NCTM Annual Conference
Got Division?

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San Antonio, Texas
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Division Across the Grades

3rd grade	4th grade	5th grade	6th grade
• interpret whole number quotients of whole numbers • determine unknown whole number in division equations • divide within 100 for word problems related to arrays, sets, & measurement quantities • understand division as an unknown factor problem	• divide to solve problems involving multiplicative comparison • find whole-number quotients and remainders with up to four-digit dividends by one-digit divisors using strategies based on place value, using equations, rectangular arrays, and/or area models	• find whole-number quotients with up to four-digit dividends by two-digit divisors using strategies based on place value, using equations, rectangular arrays, and/or area models • interpret a fraction as division of the numerator by denominator	• fluently divide whole numbers using the standard algorithm

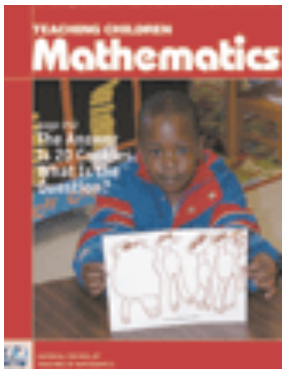
They go together…

Mulligan and Michelmore (1997) explain that the term multiplicative describes situations that lead to either multiplication or division and that every multiplication situation can lead to various division problems.

Putting Essential Understanding of Multiplication and Division into Practice 3-5, NCTM Publication

What's the Question?

Write a multiplication or division word problem that has the solution 8 something.



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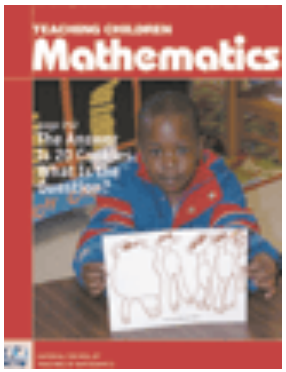

Share your problem with someone near you.

Multiplication and Division Structures

	Unknown Product	**Number of Groups Unknown (How many groups?)**	**Size of Group Unknown (How many in each group?)**
Equal Groups			
Area/Arrays			
Compare			

Multiplication and Division Structures

	Unknown Product	**Number of Groups Unknown (How many groups?)**	**Size of Group Unknown (How many in each group?)**
Equal Groups	Mark has 4 bags of apples. There are 6 apples in each bag. How many apples does Mark have altogether?	Mark has 24 apples. He put them into bags containing 6 apples each. How many bags did Mark use?	Mark has 24 apples. He wants to share them equally among his 4 friends. How many apples will each friend receive?
Area/Arrays			
Compare			

Multiplication and Division Structures

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Equal Groups	Mark has 4 bags of apples. There are 6 apples in each bag. How many apples does Mark have altogether?	Mark has 24 apples. He put them into bags containing 6 apples each. How many bags did Mark use?	Mark has 24 apples. He wants to share them equally among his 4 friends. How many apples will each friend receive?
Area/Arrays	Mark's bookshelf has 3 shelves with 6 books on each shelf. How many books does Mark have?	Mark has 18 books. They are on shelves with 6 books on each shelf. How many shelves are there?	Mark has 18 books on 3 shelves. How many books are on each shelf?
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Multiplication and Division Structures

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Compare	In June, Mark saved 5 times as much money as in May. In May, he saved $7. How much money did he save in June?	In June, Mark saved $35.00. In May, he saved $7.00. How many times as much money did he save in June as May?	In June, Mark saved 5 times as much money as he did in May. If he saved $35.00 in June, how much did he save in May?

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Now look back at the story problem you wrote for 8 something.

Which problem solving structure does your problem represent?

Multiplication and Division Structures

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The research shows…

- As Rohrer and Pashler (2010) note, varying problem structures within lessons can increase long-term retention and understanding. Diversifying the problems that you give your students compels them to focus on the deeper structure of the multiplicative situations that the problems present rather than look for key words or other superficial aspects.

Putting Essential Understanding of Multiplication and Division into Practice 3-5, NCTM Publication

2 Minute Pause
Thoughts for Curriculum and Instruction

- Which problem types are emphasized in the materials you use?
- How do the lessons in these materials help students build their understanding of the meaning of these different problem types?

Compare and Contrast these two problems

- Marcia has 40 bananas. She wants to put the bananas into bundles with 5 bananas in each bundle. How many bundles can she make?

- Tabitha has 40 bananas. She wants to share the bananas fairly between 5 friends. How many bananas will each of Tabitha's friends receive?

Let's focus on division

- Partitive, or fair-sharing division are typically problems where we are seeking *"how many in each group"?*

- Measurement division are typically situations where we are looking for *"how many groups"?* This is sometimes called repeated subtraction or quotative division.

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Let's focus on division

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- Marcia has 40 bananas. She wants to put the bananas into bundles with 5 bananas in each bundle. How many bundles can she make?

Fair-Sharing or Measurement?

- Katrina bought 18 subway tokens to share with her family. She has 4 people in her family. How many tokens will each person get? Will any be left over?

- Fair-sharing; partitive

Fair-Sharing or Measurement?

- Grace can put 6 pictures on one page of her photo album. If she has 82 pictures, how many pages will she need?

- Measurement, quotative

Fair-Sharing or Measurement?

- Kaya saved $18 and that is 3 times as much as her little sister, Ana, saved. How much did Ana save?

- Partitive

When Teaching the Concept of Division

- Start with contextual problems- use both types of division problems
- Start with small numbers, use counters, and use remainders from the beginning
- Connect division immediately to multiplication as the inverse operation.

Connect Multiplication and Division Immediately

- Just as the concepts of addition and subtraction should be taught simultaneously, so should the concept of multiplication and division.

- When students see 28 ÷ 4 = ___, they should think " ? x 4 = 28"

- We will see in a few slides how this becomes critical when we use larger numbers.

Practice: Think Multiplication

$46 \div 9 =$

Practice: Think Multiplication

46 ÷ 9 =

Nine times ___will get me close to 46?

9 x 5 = 45

5 remainder 1

Practice: Think Multiplication

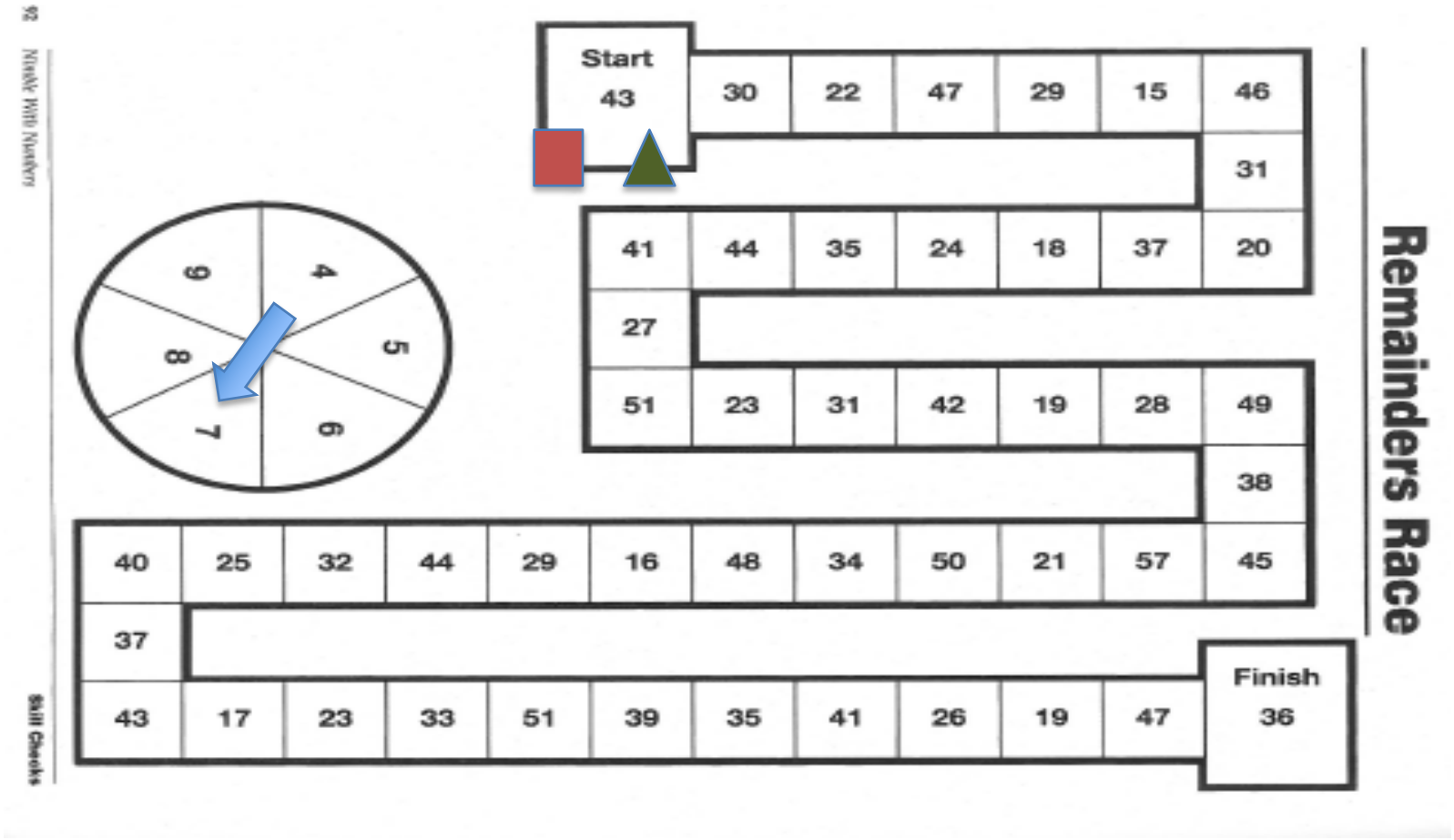
$46 \div 9 =$

$62 \div 7 =$

$53 \div 6 =$

Player 1

Spins a 7, 43 divided by 7 equals 6 with a remainder of 1, so the player moves 1 space



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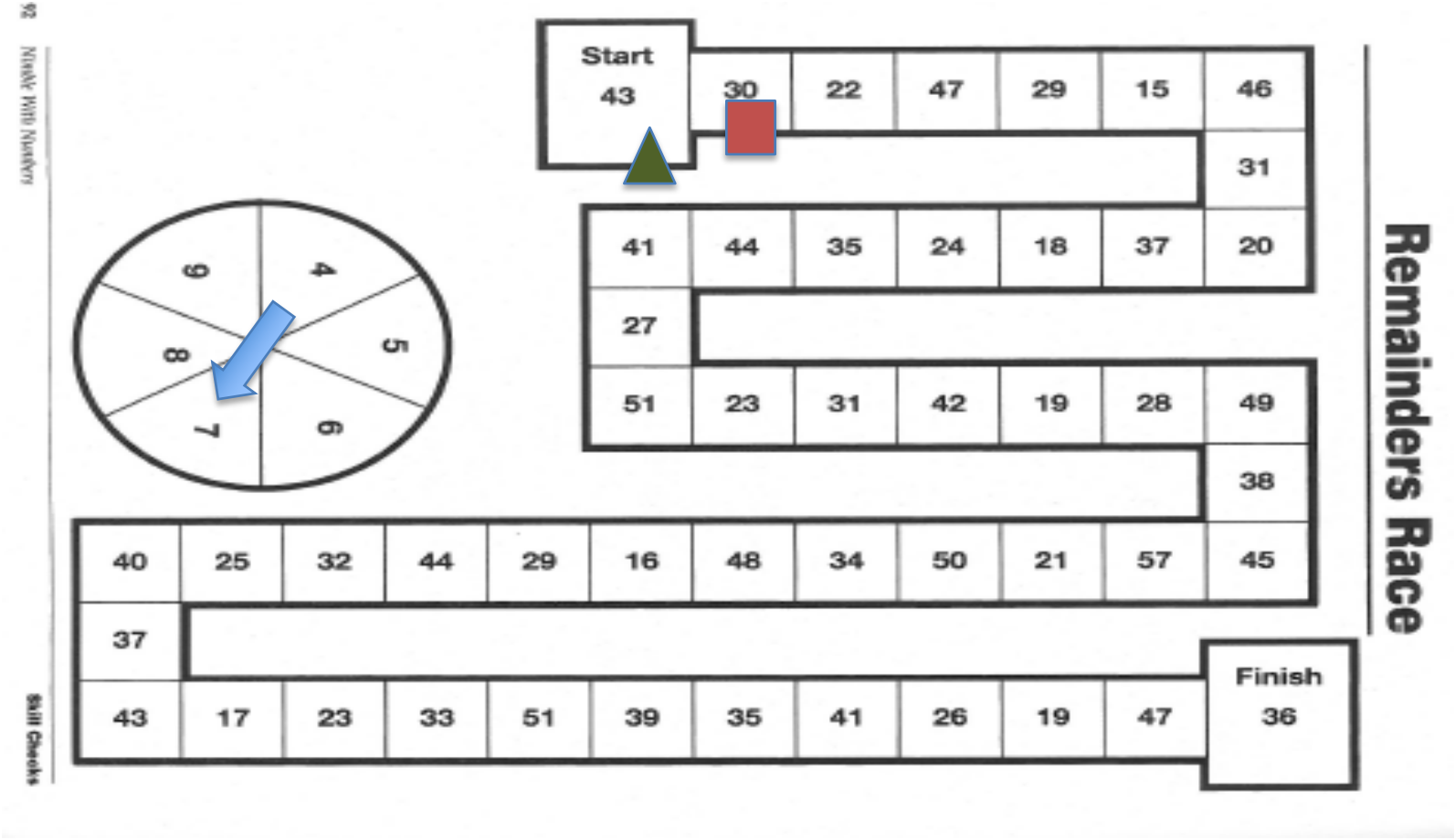
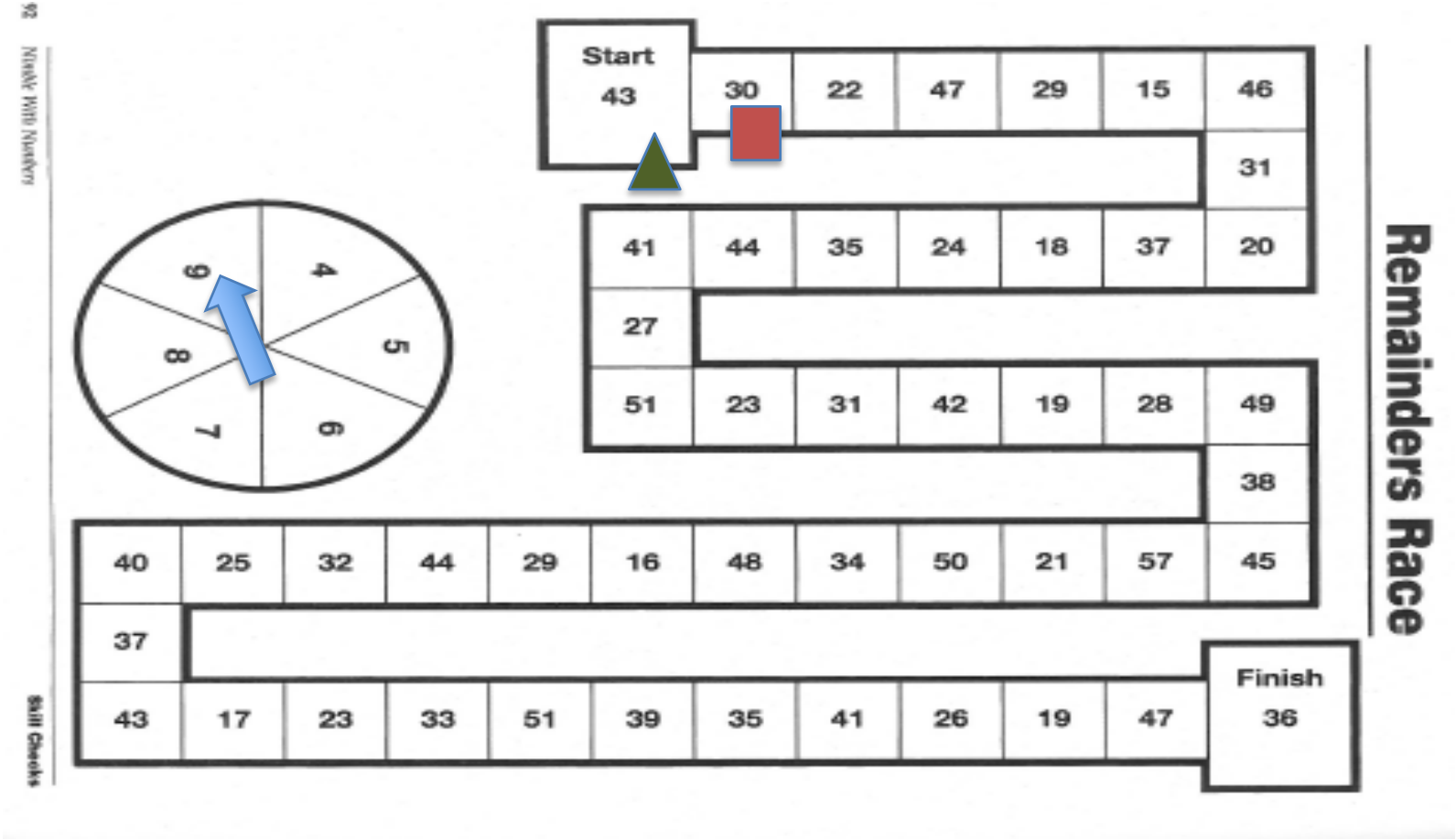


Player 2

Spins a 9, 43 divided by 9 equals 4 with a remainder of 7, so the player moves 7 spaces



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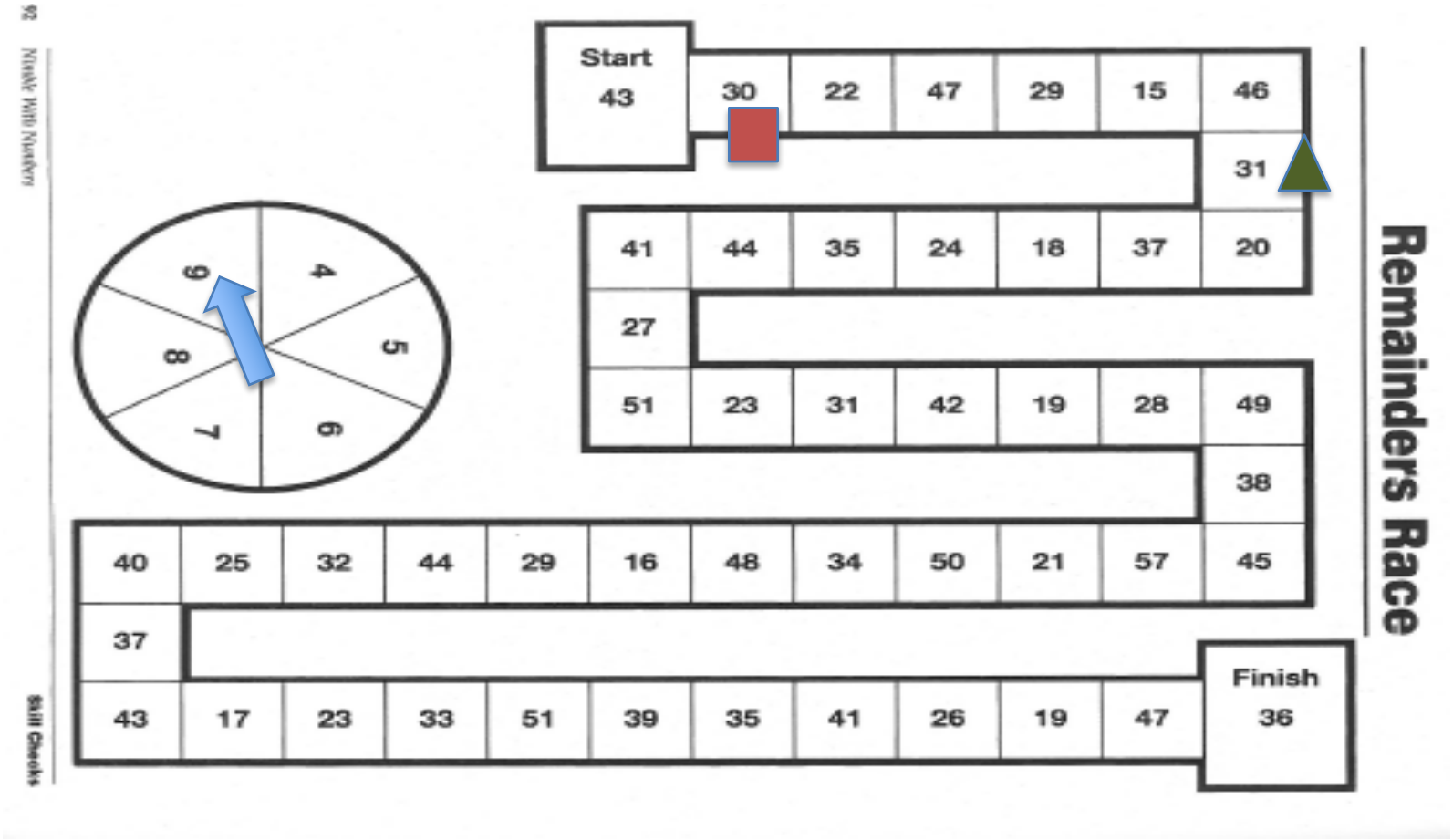
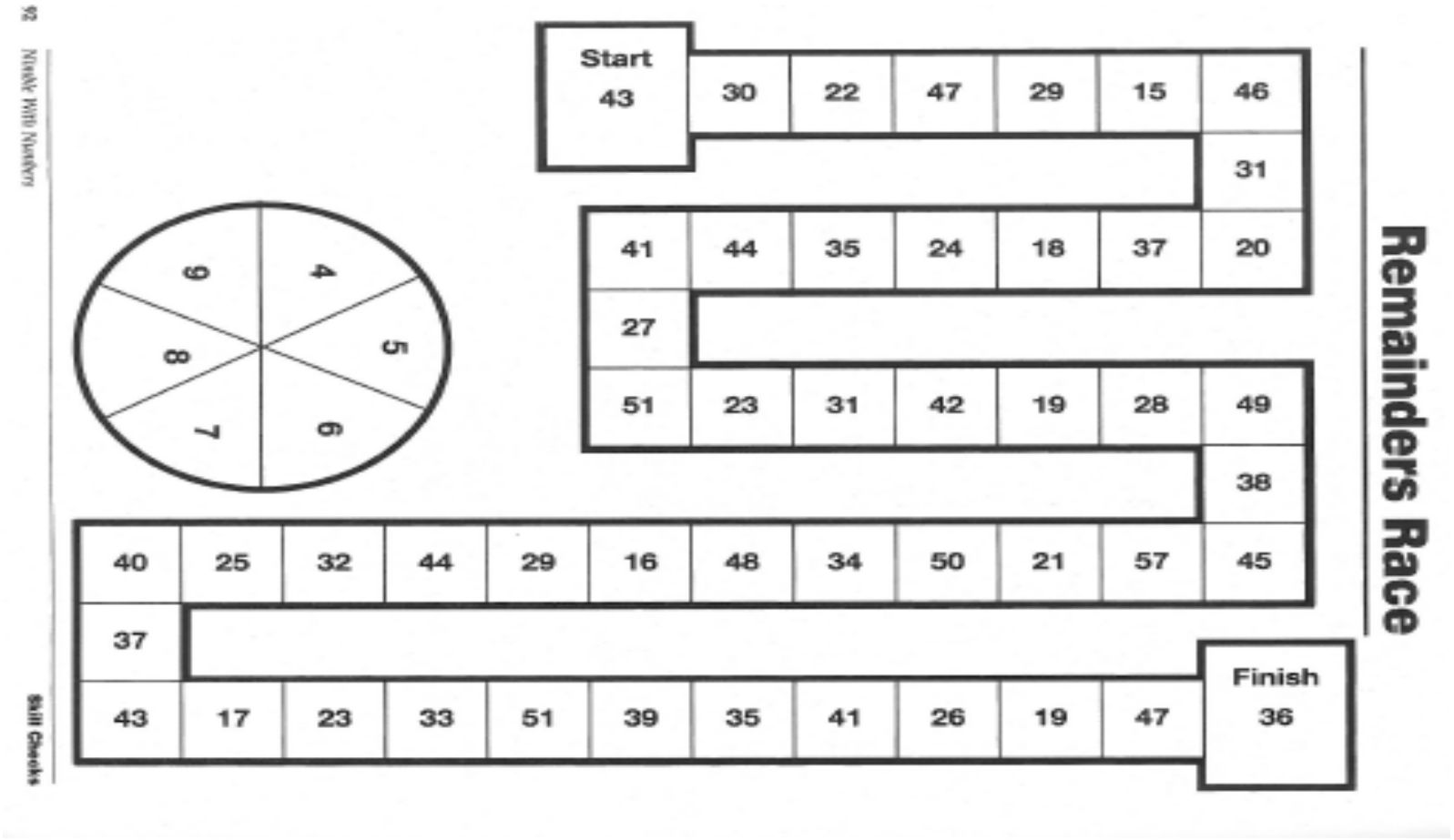


If your students need practice…



Nimble with Numbers Grades 4 & 5, pages 92-94

What's all this about remainders?

- Most things in the world do not divide equally. It seems unfair to students if we don't present problems with remainders from the beginning.

- The context of a division problem dictates the interpretation of the remainder.

For example, consider these problems

- Jennifer shares 17 cups of popcorn equally among 6 people. How many cups of popcorn will each person get?
- Otis wants to share 35 baseball cards among his 4 cousins. He wants to give the same number to each cousin. How many cards will each cousin get?
- The chorus has 22 students. For a concert, they are being driven in cars that can each hold 5 students. How many cars are needed?

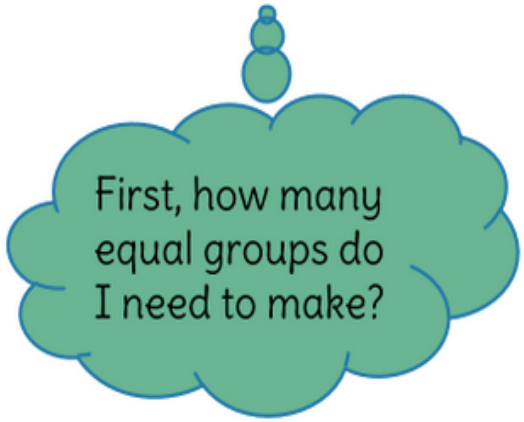
Exploring Representations

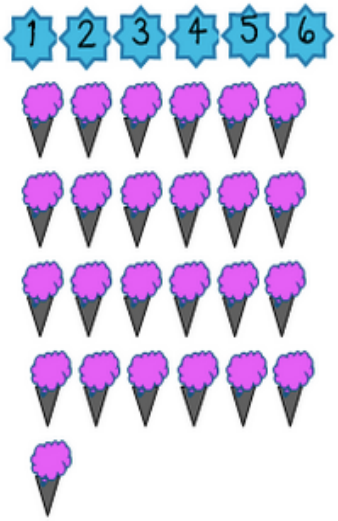
Big Ideas

Visualizing the problem is helpful when creating arrays for division.

To do this there are 3 questions to ask yourself...

$25 \div 6$





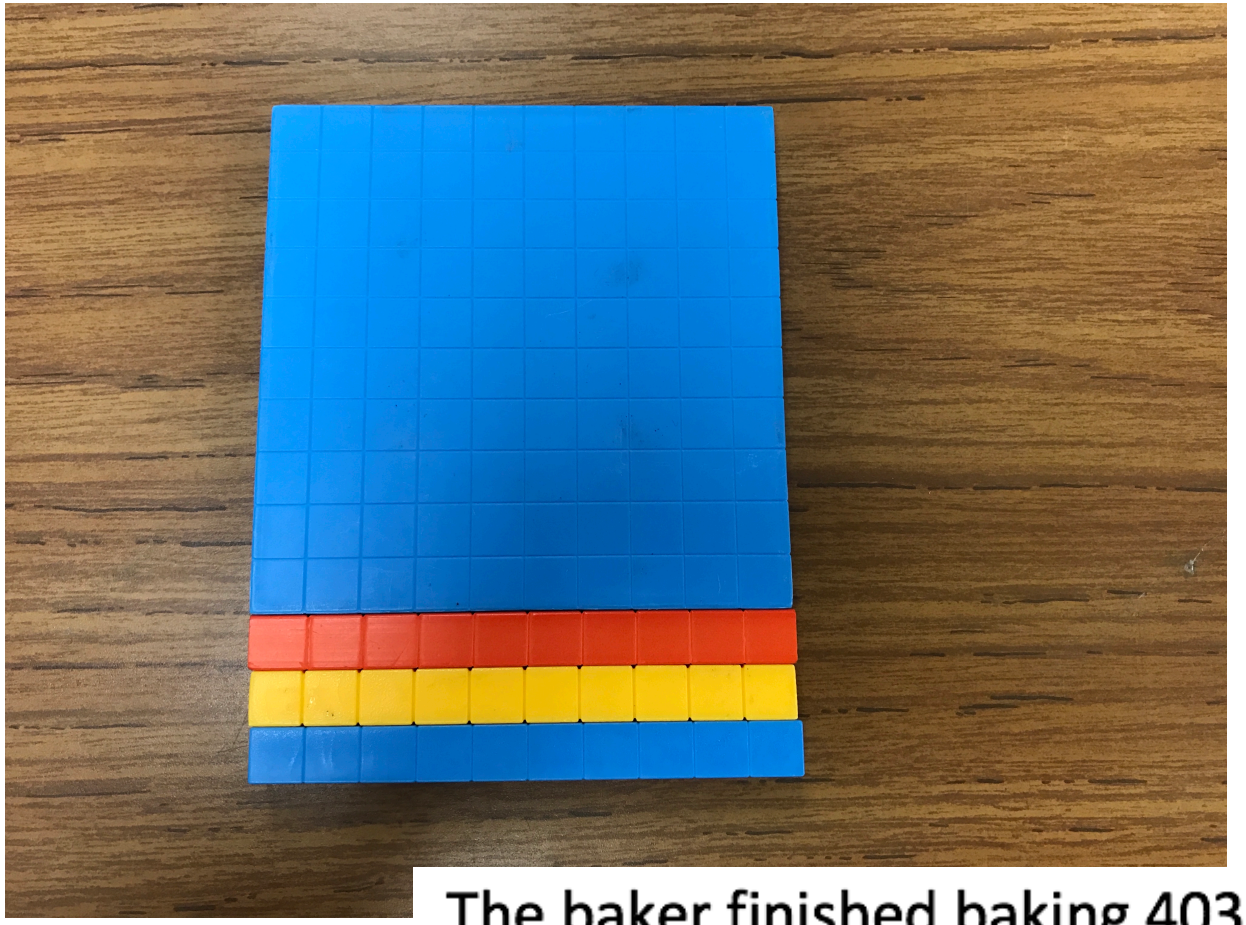
This tells me how many columns to make!

Consider This

Create a visual representation to help you solve this problem:

The baker finished baking 403 donuts for the farmers' market. If she plans to sells her donuts in sets of 13, how many bags will she need?

Area Model



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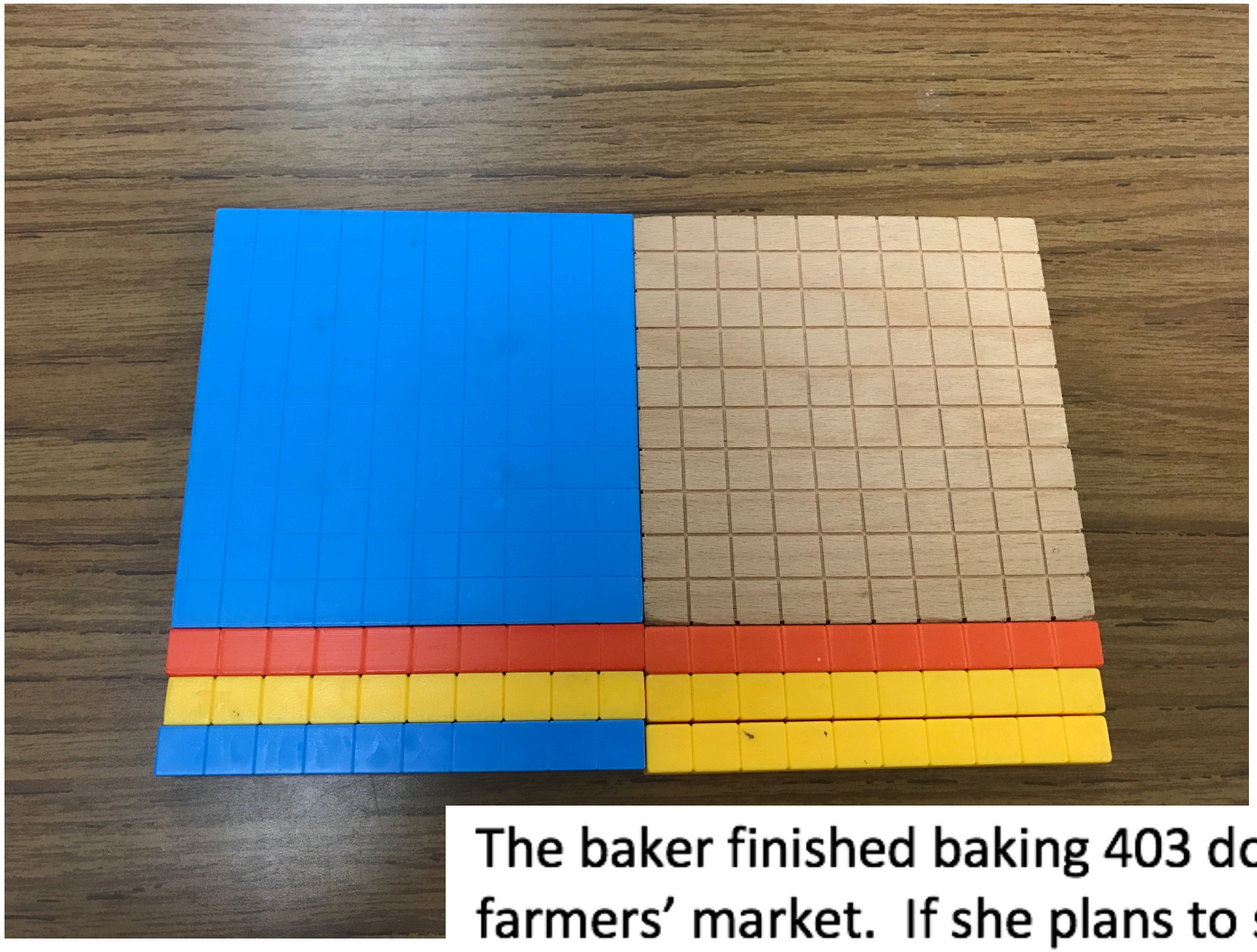

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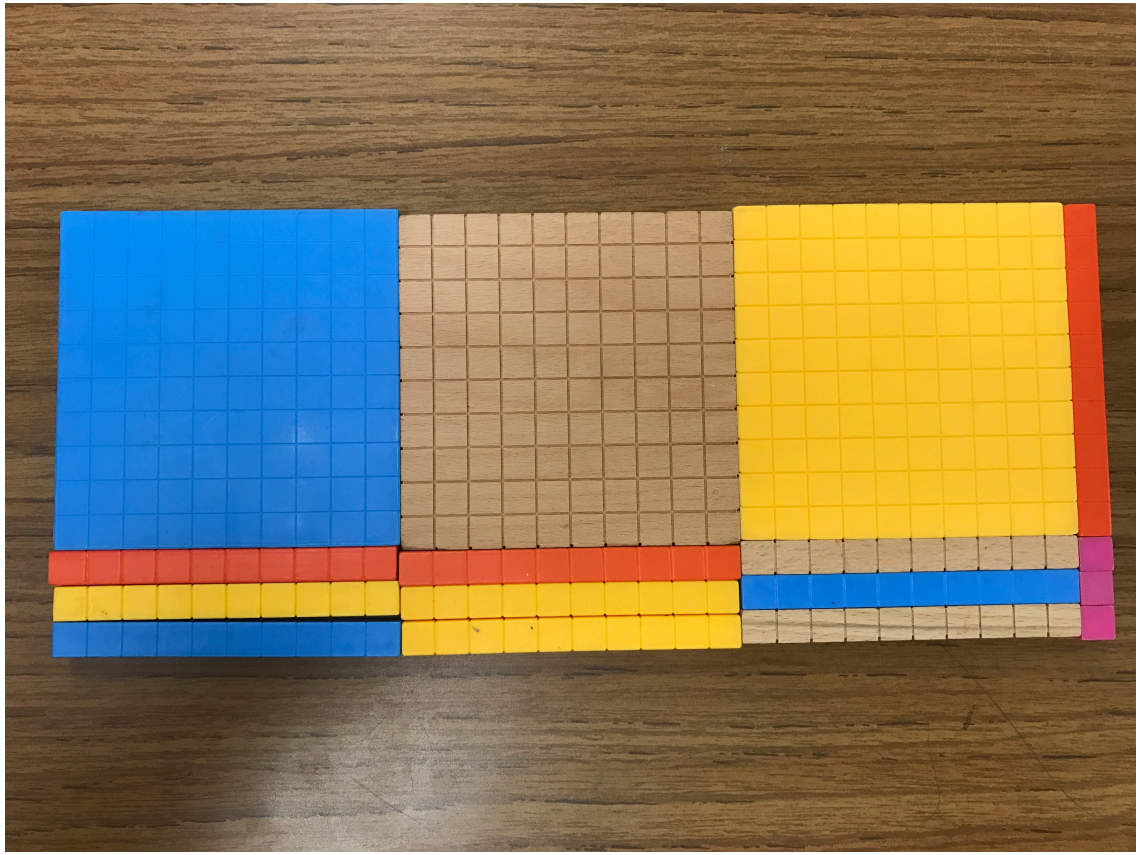

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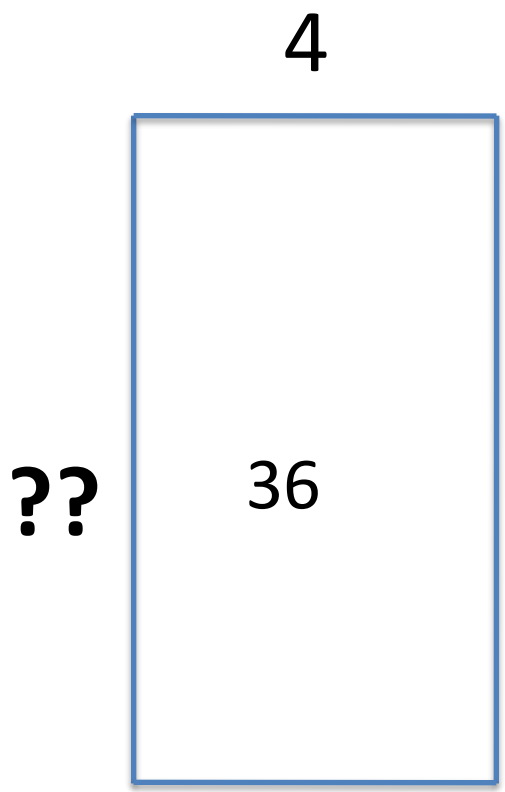
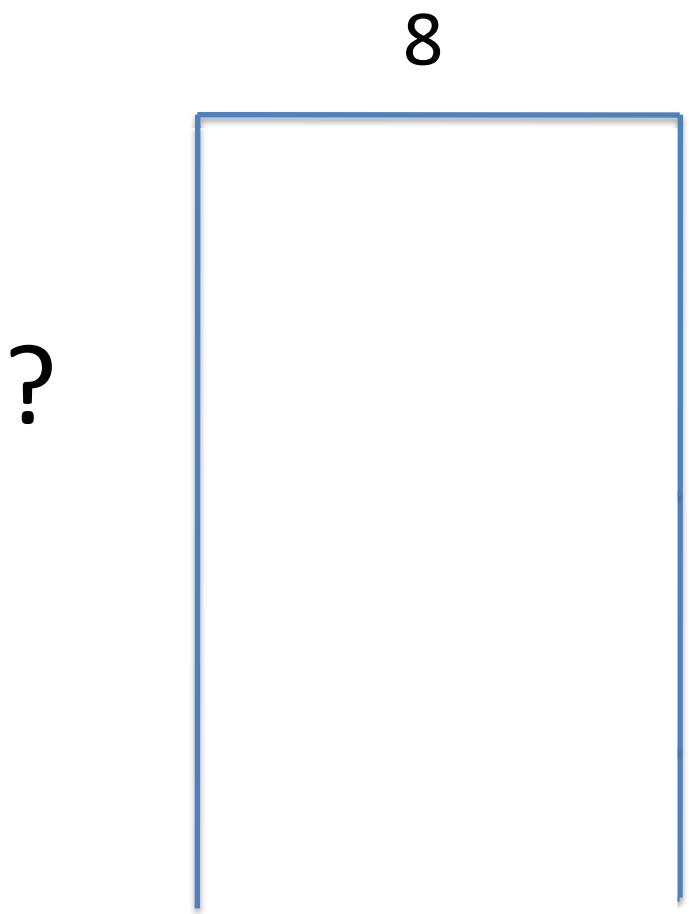
36 ÷ 4 = ?



$$623 \div 8 =$$



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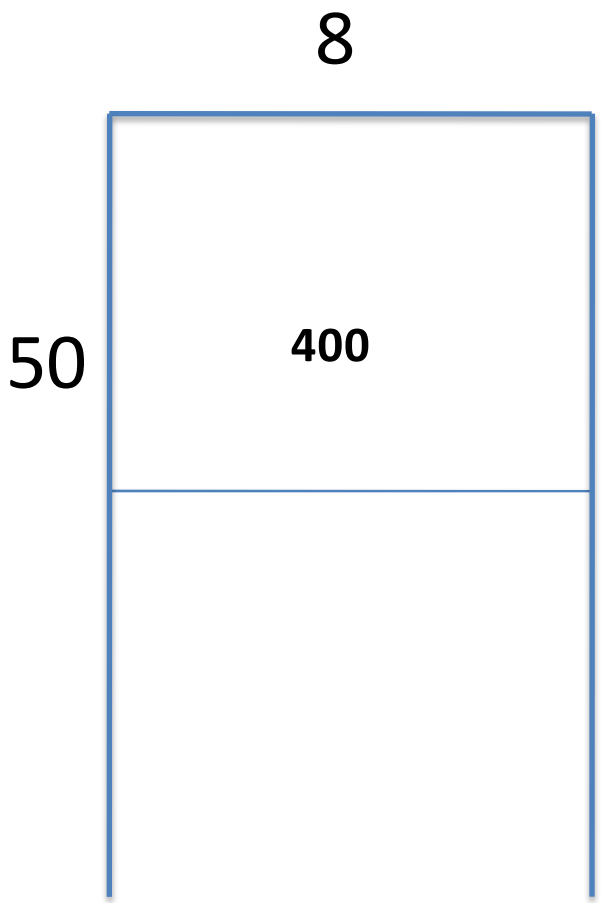
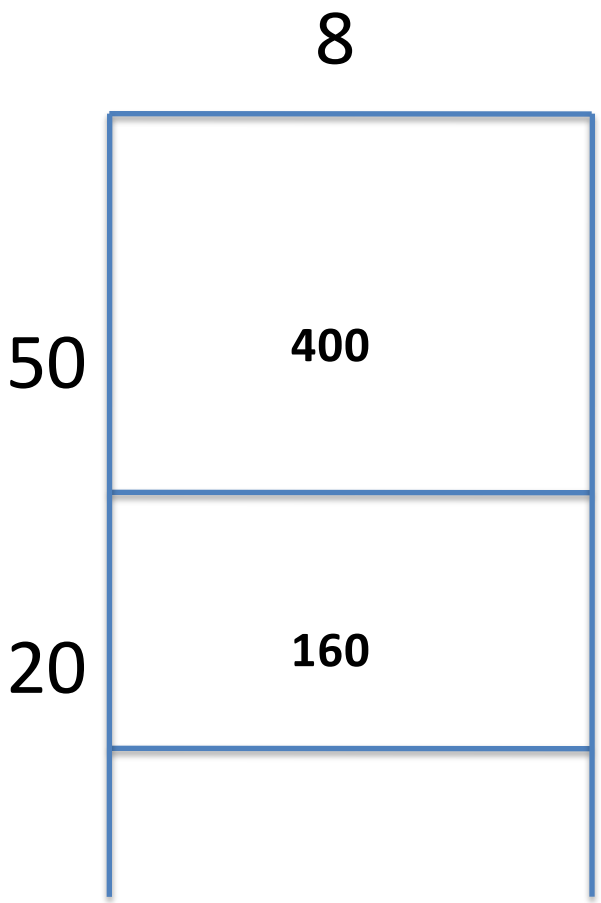


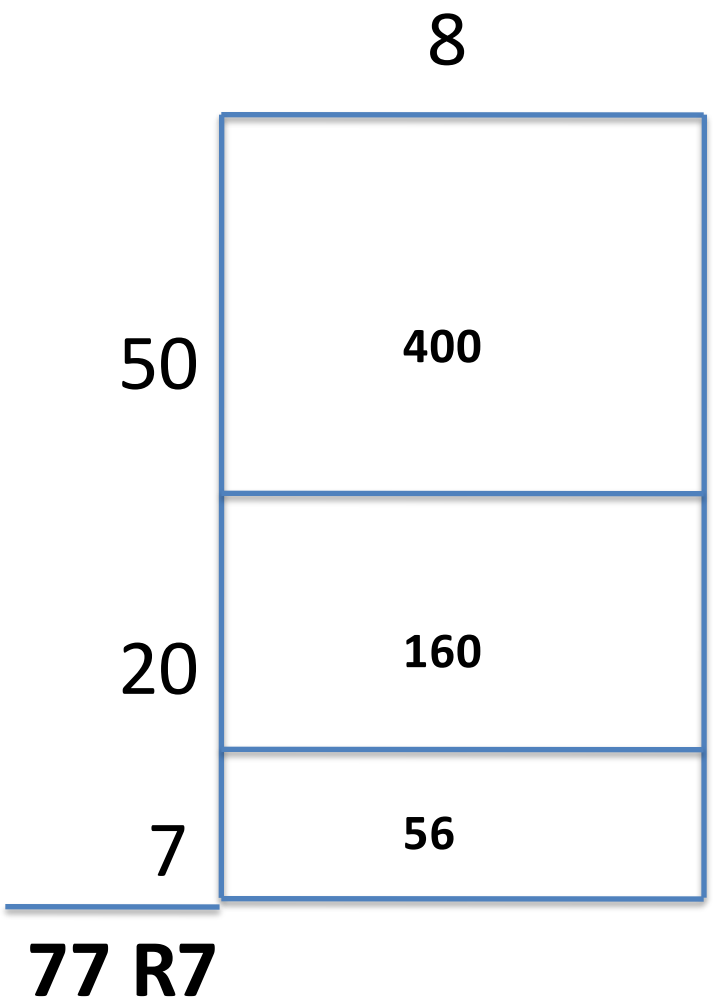
623
- 400
223

623 ÷ 8 =



623
- 400
223
- 160
63

623 ÷ 8 =



623
- 400
223
- 160
63
- 56
7

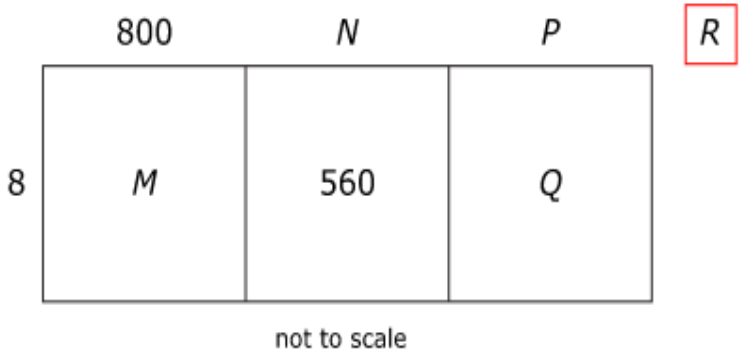
Now you try one…

Sketch an area model to solve

$1{,}875 \div 12 =$

A teacher drew an area model to find the value of $6{,}986 \div 8$.

Teacher's Model for 6,986 ÷ 8



not to scale

- Determine the number that each letter in the model represents and explain each of your answers.
- Write the quotient and remainder for $6{,}986 \div 8$.
- Explain how to use multiplication to check that the quotient is correct. You may show your work in your explanation.

Enter your answers and your explanations in the space provided.

Math symbols

+ − × ÷ (·) [·] = < > ≠ $ ° ?

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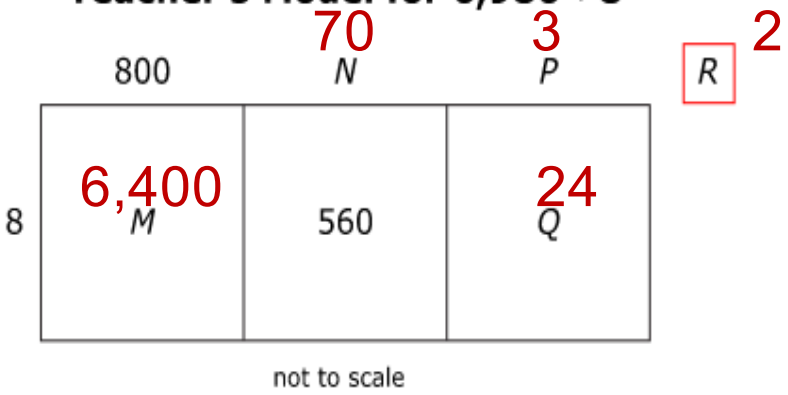

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Dividing Using Partial Quotients

- Partial quotients uses the same ideas of mental math and the relationship between multiplication and division
- Offers varied “entry points”
- Focuses on the value of the whole instead of “goes into”
- Provides a friendly lead-in to the standard algorithm

10/22/13

Mental Math

$3 \times \square = 100$

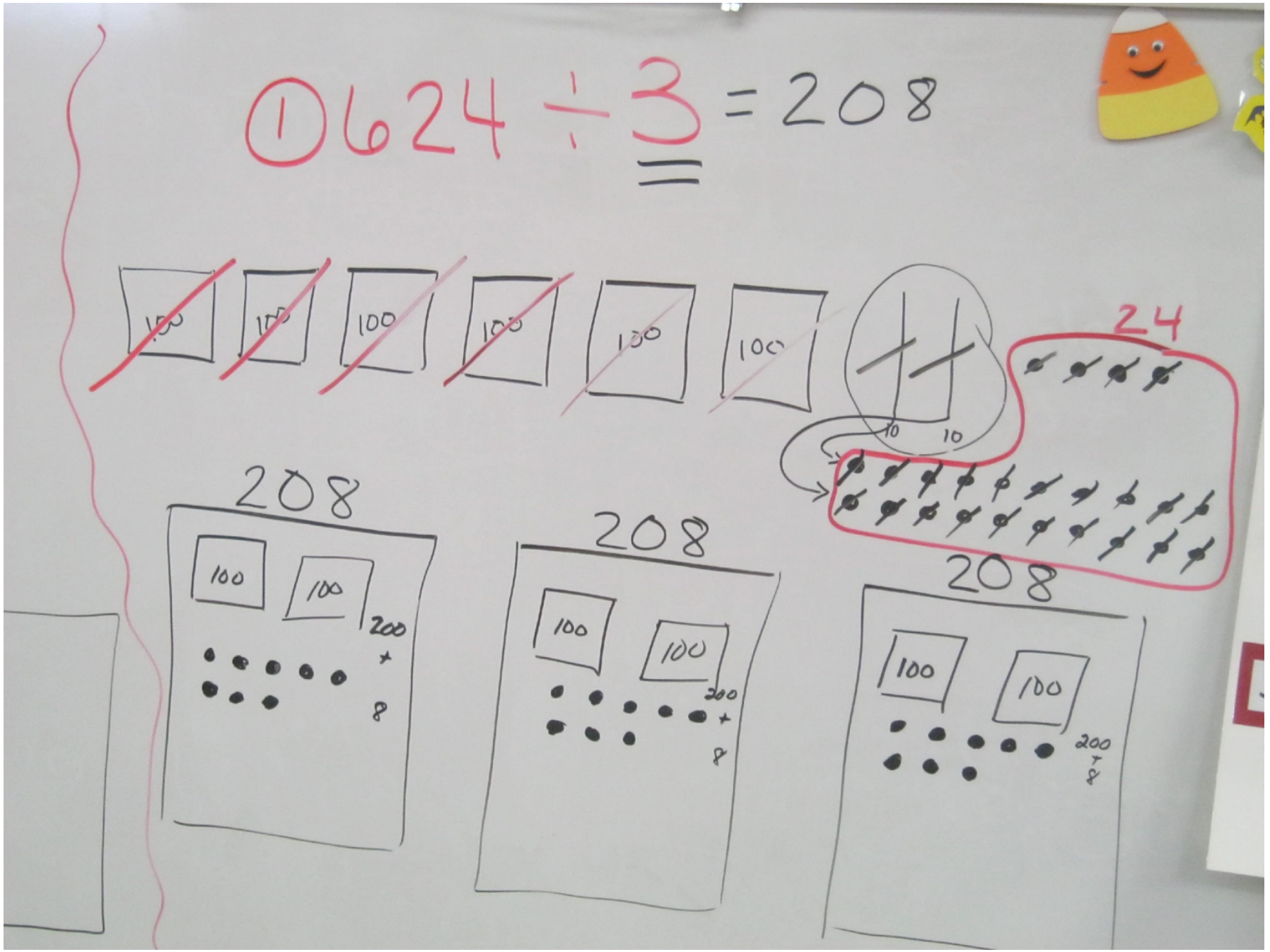
$3 \times 10 = 30$, $3 \times 20 = 60$, $\boxed{3 \times 30 = 90}$ $3 \times 40 = 120$

$5 \times \square = 138$

$5 \times 10 = 50$, $\boxed{5 \times 20 = 100}$, $5 \times 30 = 150$

$4 \times \square = 294$

$4 \times 60 = 240$, $\boxed{4 \times 70 = 280}$


①624 ÷ 3 = 208
100
100
100
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24
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200
8
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8
208
100
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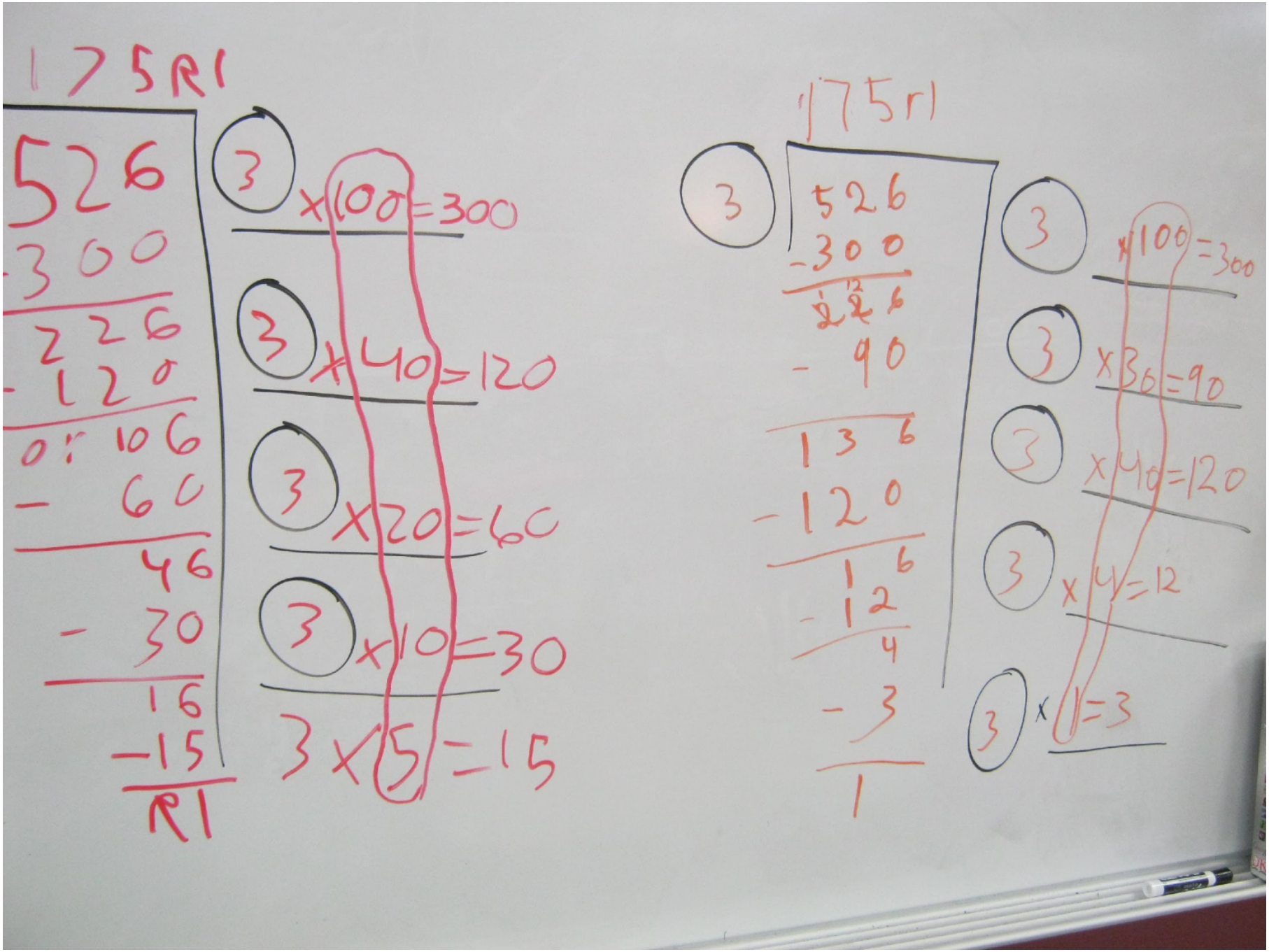
200
8


175R1
526
-300
226
-120
106
-60
46
-30
16
-15
R1
3 x 100 = 300
3 x 40 = 120
3 x 20 = 60
3 x 10 = 30
3 x 5 = 15
175r1
3
526
-300
226
-90
136
-120
16
-12
4
-3
1
3 x 100 = 300
3 x 30 = 90
3 x 40 = 120
3 x 4 = 12
3 x 1 = 3

Let's Practice Partial Quotients

$$3\overline{)4{,}190}$$

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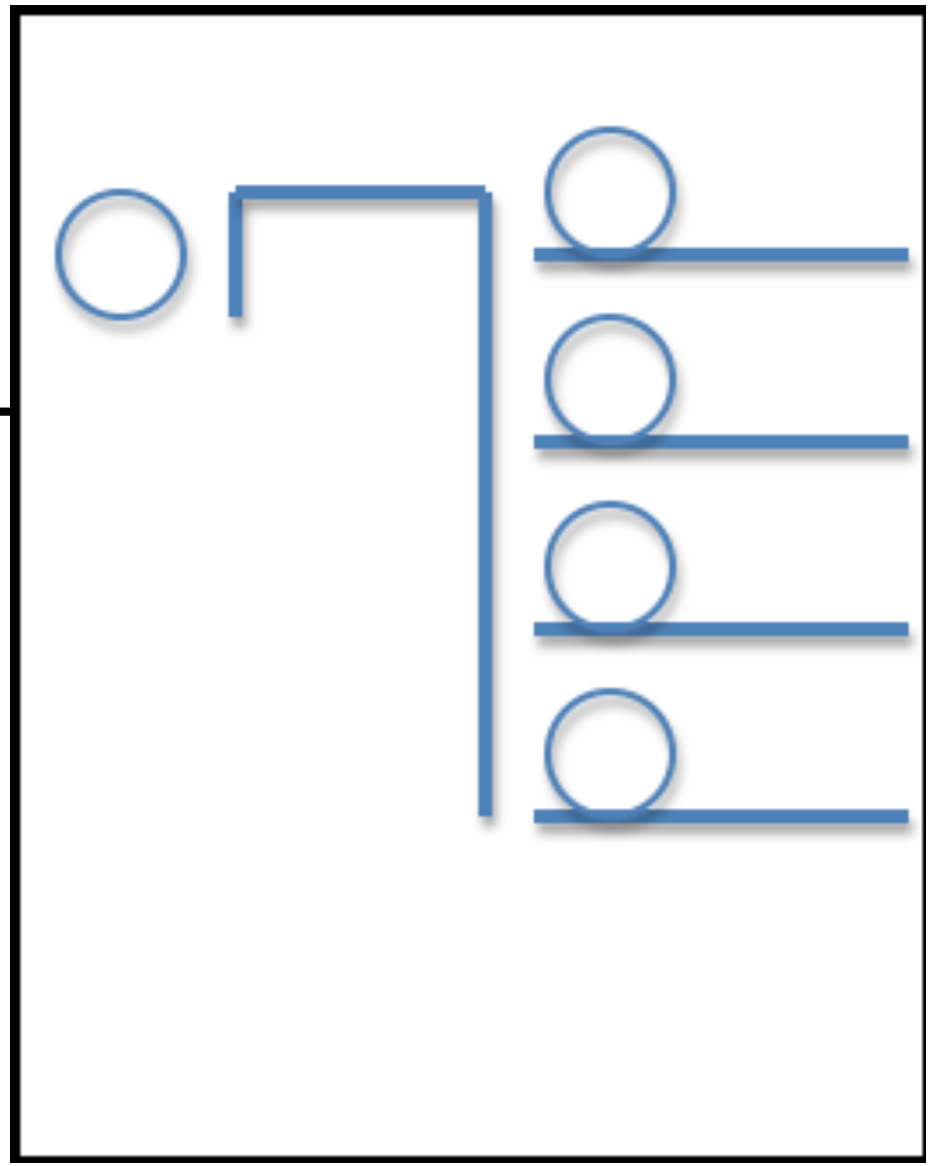

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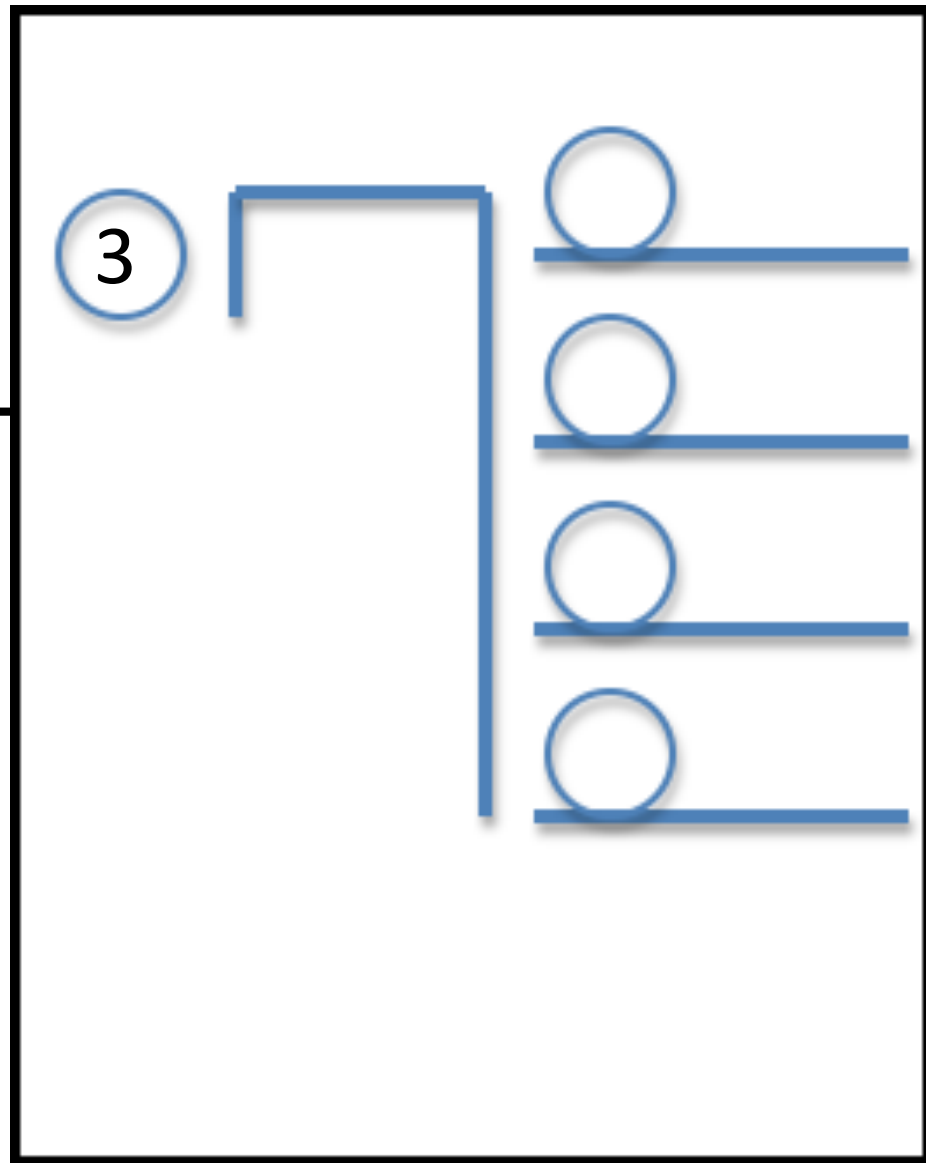

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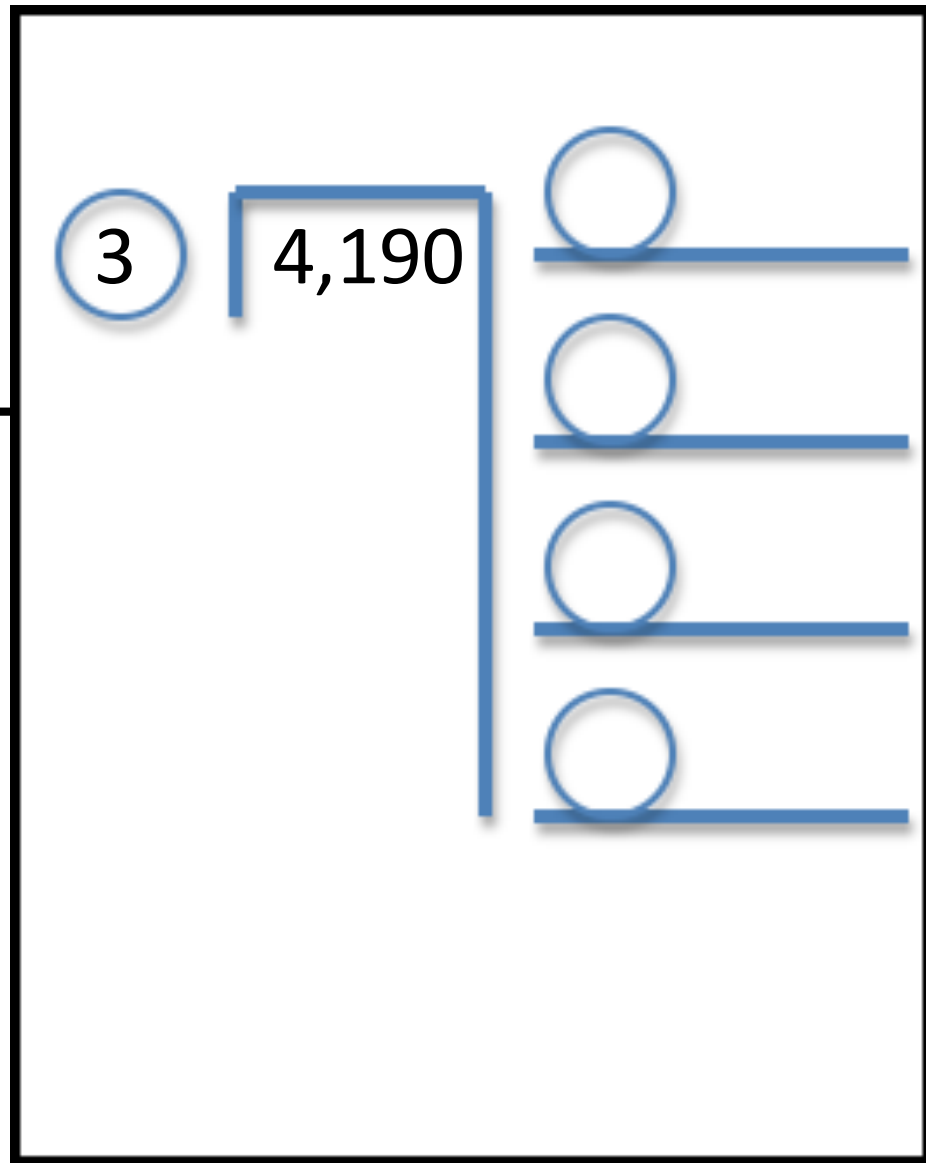

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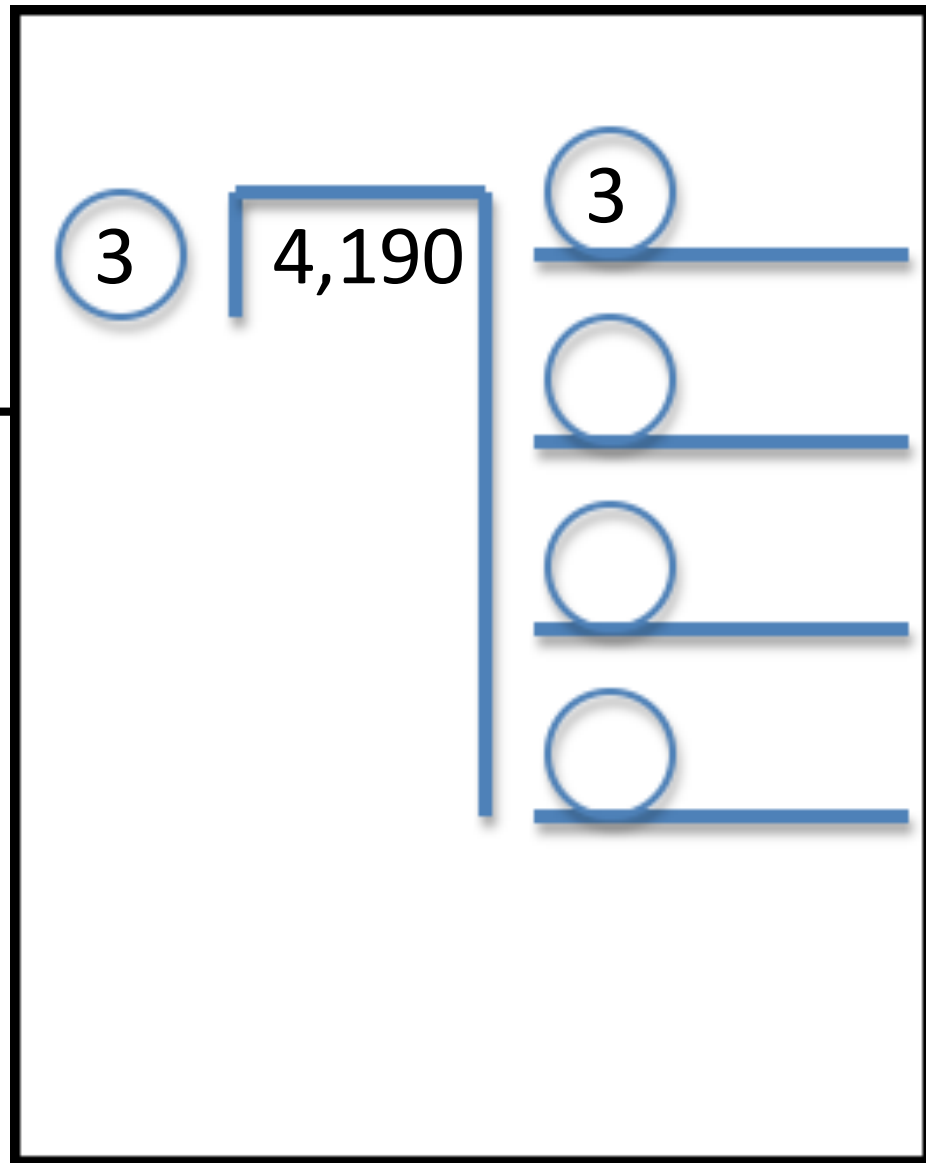

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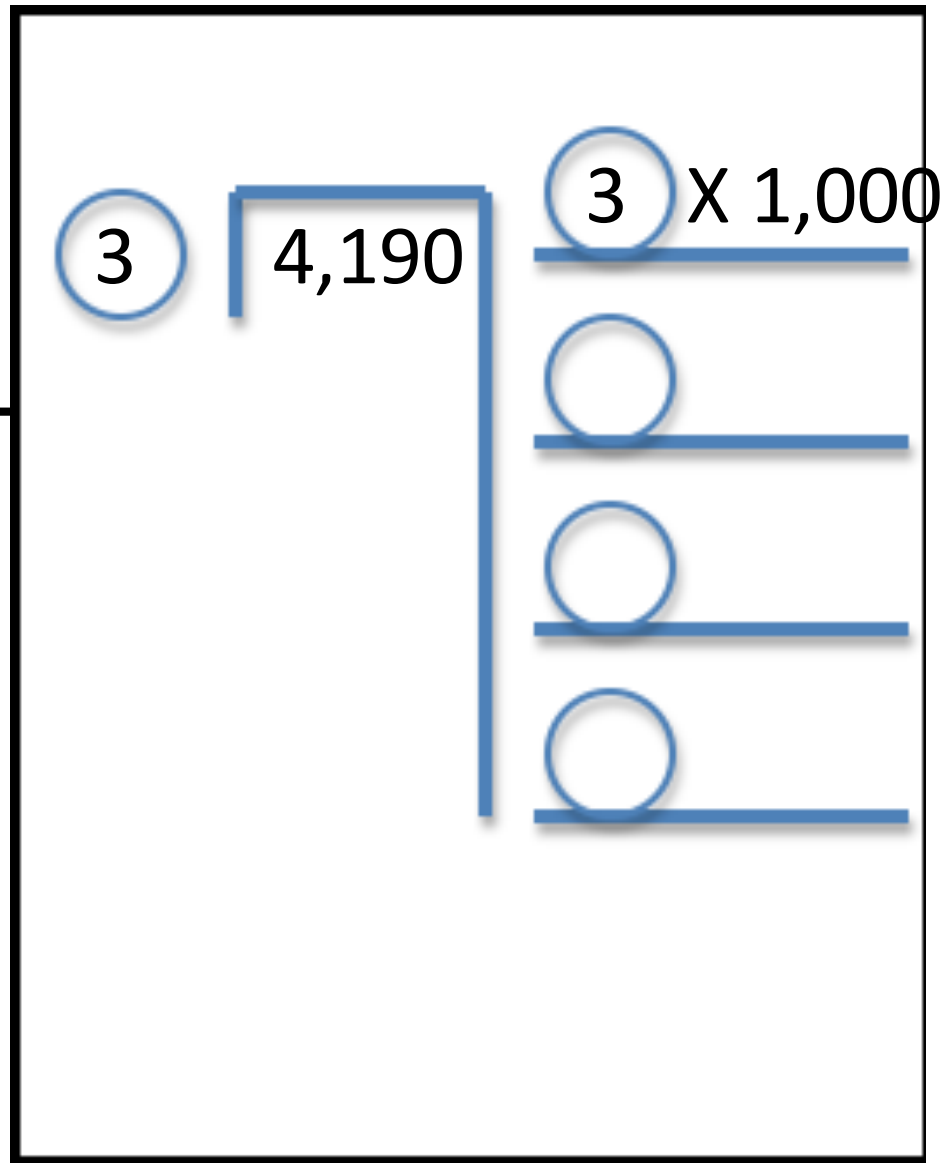

Let's Practice Partial Quotients

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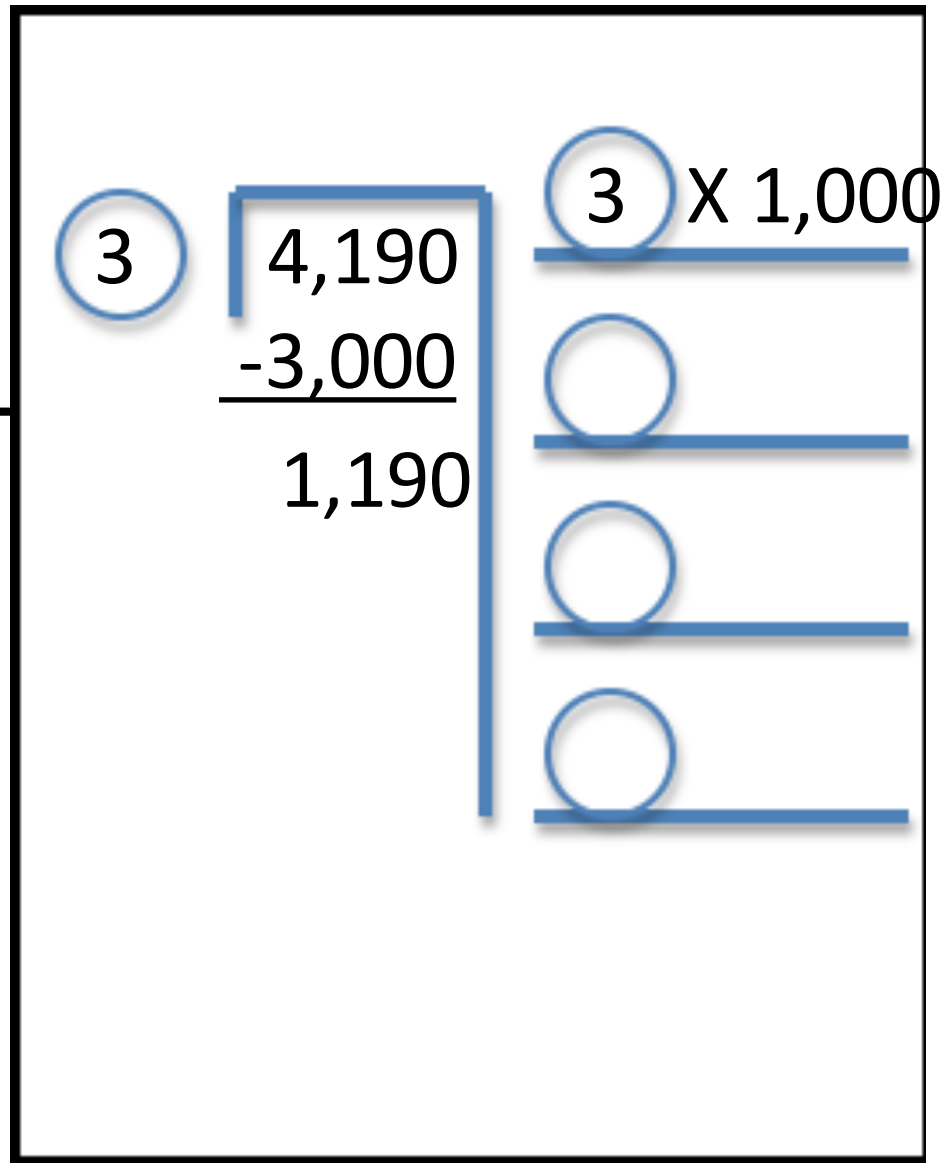

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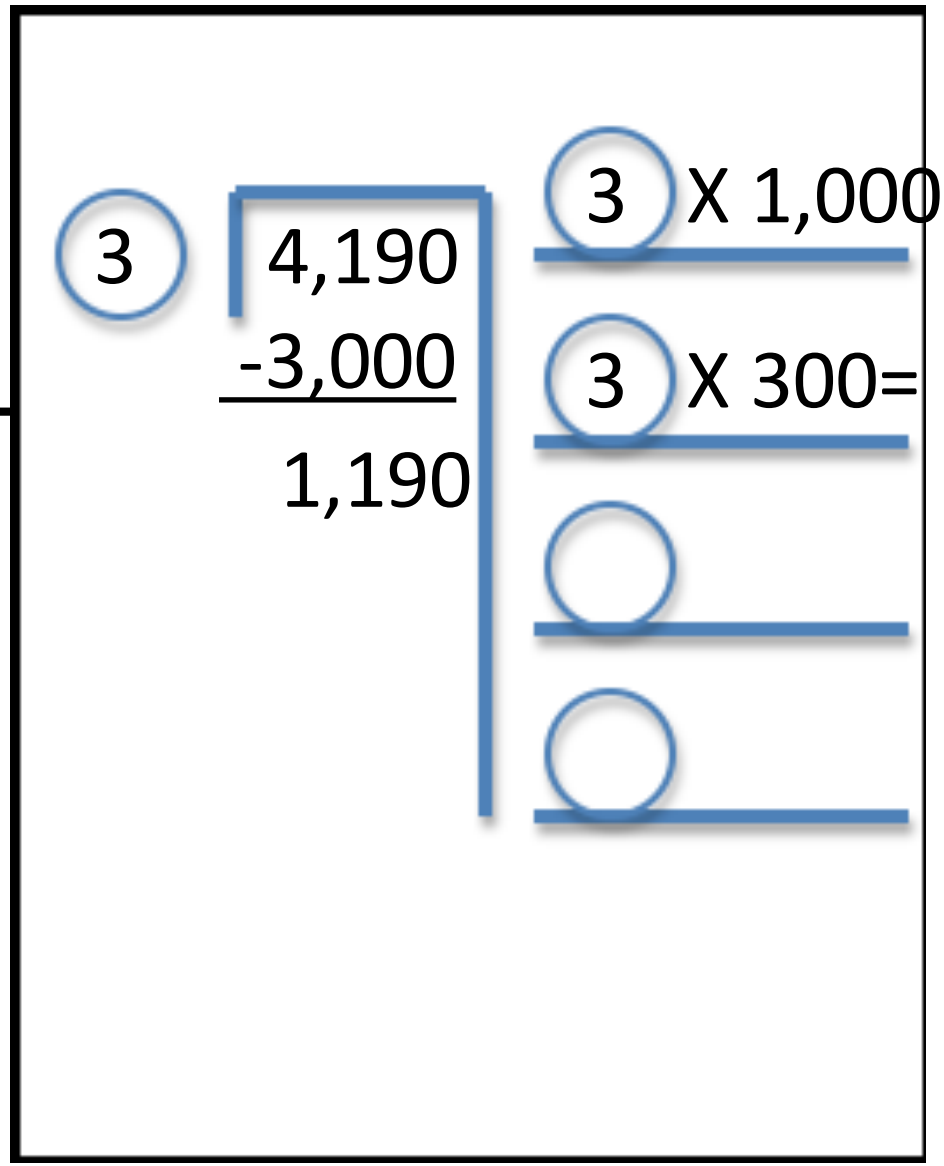

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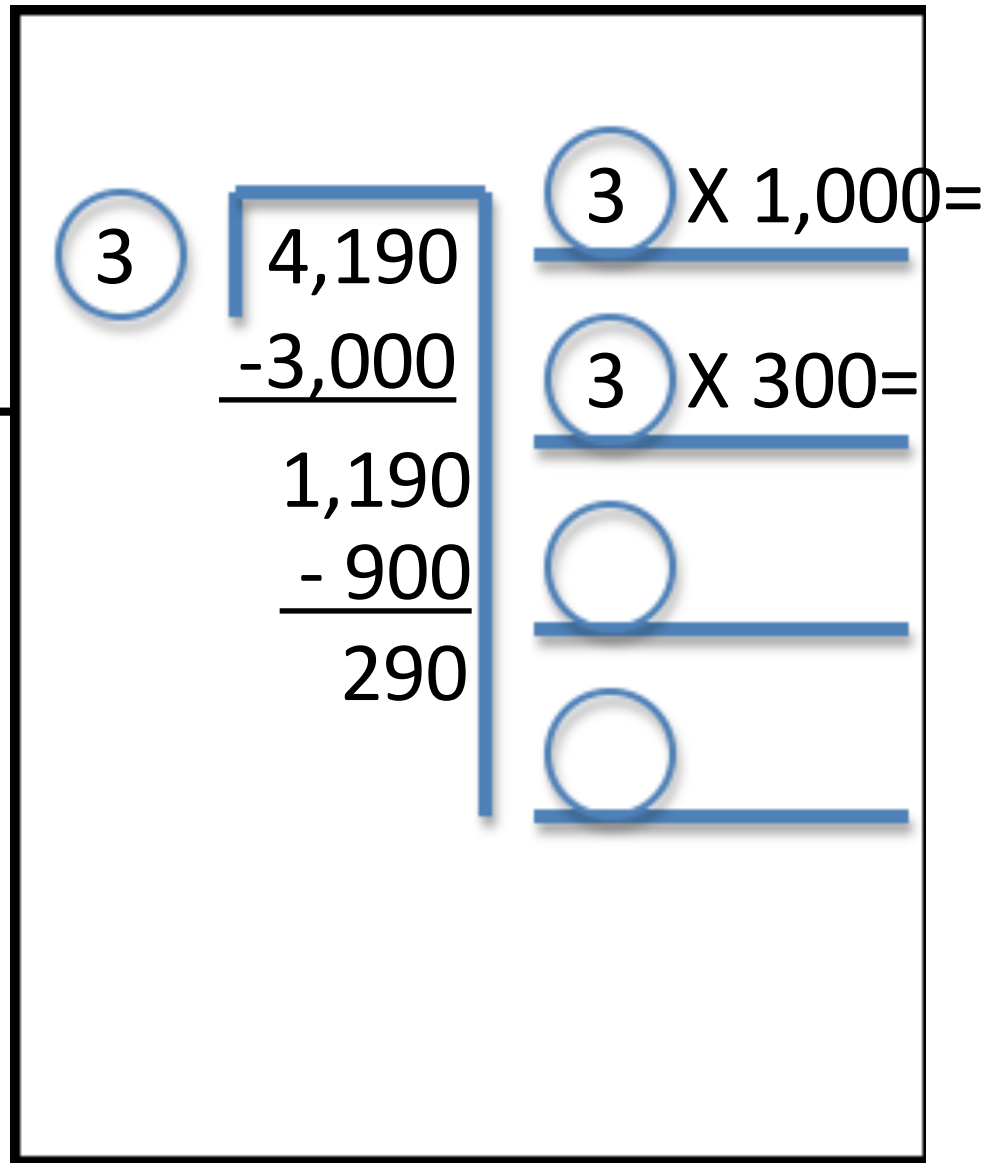

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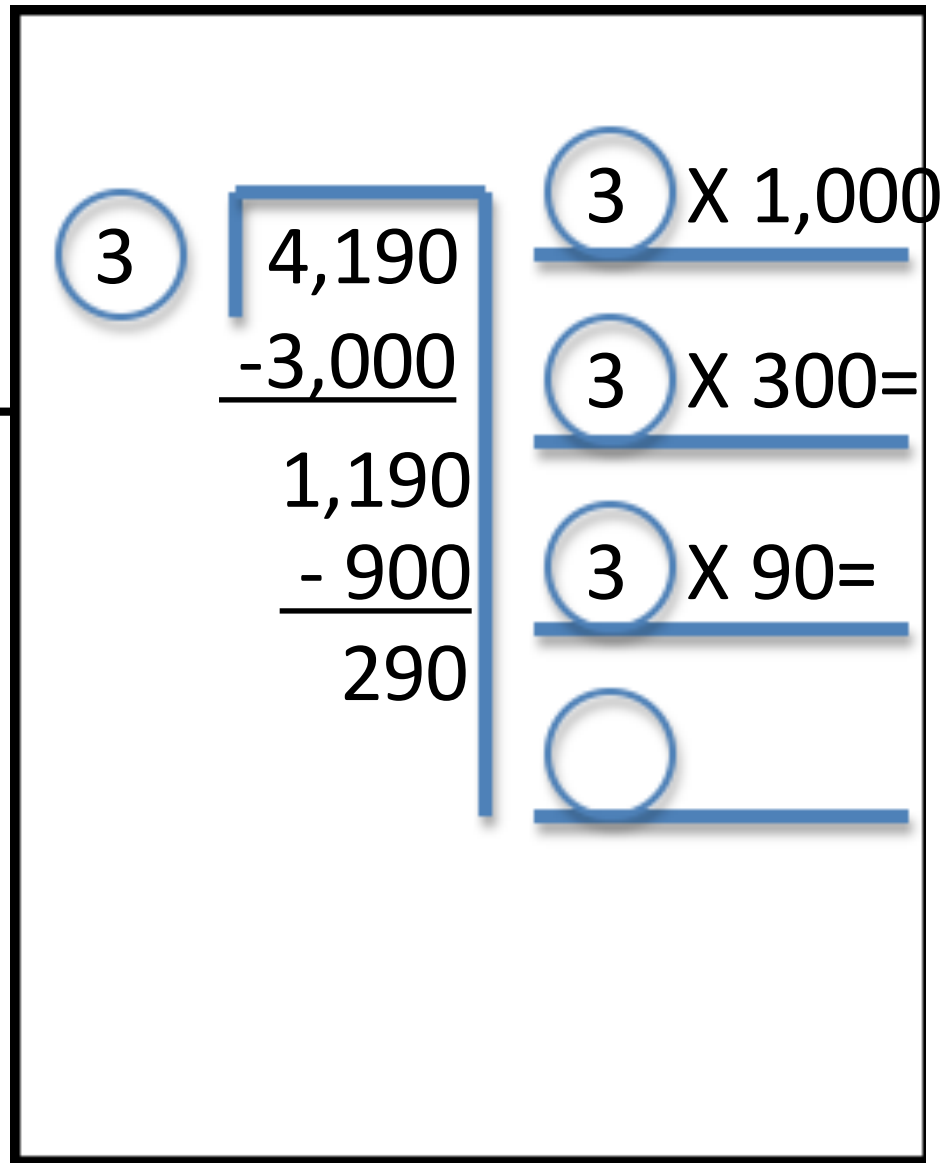

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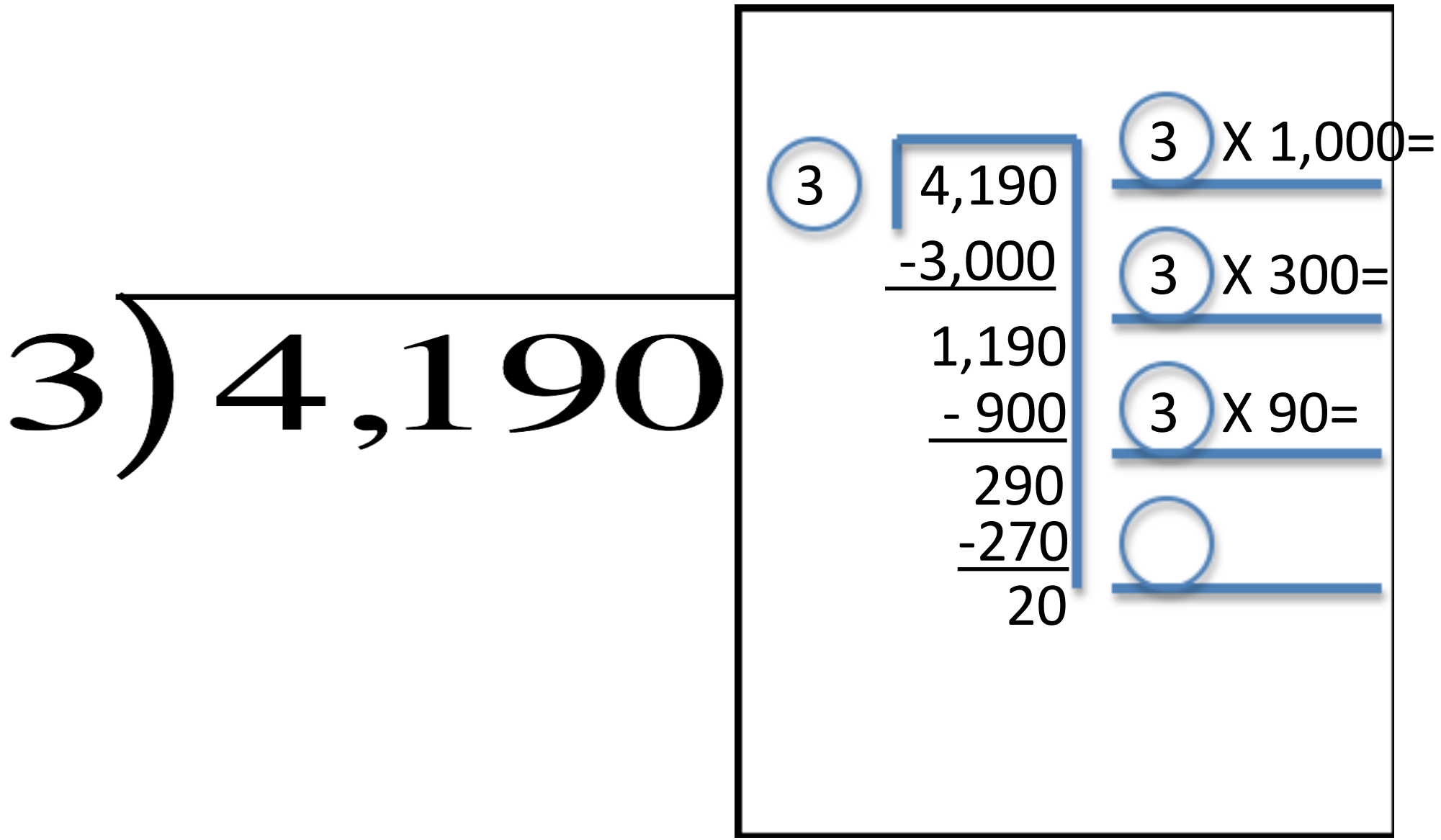


Let's Practice Partial Quotients

$$3\overline{)4{,}190}$$

3 | 4,190
-3,000 | 3 X 1,000=
1,190
- 900 | 3 X 300=
290
-270 | 3 X 90=
20 | 3 X 6=

Let's Practice Partial Quotients

$3\overline{)4{,}190}$

3) 4,190	3 X 1,000= ____
-3,000	
1,190	3 X 300= ____
- 900	
290	3 X 90= ____
-270	
20	3 X 6= ____
-18	

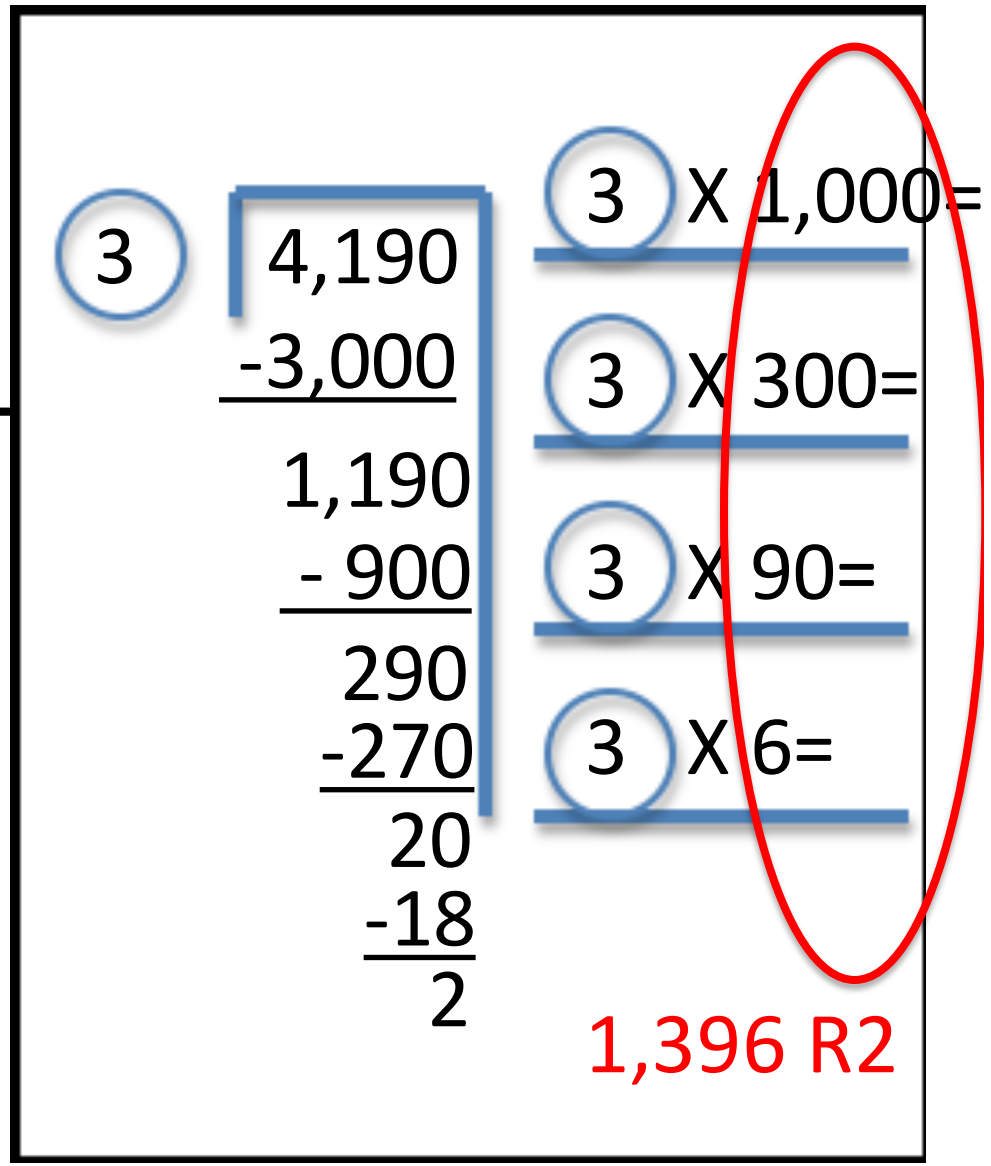
Let's Practice Partial Quotients

$3\overline{)4,190}$

$$
\begin{array}{r|l}
3\,\overline{)\,4,190} & 3 \times 1,000 = \_\_\_ \\
-3,000 & 3 \times 300 = \_\_\_ \\
1,190 & \\
-900 & 3 \times 90 = \_\_\_ \\
290 & \\
-270 & 3 \times 6 = \_\_\_ \\
20 & \\
-18 & \\
2 &
\end{array}
$$

Let's Practice Partial Quotients

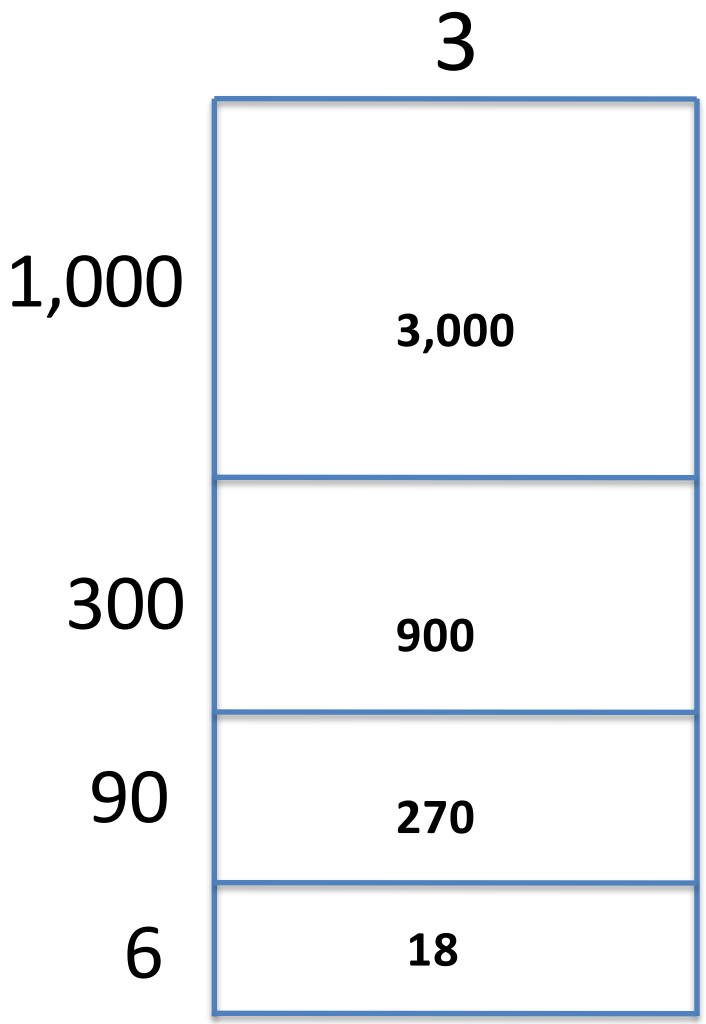
$$3\overline{)4{,}190}$$



Let’s Practice Partial Quotients

$$7\overline{)2,246}$$

Making Connections
What do you notice?



1,396 R2

3 | 4,190
-3,000
1,190
- 900
290
-270
20
-18
2

3 X 1,000=
3 X 300=
3 X 90=
3 X 6=

Dividing with the Distributive Property

- Using the distributive property to divide involves breaking the dividend into “friendlier parts”

Dividing with the Distributive Property: Examples

$721 \div 7 =$

Can be thought of as:

$700 \div 7 = \quad 100$

$21 \div 7 = \quad\quad 3$

Dividing with the Distributive Property: Examples

4,235 ÷ 5 =

Can be thought of as:

4,000 ÷ 5 = 800

200 ÷ 5 = 40

35 ÷ 5 = 7

847

Dividing with the Distributive Property: Examples

2,544 ÷ 12 =

Dividing with the Distributive Property: Examples

2,544 ÷ 12 =

Can be thought of as:

2,400 ÷ 12 = 200

144 ÷ 12 = 12

212

Be Mindful

- Expose students to both types of division problem types.
- Consider the varied representations we reviewed today.
- Monitor, discuss, and help students refine their selection of a particular representation that matches the context of the given problem.

Resources for Teaching Division

- https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-6-about-the-math-learning-targets-and-rigor
- https://hcpss.instructure.com/courses/108/pages/5-dot-nbt-dot-6-about-the-math-learning-targets-and-rigor

LearnZillion.com

Illustrativemathematics.org

Teaching Student-Centered Mathematics 3-5 by John VanDeWalle

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