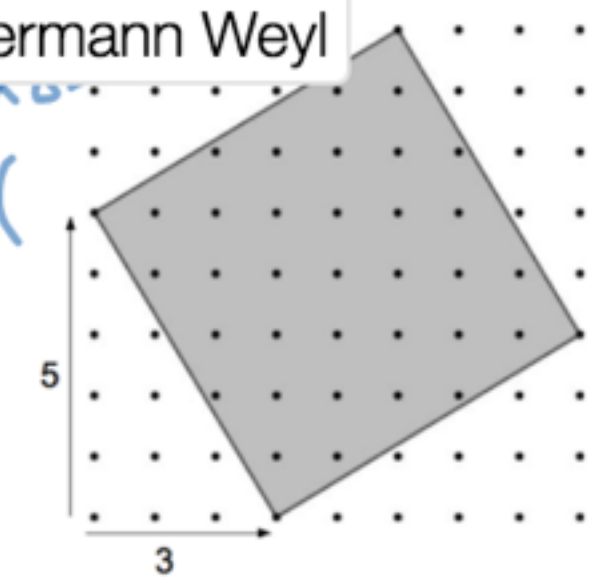
Mary Jackson



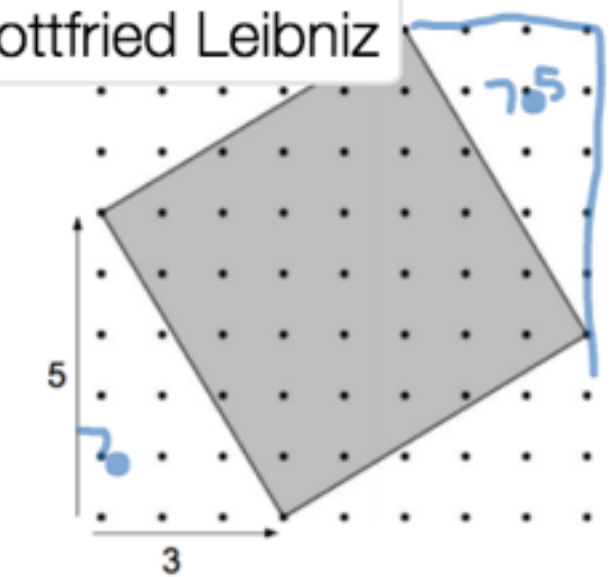
Ruth Gonzales



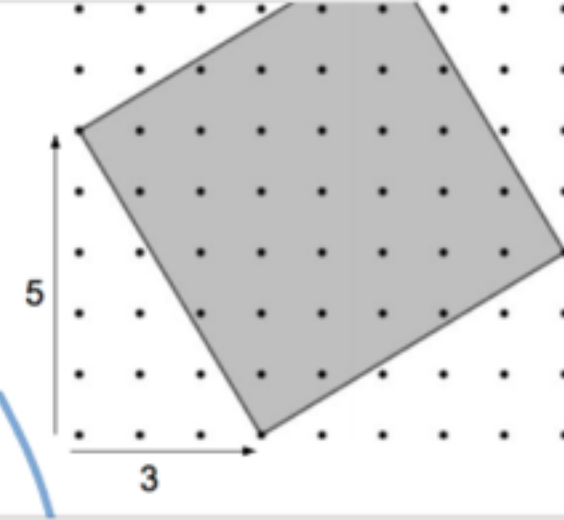
Hermann Weyl



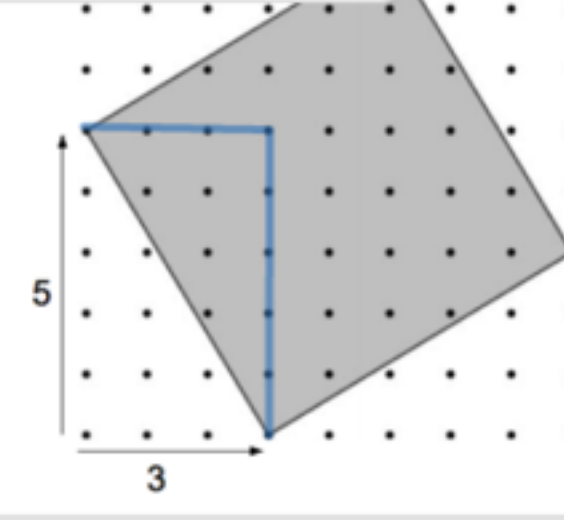
Gottfried Leibniz



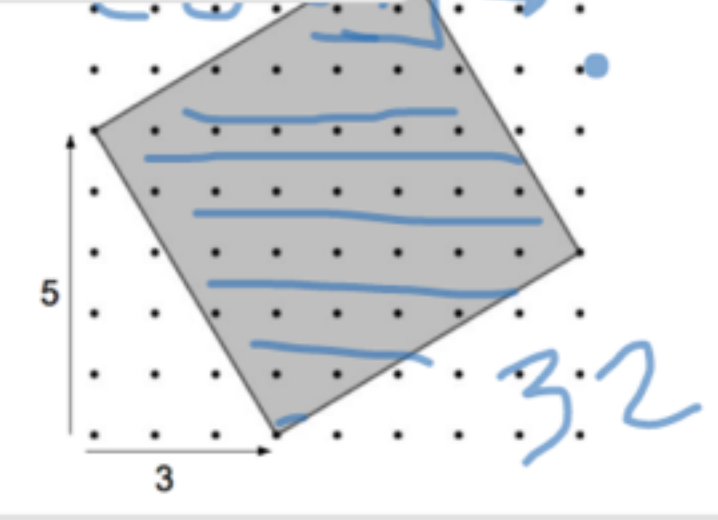
Ingrid Daubechies



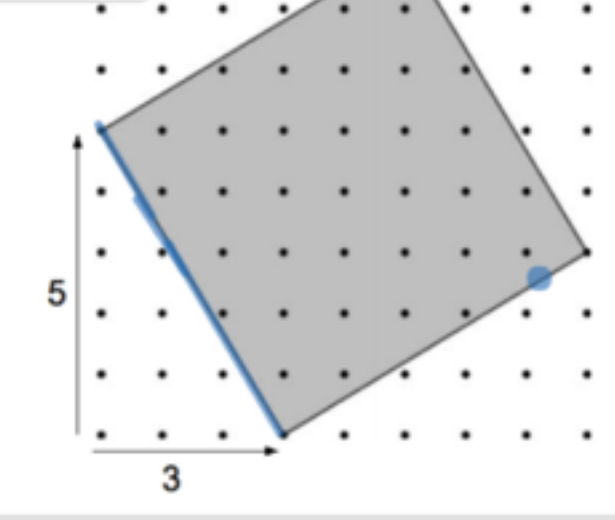
Diego Rodriguez



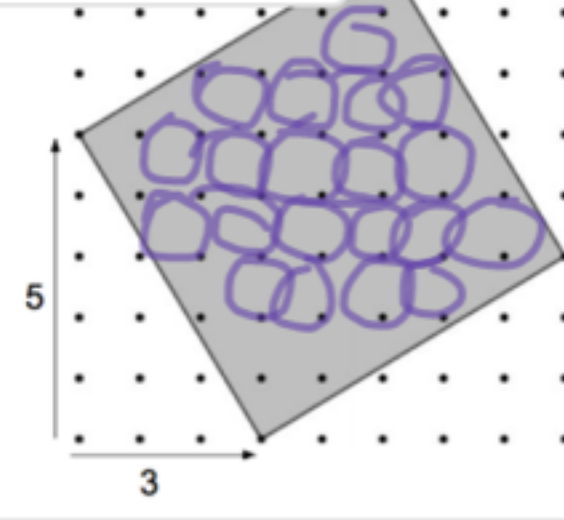
Kunihiko Kodaira



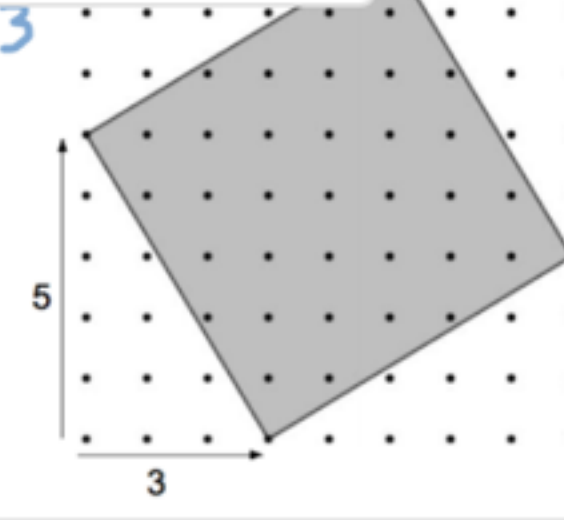
Liu Hui



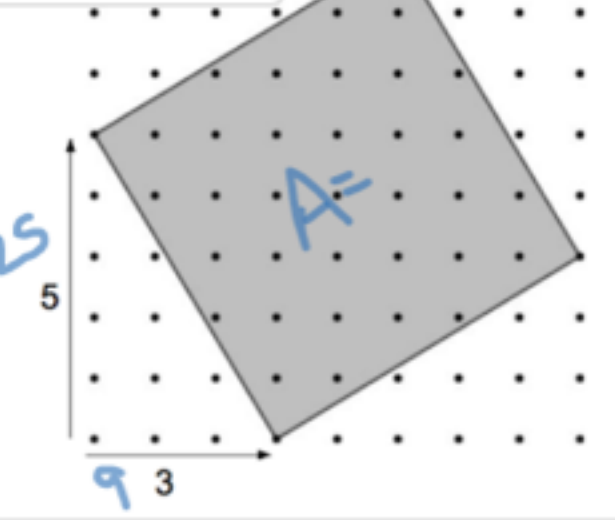
Emmy Noether



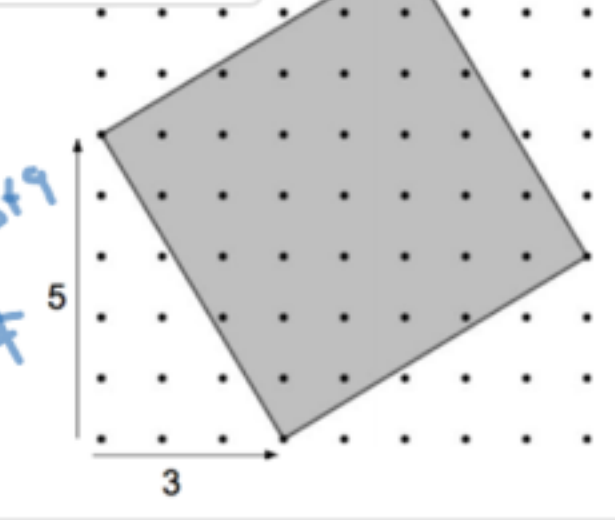
Henri Lebesgue



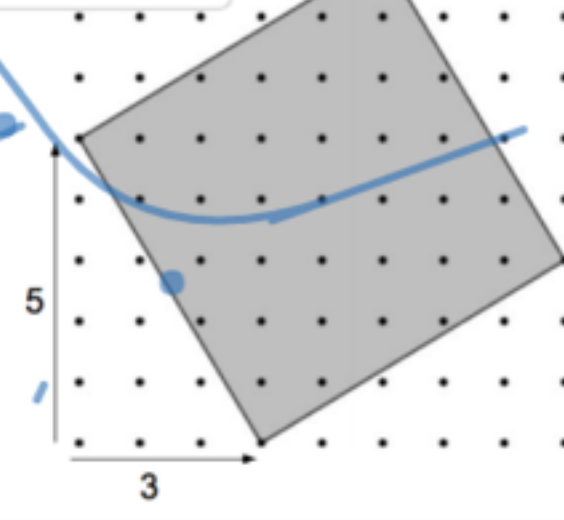
Atle Selberg



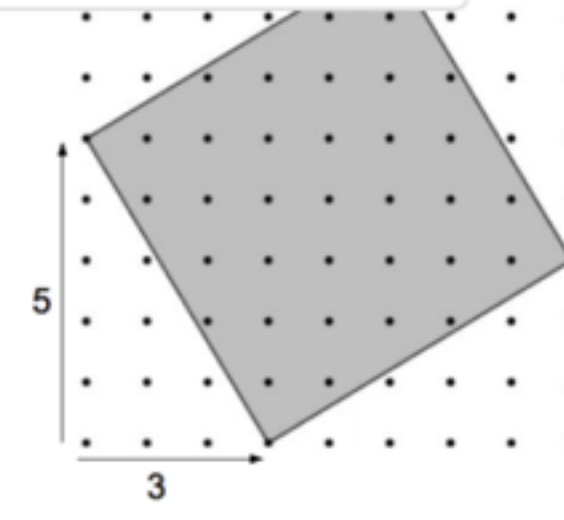
Fan Chung



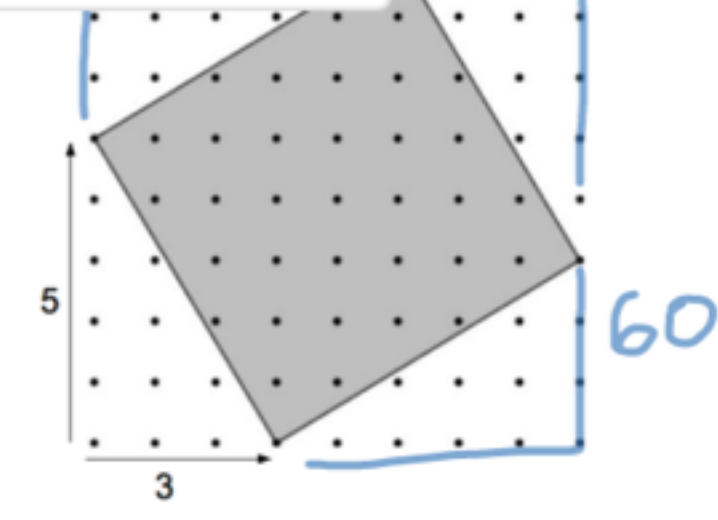
Mary Ross



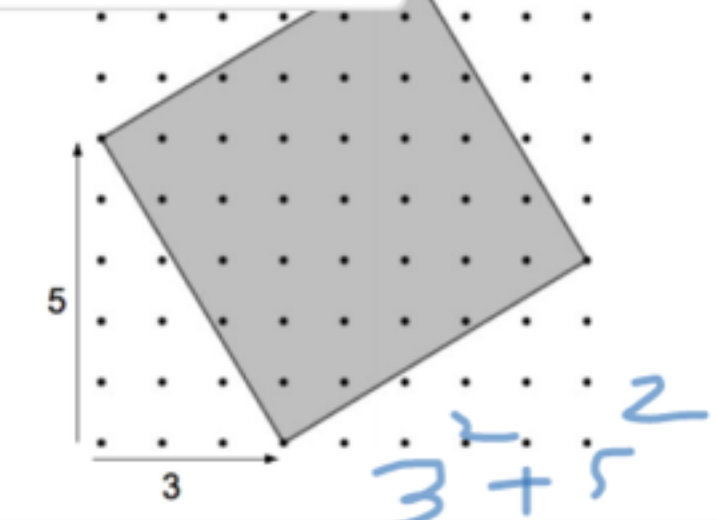
Maryam Mirzakhani



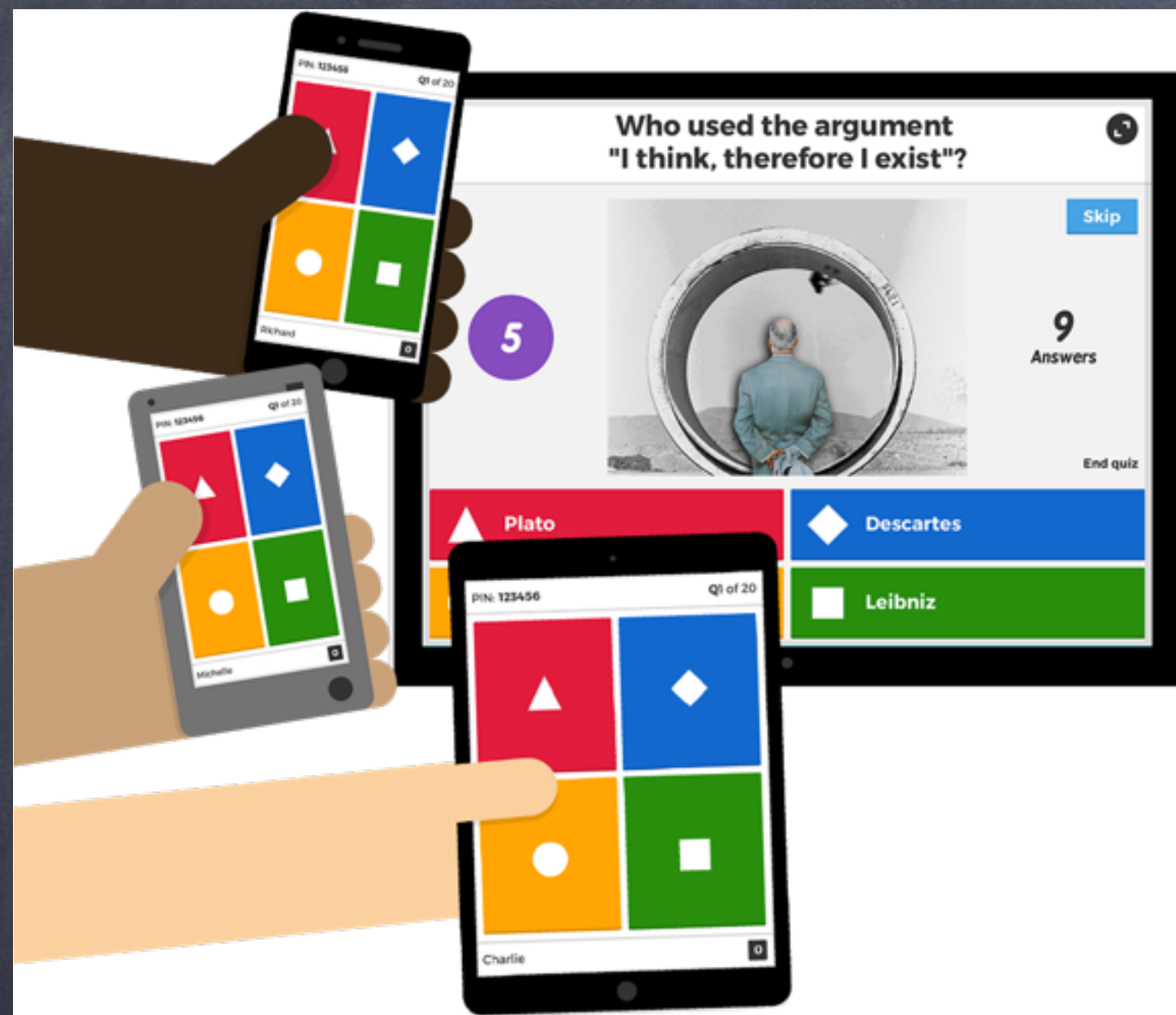
Sophie Germain



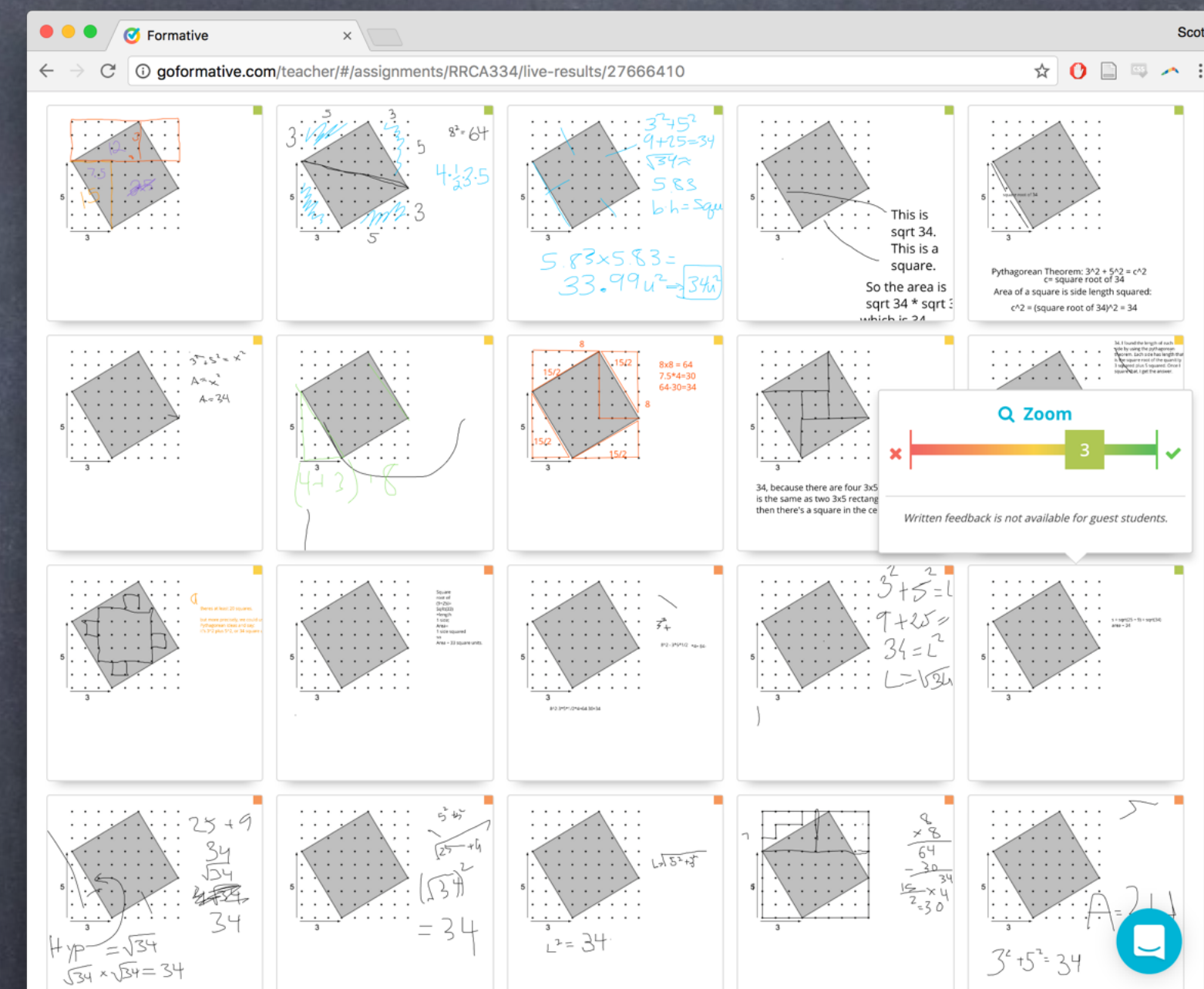
Adrien Legendre



answer only vs. open work



teacher can see answers



teacher can see work

answer only vs. open work

Submit

Farrar: Principles to Action with x

Secure <https://student.desmos.com/activitybuilder/student/58dedfbc9cccc90613e1d928>

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2 of 5

Find the area of the tilted square. Show your reasoning.

The diagram shows a tilted square on a grid. The square is shaded gray. Its side length is indicated by a vertical arrow labeled 5. The horizontal distance between the two vertices on the bottom side is indicated by a horizontal arrow labeled 3. The square is oriented such that its sides are parallel to the grid lines.

(fake mockup)

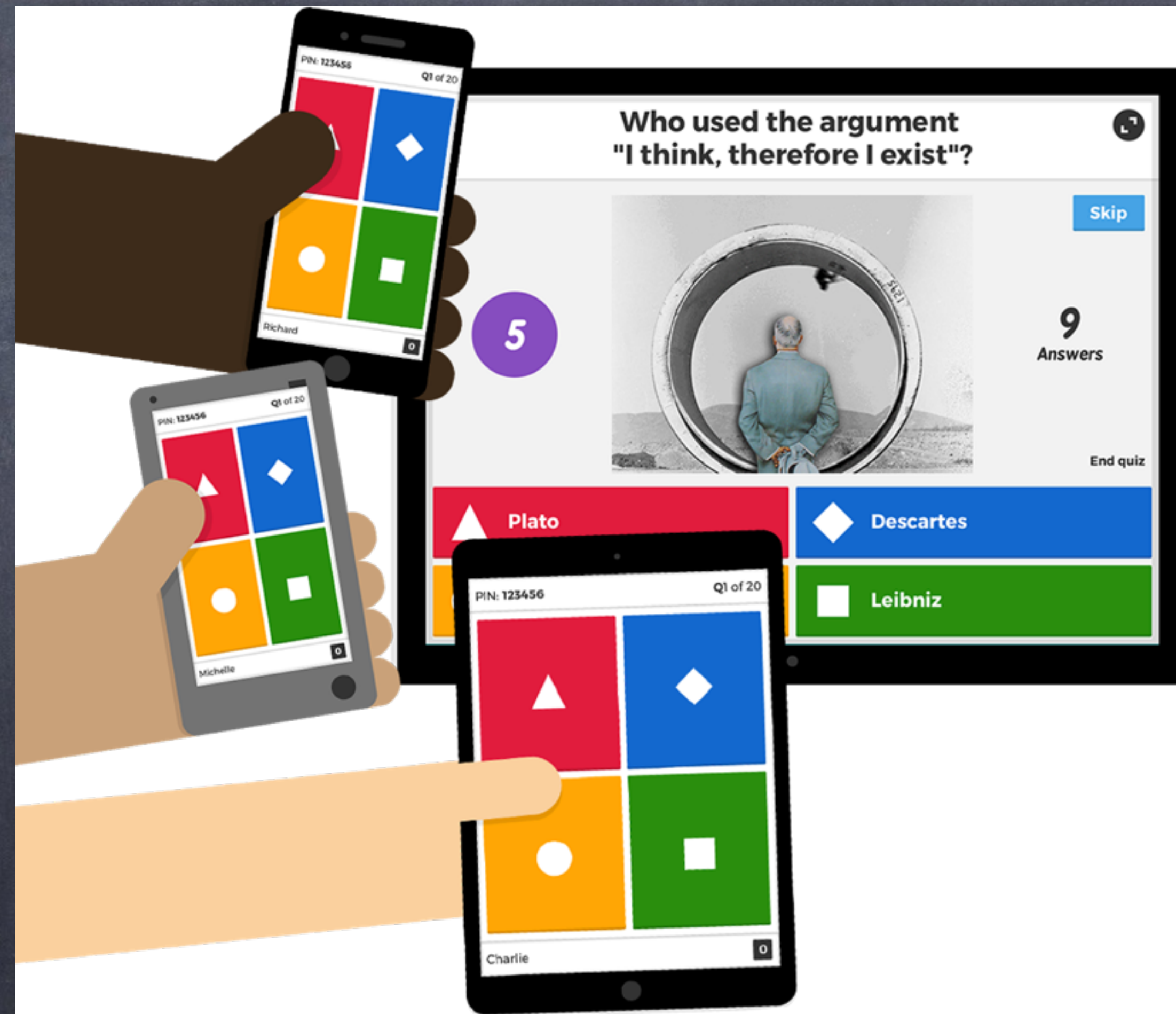
“Answer only” trims out information

correct rate?
85% correct

maybe linked to standards?
STAN.1.1 : 85%

if you're lucky, a distribution
34, 34, 34, 34, 34, 34, 34
sqrt(34)
15, 15, 15
30

“answer only” tech is for... answers only.



“open work” allows thought to be seen

[illegible]

Goals:

1. Understand what tech can/should support

aggregate student
open work

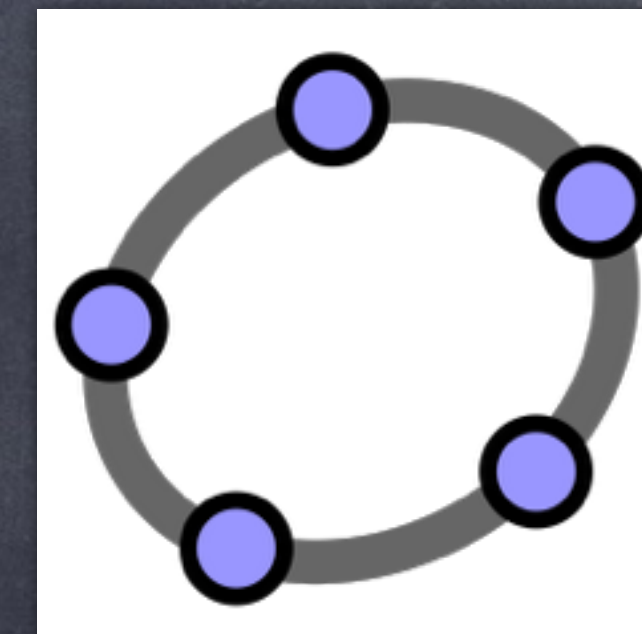


2. Have teachers leverage that support

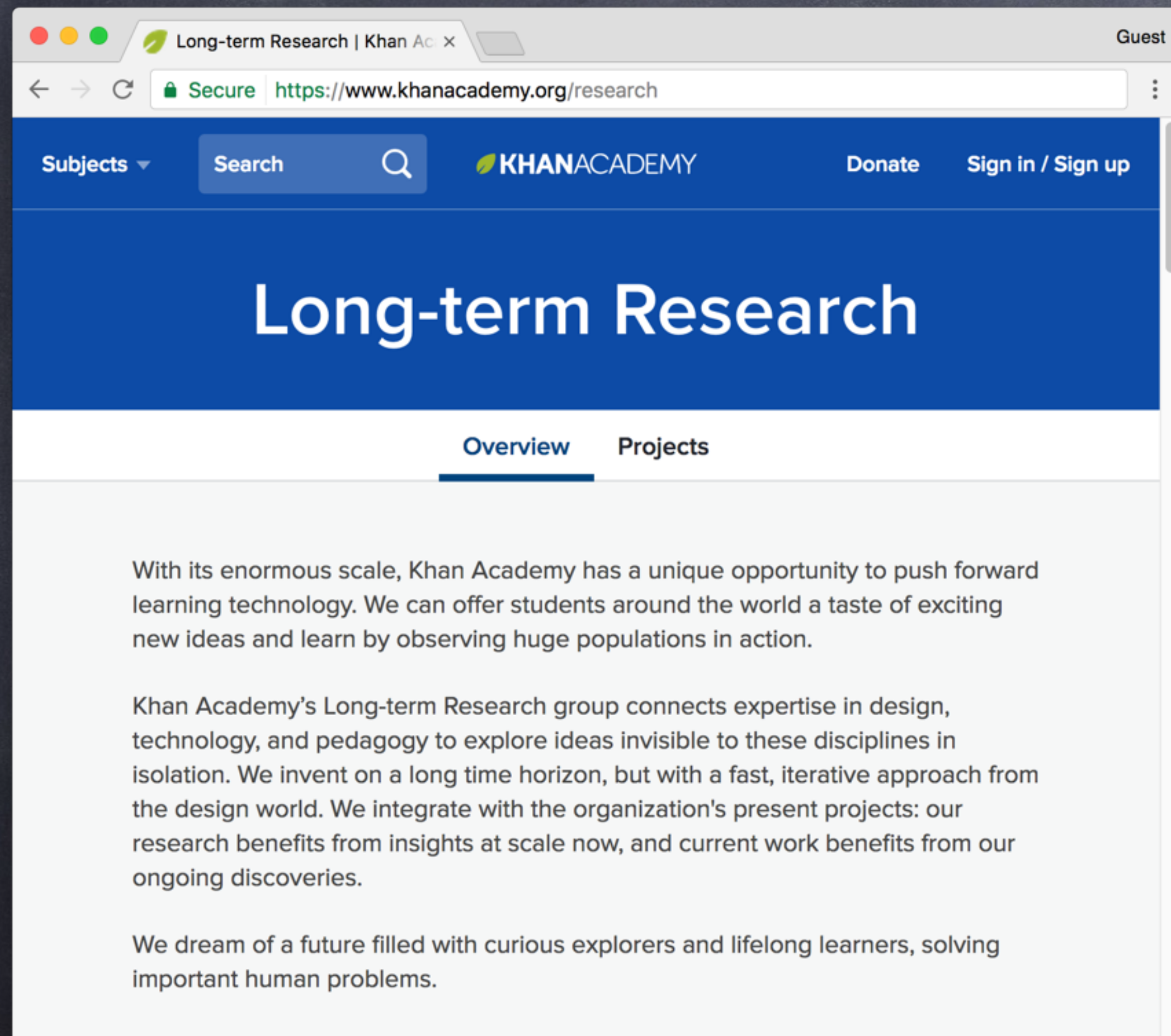


Scott Farrar

- Teaching, Leading, Coaching for 8 years
- 2014: Jumped to researching EdTech, thesis on **Desmos**
- 2016: Joined **GeoGebra** network and **Khan Academy** Long-term Research Team



khanacademy.org /research



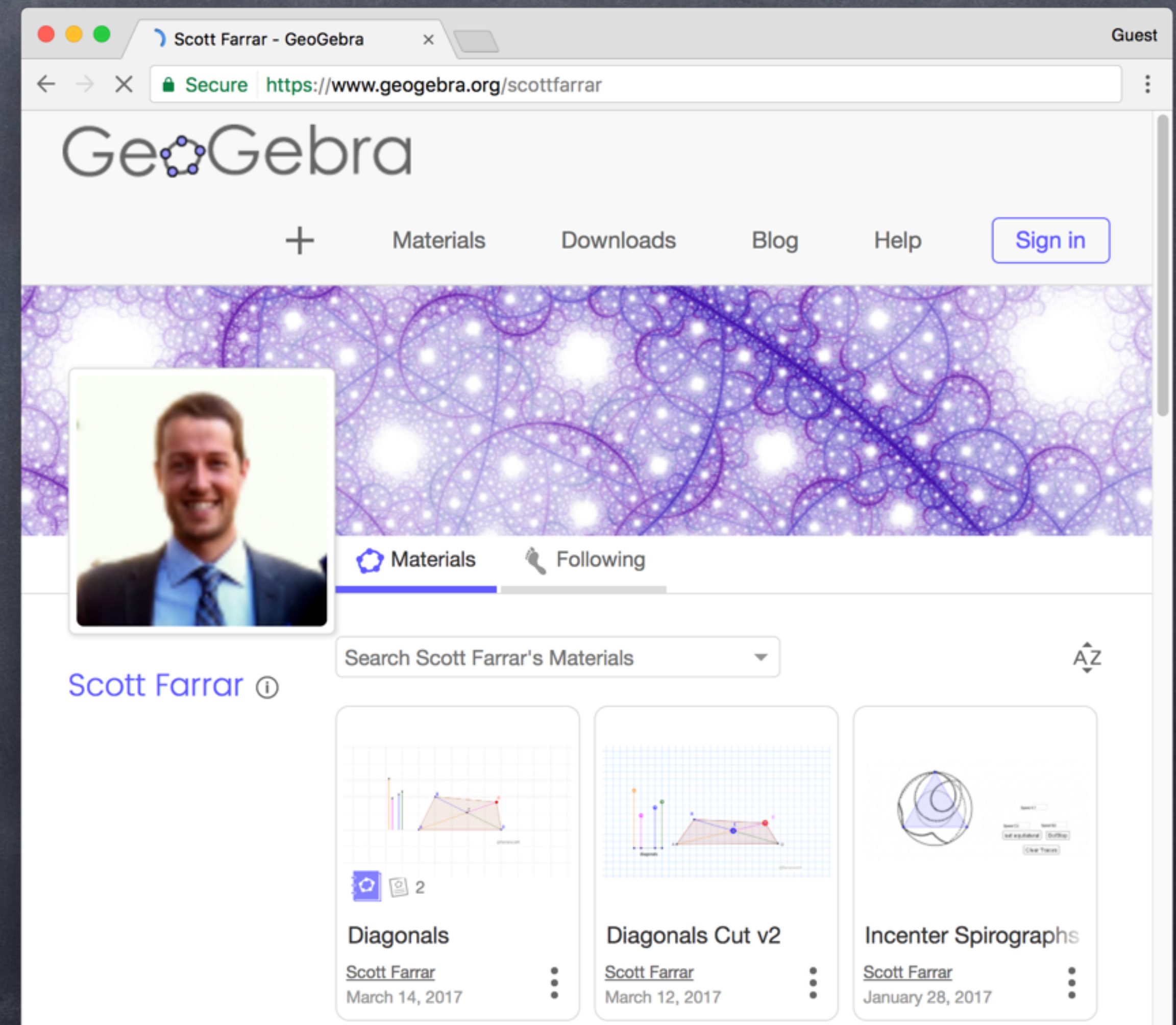
A screenshot of a web browser showing the Khan Academy Long-term Research page. The browser tab is titled "Long-term Research | Khan Academy". The address bar shows "https://www.khanacademy.org/research". The page has a blue header with "Subjects", a search bar, the Khan Academy logo, "Donate", and "Sign in / Sign up". Below the header is a large blue banner with the text "Long-term Research". Underneath the banner are two tabs: "Overview" (selected) and "Projects". The main content area has a light gray background and contains three paragraphs of text.

With its enormous scale, Khan Academy has a unique opportunity to push forward learning technology. We can offer students around the world a taste of exciting new ideas and learn by observing huge populations in action.

Khan Academy's Long-term Research group connects expertise in design, technology, and pedagogy to explore ideas invisible to these disciplines in isolation. We invent on a long time horizon, but with a fast, iterative approach from the design world. We integrate with the organization's present projects: our research benefits from insights at scale now, and current work benefits from our ongoing discoveries.

We dream of a future filled with curious explorers and lifelong learners, solving important human problems.

geogebra.org /scottfarrar



A screenshot of a web browser showing the GeoGebra profile page for Scott Farrar. The browser tab is titled "Scott Farrar - GeoGebra". The address bar shows "https://www.geogebra.org/scottfarrar". The page has a white header with the GeoGebra logo, a plus icon, "Materials", "Downloads", "Blog", "Help", and a "Sign in" button. Below the header is a large banner with a purple fractal pattern. On the left side of the banner is a profile picture of Scott Farrar. To the right of the profile picture are two tabs: "Materials" (selected) and "Following". Below the banner is a search bar labeled "Search Scott Farrar's Materials". Below the search bar are three material cards. Each card has a thumbnail image, a title, the author's name "Scott Farrar", and the date.

Diagonals

Scott Farrar
March 14, 2017

Diagonals Cut v2

Scott Farrar
March 12, 2017

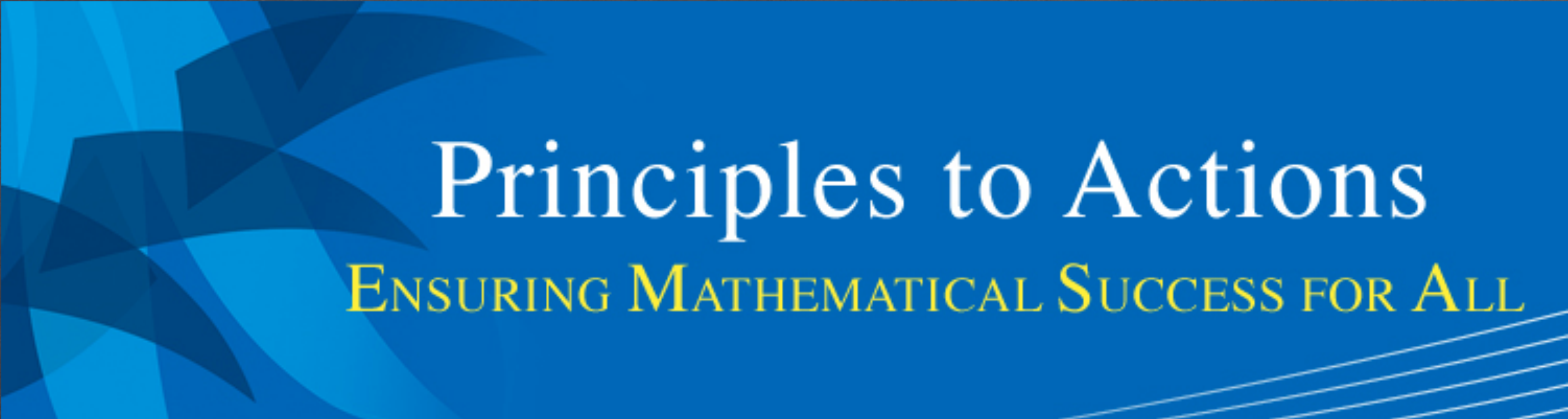
Incenter Spirographs

Scott Farrar
January 28, 2017



Principles to Actions

ENSURING MATHEMATICAL SUCCESS FOR ALL



Principles to Actions

ENSURING MATHEMATICAL SUCCESS FOR ALL

Tools and Technology. *An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.*

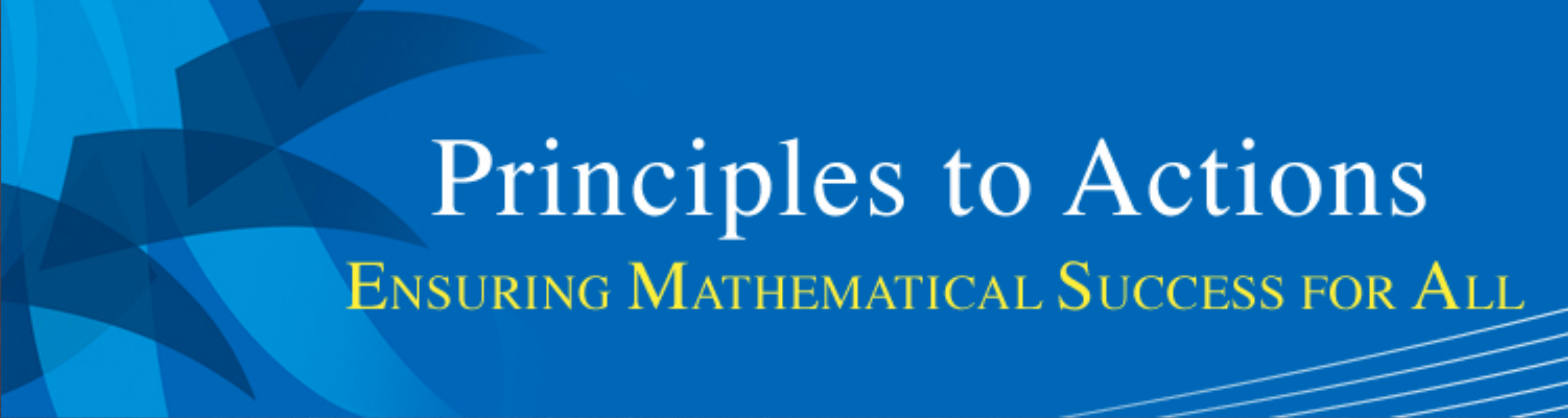
Available tools and technology help teachers and students visualize and concretize mathematics abstractions, and when these resources are used appropriately, they support effective teaching and meaningful learning.

Principles to Actions

ENSURING MATHEMATICAL SUCCESS FOR ALL

Tools and Technology. *An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.*

Available tools and technology help teachers and students visualize and concretize mathematics abstractions, and when these resources are used appropriately, they support effective teaching and meaningful learning.



Principles to Actions

ENSURING MATHEMATICAL SUCCESS FOR ALL

Tools and Technology.

1. Tech should help teachers and students visualize and concretize mathematics abstractions.
2. Tech should support effective teaching and meaningful learning.

PtA: Tech

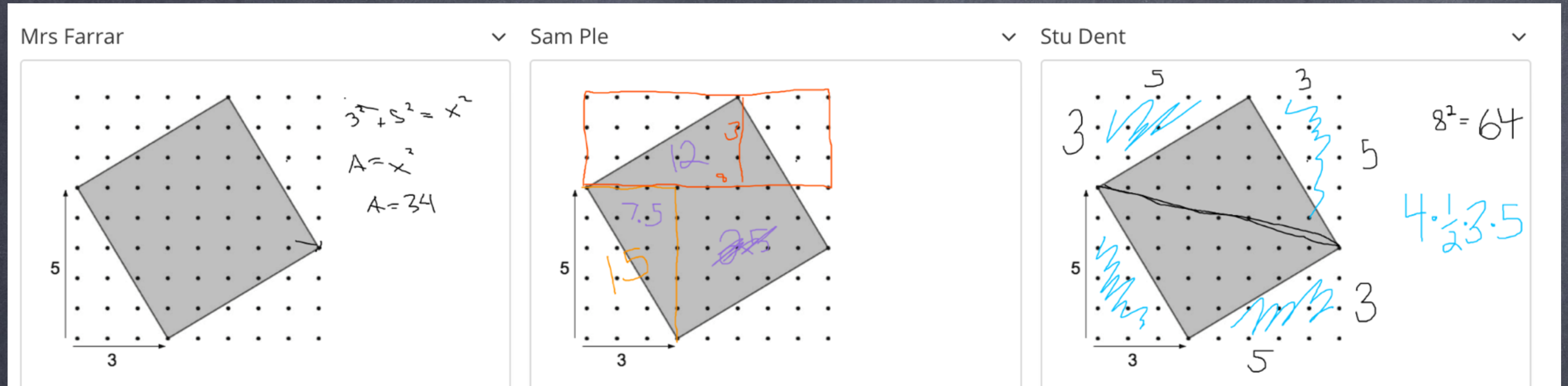
👁 visualizing
mathematics

👁 effective
teaching

👁 concretizing
abstractions

👁 meaningful
learning

Sketching Aggregation



visualizing
mathematics

concretizing
abstractions

effective
teaching

meaningful
learning

PtA Goals for Tech

👁 visualizing
mathematics

👁 effective
teaching

aggregation

👁 concretizing
abstractions

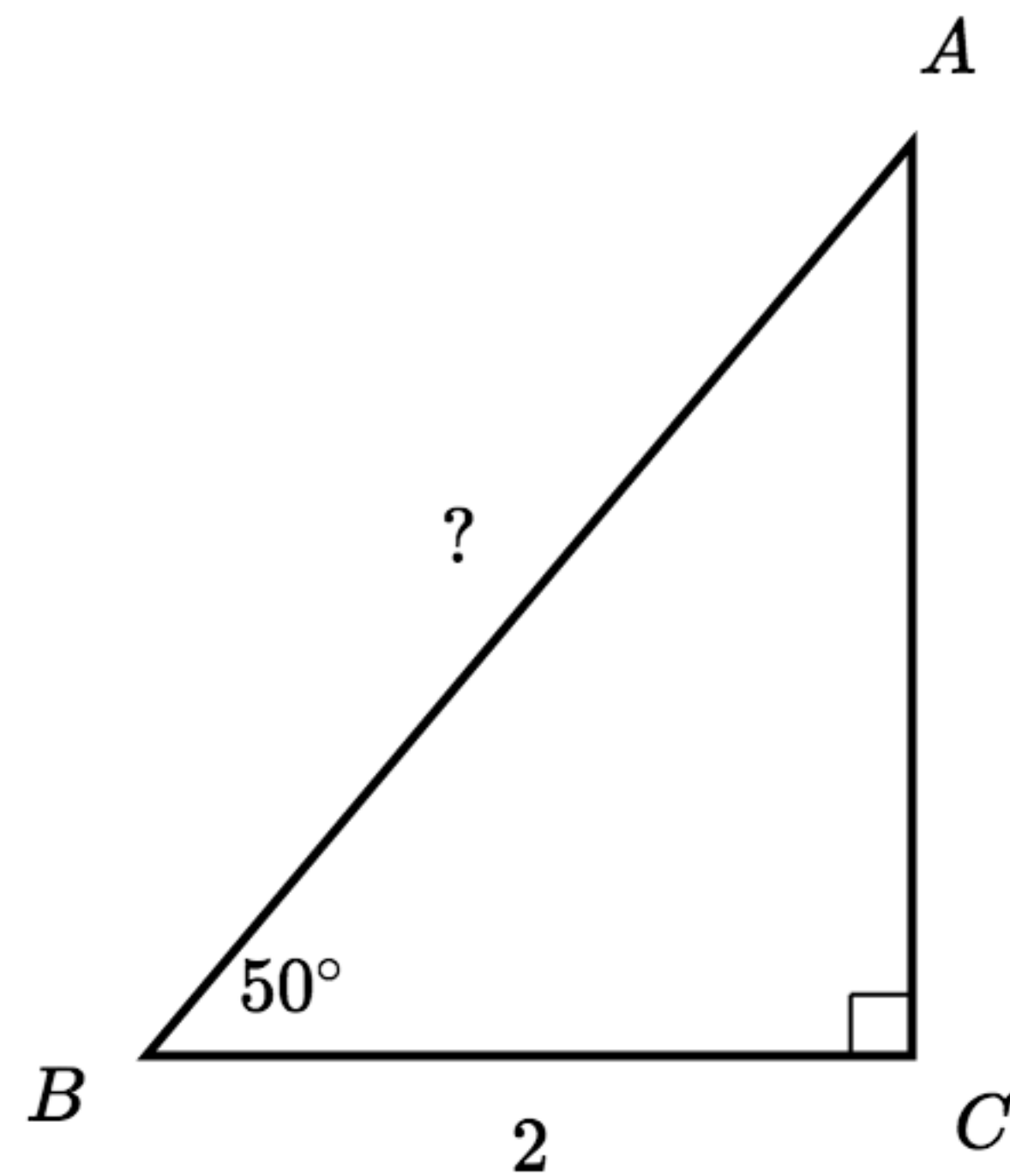
👁 meaningful
learning

Compare to:
Right/Wrong Assessment

Khan Academy Exercise

$$AB = \boxed{}$$

Round your answer to the nearest hundredth.



Right/Wrong Exercises

S2: [to S3] you forgot 50... after the sine

S3: so its 2... divided by... sin 50 ?

S2: x

S3: x?

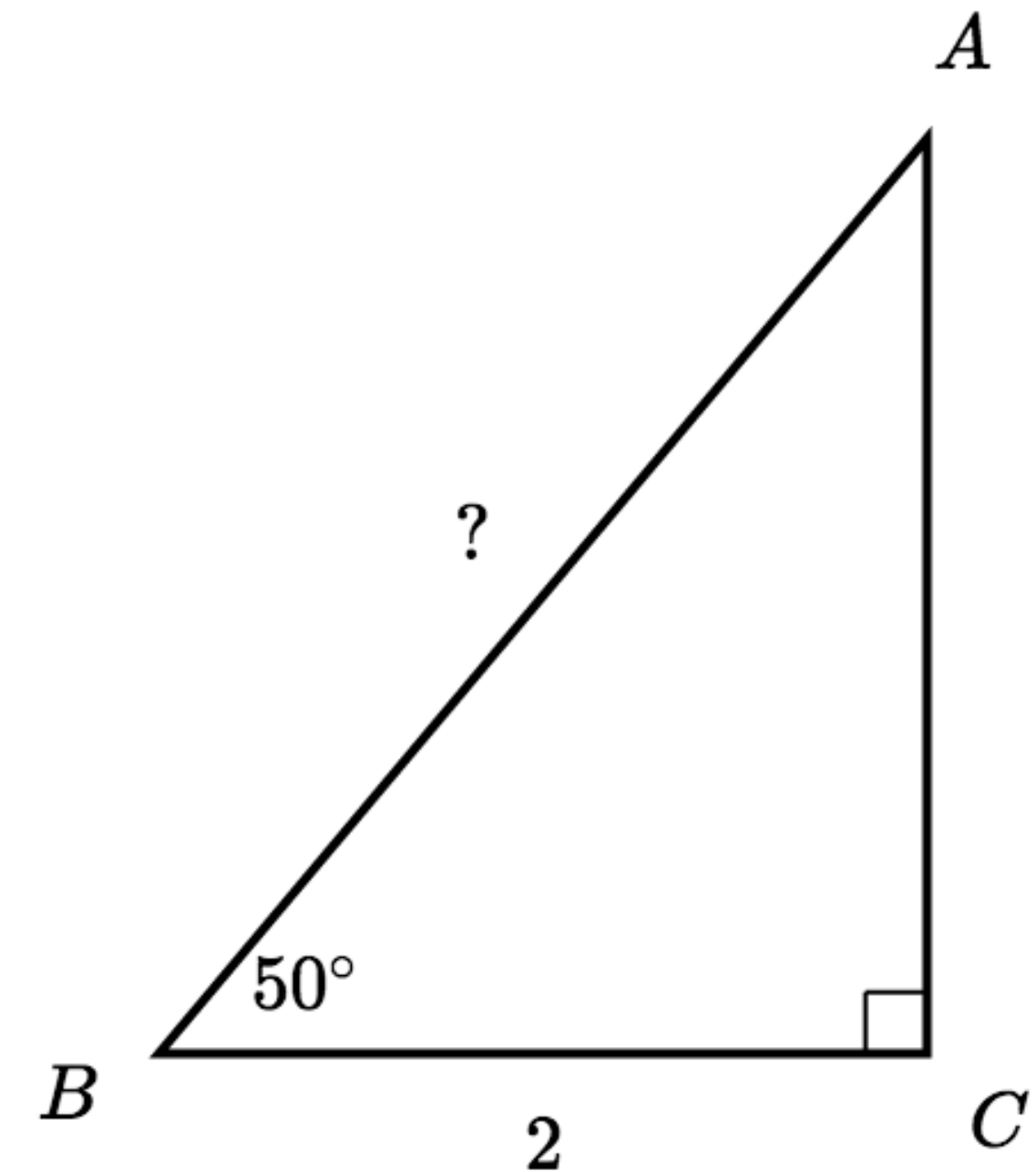
S2: this [points] is multiply

S3: so its 2 divided by sin 50 x

S2: [confused now] oh... you skipped one step. oh ok those numbers don't divide by x you cross multiply. Erase this part... keep the 2. So 2 times [inaudible] by itself right? Equals sin 50. x.

$$AB = \boxed{}$$

Round your answer to the nearest hundredth.



Right/Wrong Exercises

S2: [to S3] you forgot 50... after the sine

S3: so its 2... divided by... sin 50 ?

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S3: x?

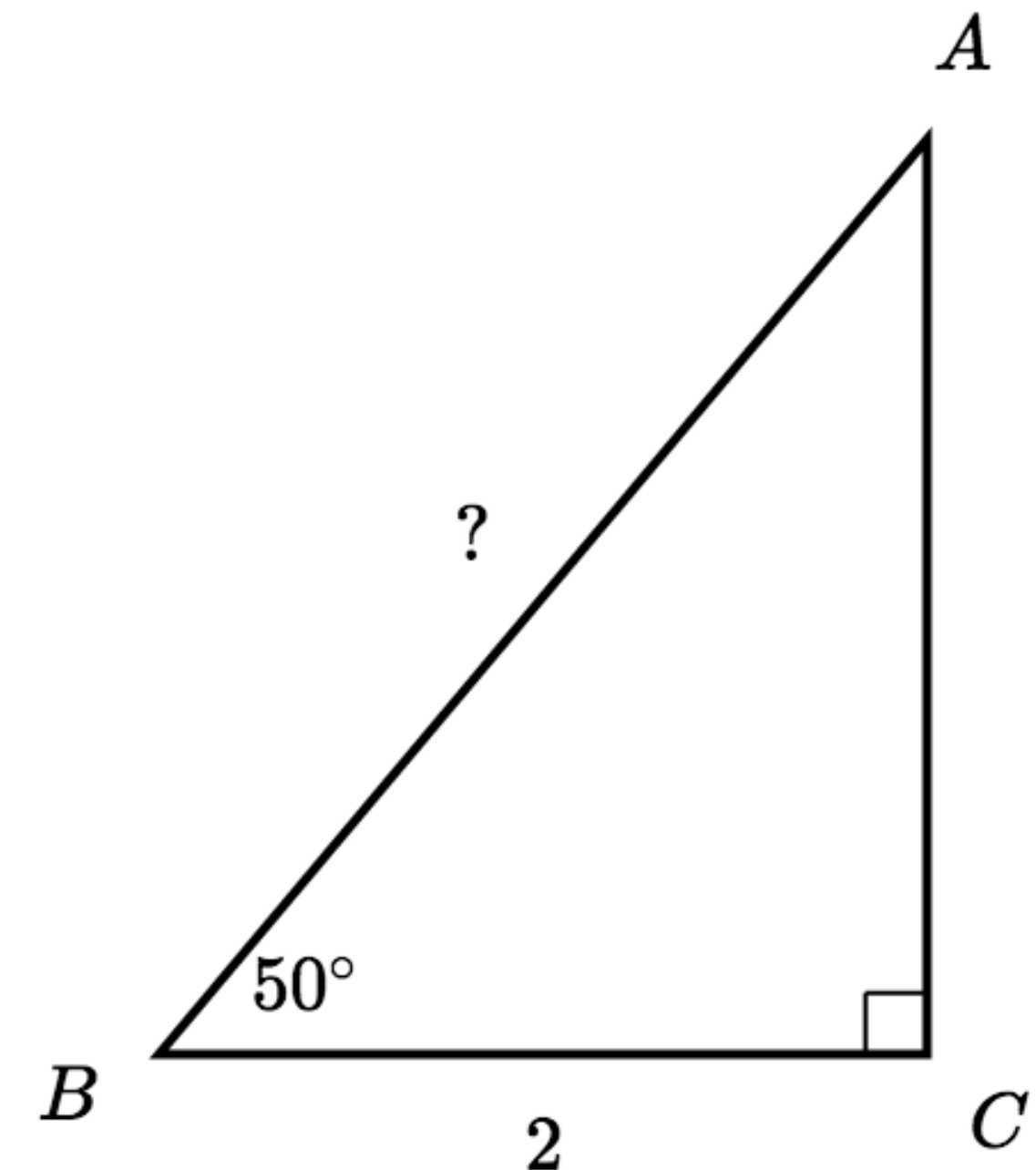
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$$AB = \boxed{}$$

Round your answer to the nearest hundredth.



The numeric input doesn't capture this thinking directly

Right/Wrong Exercises

S3: wait does x go inside the 50? where's x on the calculator?

S2: there is no x.

S1: so what's the equation for sine?

S2: sine is adjacent over hypotenuse

S1: I thought it was opposite

S2: use cosiiiiine! No its truuue

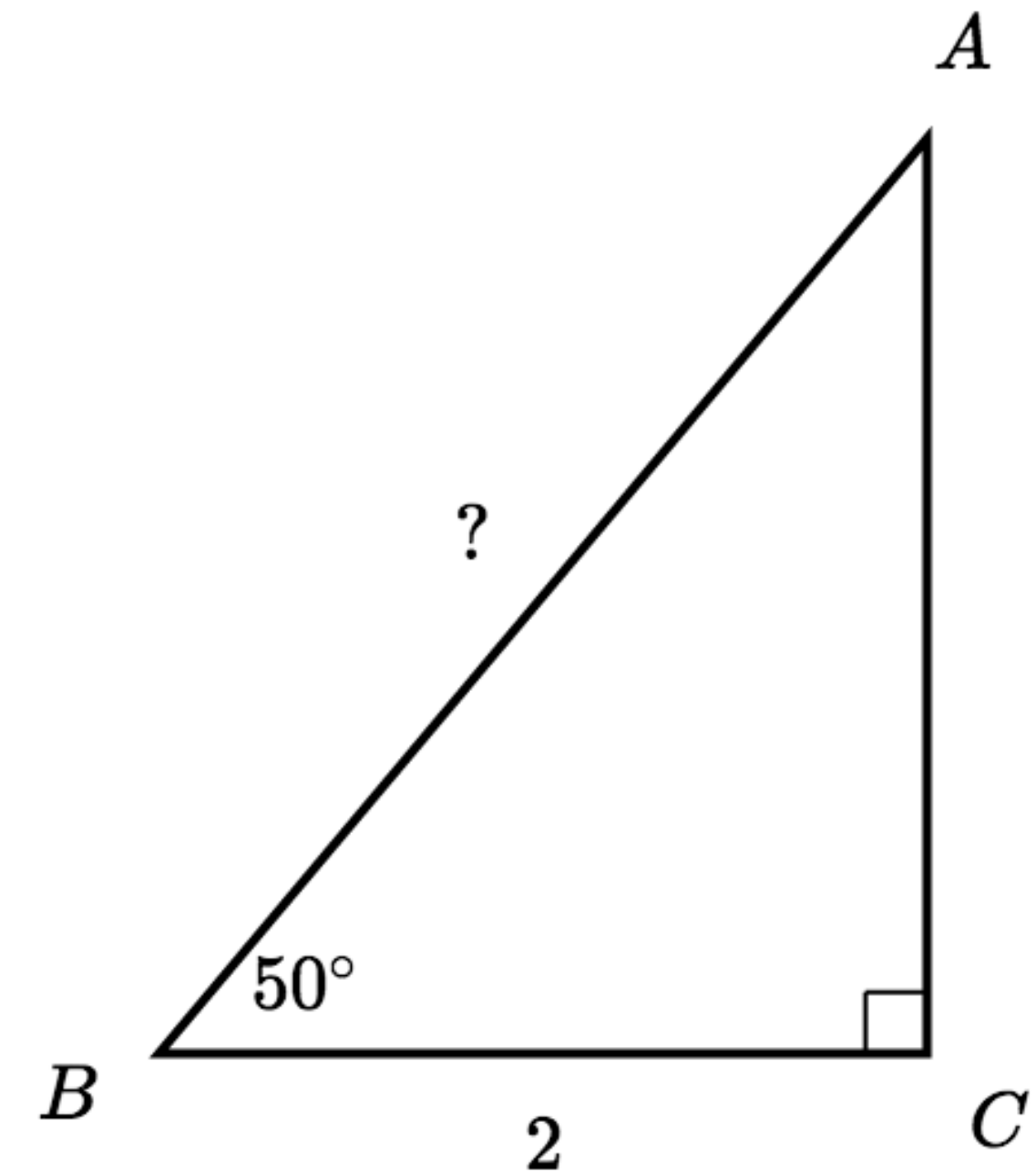
S1: you're confusing me. Look, this is the hypotenuse. Use the hypotenuse and the angle to find the opposite side. That's sine.

S2: ... then ... I did it wrong [grabs her HW from last night]

S3: sine aint working

$$AB = \boxed{}$$

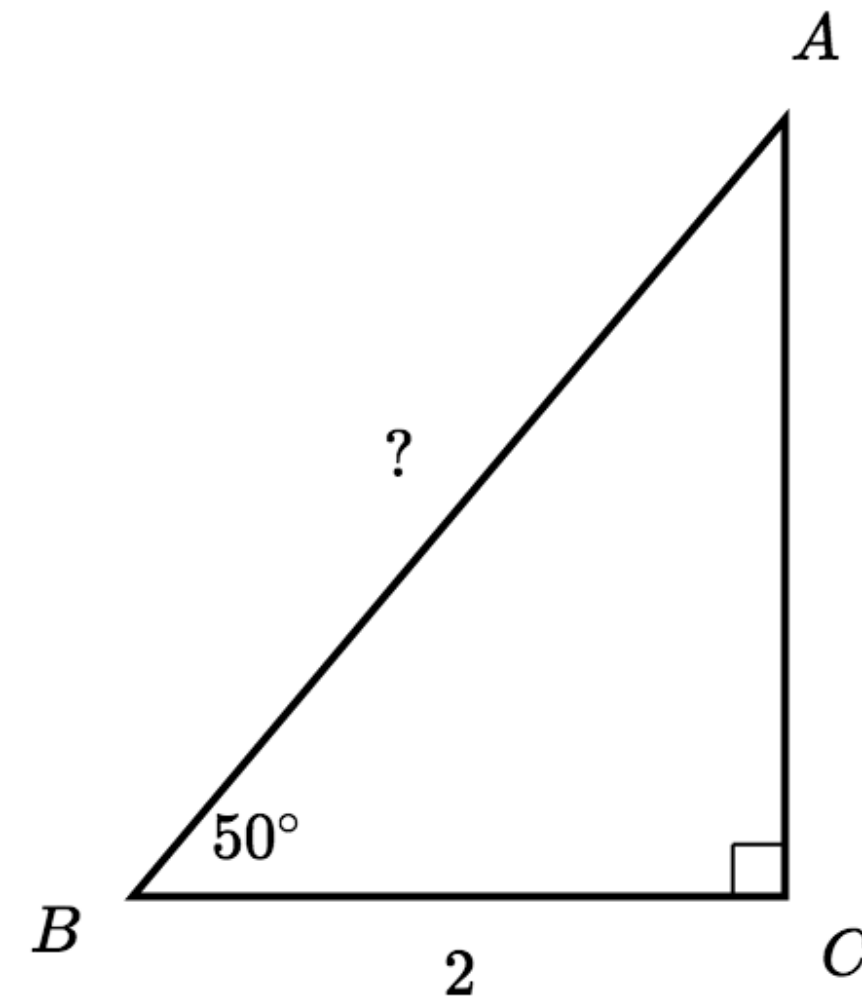
Round your answer to the nearest hundredth.



students weren't ready for right/wrong practice

$$AB = \boxed{}$$

Round your answer to the nearest hundredth.



👁 visualizing
mathematics??

👁 concretizing
abstractions??

👁 effective
teaching??

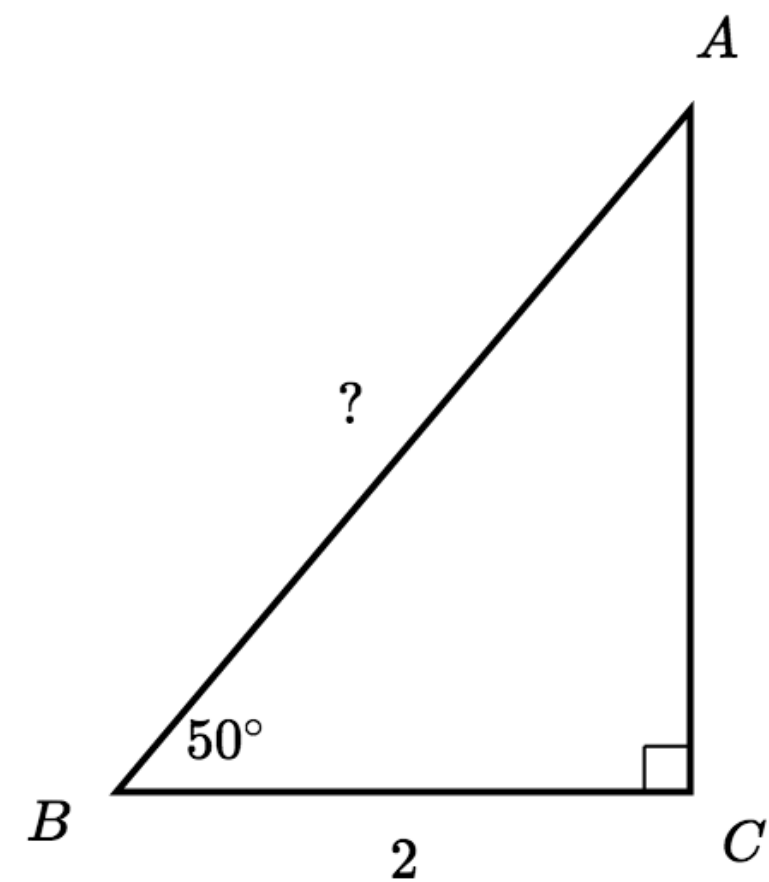
👁 meaningful
learning??

Bonus goal:
Enhance right/wrong practice by
having students coming up with
good wrong answers.

Top four common wrong answers to this exercises on Khan Academy

$AB =$

Round your answer to the nearest hundredth.



1.29

2.61

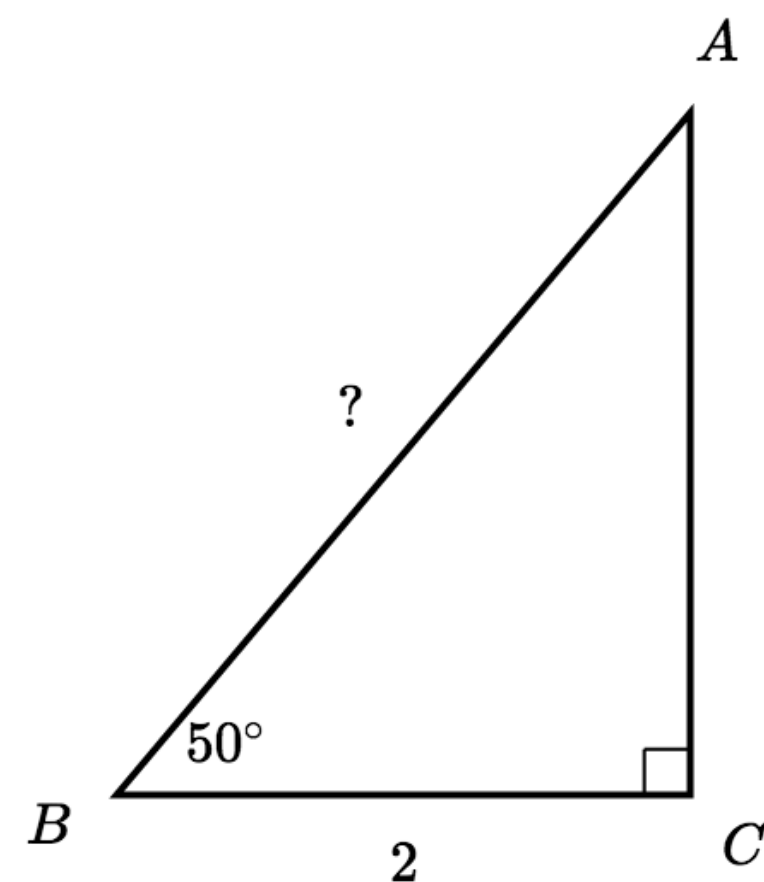
2.38

2.07

Top four common wrong answers to this exercises on Khan Academy

$AB =$

Round your answer to the nearest hundredth.



1.29

2.61

2.38

2.07

$2\cos(50)$

$2\csc(50)$

$2\tan(50)$

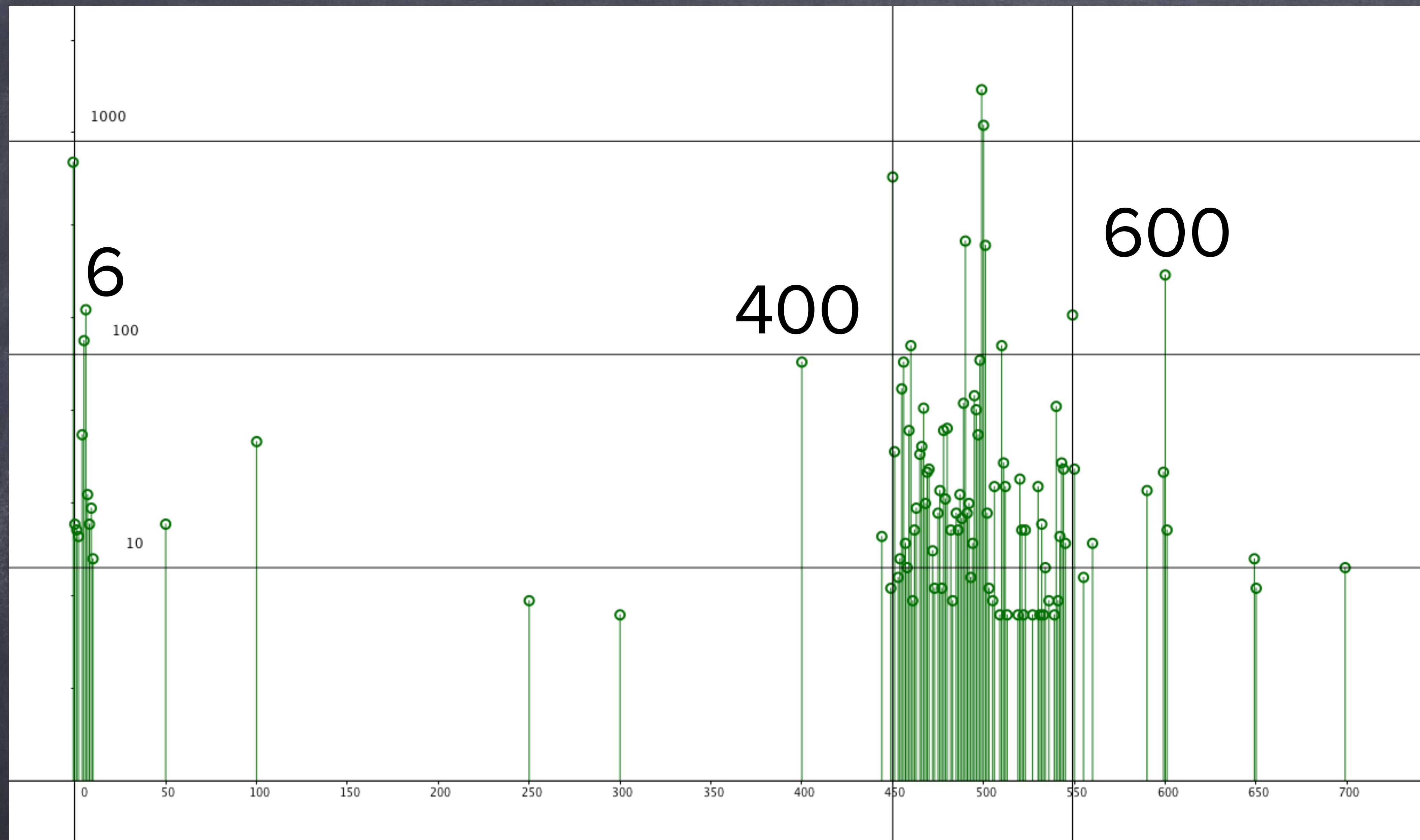
$2\sec(50 \text{ rad})$

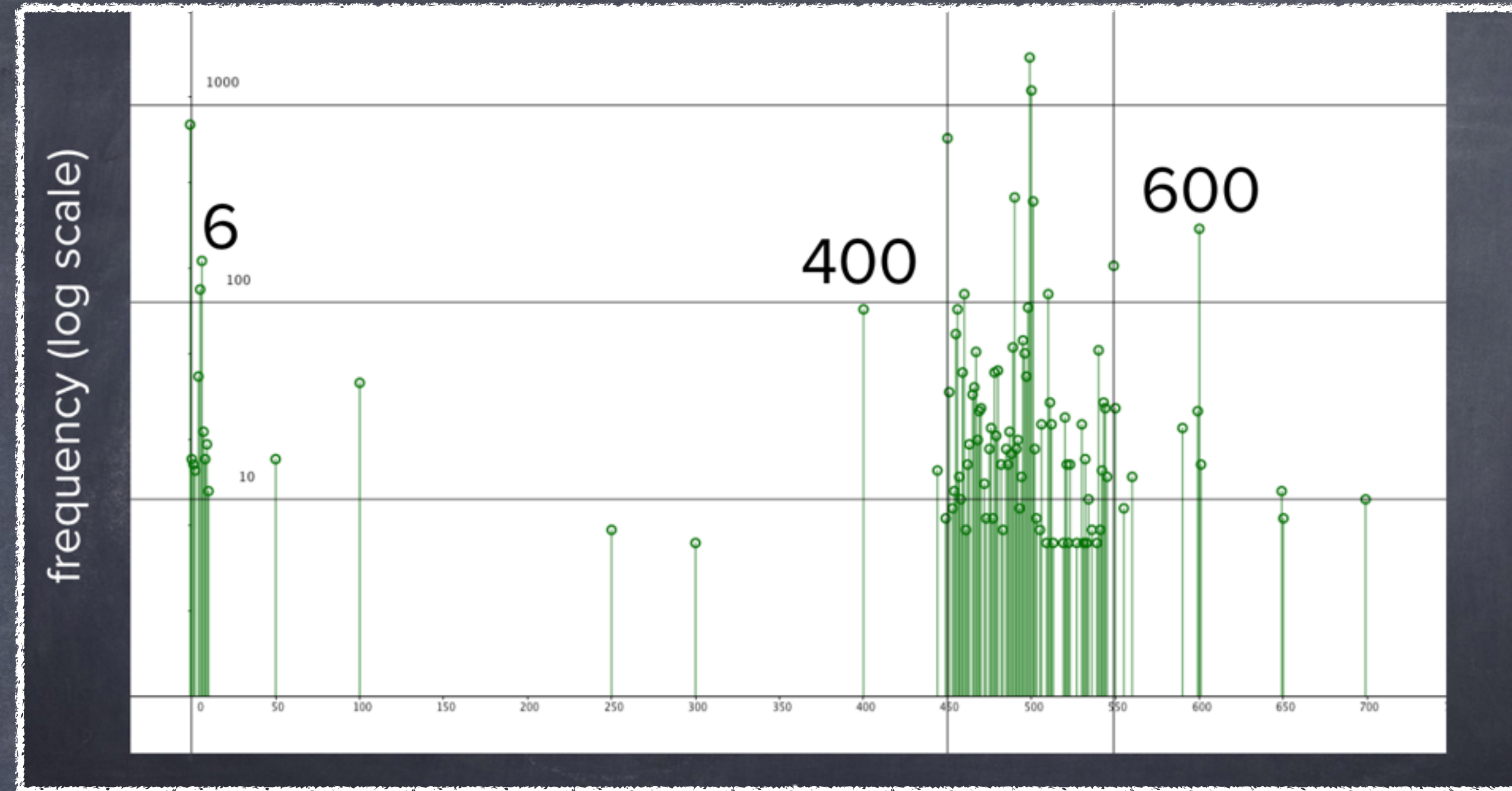
KA Research teaser...

Write a whole number that rounds to 500 if we are rounding to the nearest hundred.

What do you think are reasoned wrong answers?

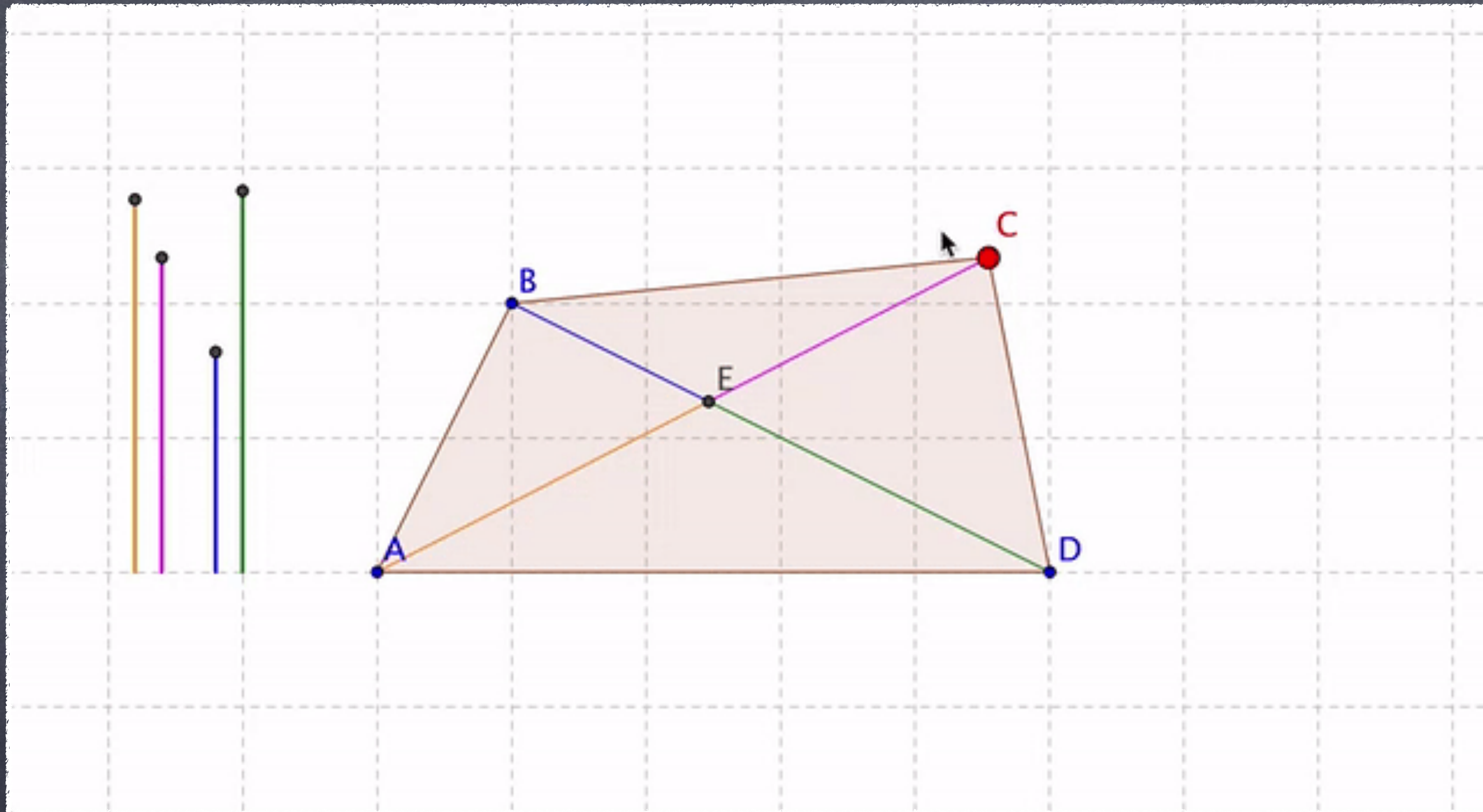
frequency (log scale)





Have your math department
(and your students!)
come up with wrong answers ... and justify them!

Constraints



abstraction:



4.5 Proving Quadrilateral Properties

Theorems

Theorem: The diagonals of a kite are perpendicular.

Theorem: The diagonals of a square are congruent and are the perpendicular bisectors of each other.



“Naive conjecture and counterexamples do not appear in the fully fledged deductive system. The zig-zag of discovery cannot be discerned in the end product.”

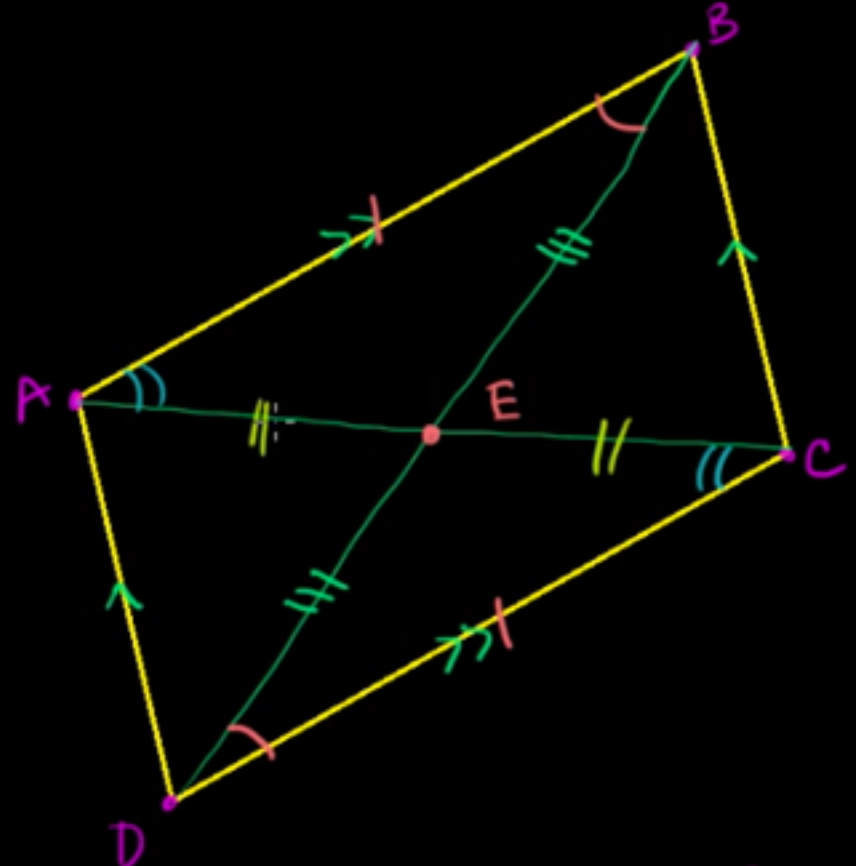
Lakatos, Proofs and Refutations 1976

deduction:

Proof: Diagonals of a parallelogram x Guest

Secure <https://www.khanacademy.org/math/geometry/hs-geo-congruence/hs-g...>

KHANACADEMY



Prove diagonals bisect each other.
 $\angle ABE \cong \angle CDE$ by alt. int. angles
 $\angle DEC \cong \angle BAE$ "
 $\triangle ABE \cong \triangle CDE$ by ASA $\cong$
 $AE = CE$ } Cor. sides of $\cong \triangle$
 $BE = DE$ }

AC is splitting DB into two segments of equal length.

3:51 / 9:06

CC HD

< THEOREMS CONCERNING QUADRILATERAL PROPERTIES

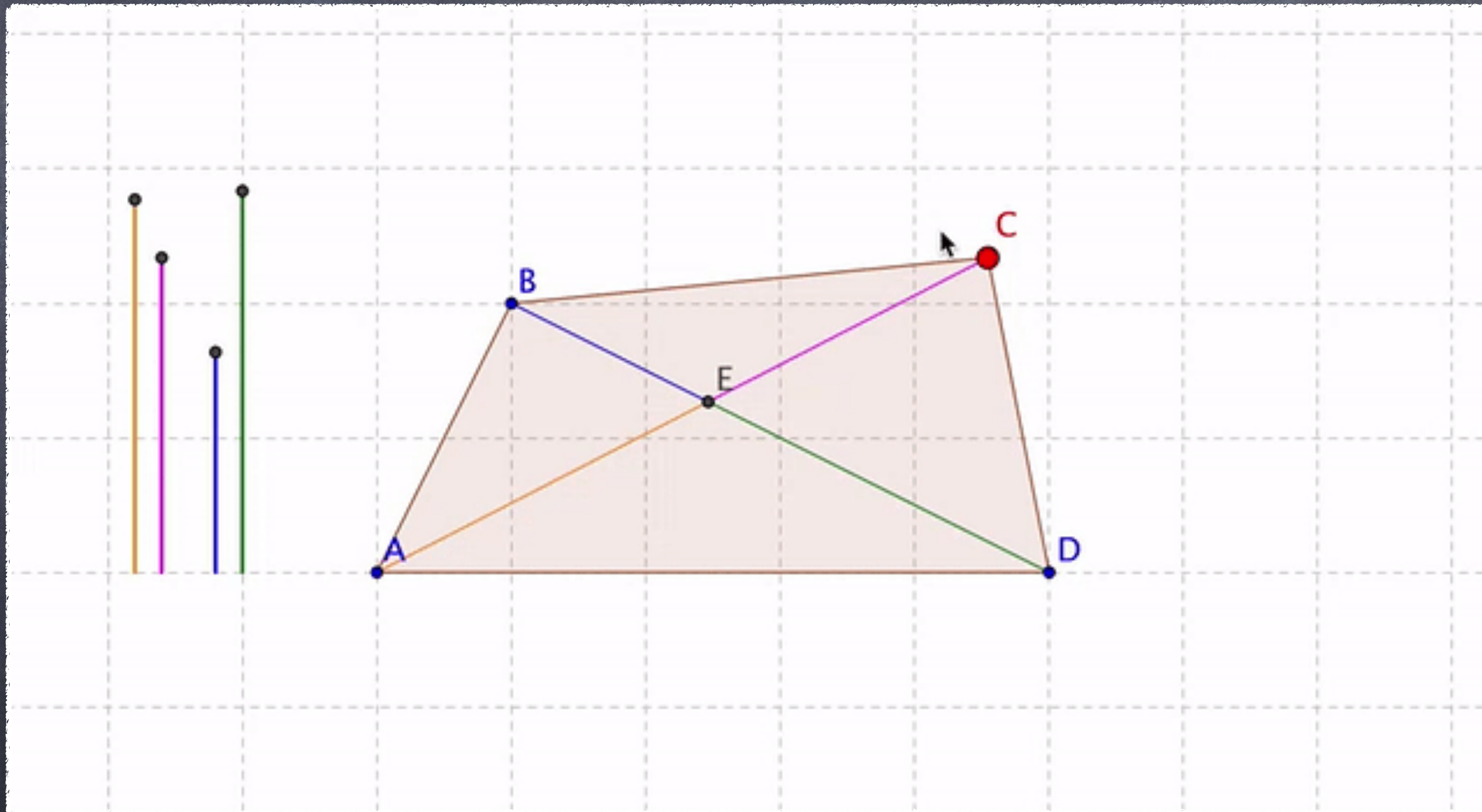
Proof: Diagonals of a parallelogram

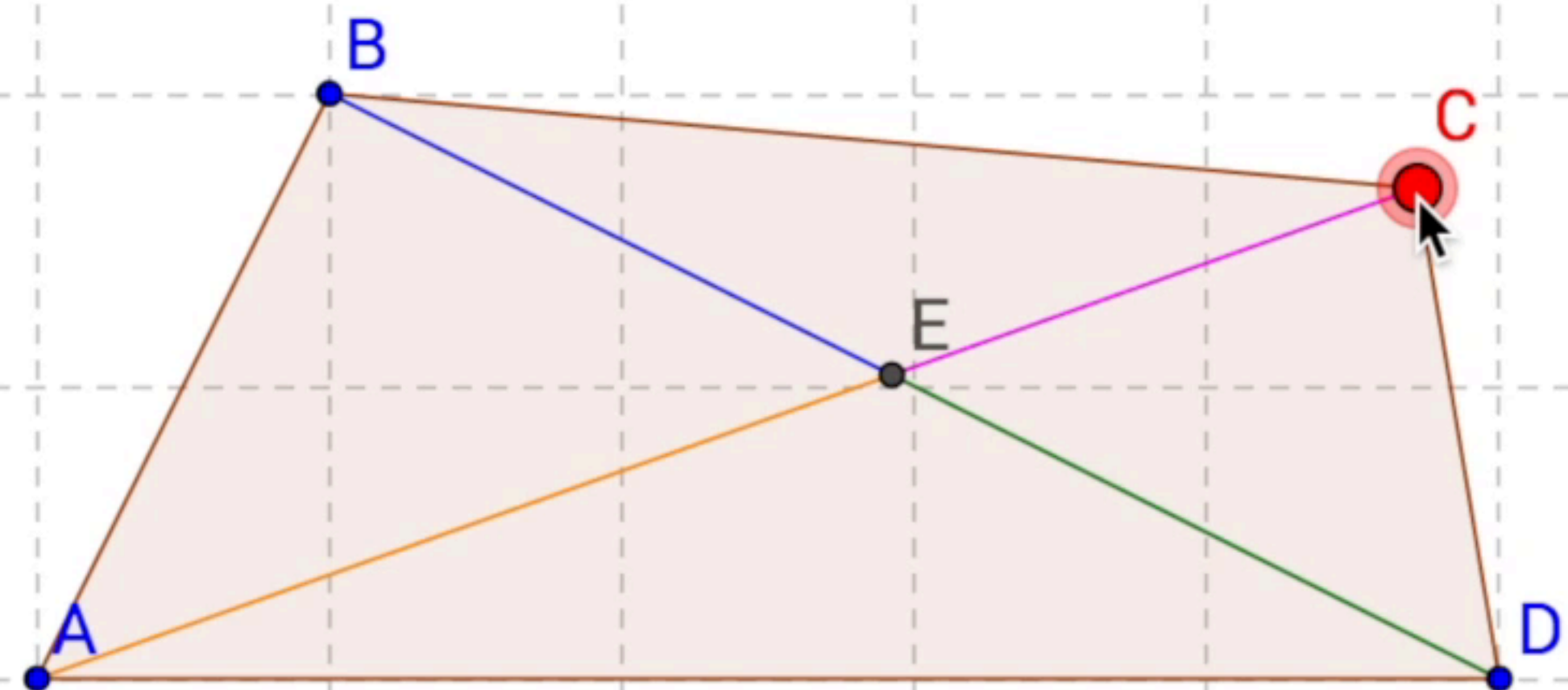
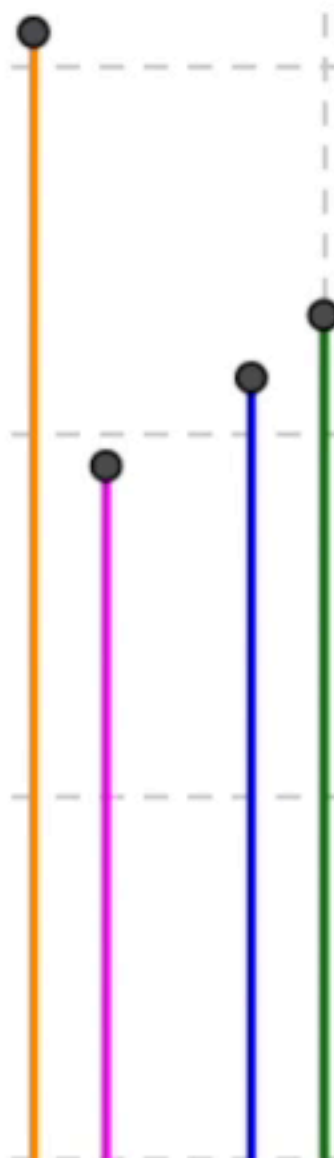
About Transcript

“The product of mathematical activity might be justified with a deductive proof, but the **product** does not represent the **process** of coming to know.”

Lampert, When the Problem is Not the Question and the Solution is Not the Answer: Mathematical Knowing and Teaching 1990

Put abstraction and deduction after
constrained exploration





@farrarscott

Alex: two the same and two the same [gestures at screen]

T: wow

Bob: heh

T2: what shape is that?

Bob: it's a-- um-- trapezoid

Alex: trapezoid
now almost

Bob: I think I know how to make...

ah how do you make a parallelogram? Ah ah ah [Bob has the mouse]

Alex: me! Me me me me

Bob: no no, no you.

Alex: I know I know... can I just try? [Bob continues dragging]
nooo

T: ok let's trade turns

Bob: ok [passes control to A]

Alex: like um A parallelogram.

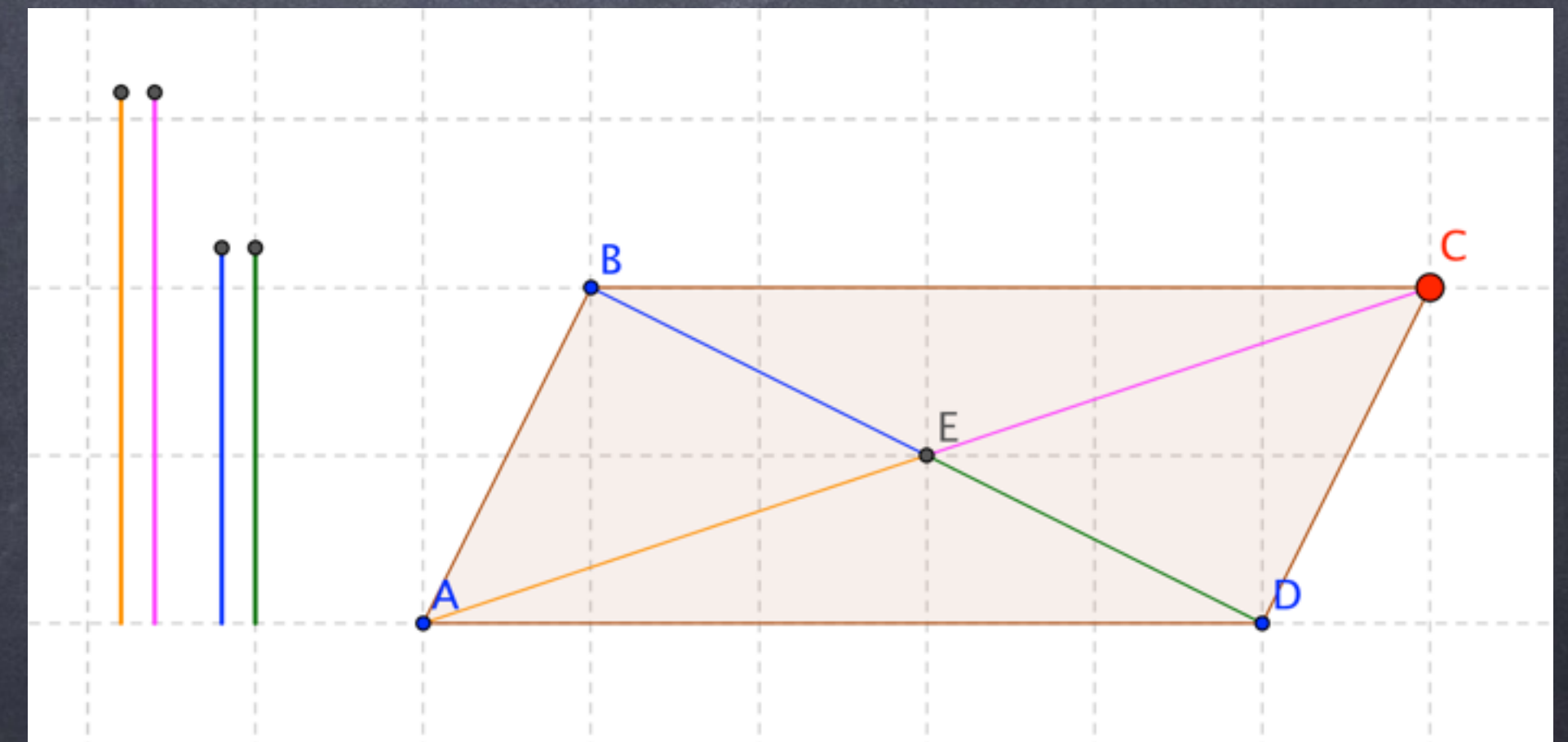
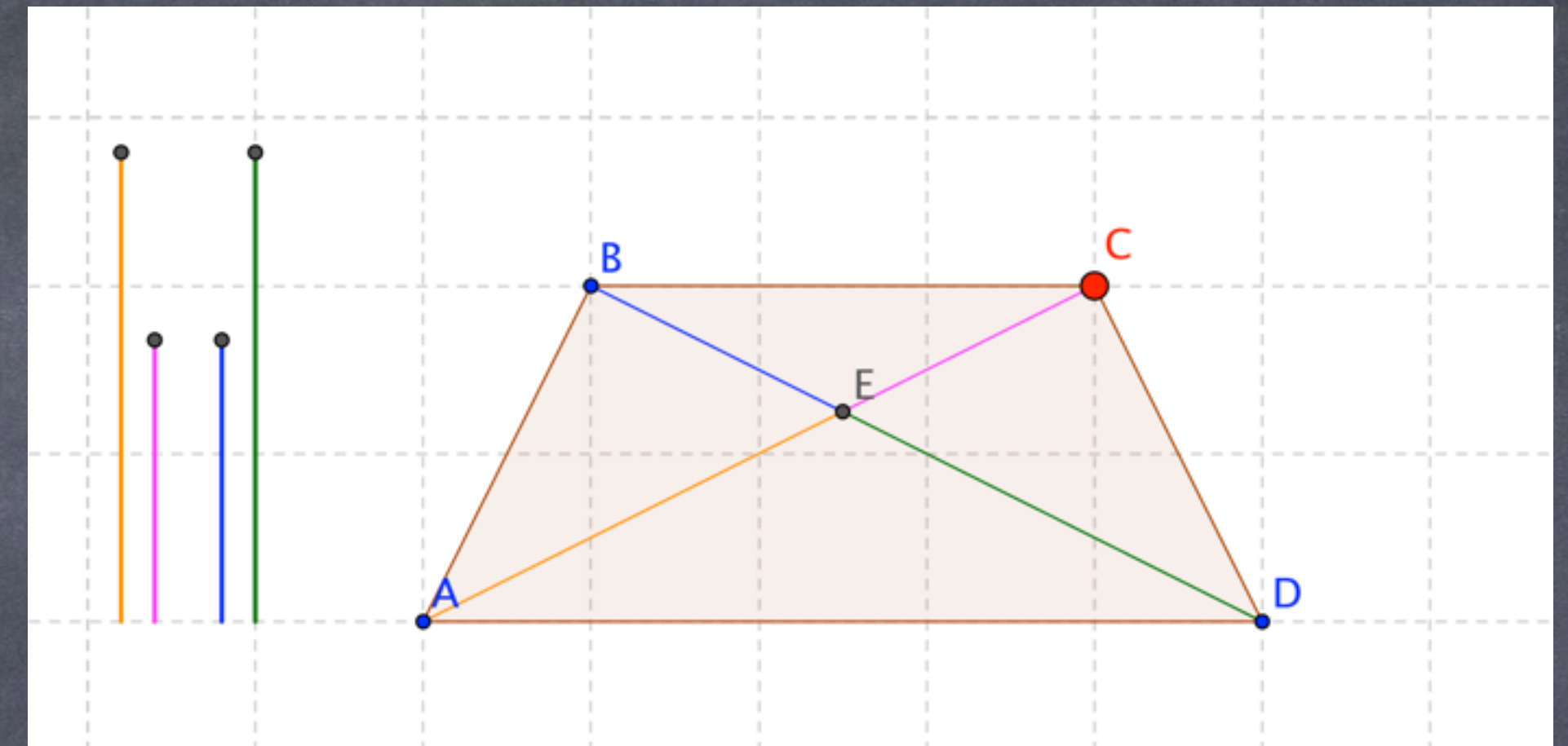
Bob: yaaaay!

Alex: oh it turns into the same...

Bob: no put that down here [points at screen]

T: ---So you found that shape, and the other shape...

Alex: ---but that's two the same two the same



Goals:

1. Understand what tech can/should support

aggregate student
open work

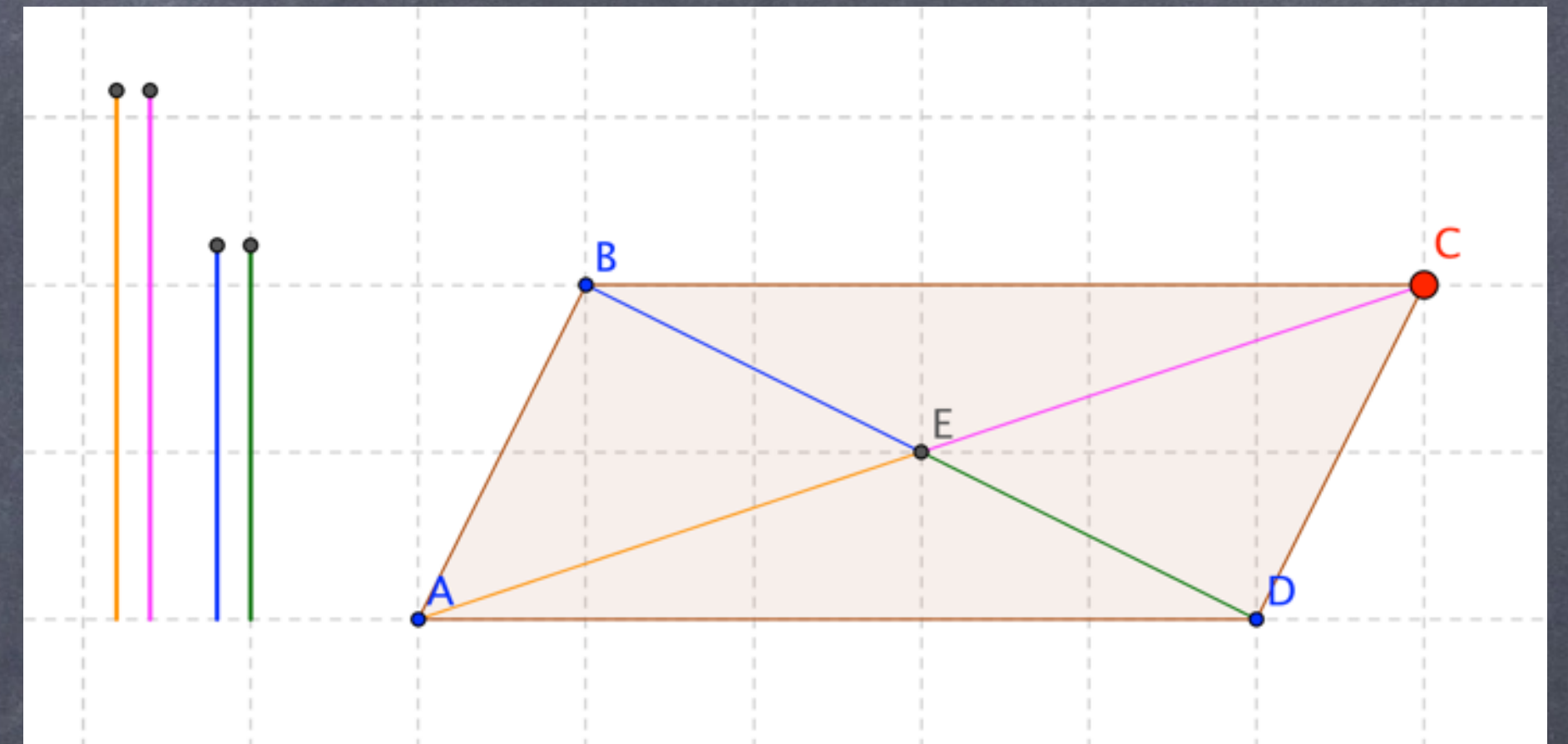
constrain student
exploration

2. Have teachers leverage that support



Constrained Exploration

- Focus interaction by limiting it.
(point C only)
- Students “breaking it” still has meaning.
(triangle)
- No rules. Instead: signals
(segments disappear for non-polygon)
- properties are apparent without words
(congruent segments more apparent)



Goals:

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aggregate student
open work

constrain student
exploration

2. Have teachers leverage that support



match constraints
to mathematics

PtA : Tech

👁 visualizing
mathematic

constraints

👁 effective
teaching

aggregation

👁 concretizing
abstractions

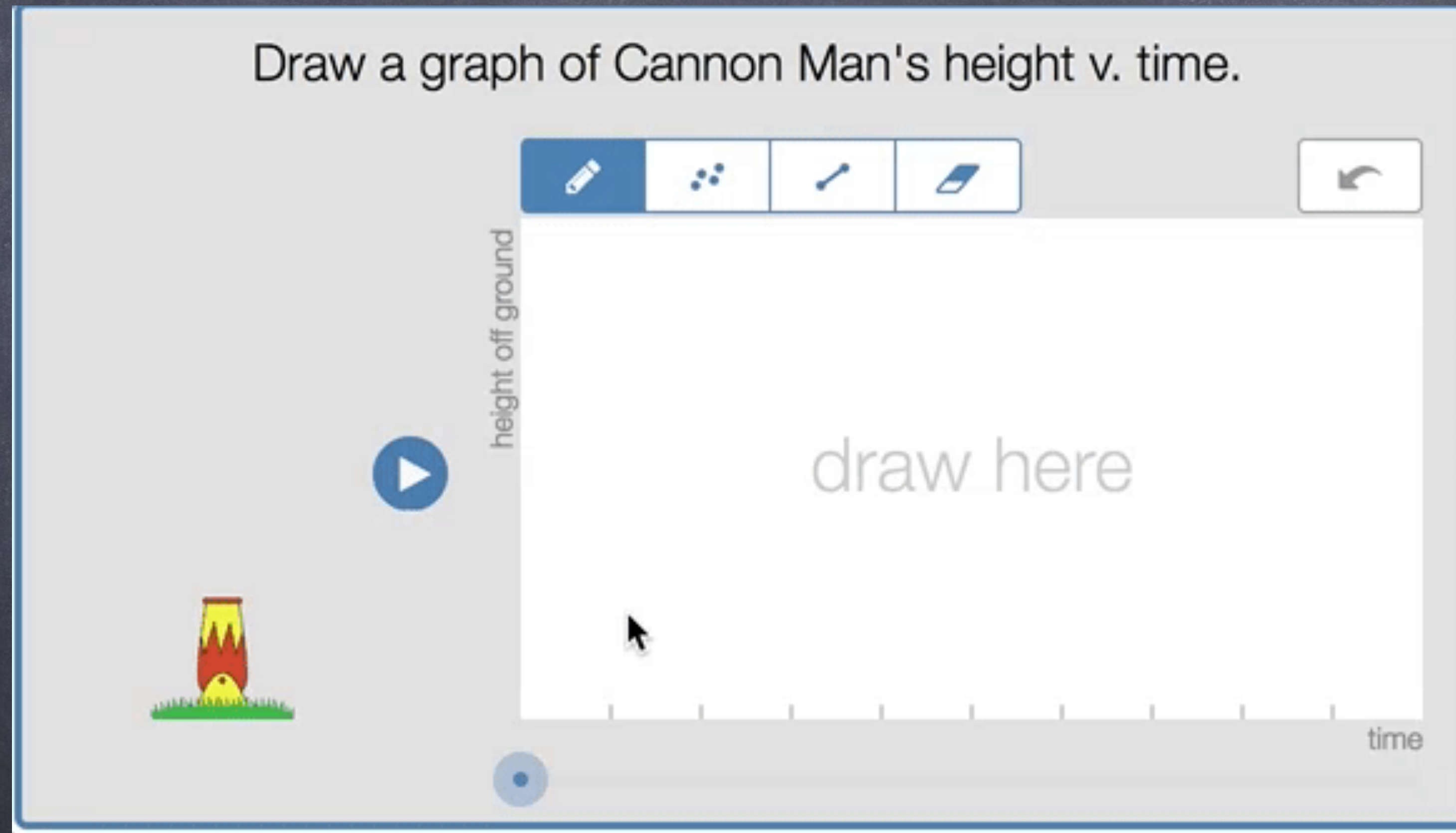
constraints

👁 meaningful
learning

?

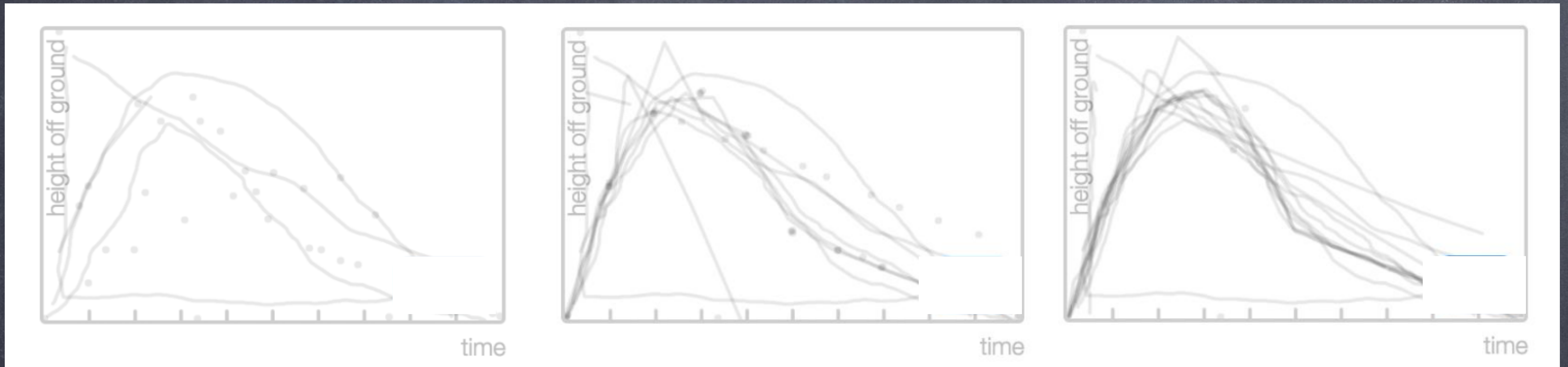
Constraints + Aggregation

concretizing a graph



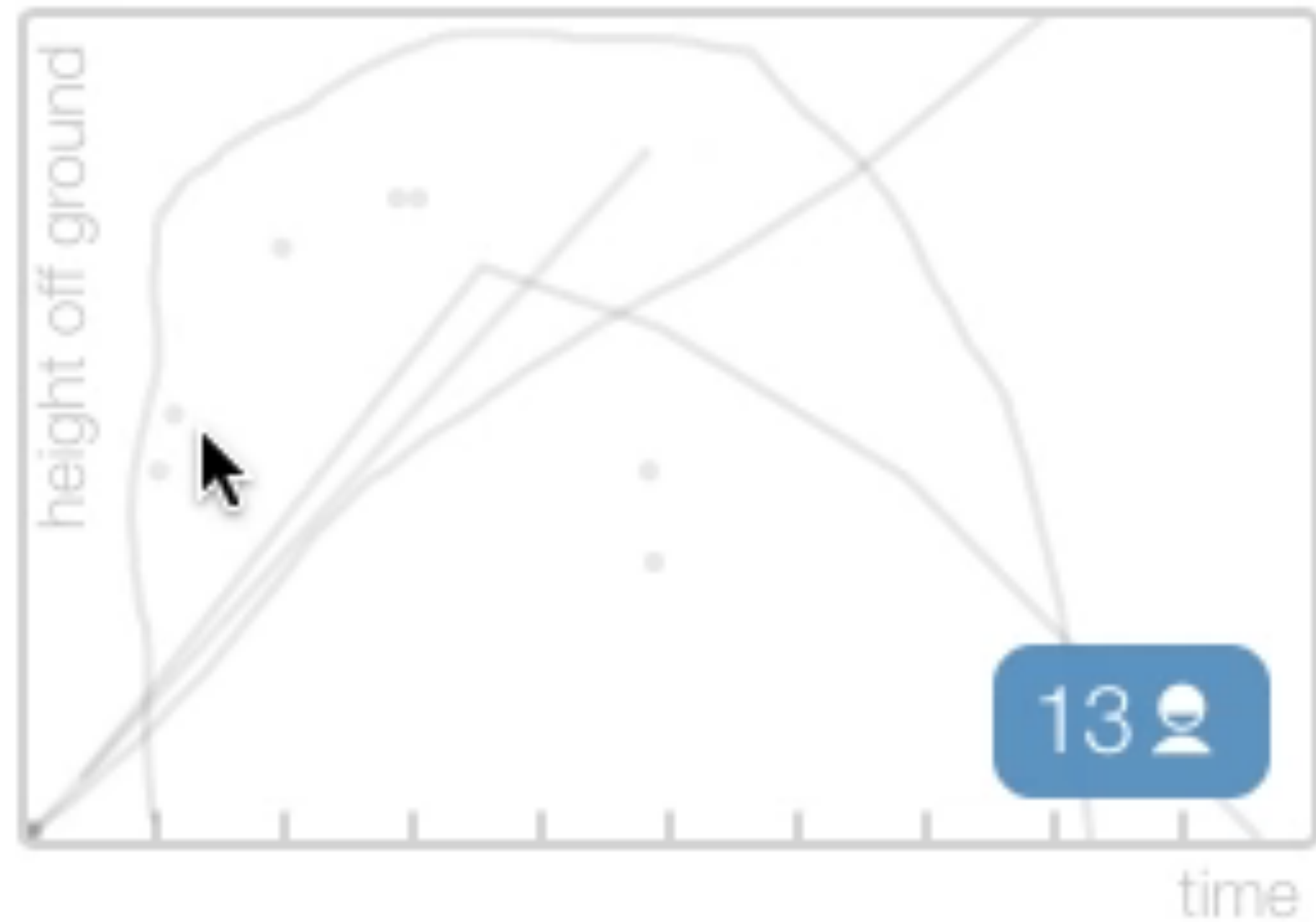
aggregation of 15 student pairs

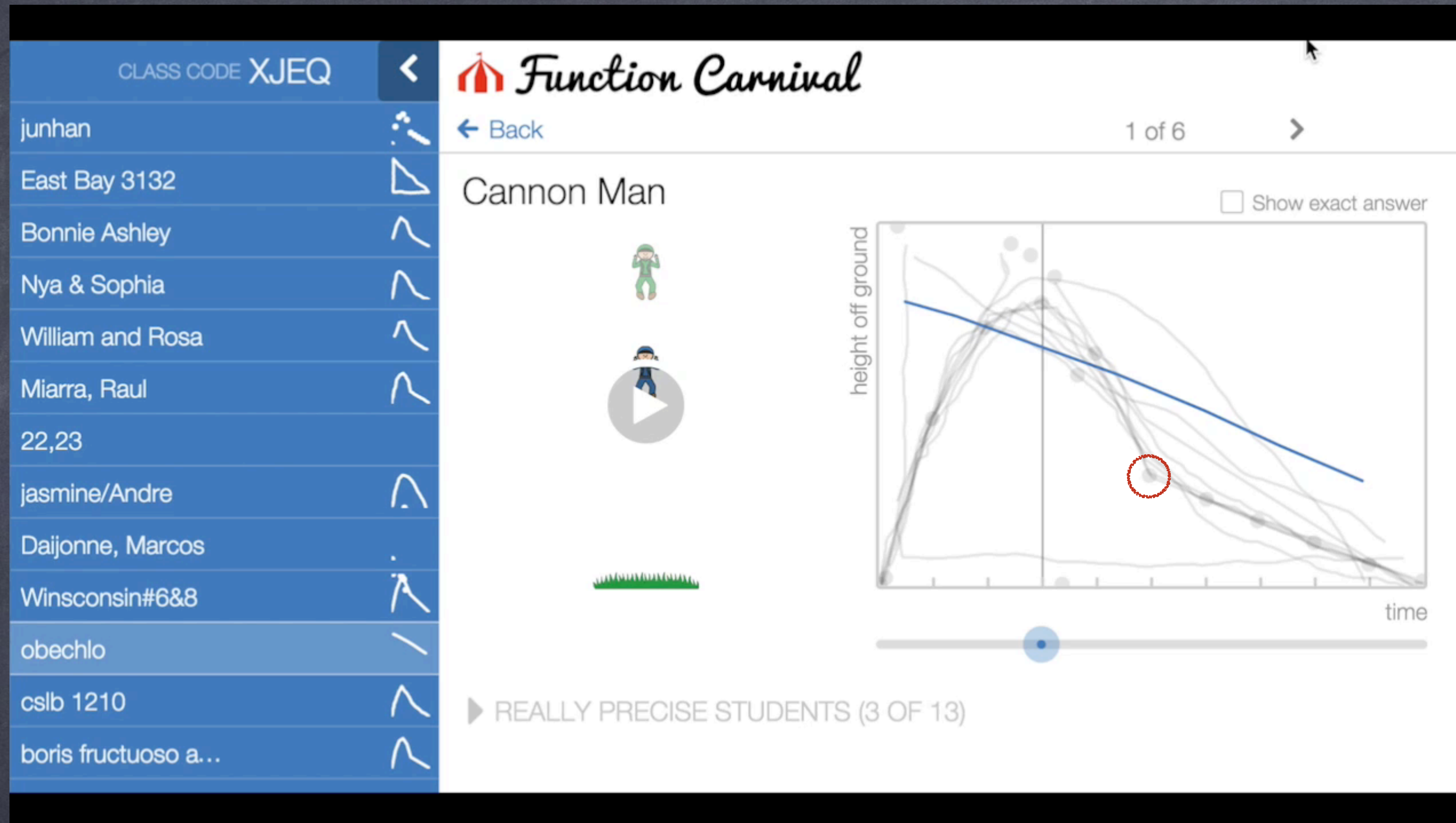
at 3 minutes, 6 minutes, 10 minutes.



timelapse of teacher's view

CANNON MAN





- visualizing mathematics
- concretizing abstractions

- effective teaching
- meaningful learning

T: most of you put this little-- I dunno-- can someone talk about this right here?

S: that dent?

T: yeah [waits] oh, did you want to talk about it?

Ss: [murmurs]

S: that's when he pulled the parachute

T: ok what happens when he pulls the parachute?

Ss: slowed down

S1: he hit a constant

T: ok I'm following, but how does this [linear part of graph] represent him slowing down?

S1: cause he was going fast and then, and then he kinda -- he doesn't stop but like it all

goes in like slow down

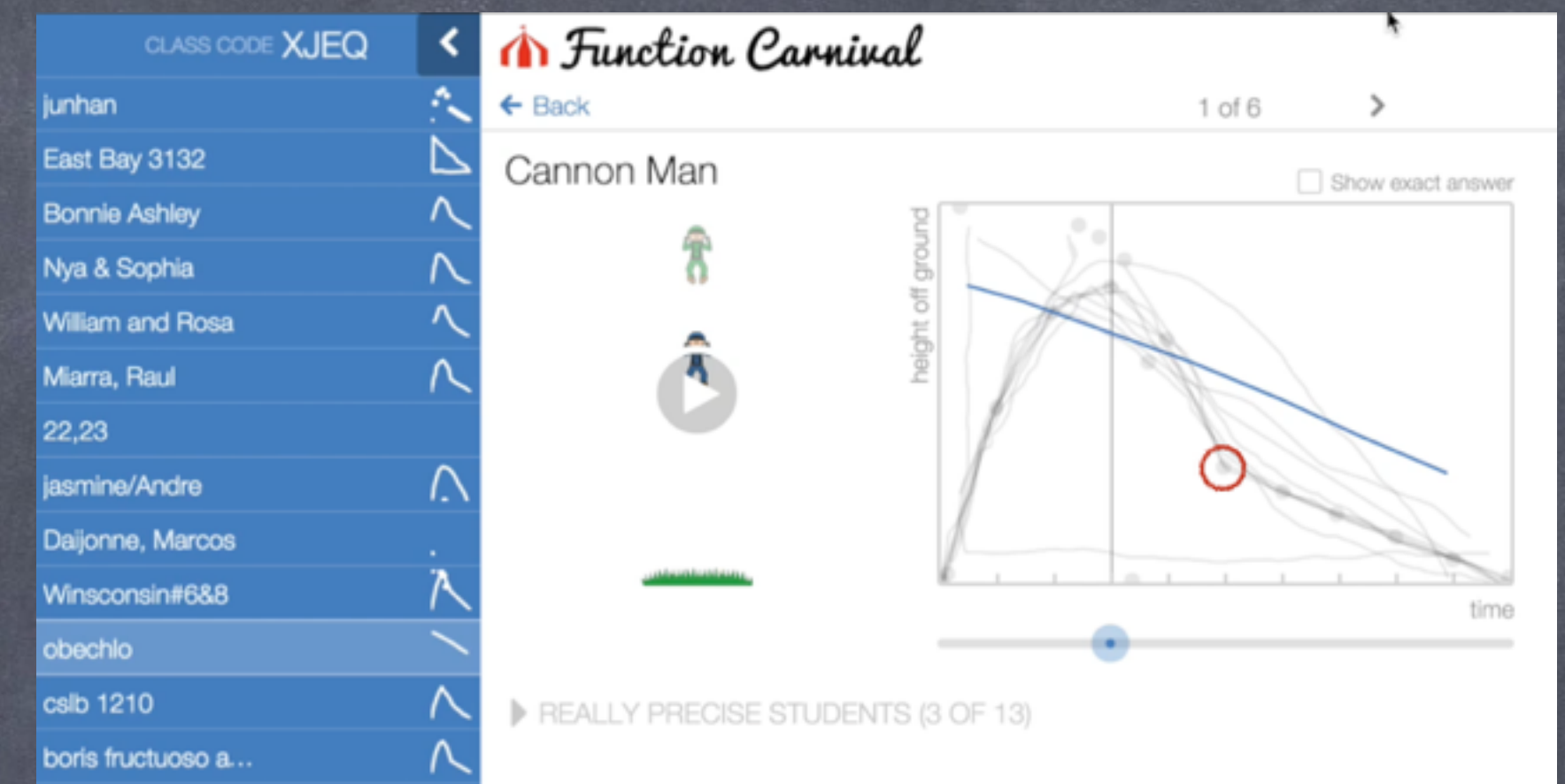
T: slow down from what?

S2: going really fast

S3: its his distance back to the ground... its taking him longer to get back down to the ground

T: so its taking him longer to get closer to the ground

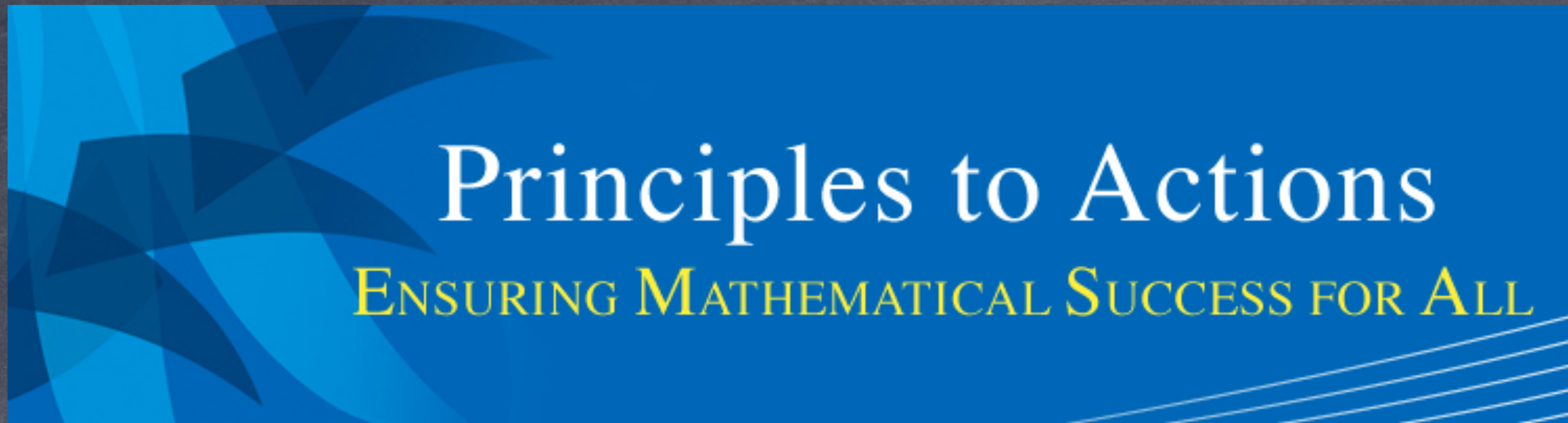
S3: yeah



Final Tech Principle:

- let students play within purposeful constraints
- collect aggregation of their play / informal work
- socially negotiate the formal knowledge as a class

• meaningful
learning



Tools and Technology.

1. Tech should help teachers and students visualize and concretize mathematics abstractions.
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Goals:

1. Understand what tech can/should support

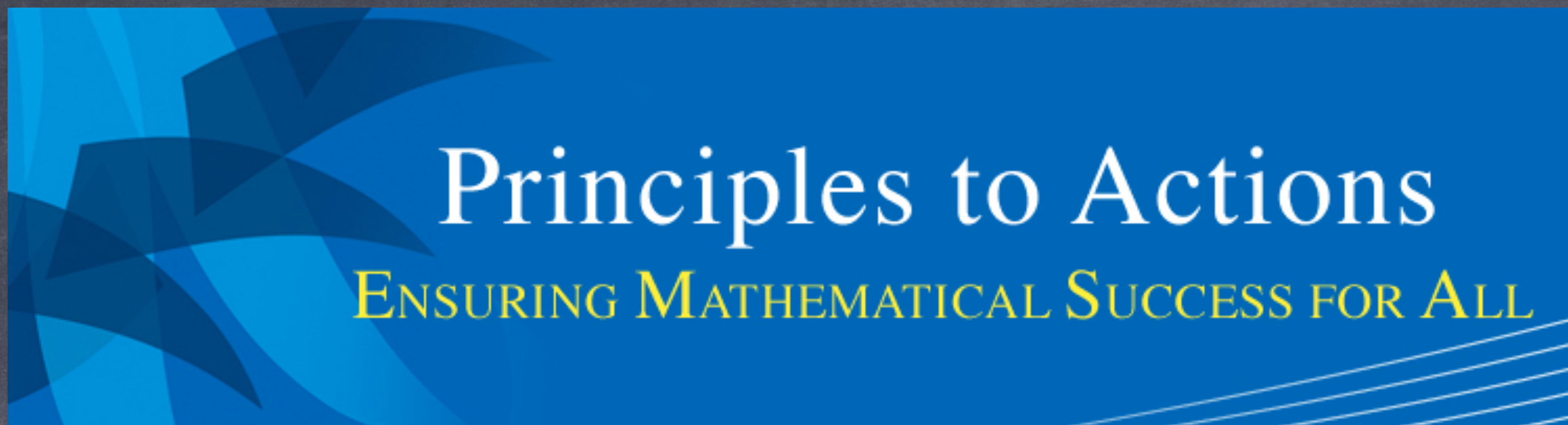
aggregate student
open work

constrain student
exploration

2. Have teachers leverage that support

connect student
work to their
understandings

match constraints
to mathematics



Is your tech Principled?

Scott Farrar

@farrarscott

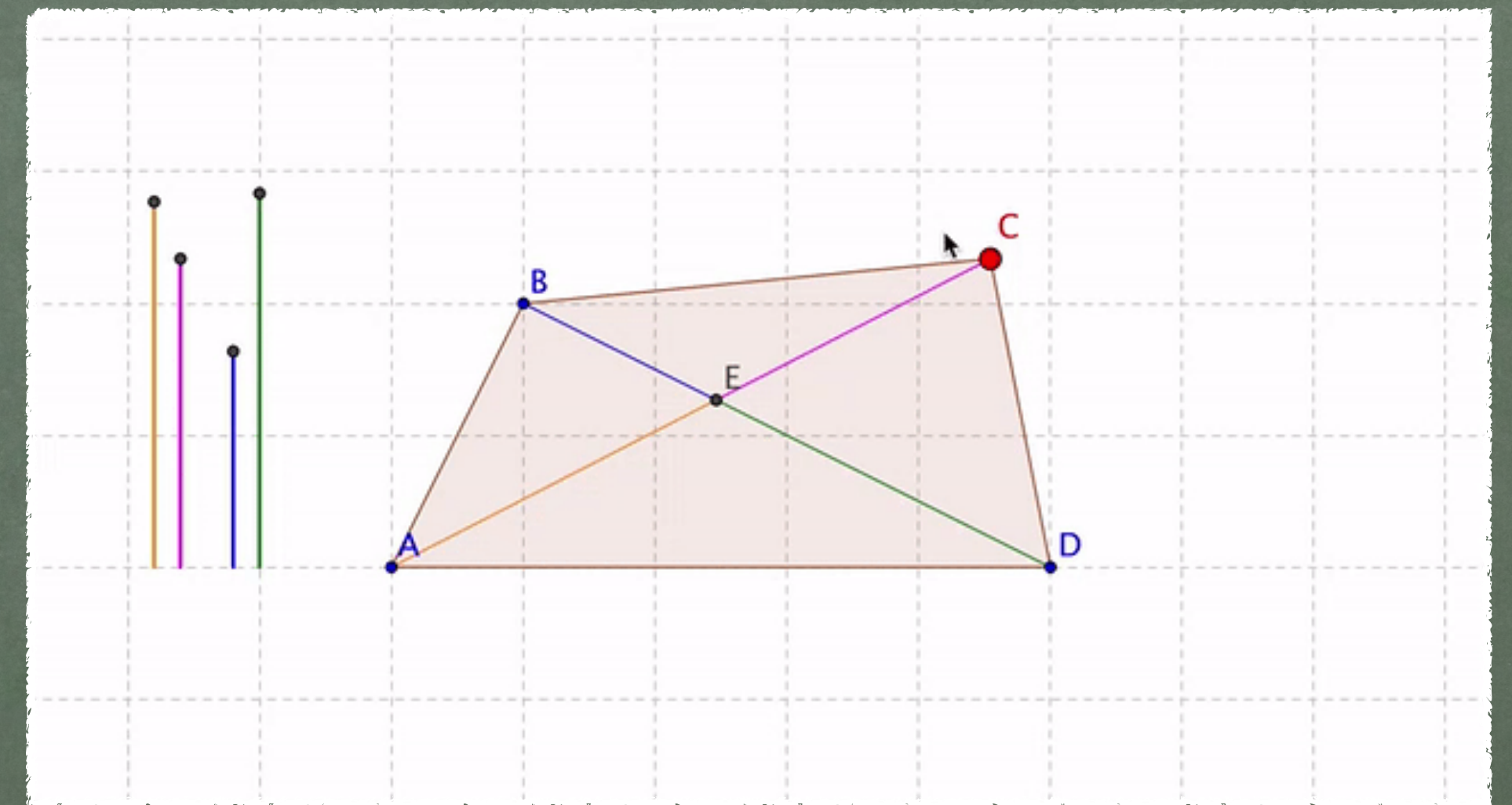
khanacademy.org/research

scottfarrar@gmail.com

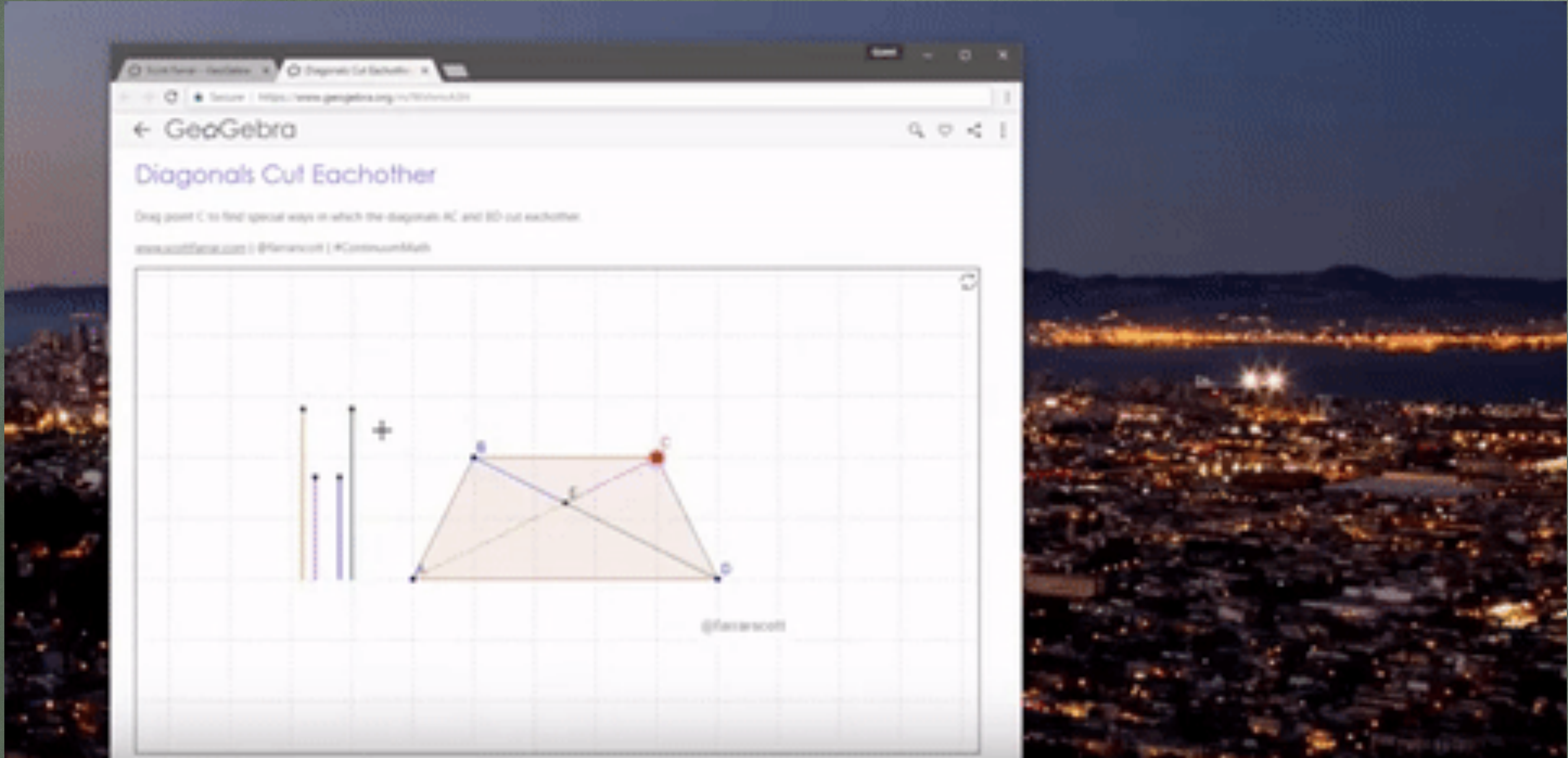
geogebra.org/scottfarrar

Extensions!

- In Function Carnival, the abstraction drives the concrete example.
- Graph drives Situation
- Make a version of Diagonals in GeoGebra that does this.
- What should drive What?



See how I did mine: <https://www.youtube.com/watch?v=CNfdOcyEYUA>



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your birth month

28

“

Each TreeTown citizen is given a crane of equal length. But the first section must be equal to their birth month.

×

29

“

Set $a =$ to your birth month to find your crane. Then, bring the crane to GrumpyCat to receive it.

×

30

▶

$a = 4$

×

1

12

31

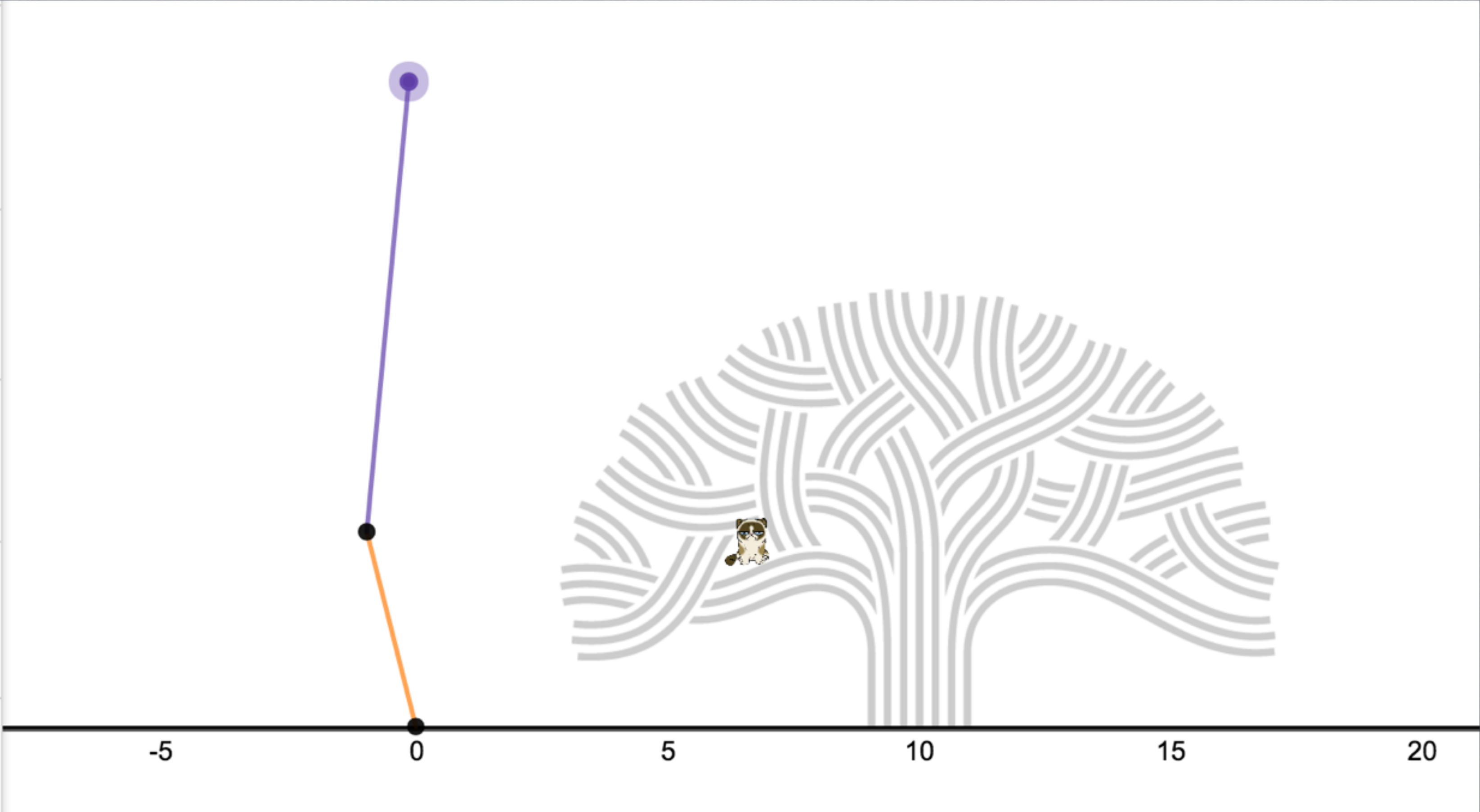
b

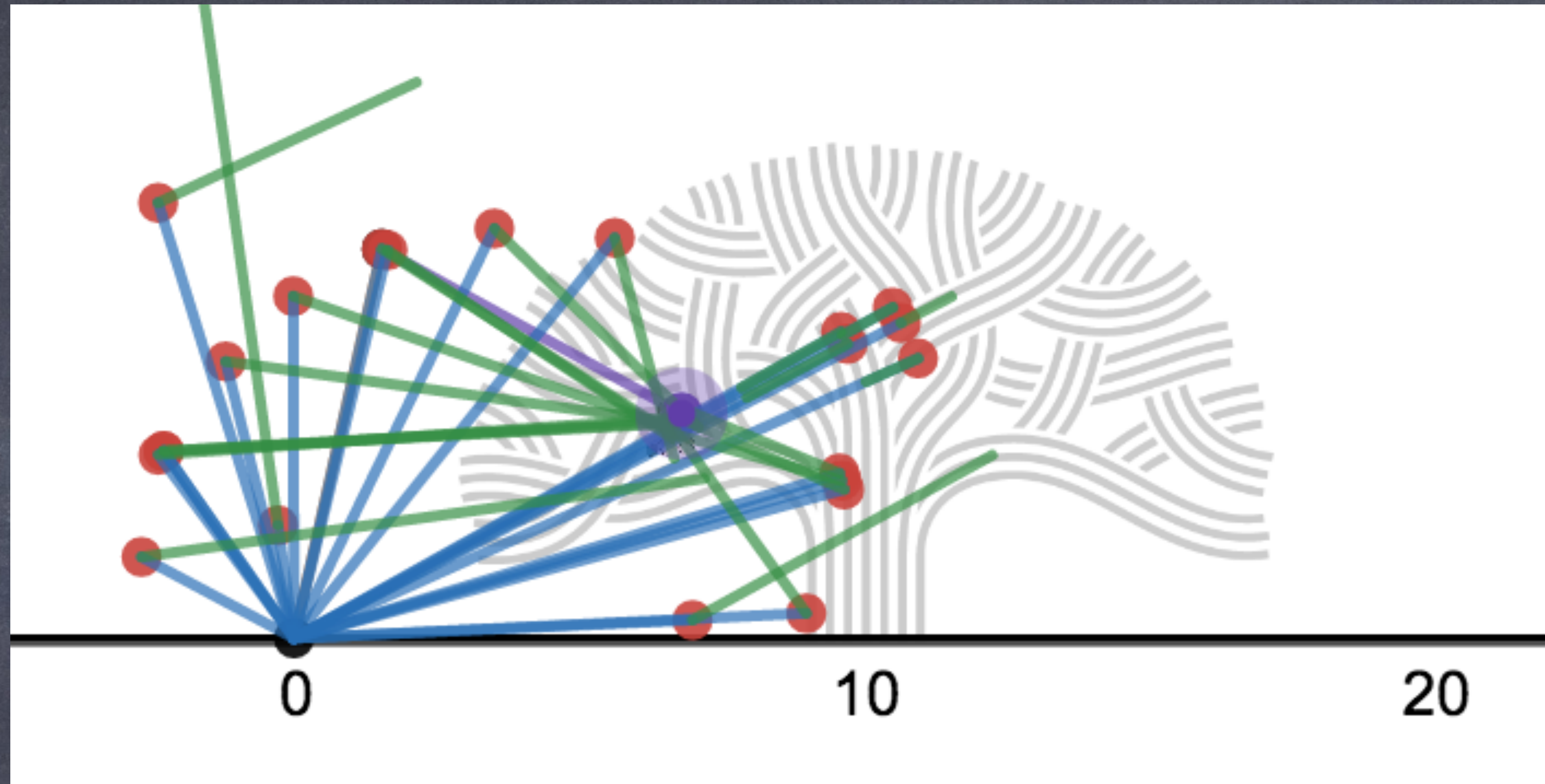
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9

32





aggregation and constraints

[grumpy cat aggregate]