FRACTIONS ON A NUMBER LINE

Making Sense of Strategic Benchmarks

Debbie Monson, University of St. Thomas
Sue Ahrendt, University of Wisconsin-River Falls
Terry Wyberg, University of Minnesota

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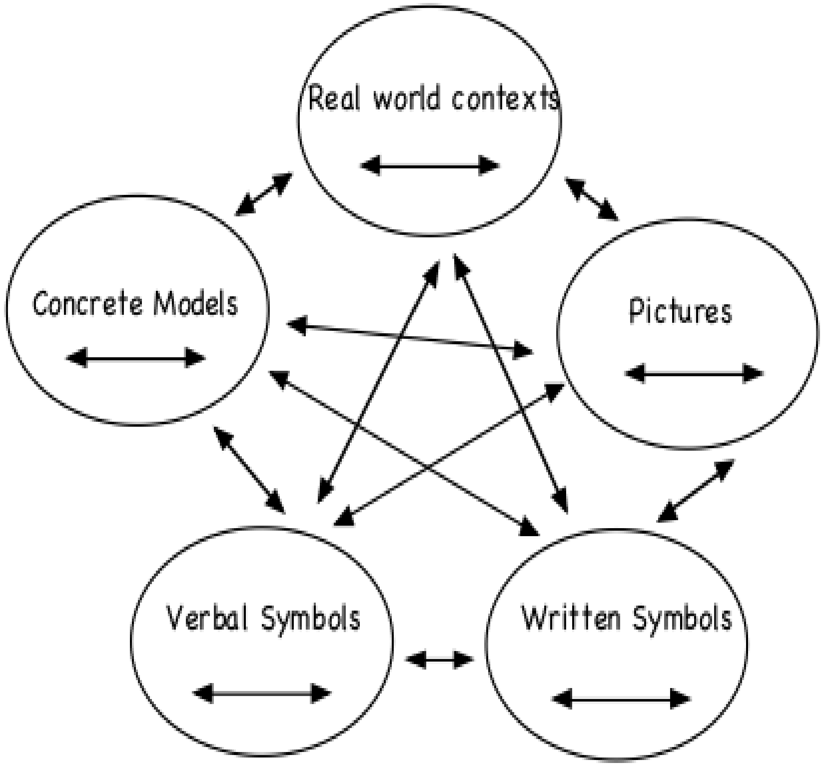
Rational Number Project

- Long term Research and Curriculum Development project
- Provided insights into students' thinking as they developed **initial ideas** about fractions and **operations** with fractions as well as instructional strategies to support student learning.
- Two curricula products: research-based fraction lessons for developing initial fraction ideas and fraction operations

Multiple Representations



Rational Number Project Insights

- RNP suggests that building mental images for fractions is an important reason for using appropriate models for fractions.
- Fraction circles have been found to build the strongest mental images for fractions. They support building an understanding of fraction as a number, its relative size and generalizations about fractions.
- Other models are important to solidify students' understanding of fractions as numbers.
- Paper folding and chips extend and enhance students' initial understanding of fractions.

Common Core Standard

- [CCSS.Math.Content.3.NF.A.2] Understand a fraction as a number on the number line; represent fractions on a number line diagram.
  - [CCSS.Math.Content.3.NF.A.2a] Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
  - [CCSS.Math.Content.3.NF.A.2b] Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

What do we know about number lines?

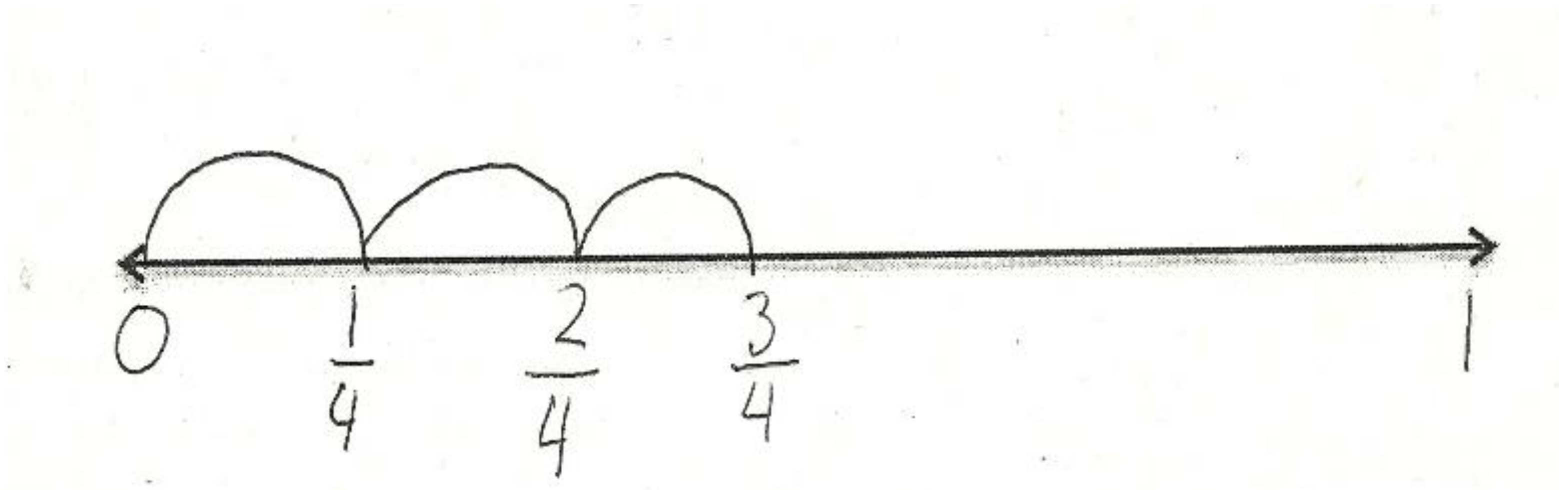
- Is it developmentally appropriate to ask third graders to place fractions on the number line?

Initial challenges Third Grade Students had when trying to make sense of fractions on a number line

- Identifying zero and one on the number line
- Identifying the unit
- Partitioning and iterating
- Ordering fractions
- Finding equivalence

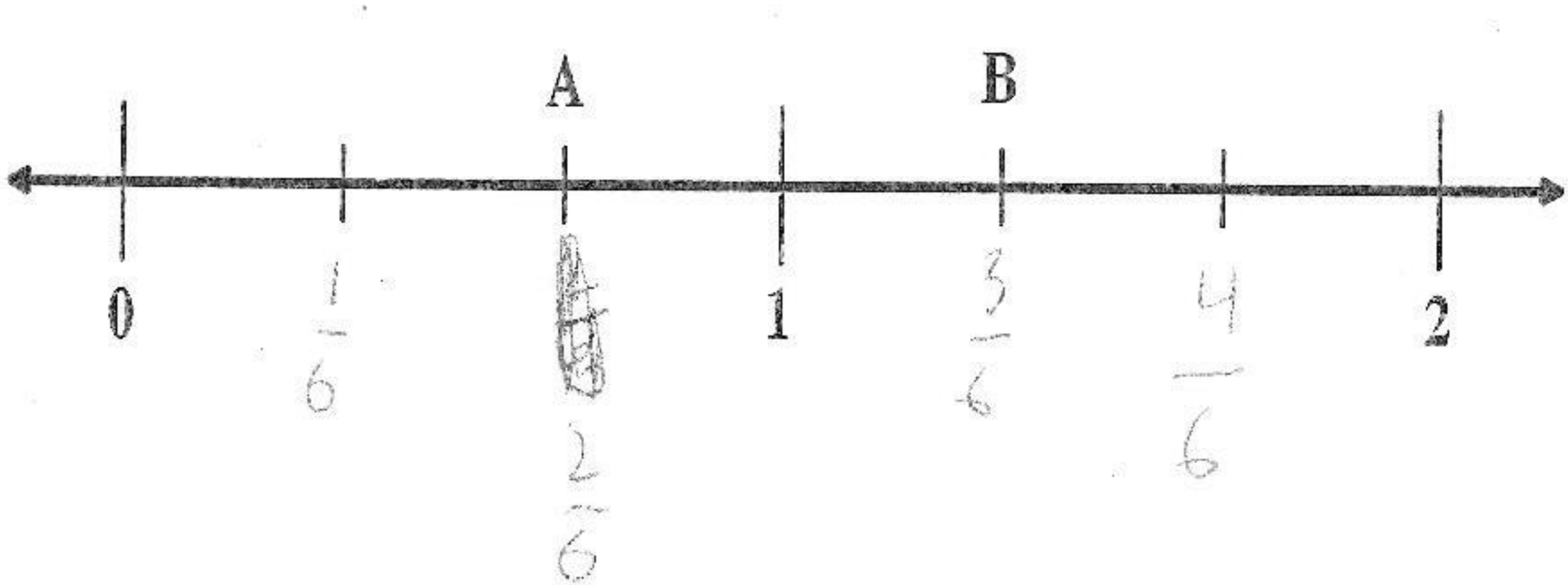
Unit Challenges

- Show ¾ given a blank number line



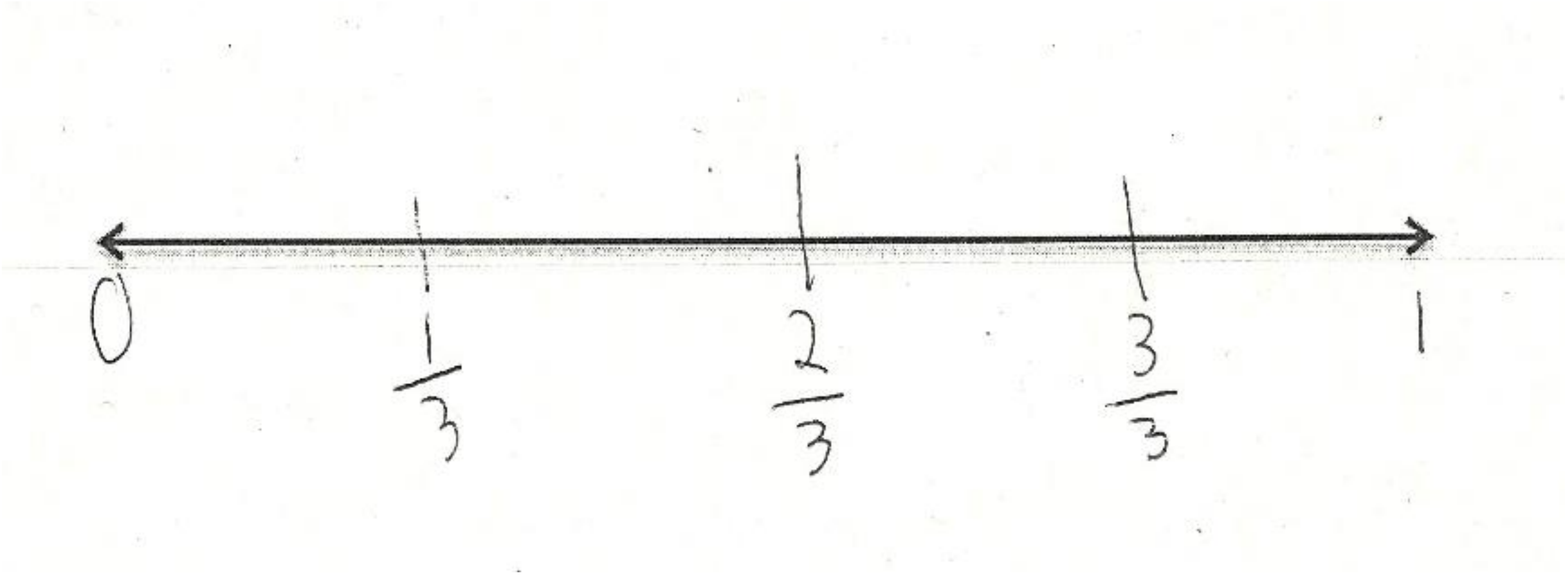
Unit Challenges

- What number names points A and B?



Where is zero? Where is one?

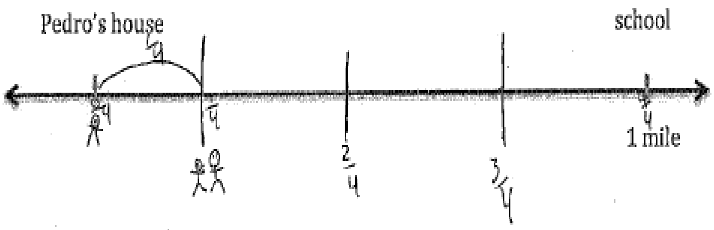
- Show 1/3, 2/3, and 3/3 on the number line



Successful 3rd grade students

- Make sense of the unit using this new model (see the need for zero and one on the number line)
- Implement a strategy for partitioning
- Interpret a fraction as a distance from zero and as a point
- Understand a fraction in relation to whole numbers
- Comfortable with common equivalences $\left(\frac{4}{4} = 1, \frac{2}{4} = \frac{1}{2}\right)$

Using Context



S: I think ½ is one-half of one mile and half of one-half is ¼ so that's how I know where ¼ is. And then if he walks that far he walks ¼ of a mile without his friend.

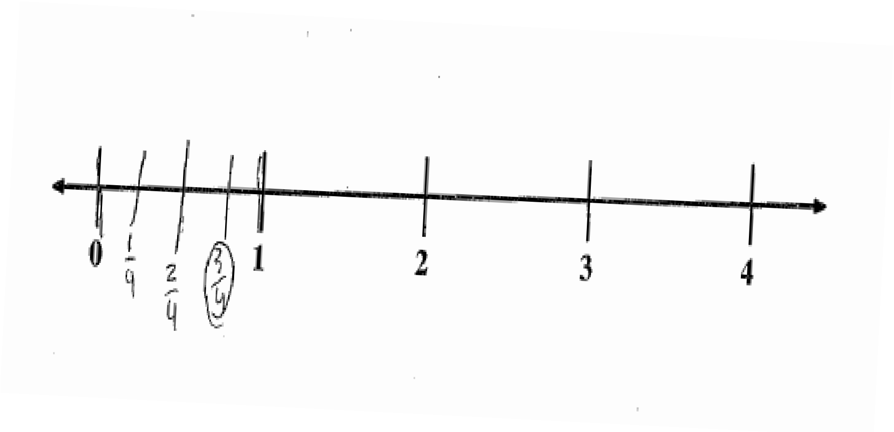
I: I understand exactly what you did. So you put a tick mark halfway in between and then you put another one half way in between that. How do you know that is ¼ (pointing to ¼ on the number line)?

S: Because that is ¼ and that's 2/4 which is equivalent to 1/2 and ¾ and 4/4 (pointing to tick marks on number line).

I: Great. And where is zero on this number line?

S: Right here. If he's at his house he hasn't walked anything to his school.

Where is ¾?

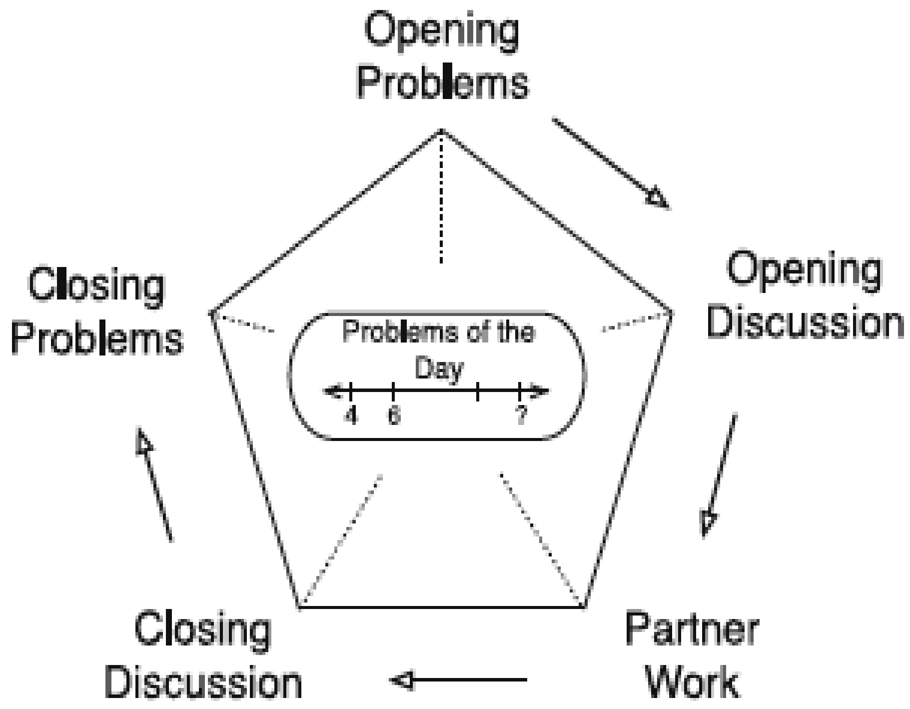


Well, because that's one, it can't get past that. Cuz it is not one and something or it's not, the numerator is not above the denominator. Or they are not the same. So 3/4 has to be between zero and one.

Next Two Phases of Teaching Experiment

- Fourth graders have experienced RNP for the initial fraction lessons taught by classroom teachers
- Number Line lessons taught by researchers

Lesson Structure (adapted from Saxe et al, 2013)



Saxe, G. B., Diakow, R., & Gearhart, M. (2013). Towards curricular coherence in integers and fractions: A study of the efficacy of a lesson sequence that uses the number line as the principal representational context. ZDM, 45(3), 343–364.

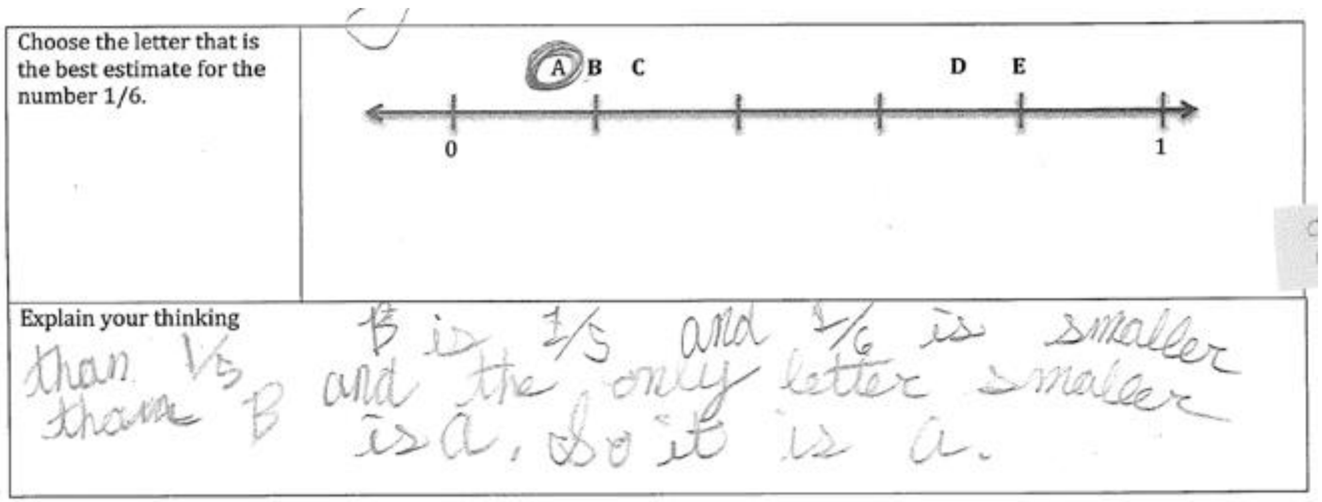
Lesson Topics with Number Line

- Order
- Equivalence
- Translations
- Reconstructing the Unit

Prior Knowledge

- Fluidity with notation connected with a variety of models
- Mental images of the magnitude of fractions (order)
- Flexibility of benchmarks of ½ and 1
- Equivlance: Connect that ¾ = 6/8

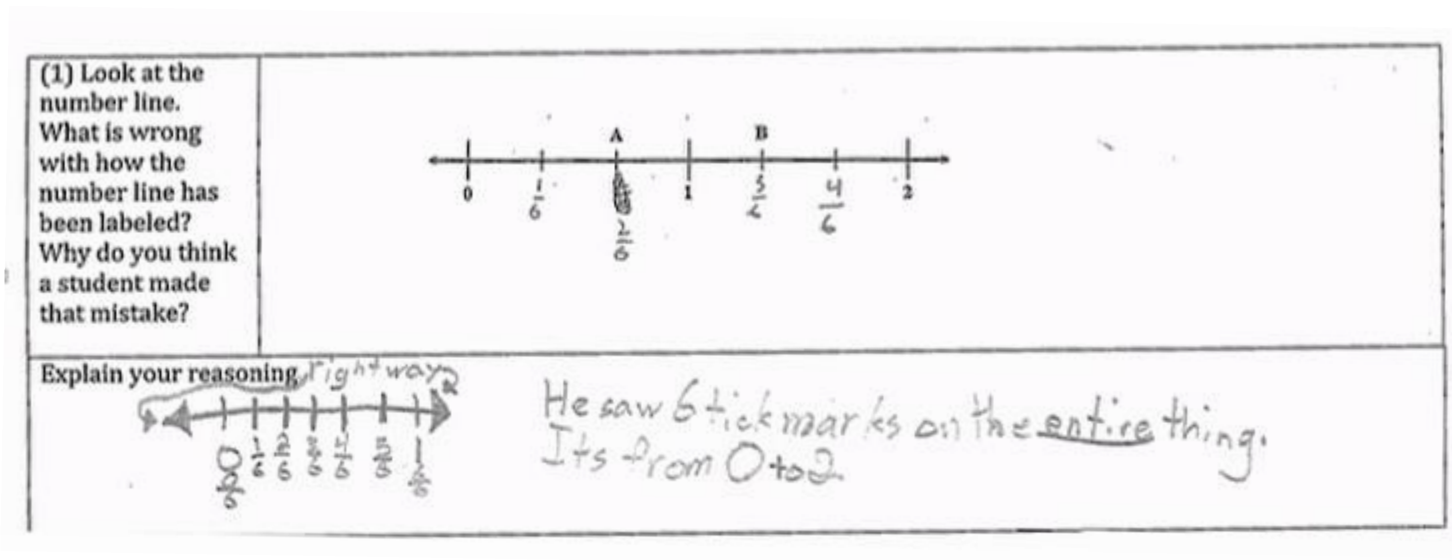
Order
Student Work Sample1


Choose the letter that is the best estimate for the number 1/6.

Explain your thinking

B is 1/5 and 1/6 is smaller
than 1/5 and the only letter smaller
than B is a, so it is a.

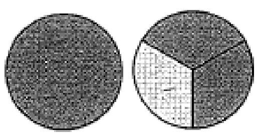
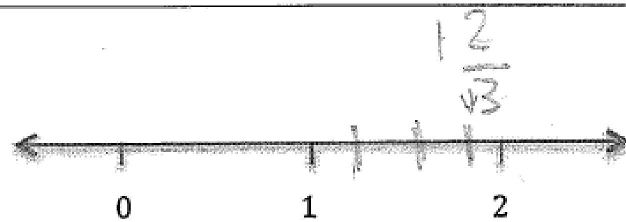
Unit
Student Work Sample 2


(1) Look at the number line. What is wrong with how the number line has been labeled? Why do you think a student made that mistake?

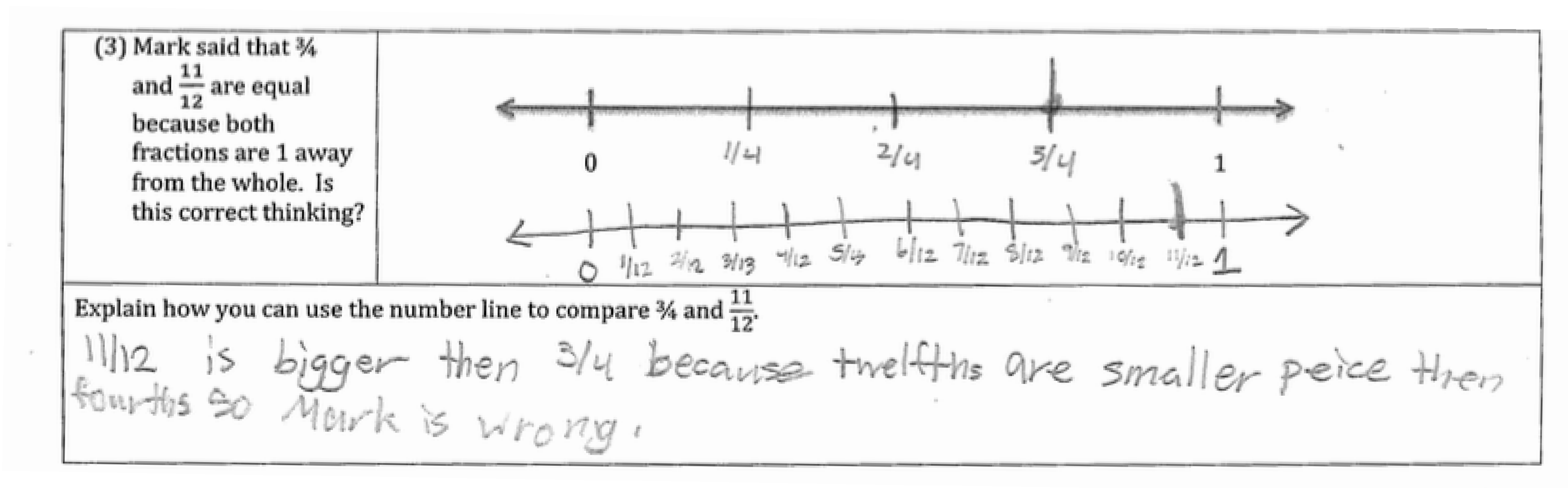
Explain your reasoning right way

He saw 6 tick marks on the entire thing. Its from 0 to 2.

Translations
Student Work Sample 3

If the whole circle is the unit, what fraction is shown? Show that fraction on the number line. 	
Explain your reasoning	 $= 1 +$  $= \frac{2}{3} = 1\frac{2}{3}$

Order
Student Work Sample 4



Equivalence
Student Work Sample 5

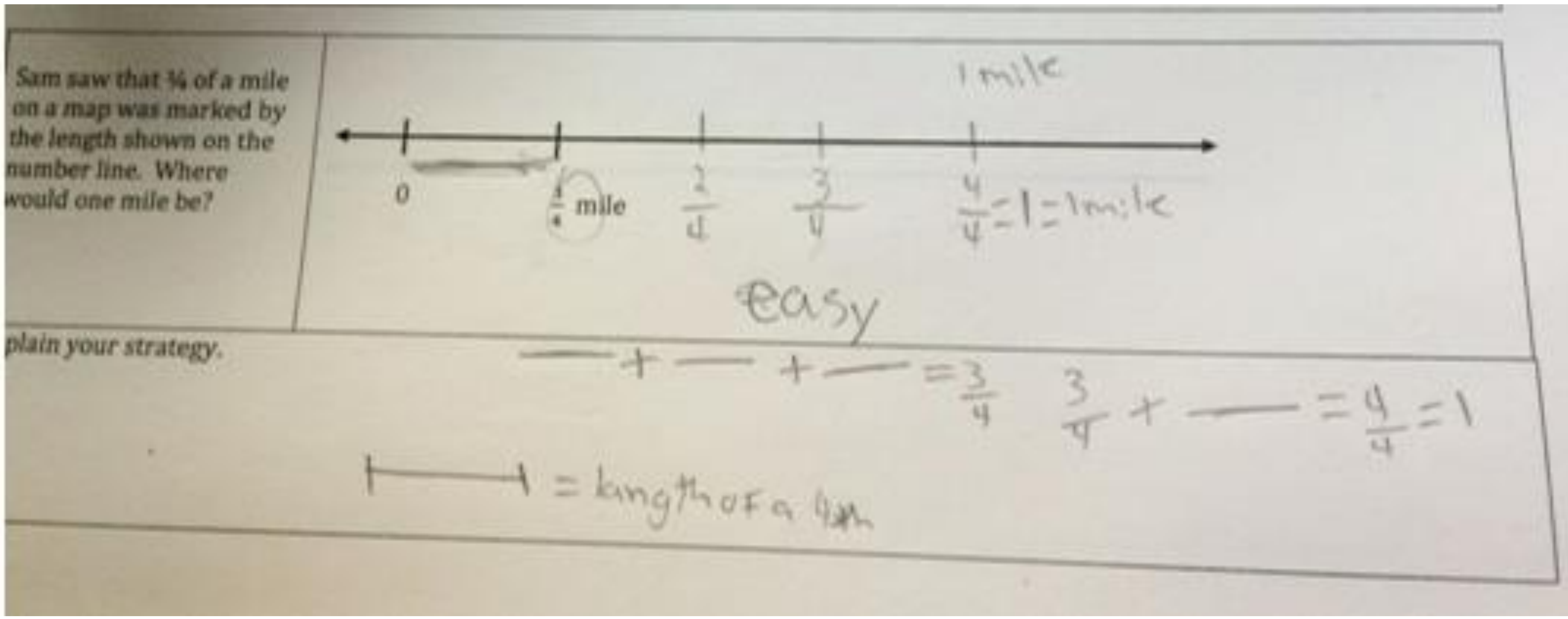
4. How many 6ths equal 3/2? Use the number line to answer this question.

$\frac{9}{6}$!

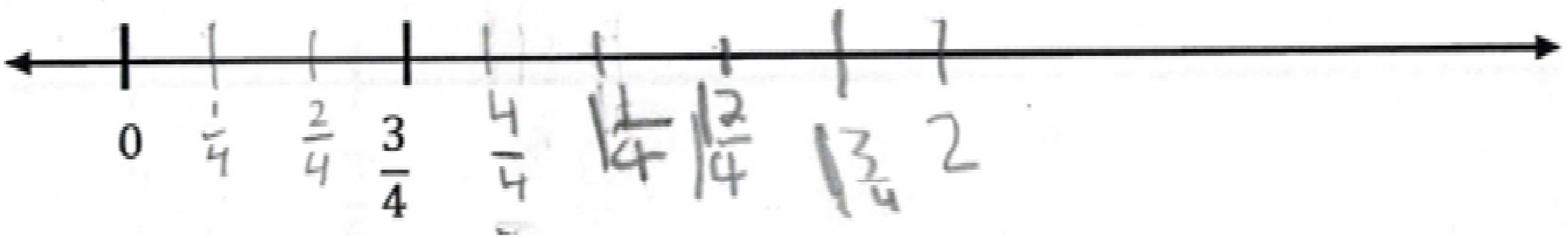
Explain you used the number line to find how many ~~8~~ 6ths equal 3/2.

first I Put $\frac{3}{2}$ on the number line then I made each side into 6ths and I Counted and the Paint that was $\frac{3}{2}$ was also $\frac{9}{6}$

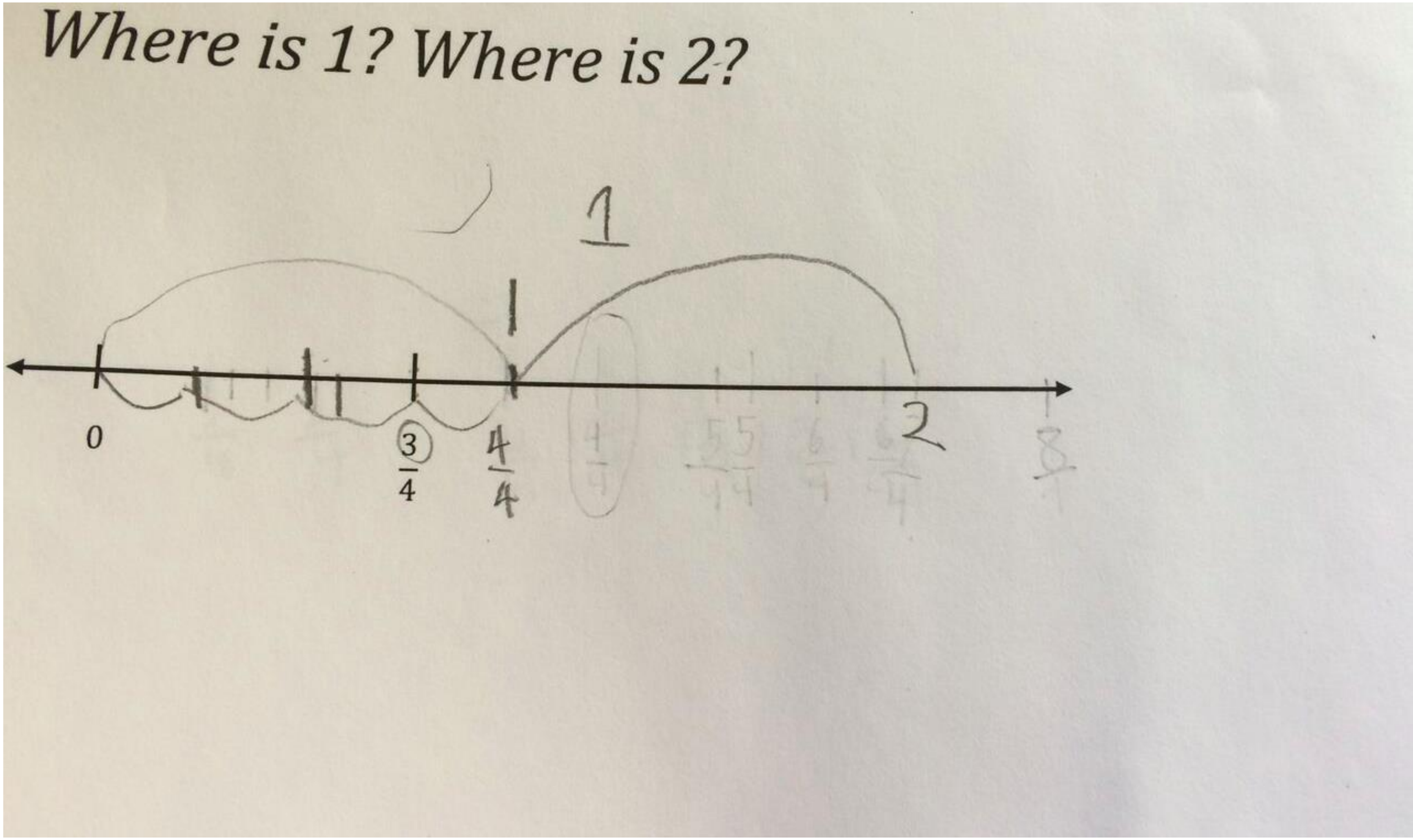
Easing into Reconstructing the Unit
Blank Problem 1



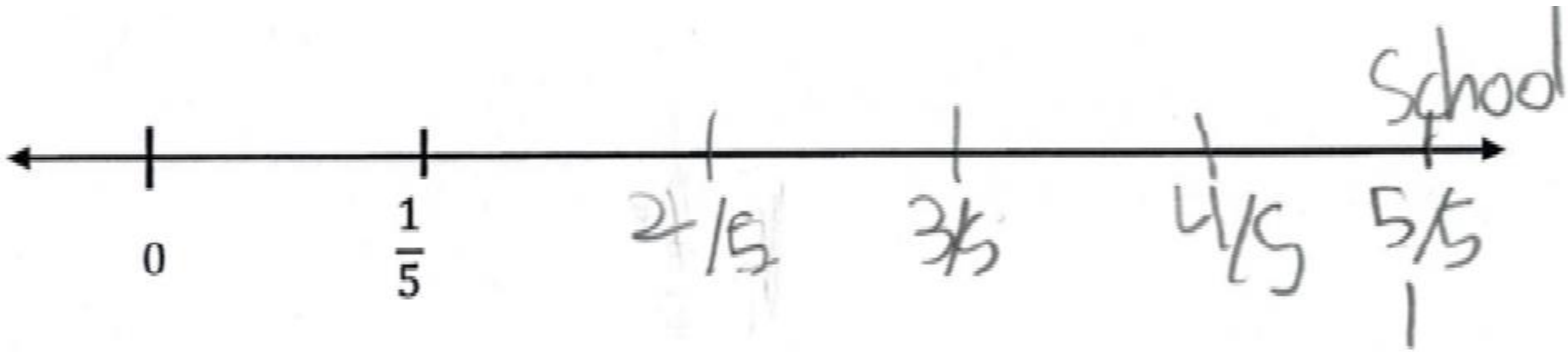
Making Sense of the numerator/denominator Blank Problem 2



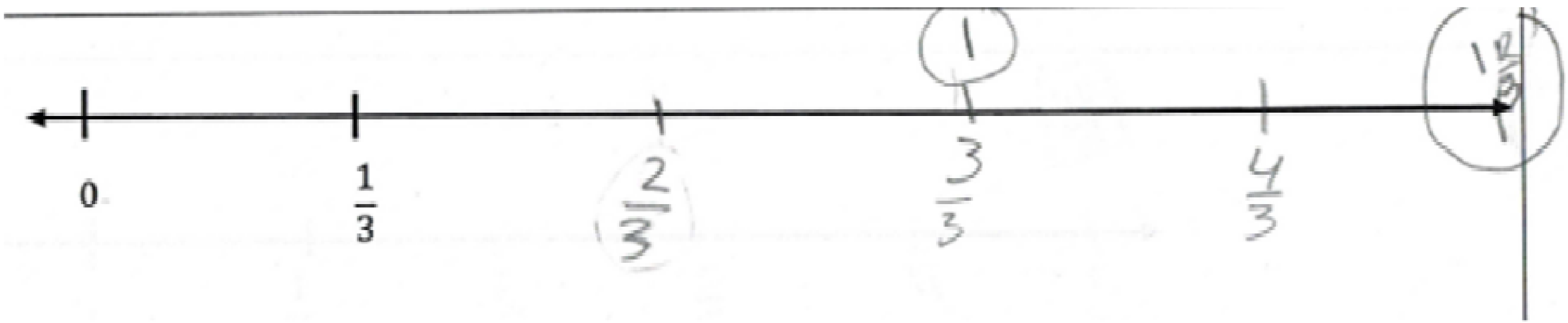
Flexibility in thinking
Blank Problem 2



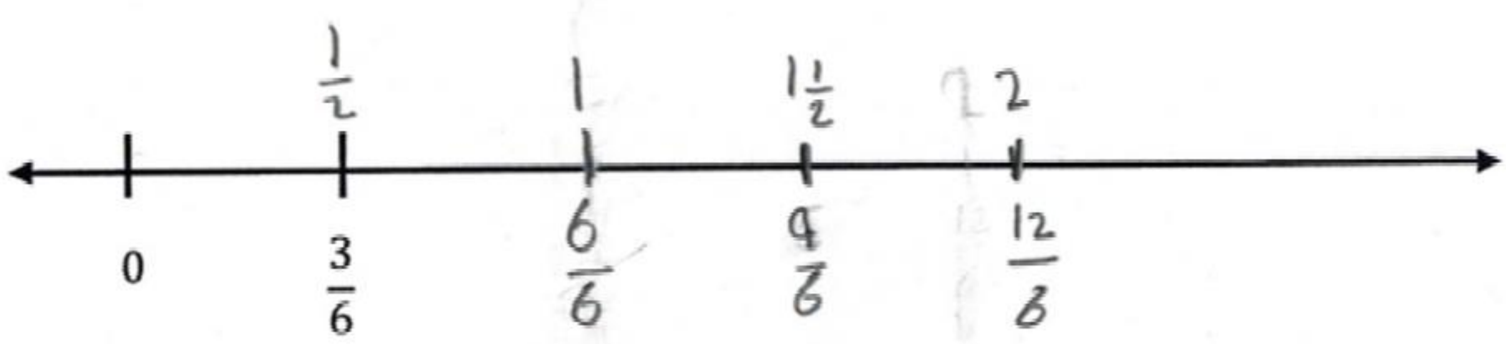
Equivalence understanding 5/5=1
Blank Problem 3

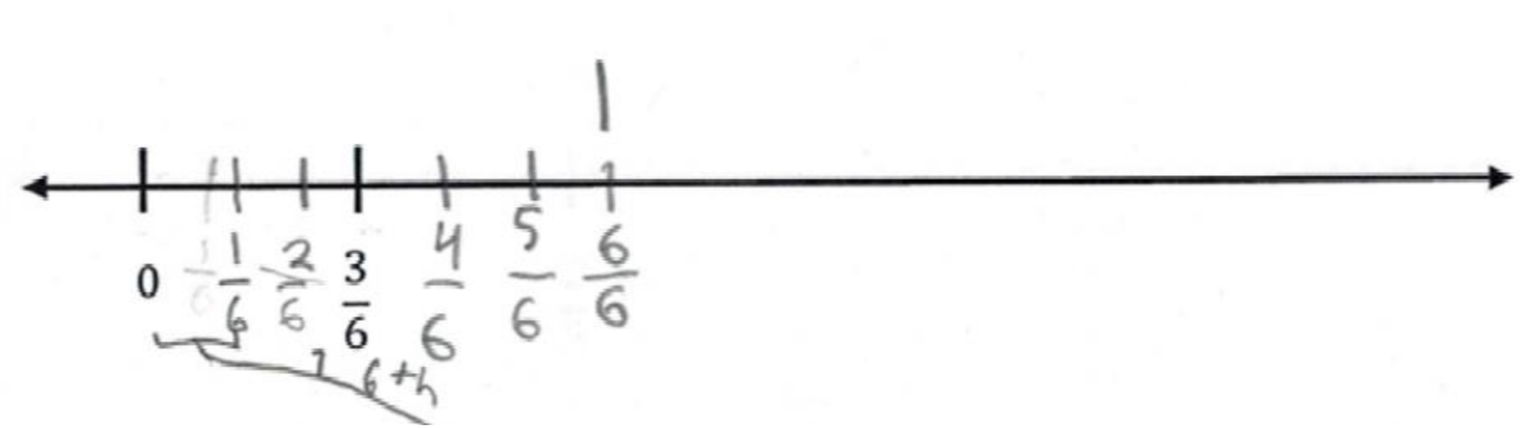


Measuring/Iterating Strategy

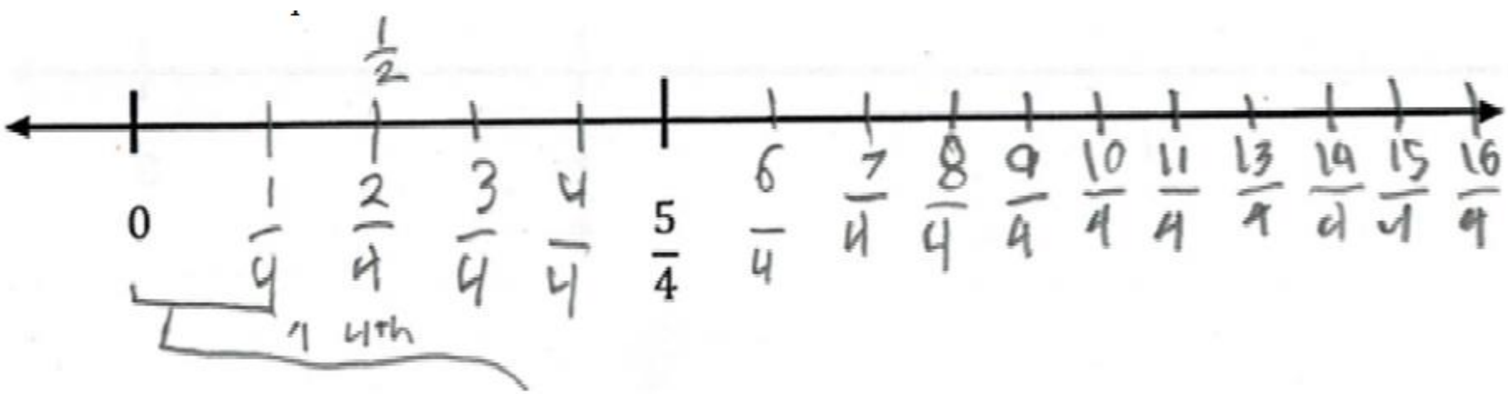


Strategies had different levels of sophistocation



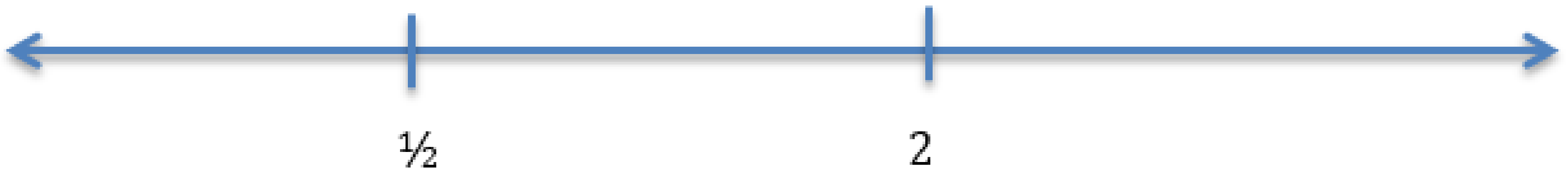


Fractions greater than 1



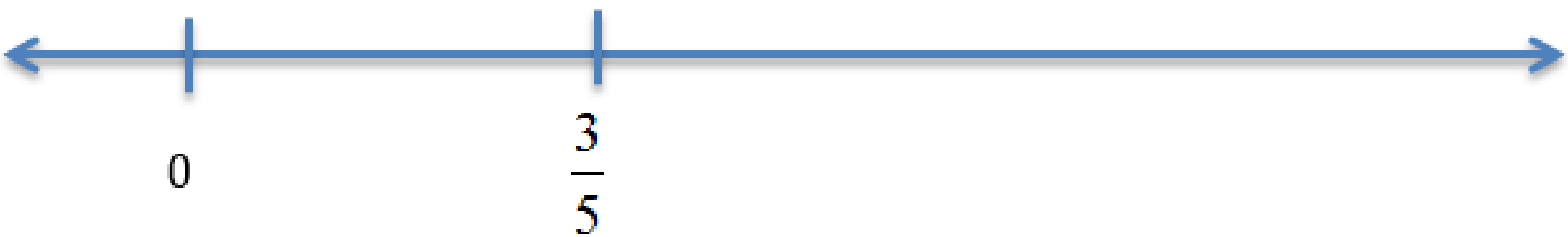
How would you do this one?
Blank Problem 4

Where are 1 and 0 on this number line?



How would you do this one?
Blank Problem 5

Show where 1 1/5 is on the number line. Explain what you are doing.



Strategic Benchmarks

- Equivalence…naming a point in more than one way
- Ability to identify zero and one
- Iterating equal distances (Move from counting tick marks to counting distances)
- Imagining tick marks that aren’t there and ignoring tick marks that are there
- Flexibility of unit (4/3 problem)
- Recognizing the role of numerator and denominator