

Welcome to

THE POWER OF UNIT FRACTION

QUANTITIES FOR BUILDING

CHILDREN'S UNDERSTANDING

Please take a handout, solve a few problems, and say hello to someone near you.

THE POWER OF UNIT FRACTION QUANTITIES FOR BUILDING CHILDREN'S UNDERSTANDING

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Big Ideas

- Importance of unit-fraction quantities in children's understanding
- Continuum of children's understanding of unit-fraction quantities
- Power of story problems to build children's understanding

What is a Unit Fraction?

$$\frac{1}{2}$$

$$\frac{1}{b}$$

$$\frac{1}{3}$$

$$\frac{1}{6}$$

$$\frac{1}{4}$$

$$\frac{1}{8}$$

How might a 5th grader solve these problems?

$$4 - 1/8 = \square$$

$$\square \times 1/4 = 2 \frac{1}{4}$$

Matthew (5th grade)

$$4 - \frac{1}{8} = \square$$

What are the mathematical details of Matthew's strategy?



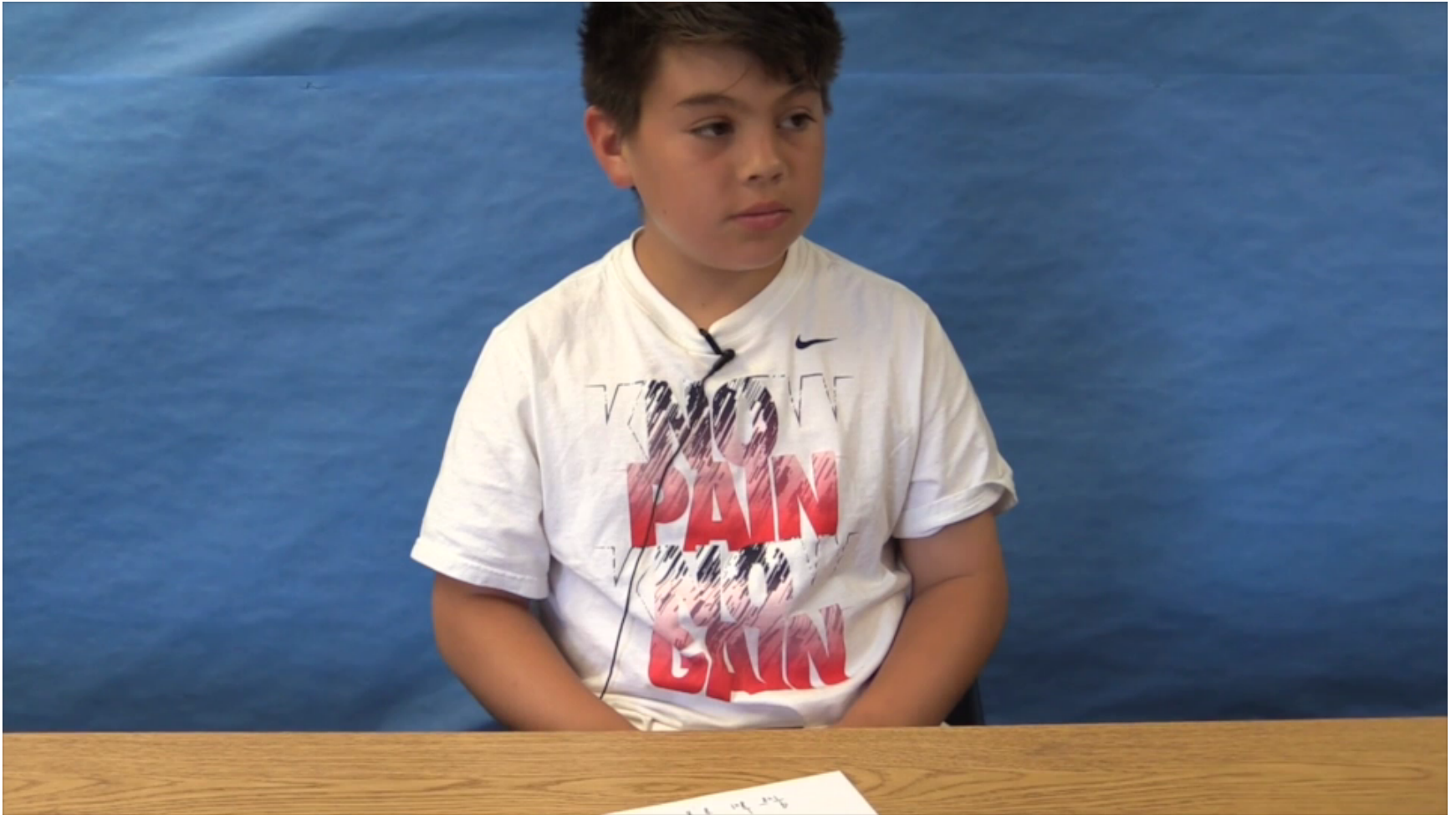
Let's discuss

- What are the mathematical details of Matthew's strategy?

$$4 - \frac{1}{8} = \boxed{3\frac{3}{8}}$$
$$\frac{4}{1 \times 8} \quad \frac{1}{8 \times 1} \quad \frac{4}{8} \quad \frac{1}{8}$$

Matthew (5th grade)

What are the mathematical details of Matthew's strategy?

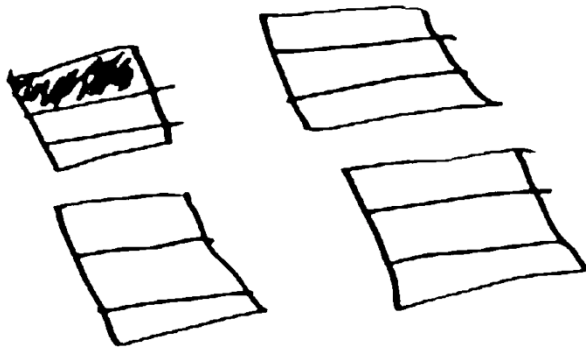


If you had 4 sandwiches, and you ate $\frac{1}{3}$ of a sandwich, how many sandwiches would you have left?

Talk to someone next to you

- What are the mathematical details of Matthew's strategy?

$$3\frac{2}{3}$$



$$4 - \frac{1}{3} = 3\frac{2}{3}$$

Rosie (5th grade)

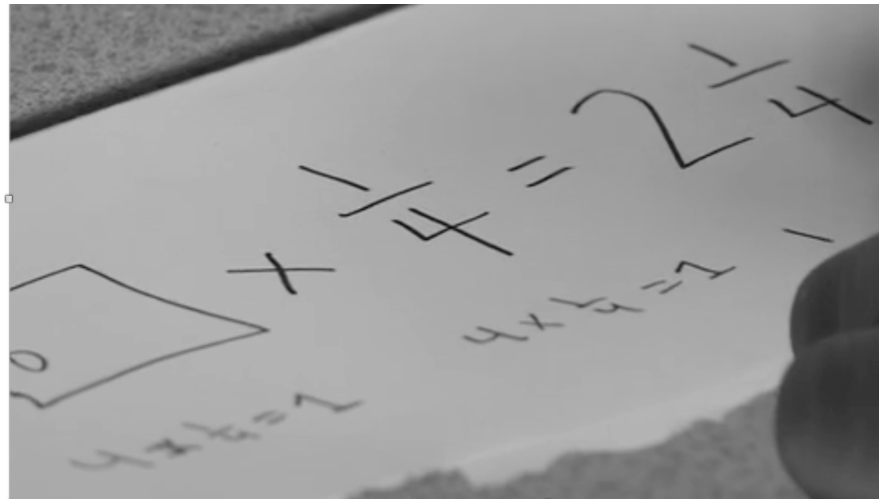
$$\square \times \frac{1}{4} = 2 \frac{1}{4}$$

What are the mathematical details of Rosie's strategy?



Talk to someone next to you

- What are the mathematical details of Rosie's strategy?



What can we say about their understanding?

Matthew

To solve the equation,
attempts to remember a
procedure

To solve the story problem,
draws models to represent
and use fraction relationship
1 whole is 3 one-thirds

Story problem was posed
at a just right level of
challenge

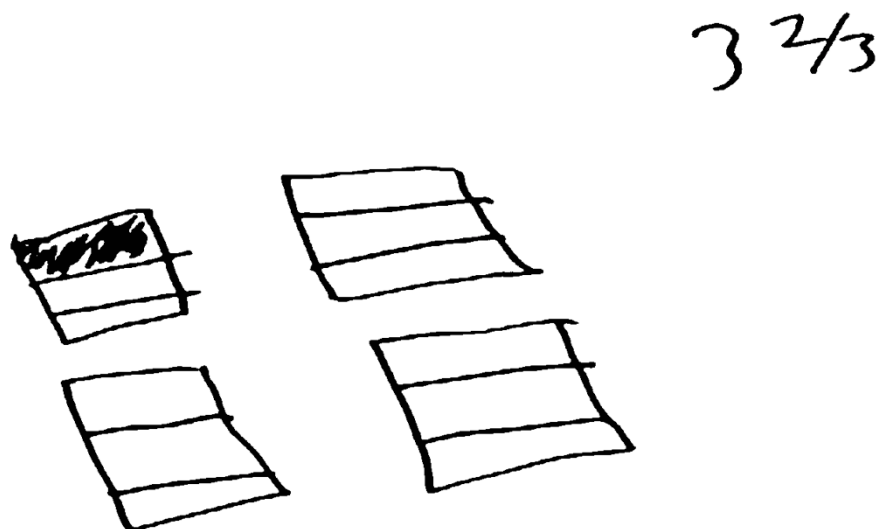
Rosie

To solve the equation, uses
fraction relationship
 $4 \times 1/4 = 1$

Equation was posed at a
just right level of challenge

What do these examples illustrate?

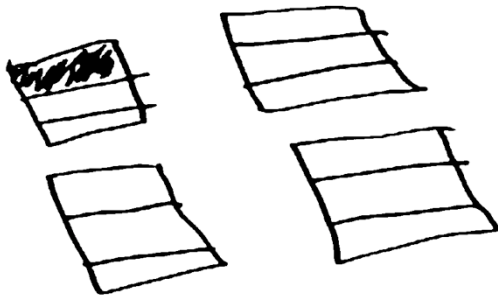
Story contexts help children make sense of fractional quantities and relationships



What do these examples illustrate?

Children's strategies are based on using a key fraction relationship

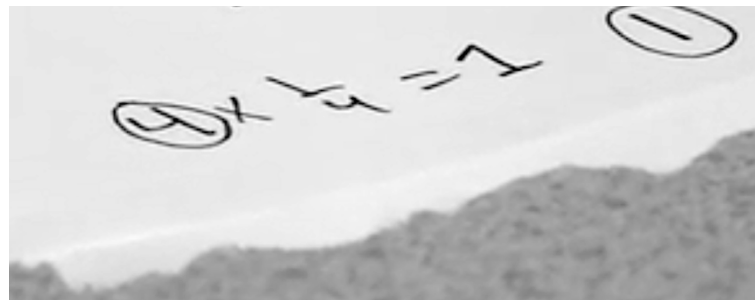
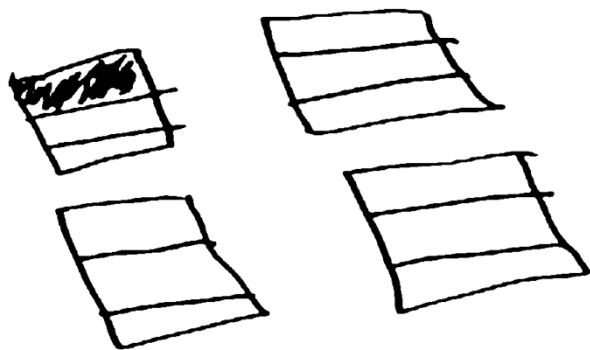
- 1 whole sandwich can be thought of as 3 one-third sandwiches
- 1 equals 4 groups of one-fourth



Key relationship: A **whole** is a grouping of **same-sized unit-fraction** quantities

What do these examples illustrate?

These fraction relationships are understood and used at different levels of abstraction



Looking at student work

Problem: A chef is making 14 pizzas. Each pizza takes $\frac{1}{6}$ of a pound of cheese. How much cheese will the chef need to make all of the pizzas?

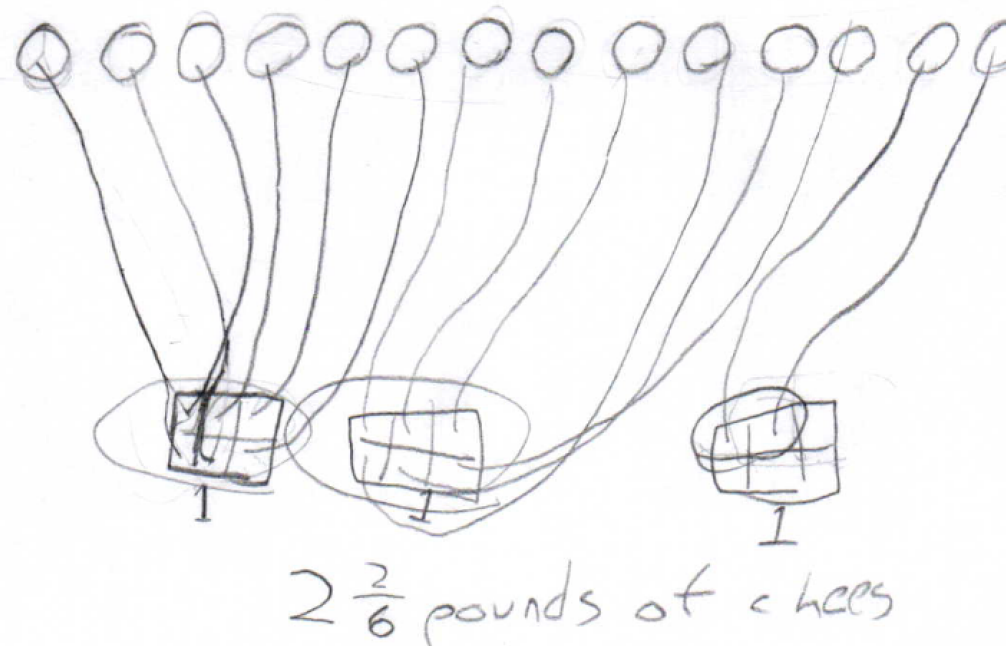
Directions: Sort strategies by similarities based on the questions below.

- Did the child represent all of the unit fractions?
- Did the child group unit fractions to make a whole?

A chef is making 14 pizzas. Each pizza takes $\frac{1}{6}$ of a pound of cheese. How much cheese will the chef need to make all of the pizzas?

- Did the child represent all of the unit fractions?
- Did the child group unit fractions to make a whole?

Maggie



A chef is making 14 pizzas. Each pizza takes $\frac{1}{6}$ of a pound of cheese. How much cheese will the chef need to make all of the pizzas?

- Did the child represent all of the unit fractions?
- Did the child group unit fractions to make a whole?

Pedro

$$\begin{array}{cccccccccccccccc} \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} & + \frac{1}{6} \\ \hline & & & & & & & & & & & & & & & \frac{2}{6} \end{array}$$

$2\frac{2}{6}$ pounds of cheese

A chef is making 14 pizzas. Each pizza takes $\frac{1}{6}$ of a pound of cheese. How much cheese will the chef need to make all of the pizzas?

- Did the child represent all of the unit fractions?
- Did the child group unit fractions to make a whole?

Karisma

6 pizzas = 1 lb of cheese
12 pizzas = 2 lb of cheese
2 pizzas = $\frac{1}{3}$ lb of cheese
 $2\frac{1}{3}$ lb of cheese

Looking at student work

Problem: A chef is making 14 pizzas. Each pizza takes $\frac{1}{6}$ of a pound of cheese. How much cheese will the chef need to make all of the pizzas?

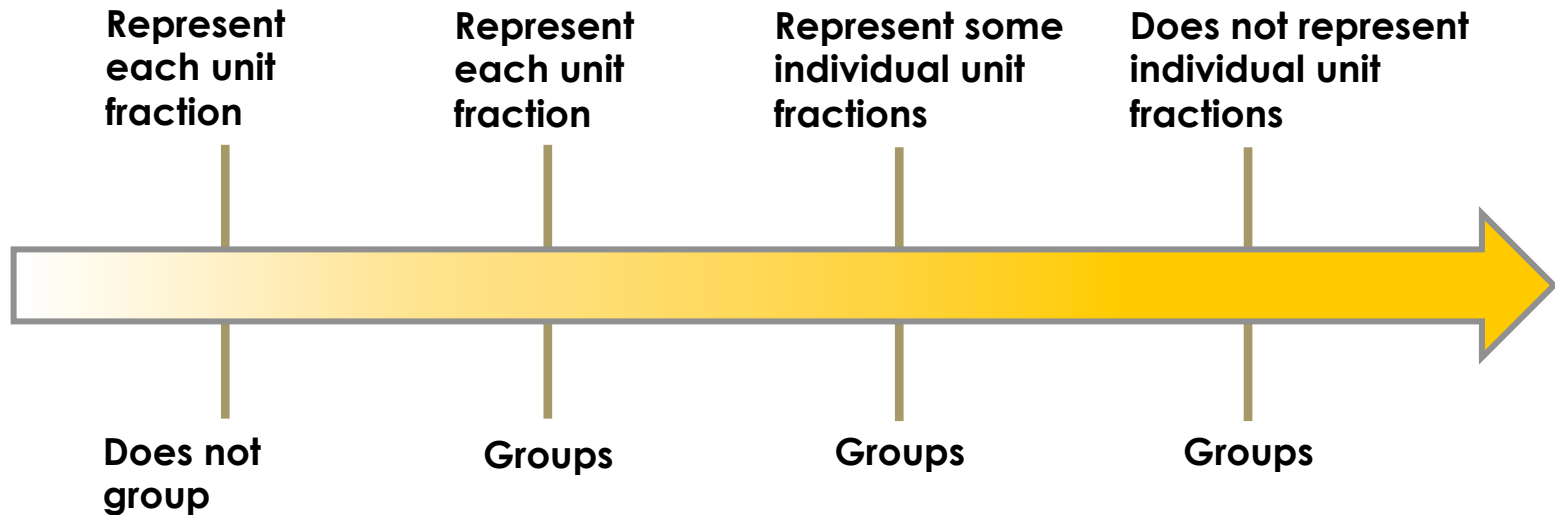
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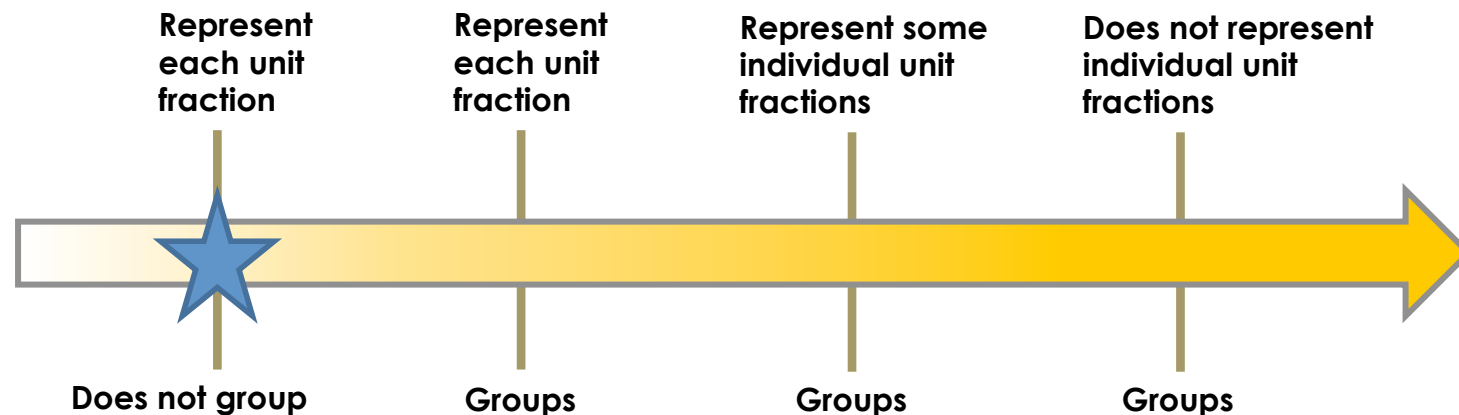
- Did the child represent all of the unit fractions?
- Did the child group unit fractions to make a whole?

Discussion

- Find another pair and
 - Share **two strategies** that you sorted into the *same* group. Why did you put them into the same group?
 - Share **two strategies** that you sorted into *different* groups? Why did you put them into different groups?

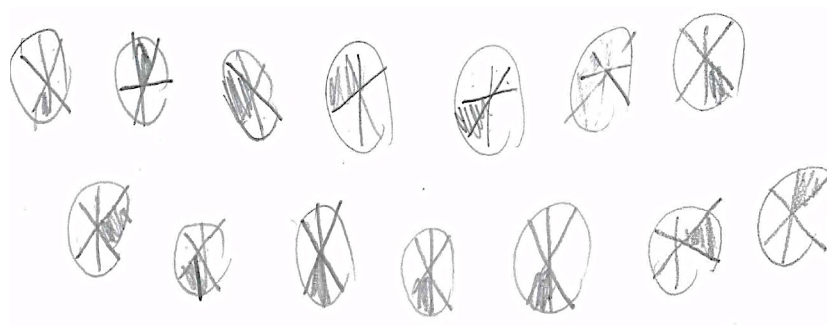
Discussion: Representing and Grouping Unit Fractions

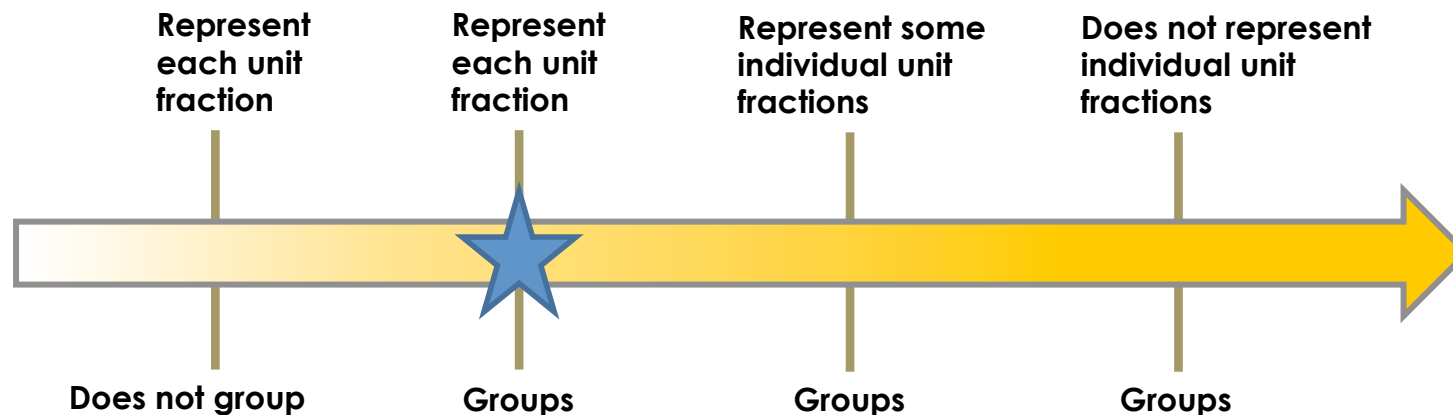




Every $\frac{1}{6}$ pound of cheese is represented individually but the grouping to one whole is not represented.

**Example:
Strategy J**

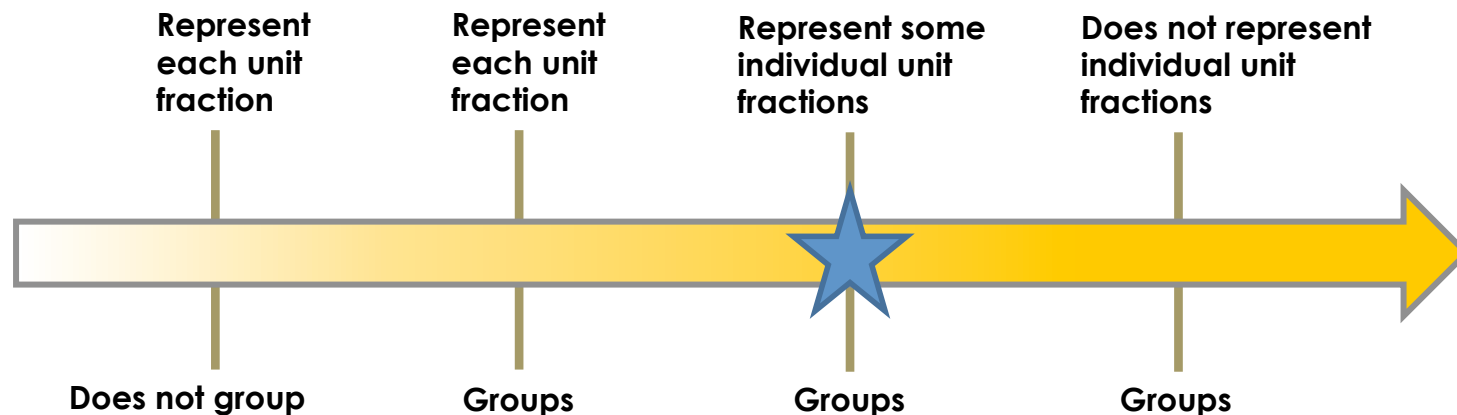




Every $\frac{1}{6}$ pound of cheese is represented individually and they are grouped to show how many make a whole pound.

Example:
Strategy B





Some of the $\frac{1}{6}$ pound of cheese are represented individually. The remaining unit fractions do not have to be represented individually.

Example:
Strategy H

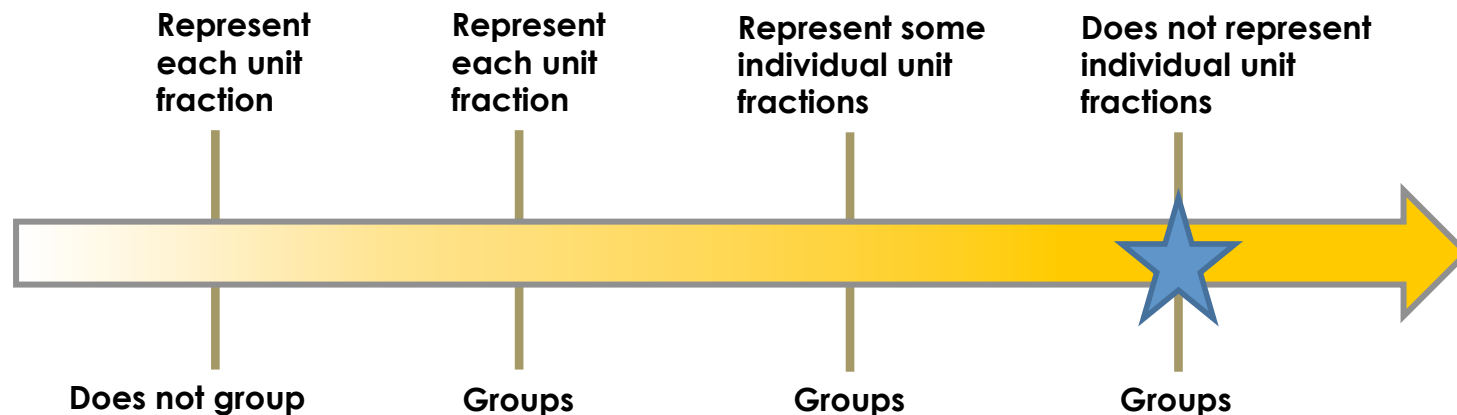
$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

1 whole = 6 pizza

2 whole = 12 pizza

$$2 + \frac{1}{6} + \frac{1}{6} = 2\frac{2}{6}$$

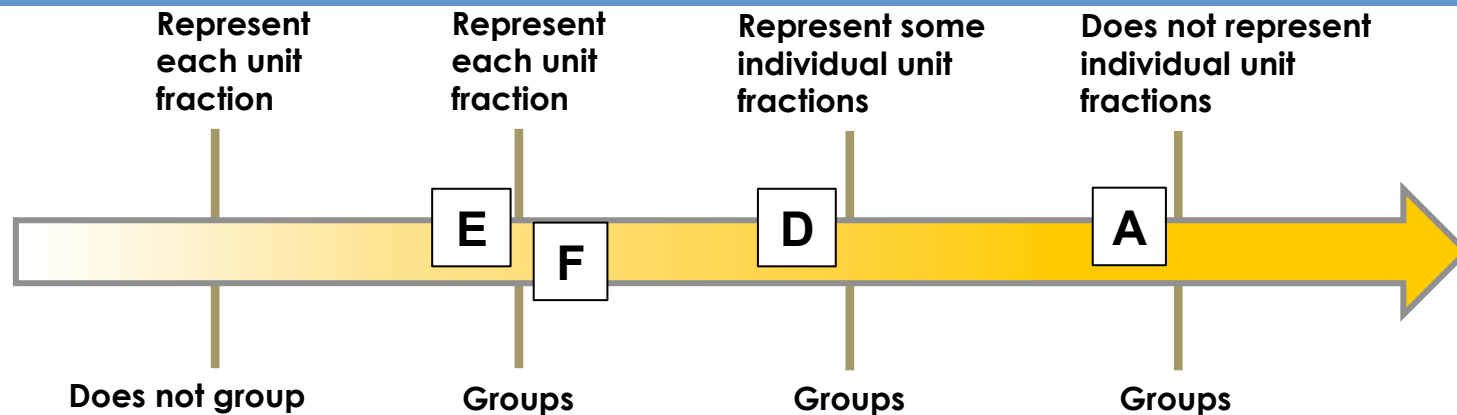
$$2\frac{2}{6} = 14 \text{ pizza}$$



The $\frac{1}{6}$ pound of cheese are not represented individually. They are grouped in a way that takes advantage of the relationship between $\frac{1}{6}$ and 1.

Example:
Strategy C

$\left(\frac{6}{6}\right)$ $\left(\frac{6}{6}\right)$ $\left(\frac{2}{6}\right)$
 $2\frac{2}{6}$ pounds of cheese



Strategy E

$$\begin{array}{cccccccccccc}
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
 \frac{1}{6} & \frac{2}{6} & \frac{3}{6} & \frac{4}{6} & \frac{5}{6} & \frac{6}{6} & \frac{1}{6} & \frac{2}{6} & \frac{3}{6} & \frac{4}{6} & \frac{5}{6} & \frac{6}{6}
 \end{array}$$

$$2\frac{13}{6} \quad 2\frac{14}{6} \quad 2\frac{1}{3} \text{ Pounds of cheese}$$

Strategy D

A chef is making 14 pizzas. Each pizza takes $\frac{1}{6}$ of a pound of cheese.
How much cheese will the chef need to make all of the pizzas?

$$\begin{array}{l}
 1 \text{ pizza} = \frac{1}{6} \\
 2 \text{ pizzas} = \frac{2}{6} \\
 3 \text{ pizzas} = \frac{3}{6} \\
 4 \text{ pizzas} = \frac{4}{6}
 \end{array}$$

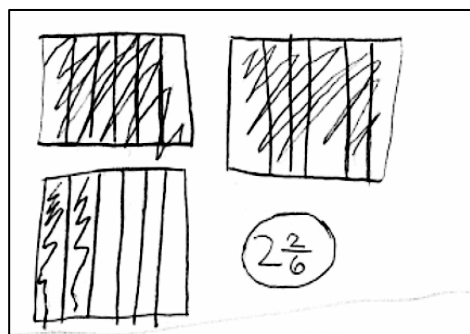
$$\begin{array}{l}
 5 \text{ pizzas} = \frac{5}{6} \\
 6 \text{ pizzas} = \frac{6}{6} = 1
 \end{array}$$

1 lb. of cheese

$$\times 2 = 2 \text{ lbs. of cheese}$$

2 $\frac{2}{6}$ lbs. of cheese needed

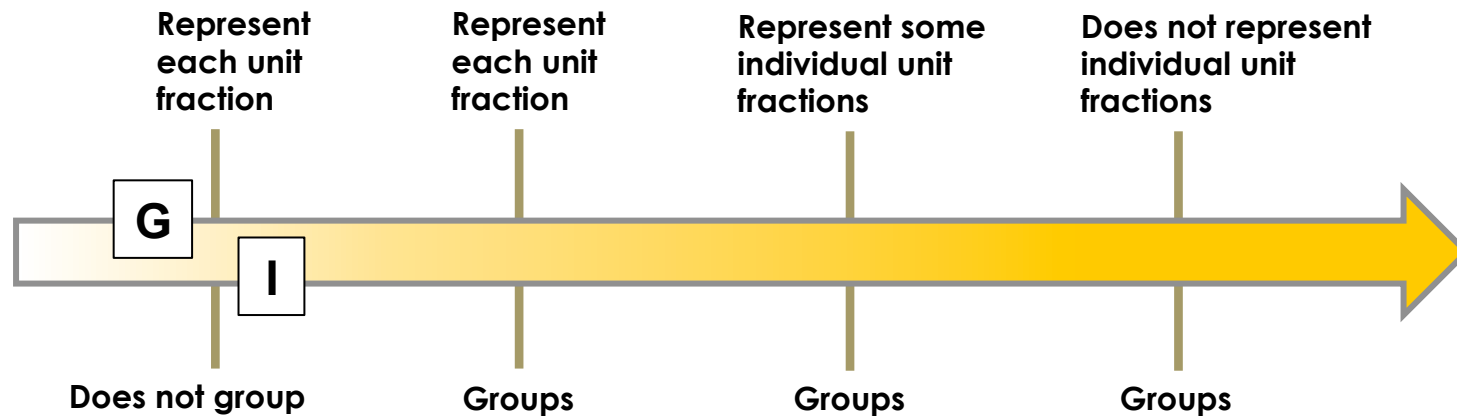
Strategy F



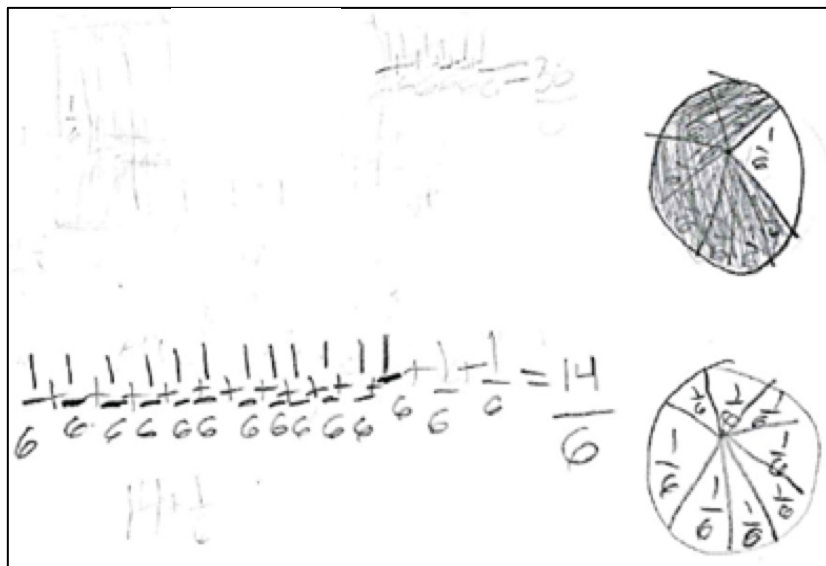
Strategy A

$$\begin{array}{l}
 1 \text{ lb} = 6 \text{ pizzas} \\
 2 \text{ lbs} = 12 \text{ pizzas} \\
 2 \frac{2}{6} \text{ lbs} = 14
 \end{array}$$

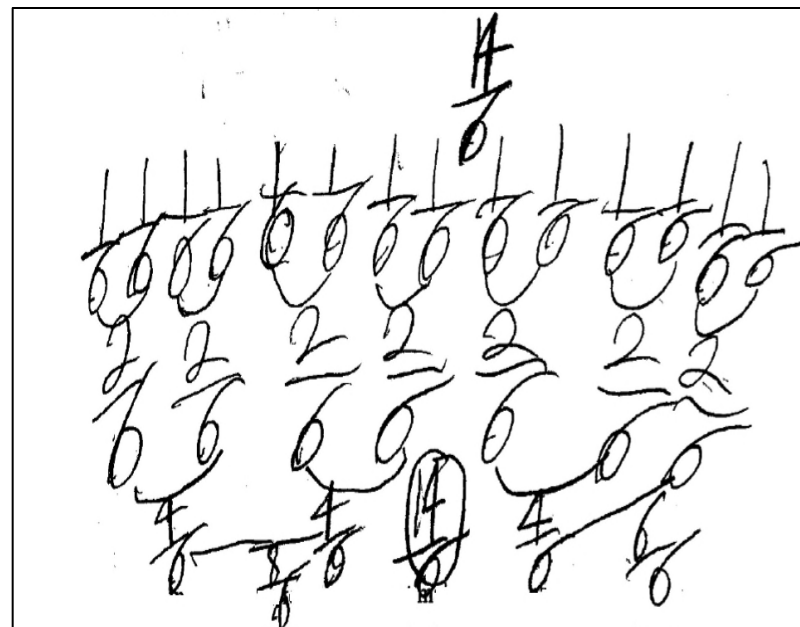
2 $\frac{2}{6}$ lbs



Strategy G



Strategy I



Teaching with story problems

- What is the role of story problems in the development of understanding fractions?

A chef is making 14 pizzas. Each pizza takes $\frac{1}{6}$ of a pound of cheese.
How much cheese will the chef need to make all of the pizzas?

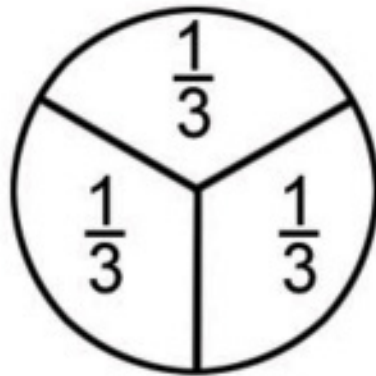
The diagram illustrates the addition of fractions to solve the problem. It shows two rows of circles, each representing a pizza. The top row contains 8 circles with the following fractions: $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{6}$ (labeled "1 whole"), $\frac{1}{6}$, and $\frac{2}{6}$. The bottom row contains 6 circles with the following fractions: $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{6}$ (labeled "1 whole"), $\frac{1}{6}$, and $\frac{2}{6}$ (labeled "2 $\frac{2}{6}$ ").

Instruction may include examples like this

Unit fractions and sharing

This pizza has been cut into three equal parts. We call these **thirds**. You can write one third as $\frac{1}{3}$.

The whole pizza cut into three equal parts gives us three thirds.



$$1 = \frac{3}{3}$$

Consider using story problems to address the same ideas

- Story problems engage children in problem solving
- Story problems help children develop understanding of the relationship between unit fractions and a whole



Examples of Story Problems

A chef is making 14 pizzas. Each pizza takes $\frac{1}{6}$ of a pound of cheese. How much cheese will the chef need to make all of the pizzas?

I have $2\frac{2}{6}$ pounds of cheese. If each pizza takes $\frac{1}{6}$ of a pound of cheese, how many pizzas can I make?

Big Ideas

- Importance of unit-fraction quantities in children's understanding
- Continuum of children's understanding of unit-fraction quantities
- Power of story problems to build children's understanding

Feel free to contact us for additional questions

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- Gladys Krause – gladyshrueda@utexas.edu
- Susan Empson – empsons@missouri.edu
- Handouts will be available on our project website
 - <http://soe.uncg.edu/rtem/>



Thank you!
Enjoy the rest of the conference!

