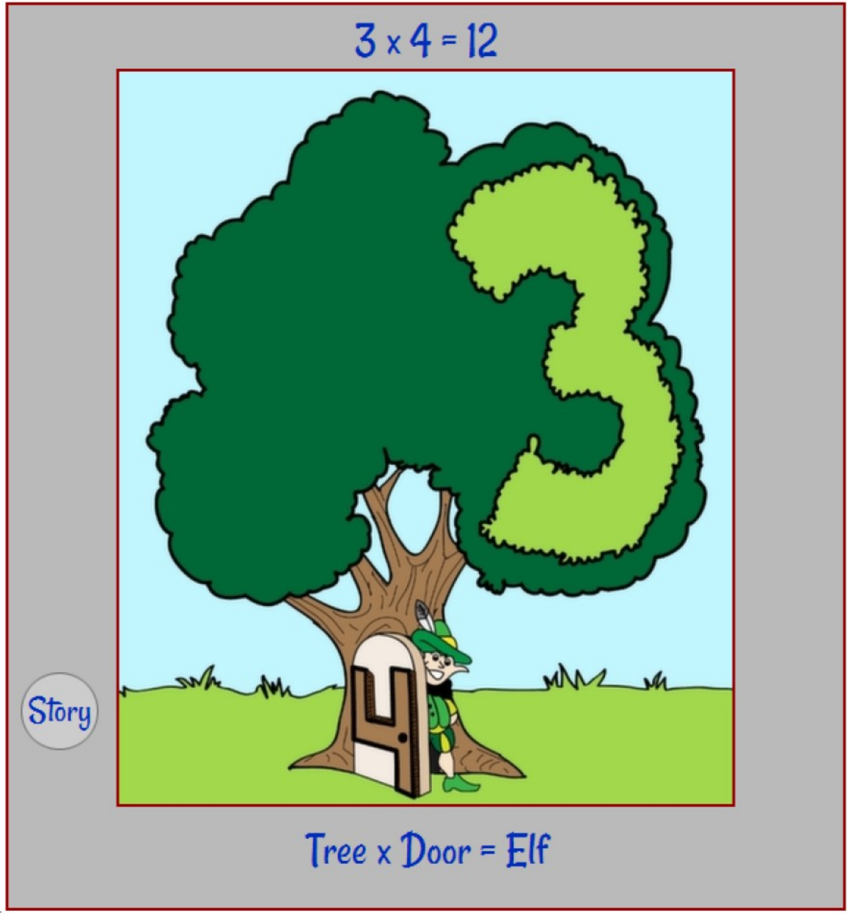
Nix the Tricks

With your table:

- Look through the cards
- Sort into piles based on any category
- Use the sticky notes to name your categories

Nix the Tricks

TINA CARDONE AND ASHLI BLACK



http://www.multiplication.com/learn/learn-fact/3/x/4

Card Sort

What did we see?

Teacher Tactics!

Card Sorts

- ✓ Color Code
- ✓ Start open
- ✓ Vocabulary
- ✓ Detail oriented
- ✓ Coupon organizer for storage

Is it a trick?

- Method
  - Kids can explain: **Shortcut**
  - “Because ___ said so”: **Trick**
- Definition
  - New term: **Mnemonic**
  - Based on previous terms: **Trick**

Types of Tricks

- Imprecise Language
- Methods Eliminating Options
- Tricks Students Misinterpret
- Math as Magic

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GLOBAL MATH DEPARTMENT

Math Makes Sense

There are 125 sheep and 5 dogs in the flock.

How old is the shepherd?

http://robertkaplinsky.com/how-old-is-the-shepherd/

Math Makes Sense

There are 125 sheep and 5 dogs in the flock.

How old is the shepherd?

https://www.youtube.com/watch?v=kibaFBgaPx4 =

http://robertkaplinsky.com/how-old-is-the-shepherd/

Math Makes Sense

There are 125 sheep and 5 dogs in the flock.

How old is the shepherd?

75% of the class of 8th graders and 100% of the class of 6th graders he asked gave a numerical answer.

http://robertkaplinsky.com/how-old-is-the-shepherd/

Let’s Do Some Math!

- How would you solve the problems on your own?
- How might a student solve these problems?
- What are some possible misconceptions/errors?
- What tricks might cause students to make mistakes?

Teacher Tactics!

Handouts

- ✓ White space
- ✓ Lines for sentences
- ✓ Grids for figures
- ✓ Accommodate for different handwriting

Math Mistakes

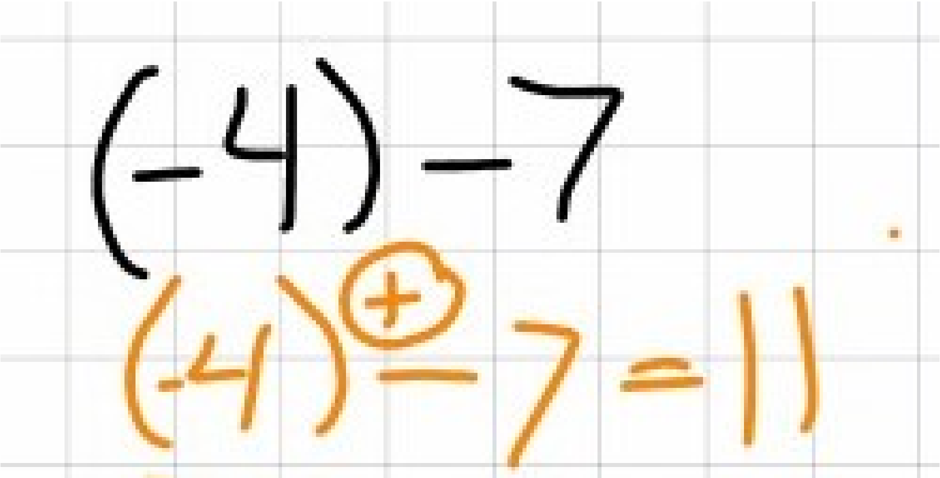
What mistakes might you expect?

1. $(-4) - 7$

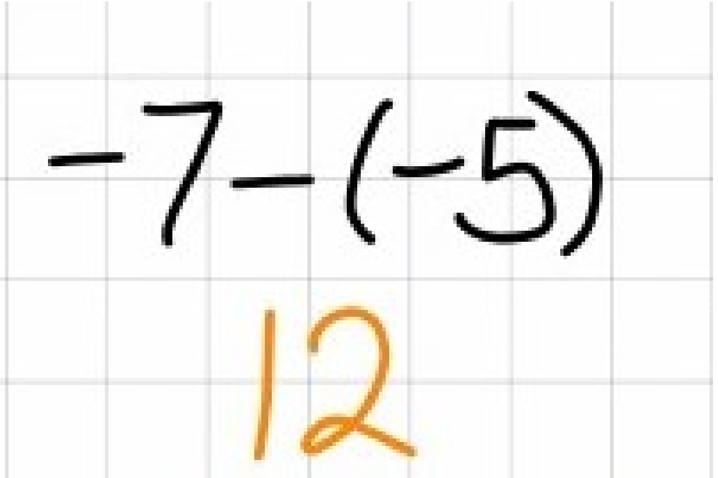
2. $-7 - (-5)$

Math Mistakes

Two negatives make a positive



1 http://mathmistakes.org/?p=328



2 http://mathmistakes.org/?p=424

Two negatives make a positive

$$2 - -5$$

$2 + 5$

or

$$2 + -5$$

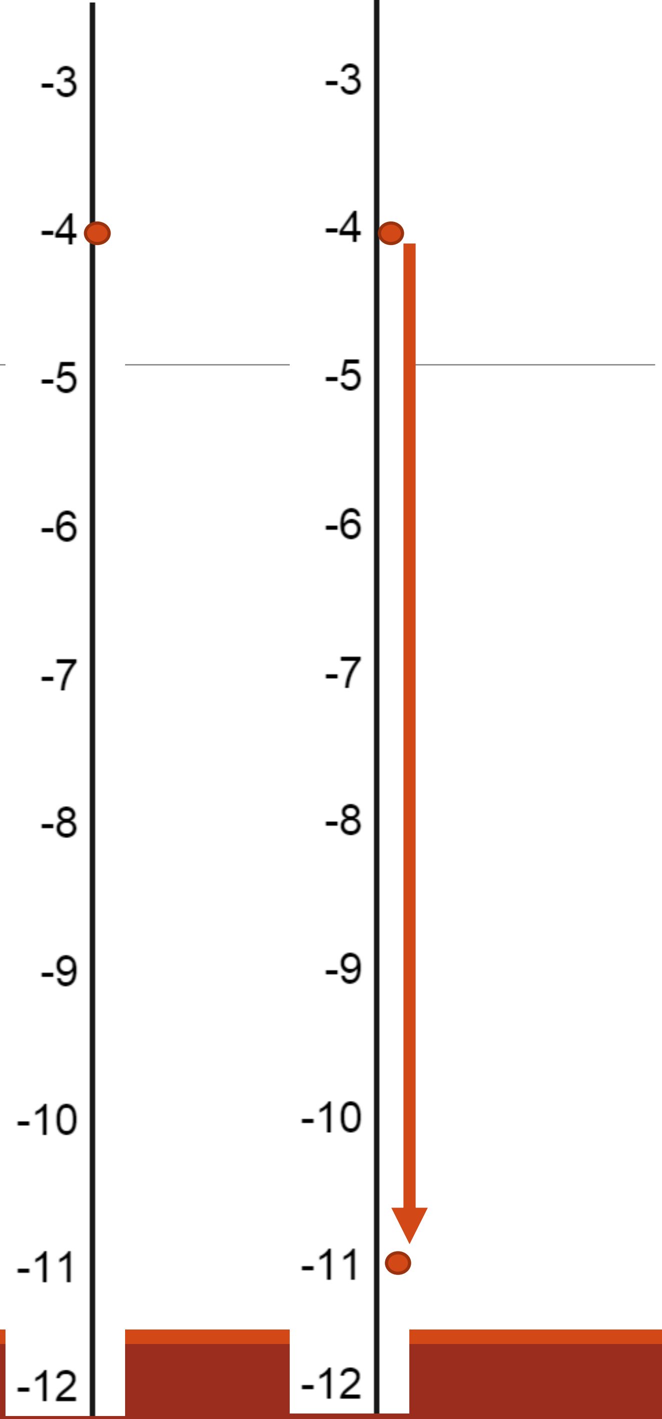
Fix:

Define subtraction as adding the opposite.

Number Lines
for Adding Integers

1. $(-4) - 7$

$(-4) + (-7)$



Teacher Tactics!

Vertical Number Lines

- ✓ More intuitive
- ✓ Many kids don’t know left from right (yet!)
- ✓ Equally valid to use y-axis or x-axis

Math Mistakes

What mistakes might you expect on this problem?

$$\frac{8x}{30} = \frac{48}{18}$$

Math Mistakes

Cross multiply… cross something!

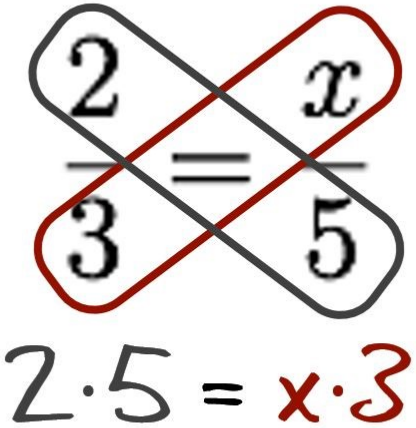
$$\frac{8x}{30} = \frac{48}{18}$$

$$\frac{\cancel{26}x}{\cancel{26}} = \frac{78}{26}$$

$$x = 3$$

http://mathmistakes.org/?p=1320

Cross Multiply



Fix:

One way to solve proportions is to treat them like any other equation – use inverse operations.

Inverse Operations

$$30 \cdot \frac{8x}{30} = \frac{48}{18} \cdot 30$$

$$\frac{1}{8} \cdot 8x = \frac{48 \cdot 30}{18} \cdot \frac{1}{8}$$

$$x = \frac{48 \cdot 30}{18 \cdot 8}$$

Math Mistakes

What mistakes might you expect on these problems?

$$7^2 + 7^3$$

$$(-8)^4 - (-8)^3$$

Math Mistakes

Exponent Rules

$$7^2 + 7^3 = 7^5 = 16807$$

$$(-8)^4 - (-8)^3 = (-8)^1 = -8$$

http://mathmistakes.org/?p=959

Exponent Rules

The rules are **not** tricks, except for students who think they are magic.

Fix:

Teach students to go back to examples and the definition of exponent when they forget a generalization.

Understanding Exponents

Julia Robinson

4^3	$4 \bullet 4 \bullet 4$	64	PATTERN? $\div 4$
4^2	$4 * 4$	16	$\div 4$
4^1	4		$\div 4$
4^0	1		$\div 4$

https://teacher.desmos.com/activitybuilder/custom/57d88ac29775ad6a0d6b878d

Understanding Exponents

$$7^2 + 7^3$$

$7 \cdot 7 + 7 \cdot 7 \cdot 7$

$49 + 343$

392

Math Mistakes

What mistakes might you expect on this problem?

$$4x - 4(x + 2)$$

Math Mistakes

Switch the side, switch the sign

$$4x - 4(x+2)$$

$$4x - \cancel{4x} + 8$$

$$+4x + \cancel{4x}$$

$$8x + 8$$

http://mathmistakes.org/?p=639

Switch the Side, Switch the Sign

$$-4 + x = 12 + 4$$

Fix:

Inverse operations

Teach the meaning of the equality symbol

$$4x - 4(x + 2)$$

$$4x + (-4)(x+2)$$

$$4x + (-4)x + (-4)2$$

$$0x + (-8)$$

$$-8$$

Research Says

"Data from the 13 million students who took PISA tests showed that the lowest achieving students worldwide were those who used a memorization strategy – those who thought of math as a set of methods to remember and who approached math by trying to memorize steps. The highest achieving students were those who thought of math as a set of connected, big ideas."

http://hechingerreport.org/memorizers-are-the-lowest-achievers-and-other-common-core-math-surprises/

Nixing Tricks

Is HARD!

Nixing Tricks

Is HARD!

Hard for kids who have already learned them

Nixing Tricks

Is HARD!

Hard for kids who have already learned them

Hard for kids who are used to learning them

Nixing Tricks

Is HARD!

Hard for kids who have already learned them

Hard for kids who are used to learning them

Hard for teachers who have them in their vocabulary

Nixing Tricks

$\sin^2(x)+\cos^2(x)=1$	Identity
$\sin^2(x)=1-\cos^2(x)$	Subtract $\cos^2(x)$
$\sin^2(x)=[1+\cos(x)][1-\cos(x)]$	Opposite of FOIL
$\dfrac{\sin(x)}{1-\cos(x)}=\dfrac{1+\cos(x)}{\sin(x)}$	Uncross multiply

Nixing Tricks

$\sin^2(x)+\cos^2(x)=1$	Identity
$\sin^2(x)=1-\cos^2(x)$	Subtract $\cos^2(x)$
$\sin^2(x)=[1+\cos(x)][1-\cos(x)]$	Opposite of FOIL
$\dfrac{\sin(x)}{1-\cos(x)}=\dfrac{1+\cos(x)}{\sin(x)}$	Uncross multiply

Students have no idea how to undo a trick – because they don't know what they are doing!

Understanding something includes having an idea of the inverse.

Attend to Precision

Operation	Inverse
Add	Subtract
Multiply	Divide
Cross Multiply	?
FOIL	??

Attend to Precision

Imprecise language	**Precise mathematical language**
Take out the *x*.	Factor *x* from the expression. Divide both sides of the equation by *x*, with a caution about the possibility of dividing by 0.
Move the 5 over.	Subtract 5 from both sides of the equation.
Use the rainbow method. Use FOIL.	Use the distributive property.
Plug in the 2.	Substitute 2 for x.
The numbers cancel out.	The numbers add to zero. The numbers divide to one.

FOIL

$$(2x + 3)(x - 4)$$

First Inside

$$2x^2 - 8x + 3x - 12$$

Outside Last

Fix:

Use the distributive property.

FOIL

[redacted] $(x - 4)$

$= $ [redacted] $(x) +$ [redacted] (-4)

	2x	**3**
x		
−4		

FOIL

$$(2x+3)(x-4)$$

$$=(2x+3)\text{[redacted]} + (2x+3)\text{[redacted]}$$

$$=2x^2+3x-8x-12$$

	2x	**3**
x	$2x^2$	$3x$
-4	$-8x$	-12

FOIL

$$(2x + 3)(x - 4)$$

$$= (2x + 3)(x) + (2x + 3)(-4)$$

$= 2x^2$ [redacted] $- 12$

$$= 2x^2 - 5x - 12$$

	2x	**3**
x	$2x^2$	$3x$
−4	$-8x$	-12

Teacher Tactics!

Offer Students Choice

- ✓ Show multiple strategies
- ✓ Share the benefits of each
- ✓ Don’t make it a false choice

Focus on Understanding

With Tricks Students:

- Are stuck if they forget

$$2 + (-5)$$

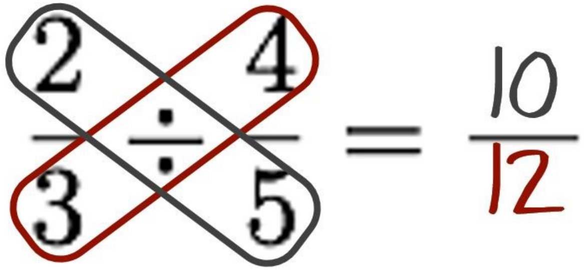
Keep Change Change

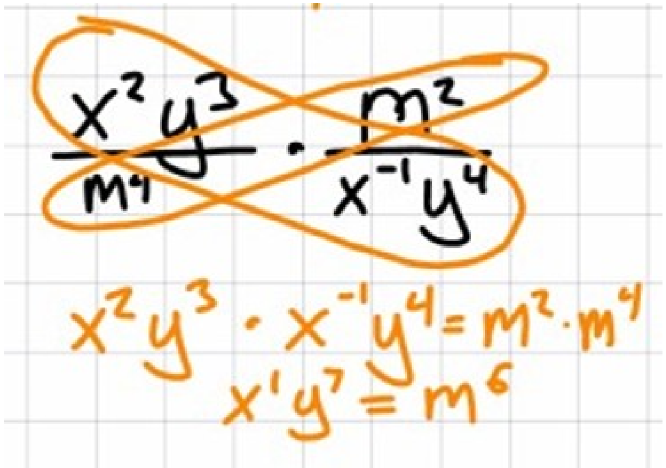
$$2 - (+5)$$

Focus on Understanding

With Tricks Students:

- Are stuck if they forget
- Generalize to dissimilar contexts





Focus on Understanding

With Tricks Students:

- Are stuck if they forget
- Generalize to dissimilar contexts
- Don't generalize to similar context

$$(2x + 3)(x - 4)$$

First Inside

$$2x^2 - 8x + 3x - 12$$

Outside Last

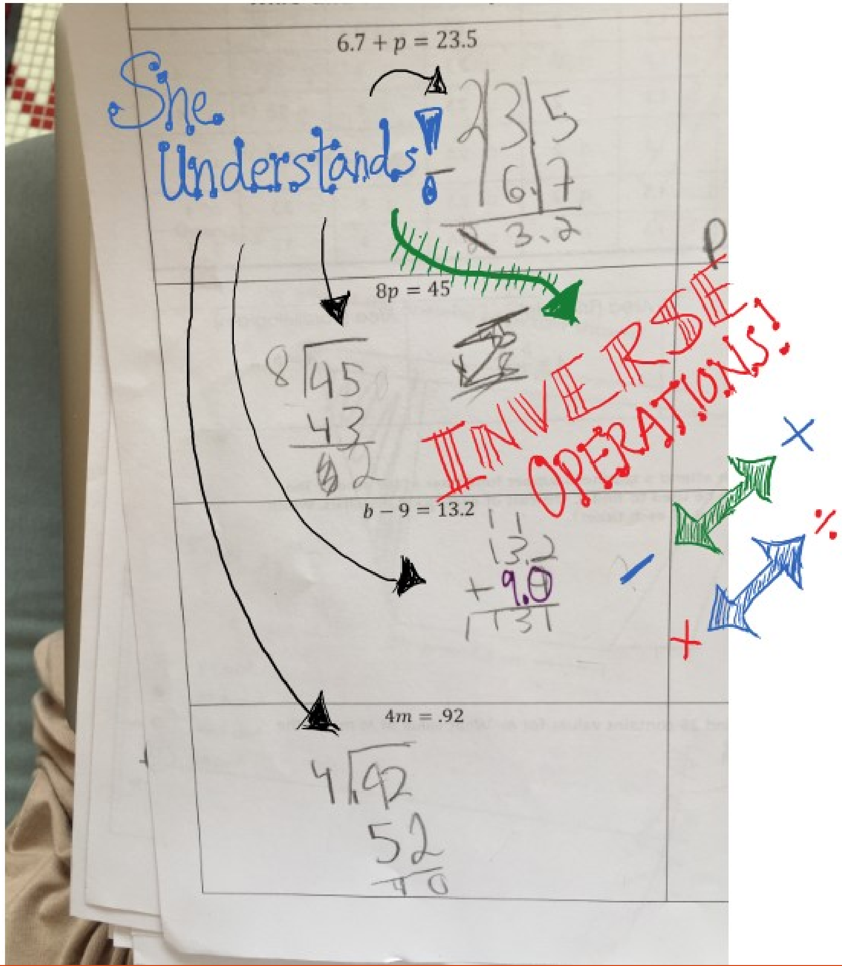
$$(2x + 3)(4x^2 + x - 4)$$

Focus on Understanding

With Tricks Students:

- Are stuck if they forget
- Generalize to dissimilar contexts
- Don’t generalize to similar context

It’s Not Too Late



See mistakes?
Focus on the positive.

Even if they learned tricks before they can still relearn that math makes sense.

Interesting Methods

More Math!

- How would you solve the problems on your own?
- What insights could you/students use to solve these problems?
- How do those methods compare to traditional ones?

Interesting Methods

$$12(2x + 4) - 3 = 141$$

Interesting Methods

$$\boxed{12(2x + 4)} - 3 = 141$$

$$12\boxed{(2x + 4)} = 144$$

$$\boxed{2x} + 4 = 12$$

$$2\boxed{x} = 8$$

$$x = 4$$

Interesting Methods

Jill had 72 candies.
She gave the same number to each of her three brothers.
There were 48 candies left over.

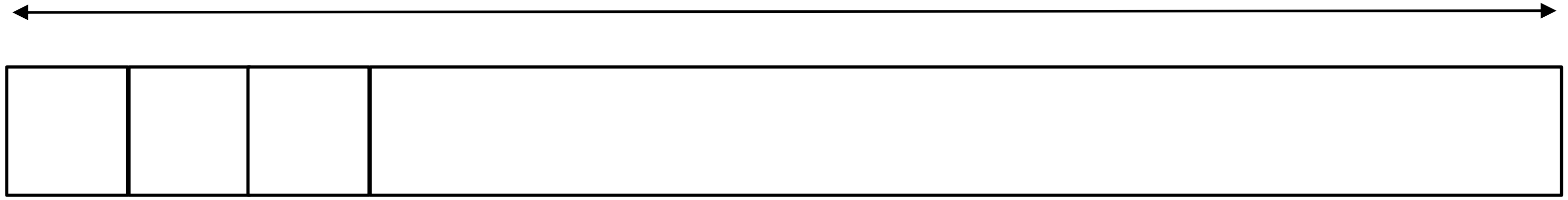
Interesting Methods

Visualpatterns.org

Jill had 72 candies.
She gave the same number to each of her three brothers.
There were 48 candies left over.

What do you know?

Label the parts of the diagram with information from the problem.



Write an equation based on the model.

Solve the equation.

Write your answer in a sentence.

Interesting Methods

You have seven cups of dog food. You use two-thirds of a cup of food at each meal.

Interesting Methods

You have seven cups of dog food. You use two-thirds of a cup of food at each meal.

https://www.youtube.com/watch?v=mUKbL_OL_-Q

Interesting Methods

Fill in the blank with $<$, $>$ or $=$.

$$\frac{4}{9} \qquad \frac{4}{5}$$

Interesting Methods

Fill in the blank with $<$, $>$ or $=$.

$$\frac{4}{9} \qquad \frac{4}{5}$$

$$\frac{1}{9} < \frac{1}{5}$$

$$\frac{4}{9} < \frac{4}{5}$$

Interesting Methods

Fill in the blank with <, > or =.

$$\frac{7}{9} \qquad \frac{11}{13}$$

Fill in the blank with $<$, $>$ or $=$.

$$\frac{7}{9} \qquad \frac{11}{13}$$

$$\frac{2}{9} > \frac{2}{13}$$

$$1-\frac{2}{9} < 1-\frac{2}{13}$$

$$\frac{7}{9} < \frac{11}{13}$$

Interesting Methods

$$\frac{4}{10} = \frac{32}{x}$$

$$\frac{15}{5} = \frac{y}{4}$$

Interesting Methods

$$\frac{4}{10} = \frac{32}{x}$$

$\cdot 8$

$\cdot 8$

$$\frac{15}{5} = \frac{y}{4}$$

$\cdot 3$

$\cdot 3$

Interesting Methods

$$\frac{1}{2} = \frac{x + 20}{50}$$

Interesting Methods

$$\frac{1}{2} = \frac{\boxed{}}{50} \quad =25$$

DRT vs. TRD, Prove Your Tricks

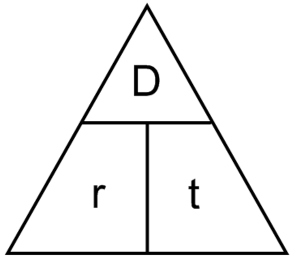
Students: “This is easy! Just use the DRT triangle”

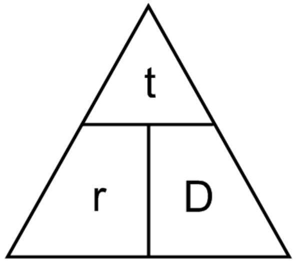
Mr. Cox: “I learned the TRD triangle.”

S1: “No, that won't work. That's not what he told us.”

S2 : “He said it didn't matter how we wrote it.”

Mr. Cox: “So which is it; does one work or are they the same? Make your case and be ready to defend it.”





http://coxmath.blogspot.com/2014/04/dirty-triangles.html

Why Am I Teaching This?
How Did They Get Here?

http://whyamiteachingthis.weebly.com/

Progressions videos:

Counting to Cardinality: https://vimeo.com/157838163

Addition and Subtraction: https://vimeo.com/157768846

Multiplication: https://vimeo.com/149428217

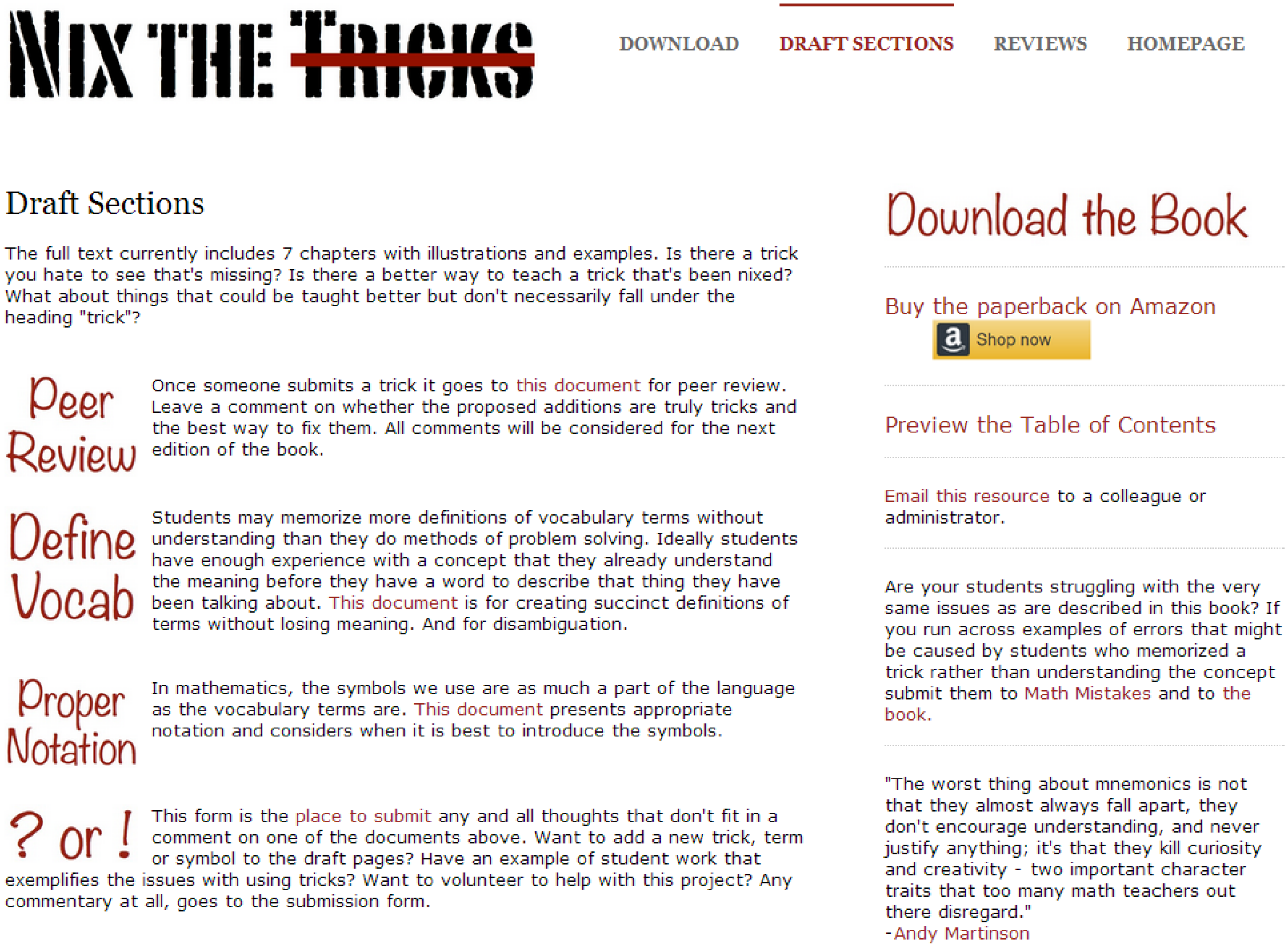
Division: https://vimeo.com/153668928

NixTheTricks.com

See other’s ideas

Make a suggestion

Have a debate



NIX THE ~~TRICKS~~

DOWNLOAD DRAFT SECTIONS REVIEWS HOMEPAGE

Draft Sections

The full text currently includes 7 chapters with illustrations and examples. Is there a trick you hate to see that's missing? Is there a better way to teach a trick that's been nixed? What about things that could be taught better but don't necessarily fall under the heading "trick"?

Peer Review Once someone submits a trick it goes to this document for peer review. Leave a comment on whether the proposed additions are truly tricks and the best way to fix them. All comments will be considered for the next edition of the book.

Define Vocab Students may memorize more definitions of vocabulary terms without understanding than they do methods of problem solving. Ideally students have enough experience with a concept that they already understand the meaning before they have a word to describe that thing they have been talking about. This document is for creating succinct definitions of terms without losing meaning. And for disambiguation.

Proper Notation In mathematics, the symbols we use are as much a part of the language as the vocabulary terms are. This document presents appropriate notation and considers when it is best to introduce the symbols.

? or ! This form is the place to submit any and all thoughts that don't fit in a comment on one of the documents above. Want to add a new trick, term or symbol to the draft pages? Have an example of student work that exemplifies the issues with using tricks? Want to volunteer to help with this project? Any commentary at all, goes to the submission form.

Download the Book

Buy the paperback on Amazon

Shop now

Preview the Table of Contents

Email this resource to a colleague or administrator.

Are your students struggling with the very same issues as are described in this book? If you run across examples of errors that might be caused by students who memorized a trick rather than understanding the concept submit them to Math Mistakes and to the book.

"The worst thing about mnemonics is not that they almost always fall apart, they don't encourage understanding, and never justify anything; it's that they kill curiosity and creativity - two important character traits that too many math teachers out there disregard."
-Andy Martinson

Now What?

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