

High Impact Games and Meaningful Mathematical Dialog

**Teaching Integrated Math and Science Project
University of Illinois at Chicago**



Elizabeth Cape
Jennifer Leimberer
Sandra Niemiera

UIC



www.mathtrailblazers.uic.edu



High-Impact Games and Meaningful Mathematical Dialog

Welcome!

Elizabeth Cape

Math Consultant

Teaching Integrated Math and Science Project, Metro Chicago Mathematics Initiative
Learning Sciences Research Institute, University of Illinois at Chicago

“Mastery can best be achieved by purposeful, meaningful inquiry-based instruction - instruction that promotes number sense.”

-Arthur J. Baroody

Why Children Have Difficulties Mastering the Basic Number Combinations & How to Help Them

“...Research evidence points in one direction: The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense.

-Jo Boaler

Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts

“Decades of drill and timed testing have failed our students, often leading to a lack of fluency and a negative disposition toward mathematics.”

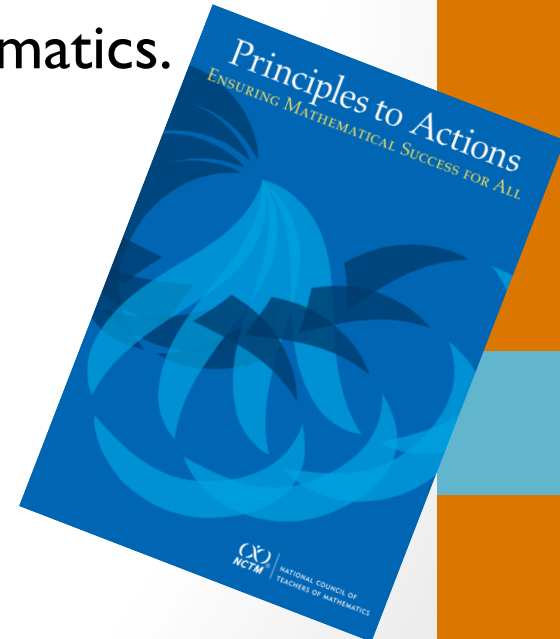
Gina Kling and Jennifer M. Bay-Williams

Three Steps to Mastering Multiplication Facts

Mathematics Teaching Practices

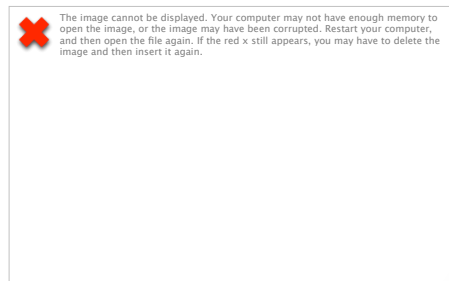
- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

-NCTM's *Principles to Actions*



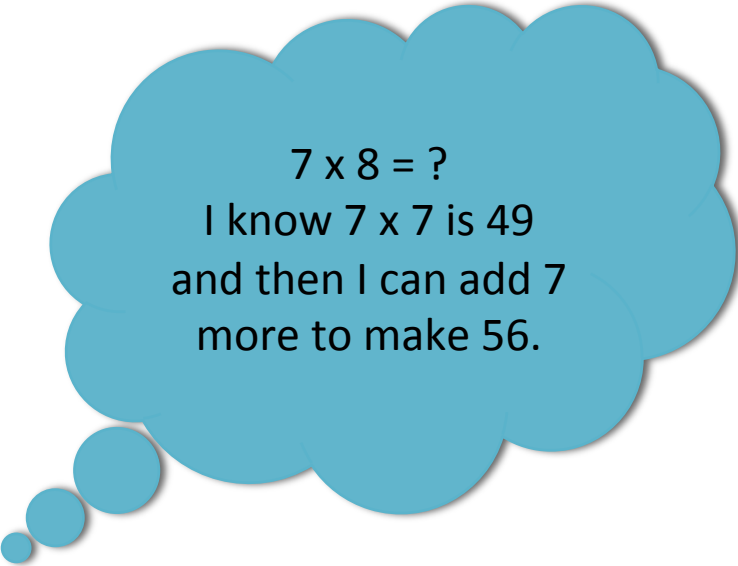
Session Goals

- Articulate a problem solving and number sense approach to multiplication.
- Discuss how to use games to develop models, strategies, and flexibility with numbers and operations.
- Learn how to analyze games for cognitive demand and select those of high impact to build procedural fluency from conceptual understanding.
- Explore ways to facilitate meaningful mathematical discourse during and after game play to highlight strategies and misconceptions, make connections, and support students as they learn.

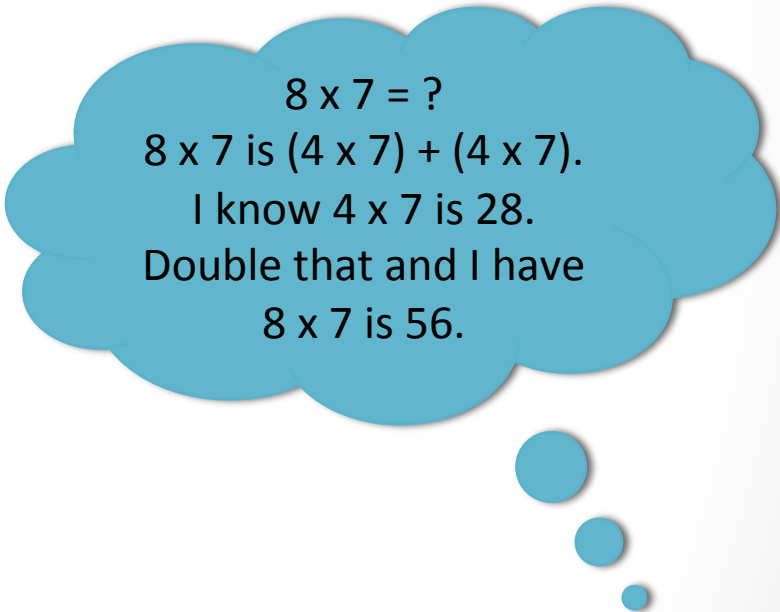


What is number sense?

- Research tells us that the best mathematics classrooms are those in which students learn number facts and number sense through **engaging activities that focus on mathematical understanding rather than rote memorization.**


$$7 \times 8 = ?$$

I know 7×7 is 49
and then I can add 7
more to make 56.


$$8 \times 7 = ?$$

8×7 is $(4 \times 7) + (4 \times 7)$.

I know 4×7 is 28.

Double that and I have

8×7 is 56.

What do we mean by fluency?

Procedural fluency is **skill in carrying out procedures:**



-National Research Council
-CCSSM, 2010

Fluency is

- Fluency comes about when students develop **number sense** - when they are mathematically confident because they **understand numbers**.
- Students have *mastered* their facts when they are so fluent at applying their strategies that they do so *automatically*.

-Gina Kling and Jennifer M. Bay-Williams
Three Steps to Mastering Multiplication Facts

Students can apply strategies, develop number sense, and learn and remember math facts as they play games.

How do students learn facts?

Phase 1: Modeling and/or counting to find the answer

- Solving 6×4 by drawing 6 groups of 4 dots and skip counting the dots



Phase 2: Deriving answers using reasoning strategies based on known facts

- Solving 6×4 by thinking $5 \times 4 = 20$ and adding one more group of 4



Phase 3: Mastery (efficient production of answers)

- Knowing that $6 \times 4 = 24$

-Arthur J. Baroody

Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them

Multiplication Facts Strategies

The authors suggest this sequence and these strategies for teaching children fluency, flexibility, and reasoning strategies with multiplication facts.

Sequence and strategies for teaching multiplication facts

Foundational facts*	
1. 2s, 5s, and 10s (begin these late in second grade)	Use story problems, arrays, skip counting, and patterns on a hundred chart and a multiplication table to learn these facts.
2. 0s* 1's, multiplication squares (2×2 , 3×3 , etc.)	

Derived fact strategies

3. Adding or subtracting a group

Start with a nearby 2s, 5s, or 10s fact, then subtract (or add) the group.

Example: I don't know 9×6 , so I think " $10 \times 6 = 60$ " and subtract one group of 6 to get 54.

4. Halving and doubling

Look for an even factor. Find the fact for half of that factor, then double it.

Example: I don't know 6×8 , so I think " $3 \times 8 = 24$ " and double that to get 48.

5. Using a square product

Look for a nearby square. Find that fact and add on or subtract off the extra group.

Example: I don't know 7×6 . I use $6 \times 6 = 36$ and add one more 6 to get 42.

6. Decomposing a factor

Partition one of the factors into a convenient sum of known facts, find the two known facts, and combine the products.

Example: I don't know 7×6 . I break the 7 into 2 and 5, because I know 2×6 and 5×6 . Then I add 12 and 30 to get 42.

Why games?

Engaging

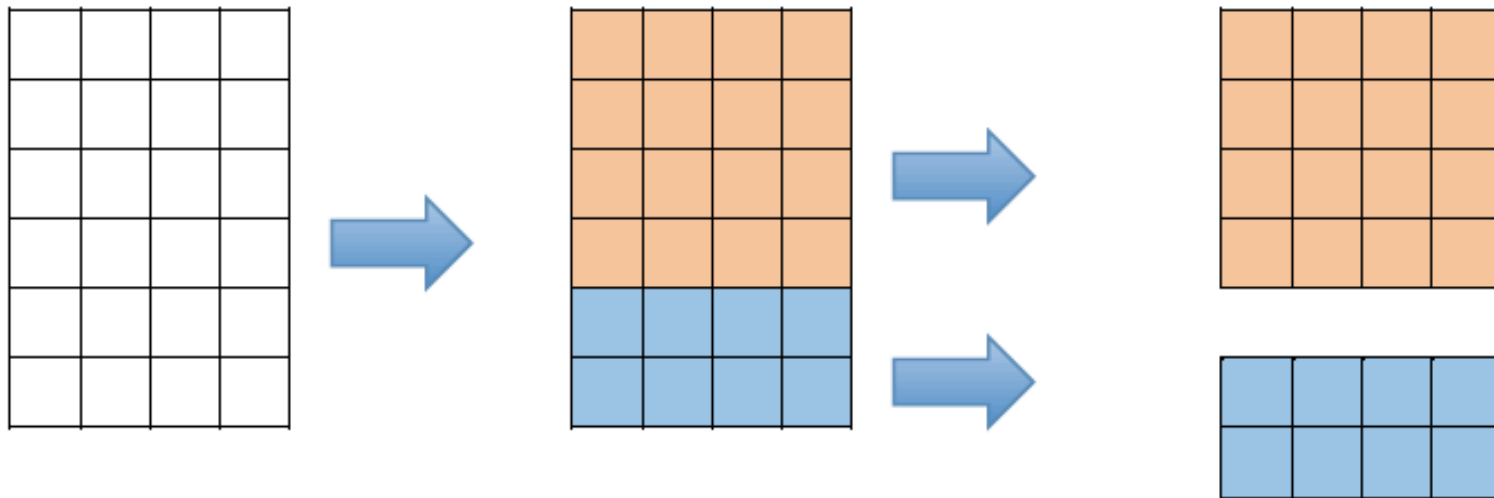


Time spent
reasoning
and problem
solving

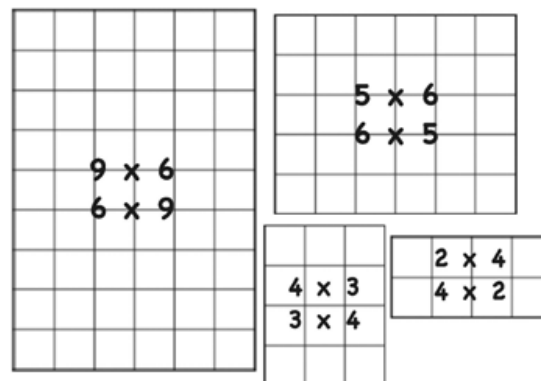
Low
stress

Cover It (p. 3)

(b) Using an array representation to decompose the fact 6×4

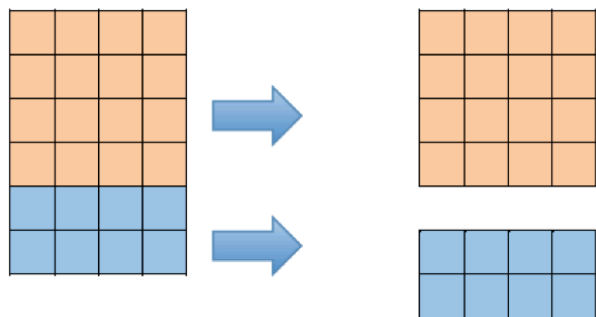


I can split my 6×4 array into two smaller arrays, one that is 4×4 and one that is 2×4 . I know that $4 \times 4 = 16$ and $2 \times 4 = 8$. I then add the smaller products of 16 and 8 and get 24 for my answer.



Meaningful Practice

“...meaningful practice involves helping students learn their facts through rich, engaging mathematical activities that provide the additional benefits of promoting **problem solving, reasoning, and communicating mathematical thinking.**”



$$6 \times 4 = (4 \times 4) + (2 \times 4)$$

From Gina Kling and Jennifer M. Bay-Williams
Three Steps to Mastering Multiplication Facts

Incorporate meaningful practice that

- ✓ does not feel like mindless, repetitive exercises
- ✓ presents novel problems for students to solve
- ✓ requires the application of what students have learned in new ways
- ✓ stretches students to the next level

2 Flashcard Activities

Around the World

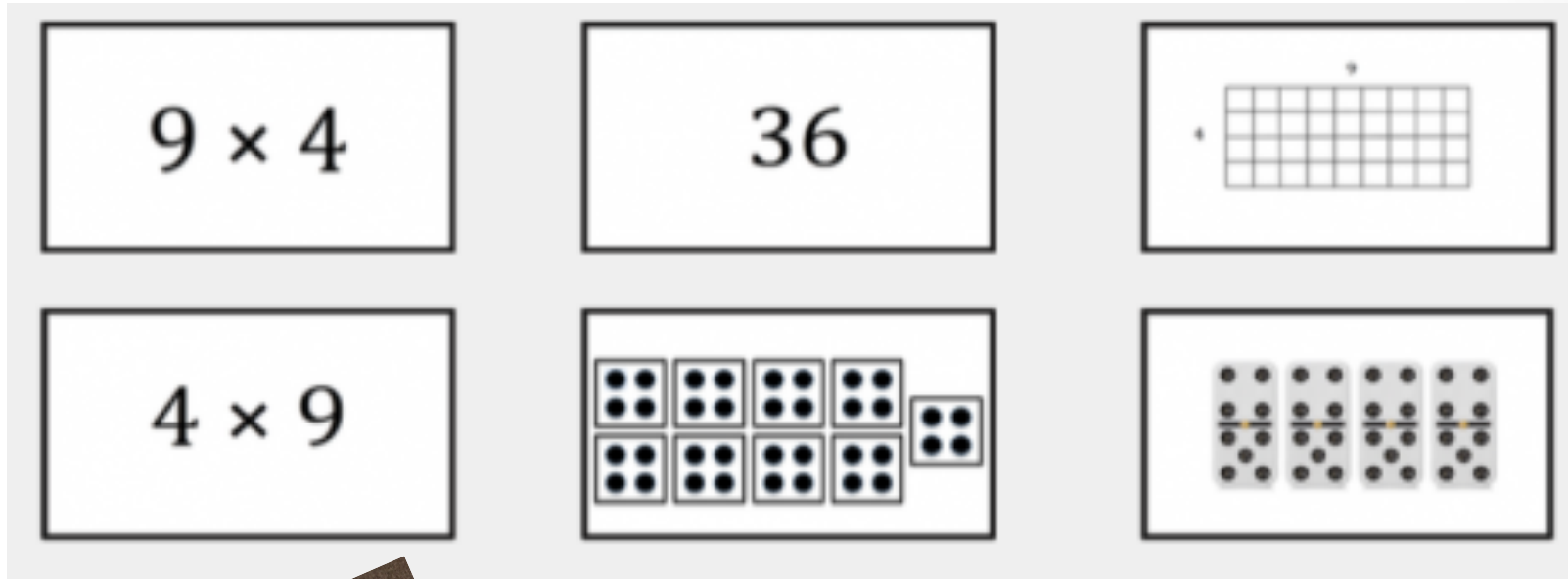
- Stand in a circle.
- One player stands next to another player.
- These 2 players call out the answer to a displayed math fact problem as quickly as possible.
- The first player to correctly answer moves around the world to the next player.



$$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times 8 \\ \hline \end{array}$$

Math Cards promote student understanding of facts. (p. 9)



Which game...

- used models? How?
- was less stressful? Why?
- was more engaging for every player? Why?
- fostered flexible thinking? How?
- encouraged reasoning and problem solving?
How?
- provided opportunity for dialog?

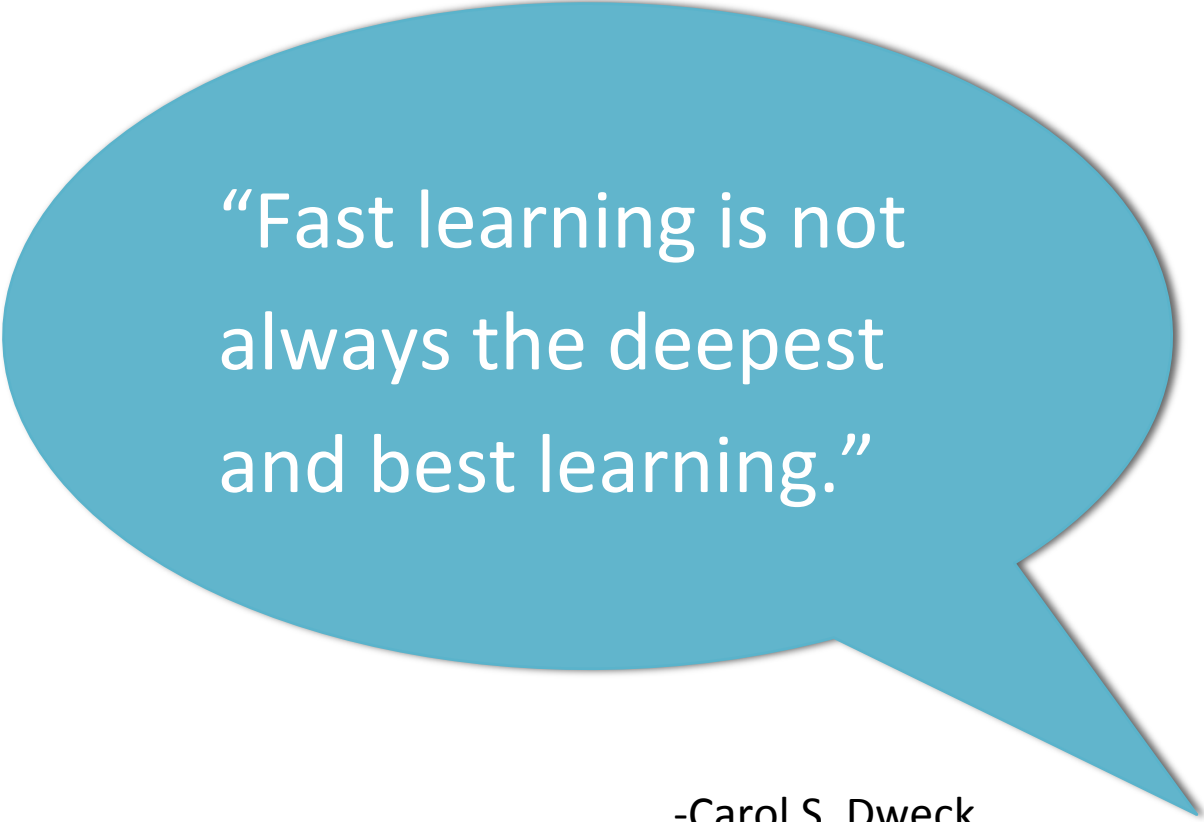
Game Analysis Tool (p. 13)

Game Analysis Tool

Name of Game
Math Involved in the Game
This game supports students' development of (circle all that apply): models informal strategies counting strategies reasoning strategies flexibility connections recall

1. What strategies do you anticipate your students will use when playing the game?
2. Where do you anticipate your students will be successful?
3. Where do you anticipate they will be challenged?
4. What questions would you ask as the game progresses to promote conceptual development?

Foster a “Growth Mindset”

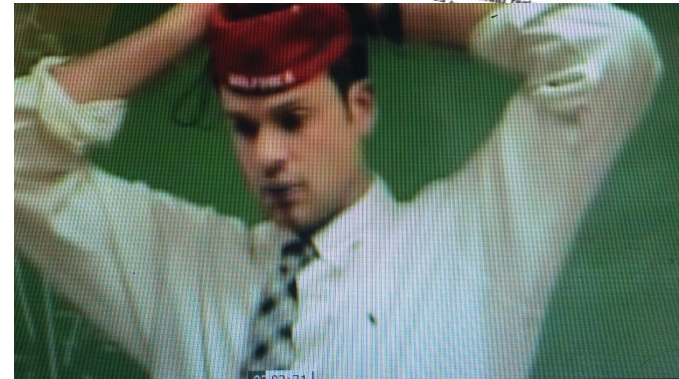


“Fast learning is not
always the deepest
and best learning.”

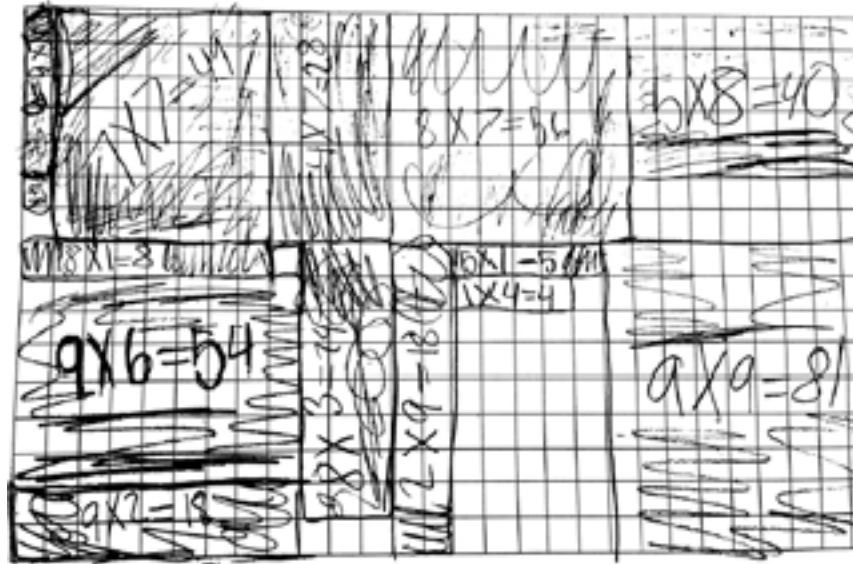
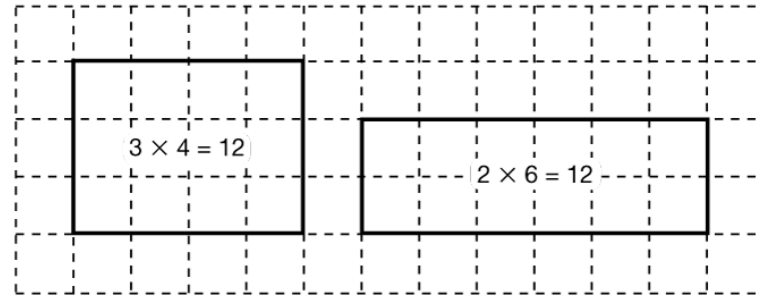
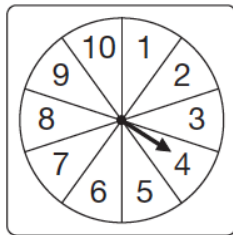
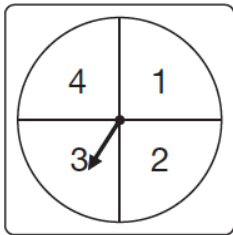
-Carol S. Dweck
Even Geniuses Work Hard

Dr. Jo Boaler on Timed Tests:

- Cause math fear, stress, & early onset math anxiety
- Can lead to low achievement, math avoidance, & lifelong negative math experiences
- Mislead students to believe that:
 - they are to perform in math rather than enjoy and learn
 - a good math performance is a fast math performance
 - memorizing facts is the most important part of math



Floor Tiler



Let's play! (pp. 4 – 8)



- The first player makes two spins so that he or she has two numbers. The player may either spin one spinner twice or spin each spinner once.
- The player must then find the **product** of the two numbers he or she spun.
- The player colors in a rectangle with that number of grid squares on the grid paper. He or she can use any two factors of the product to make a rectangle, not just the facts on the spinner.
- Once the player has made his or her rectangle, the player draws an outline around it and writes its number sentence inside.
- Players take turns spinning and filling in their grids.
- If a player is unable to fill in a rectangle for his or her spin, that player loses the turn, and the next player takes a turn.
- The first player to completely fill in his or her grid paper wins the game.

As we promote computational fluency, we must also

foster
conceptual
understanding

help develop
reasoning and
problem-
solving skills

encourage
hard work and
perseverance

Phase 1: Modeling and/or counting to find the answer

- Solving 6×4 by drawing 6 groups of 4 dots and skip counting the dots



Phase 2: Deriving answers using reasoning strategies based on known facts

- Solving 6×4 by thinking $5 \times 4 = 20$ and adding one more group of 4



Phase 3: Mastery (efficient production of answers)

- Knowing that $6 \times 4 = 24$

Post Game Re-engagement: Meaningful Mathematical Discourse

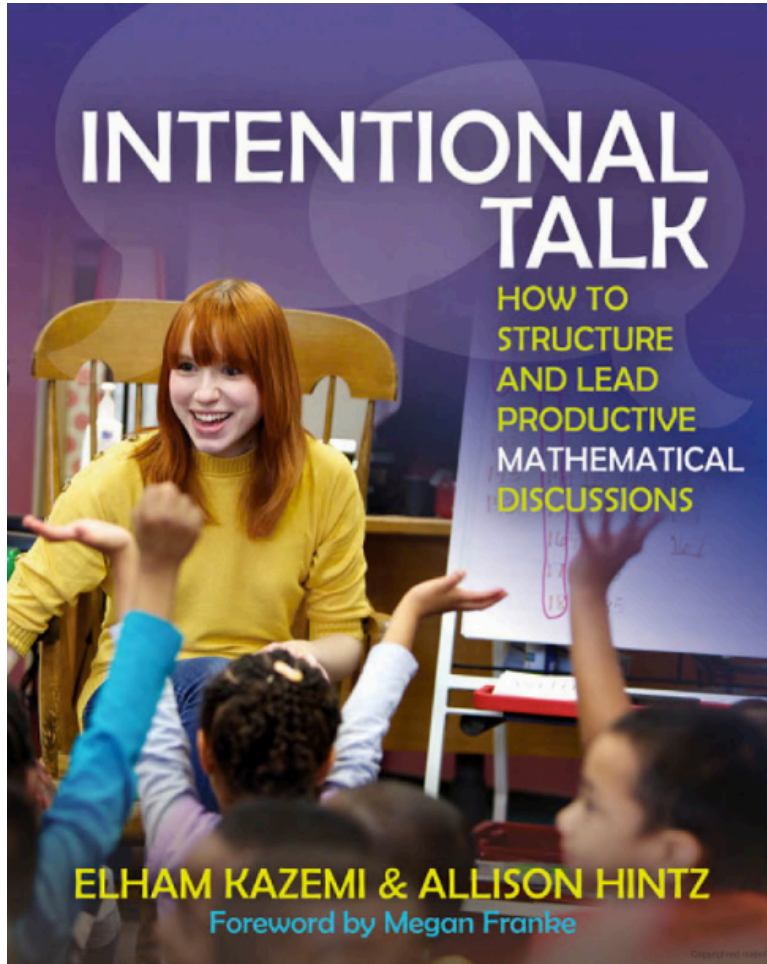
- ✓ Revisit student thinking
- ✓ Address conceptual understanding
- ✓ Examine task from different perspectives
- ✓ Critique student approaches/solutions to make connections
- ✓ Engage entire class in the mathematics at high cognitive level

Meaningful Mathematical Discourse

“Discourse is the mathematical communication that occurs in a classroom. Effective discourse happens when students articulate their own ideas and seriously consider their peers’ mathematical perspectives as a way to construct mathematical understandings.”

— National Council of Teachers of Mathematics
(2010).

4 Principles



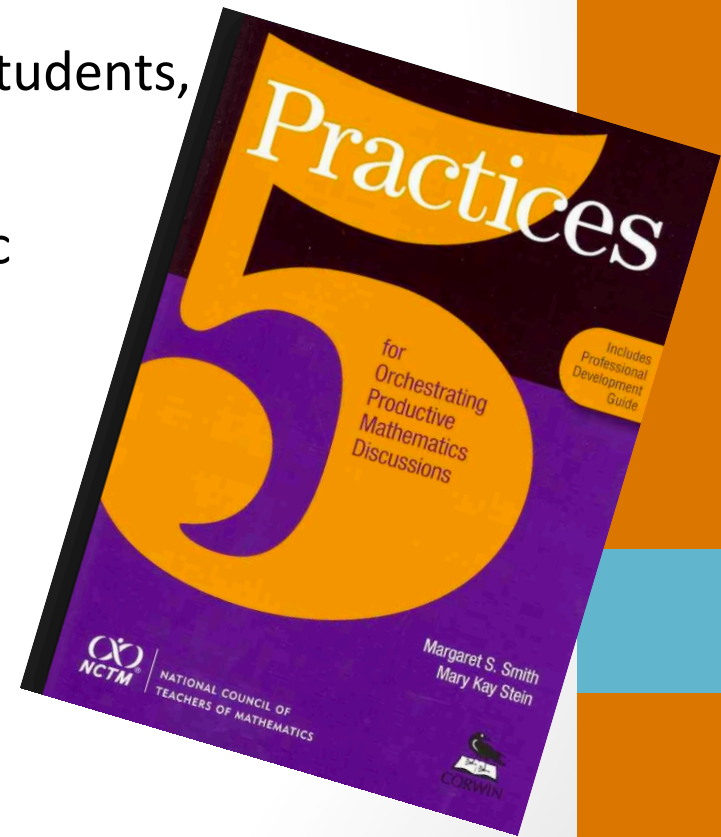
- ✓ 1. Discussions should achieve a mathematical goal
- ✓ (Put them all here?)

What is your goal?

Targeted Discussion Structure	Goal
Compare and Connect	To compare similarities and differences among strategies
Why? Let's Justify	To generate justifications for why a particular mathematical strategy works
What's Best and Why?	To determine a best (most efficient) solution strategy in a particular circumstance
Define and Clarify	To define and discuss appropriate ways to use mathematical models, tools, vocabulary, or notation
Troubleshoot and Revise	To reason through which strategy produces a correct solution or figure out where a strategy went awry

Orchestrating Productive Mathematical Discourse

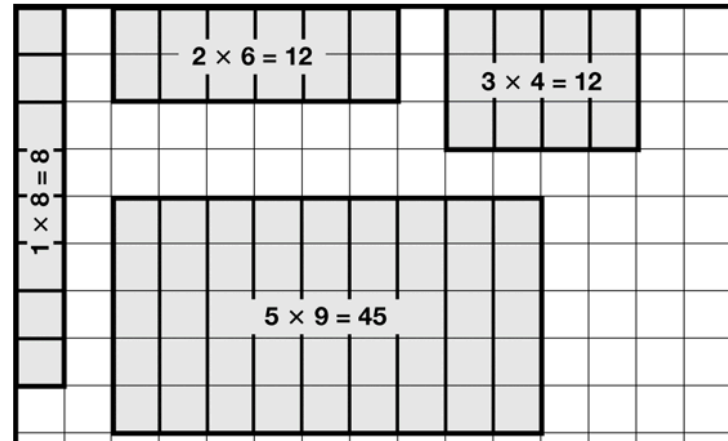
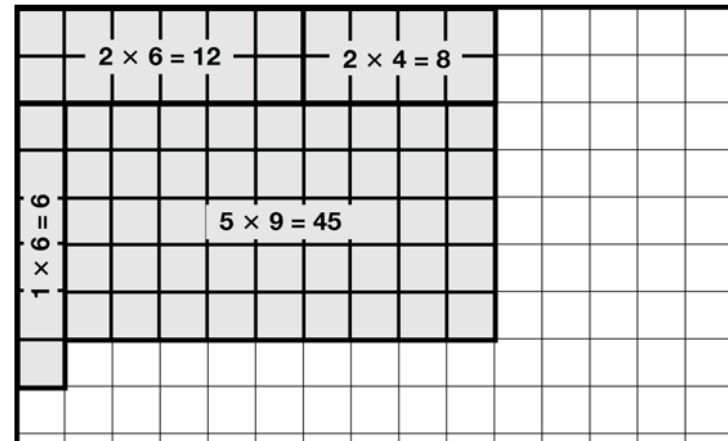
- **Anticipating** student responses prior to the lesson
- **Monitoring** students' work on, and engagement with, the task
- **Selecting** particular students, or groups of students, to present their mathematical work
- **Sequencing** students' responses in a specific order for discussion
- **Connecting** different students' responses, and connecting the responses to key mathematical ideas



Let's Talk: Floor Tiler

- **After the game...**
- **Did you usually use the numbers that you spun or did you often try to use other factor pairs for the same product?**
- **What were your reasons for your choices? Do you think some choices were better than others? Why?**
- **During game, ask: David spun $4 \times 9 = 36$, but he has no room on his board for any of the rectangles for 36. So he lost his turn. What could he do on his next turn? (He could use the small spinner so his product isn't as big.)**

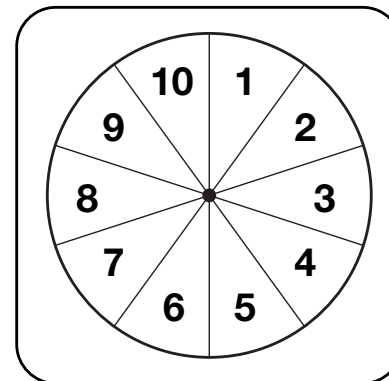
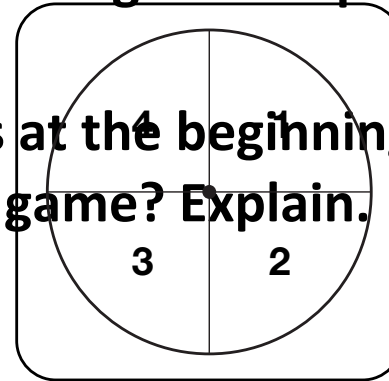
How did you arrange your rectangles? Did you try to use as few gaps as possible (as in Figure 2) or did you spread out your rectangles (as in Figure 3)? Do you think one method is better than the other? Why?



Name _____ Date _____

Spinners 1-4 and 1-10

- Did you develop a strategy for choosing which spinner to use?
- Did you choose the same spinners at the beginning of the game as you did at the end of the game? Explain.

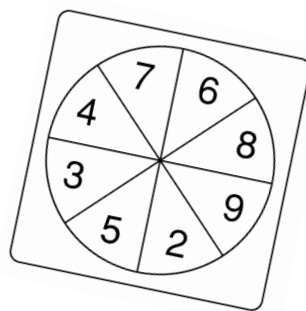


DRAFT Copyright © 2012 • TIMS Project, Inc. • DO NOT DISTRIBUTE

Which has more cognitive demand?

- Math Bingo

Cover 2 x 7.



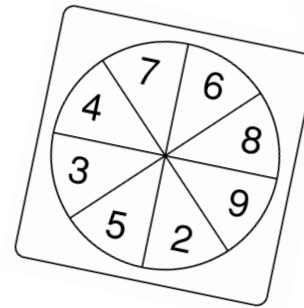
B	I	N	G	O
84	120	8	88	28
90	70	63	36	48
6	108	Free Space	14	44
30	18	9	4	121
50	132	35	32	16

- Product Bingo (pp.)

Board 1				Board 2			
40	72	10	35	9	P	22	81
42	28	20	27	64	13	25	32
15	45	P	6	15	14	56	29
30	48	14	56	7	10	4	49

Board 3				Board 4			
8	24	27	54	4	45	25	81
20	12	21	32	49	56	6	32
36	P	14	45	9	64	P	10
63	18	72	16	15	8	42	48

Post Game Dialog



- Which game board had the most winning games?
- Which game board had the fewest winning games?
- Why do you think one game board won more than others? Explain.
- Are there products that come up more often than others? Which ones? Why?
- Which products come up the least often? Why?
- Are there products that never come up by spinning the spinner?

Board 1

40	72	10	35
42	28	20	27
15	45	P	6
30	48	14	56

Board 2

9	P	22	81
64	13	25	32
15	14	56	29
7	10	4	49

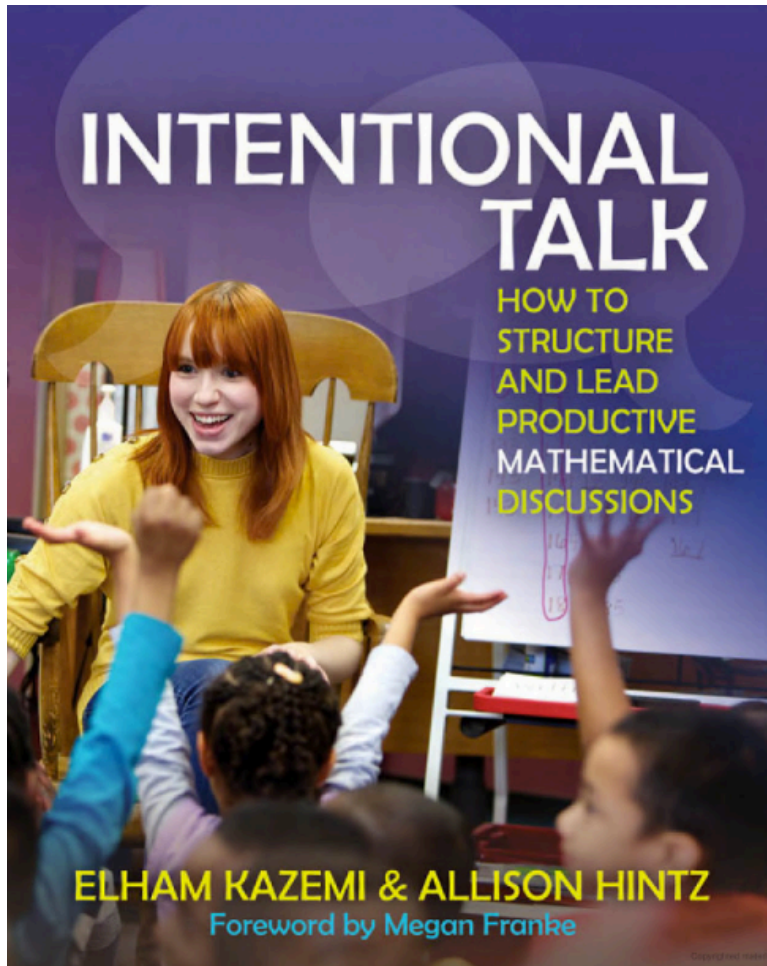
Board 3

8	24	27	54
20	12	21	32
36	P	14	45
63	18	72	16

Board 4

4	45	25	81
49	56	6	32
9	64	P	10
15	8	42	48

4 Principles



- ✓ 1. Discussions should achieve a mathematical goal
- ✓ 2. Students need to know what and how to share

Classroom Norms and Behaviors

Discuss:

- rules for listening
- ways to agree and disagree respectfully
- how to critique the reasoning of others rather than the person
- personal responsibilities
- how to take turns
- ways to support facts or opinions

Talk Moves to Support Classroom Discussion

Revoicing: (“So, you are saying...”)

Repeating: (“Can you repeat what he just said using your own words?”)

Reasoning: (“Do you agree or disagree? Why?”)

Adding On: (“Would someone like to add something more to this?”)

Wait Time: (“Take your time...”)

Turn-and-Talk: (“Turn and talk to your neighbor...”)

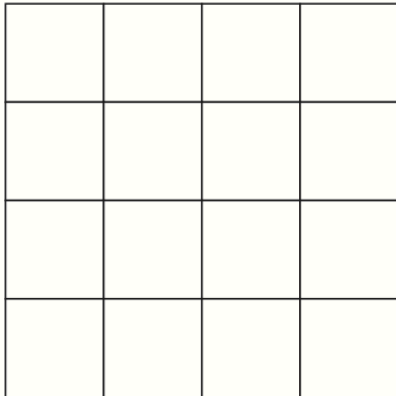
Revise: (“Has anyone’s thinking changed?”)

But wait...There's more!

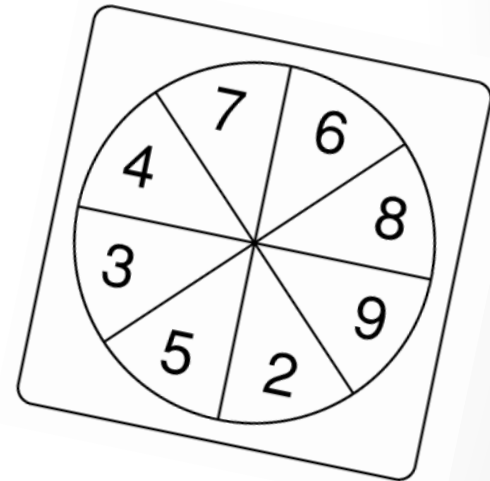
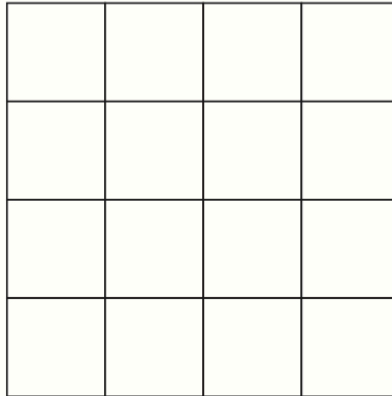
Design Your Own Boards

1. Design your own Product Bingo game boards. The spinner is labeled with the numbers 2 through 9. Design a "Best" game board that is highly likely to win and a "Worst" game board that is unlikely to win.

Best



Worst



Sample Dialog: Terms

Teacher: Why were those good products?

Maya: Because you could get them lots of ways. Twelve, you could get with 2×6 and with 3×4 .

Teacher: 12 and 24 are products. Do you have a name for those other numbers you said, 2, 6, 3, 4?

Maya: They're the factors. Factors of 12.

Teacher: Good. Yes, 2, 3, 4, and 6 are all factors of 12. What is the difference between Board 3 and Board 2? Look at the products on those boards.

Ana: On Board 2, you have products that are never going to win, like 22 and 7.

Teacher: Why are those products never going to win? Don't they have factors?

Ana: I mean 7's only got two factors, 1 and 7. It's a prime number.

Teacher: That's right, Ana, it's a prime number, but why do you say it can never win?

Ana: Because 1 isn't on the spinner so you can never spin it.

Teacher: What about 22? Why will that never win? Is it a prime number too?

Luis: I don't think it is a prime number.

Teacher: Why don't you think it is prime?

Luis: Because you can divide it. It ends in 2.

Nila: It's 11 times 2.

Teacher: What did you say, Nila? What are the factors of 22?

Nila: 11 and 2. Also 1 and 22.

Teacher: Okay, so it's not a prime number. So why will it never win?

Nila: Because 2 is the only one that is on the spinner. The spinner doesn't have 1 or 11 or 22, so you can never spin a 22.

Meaningful Dialog

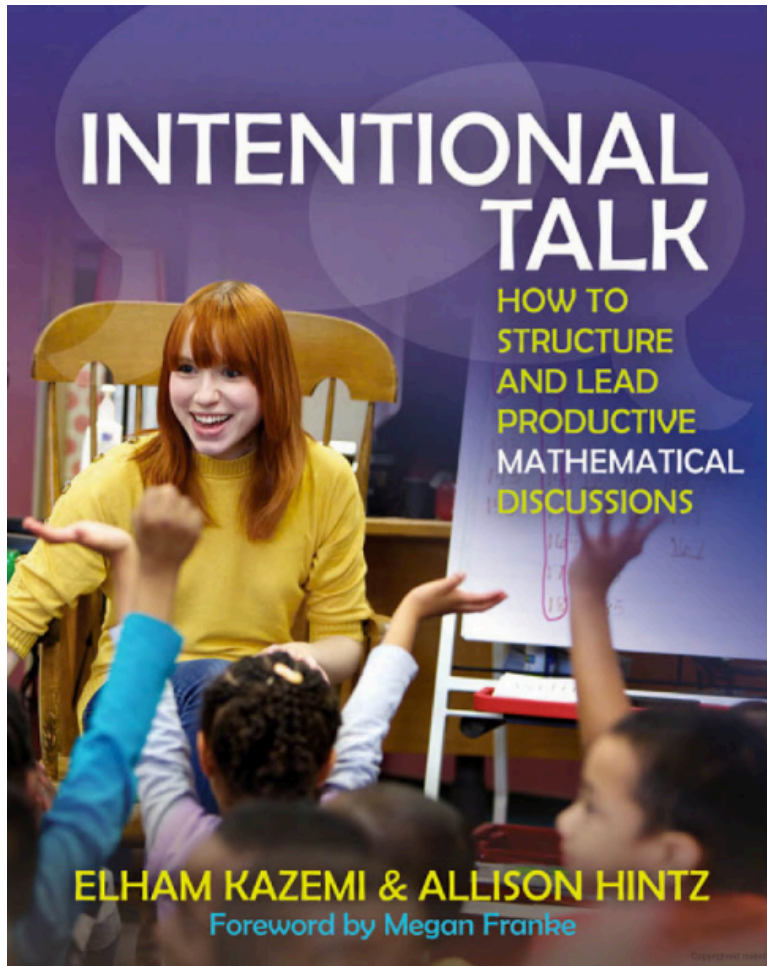
- Describe how you chose numbers for the “Best” game board. Use terms such as “factors,” “multiples,” and “prime” in your description.
- **I used all even numbers and no prime numbers. I also made sure the factors on the board are on the spinner. First, I thought about which numbers can happen on the spinner. It was the numbers with both factors on the spinner like 10, 12, and 20. Then I thought some numbers have more factors on the spinner, like 12 could be 3 4 or 6 2. I put a lot of those numbers I could think of on my board.**
- Describe how you chose numbers for the “Worst” game board. Use terms such as “factors,” “multiples,” and “prime” in your description.
- **I made a board that was horrible because it has mostly prime numbers and 1 isn’t even on the spinner. Just the opposite of the best board. I picked numbers that don’t have factors on the board. Like 22 is 2 11, but 11 isn’t on the board so it’s bad. Or primes don’t work either since 1 isn’t there, so I picked them a lot.**

Compare strategies with scenarios:

Play Product Bingo using the game boards you designed. Did your “Best” game board win or not? Explain why.

- **My best board won twice and lost some too. It had a lot of good numbers with factors on the spinner. My worst board lost all the time. It hardly got any beans at all.**

4 Principles



- ✓ 1. Discussions should achieve a mathematical goal
- ✓ 2. Students need to know what and how to share
- ✓ 3. Teachers need to orient students to one another and the mathematical ideas
- ✓ 4. Teachers must communicate that all students are sense makers and that their ideas are valued

I ♥ Mistakes

“Tell students that you love mistakes and that they will be valued at all times. Tell them that it is good to make mistakes as we know that when people make mistakes, their brains are growing. This single message can be incredibly liberating for students.”

-Jo Boaler

Praise & Mindset



-Carol Dweck

Multiplication Digits

Multiplication Strategies Menu

Breaking into tens and ones

Using Expanded Form

$$\begin{array}{r} 23 = 20 + 3 \\ \times 6 \\ \hline 120 + 18 = 138 \end{array}$$



Nila

or

20	3
$6 \times 20 = 120$	$6 \times 3 = 18$

$$\begin{array}{r} 120 \\ + 18 \\ \hline 138 \end{array}$$



Sara

Using All-Partials

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 18 \\ + 120 \\ \hline 138 \end{array} \quad \text{or} \quad \begin{array}{r} 23 \\ \times 6 \\ \hline 120 \\ + 18 \\ \hline 138 \end{array}$$



Jacob

Compact Method

$$\begin{array}{r} 1 23 \\ \times 6 \\ \hline 138 \end{array}$$



Ming

Other ways to use simpler problems

Thinking About Money

$$27 \times 4$$



Jerome

$$\begin{aligned} 27 \times 4 &= 25 \times 4 + 2 \times 4 \\ &= 100 + 8 \\ &= 108 \end{aligned}$$

Using Simpler Numbers

$$48 \times 6$$



Tanya

I know $48 + 2 = 50$.
So, $50 \times 6 = 300$ and $2 \times 6 = 12$.
Then I subtracted $300 - 12 = 288$.

Another Strategy: _____

Choose a game to evaluate.

Game directions can be found in your packet.

Game Analysis Tool

Name of Game
Math Involved in the Game
This game supports students' development of (circle all that apply): models informal strategies counting strategies reasoning strategies flexibility connections recall

1. What strategies do you anticipate your students will use when playing the game?
2. Where do you anticipate your students will be successful?
3. Where do you anticipate they will be challenged?
4. What questions would you ask as the game progresses to promote conceptual development?

- List game choices here



Play a game
and fill out an
analysis tool.

Jigsaw: Share What You Know

Share information about games using
game analysis tool to discuss a game.
Please do for all games played.

How do you choose a good game?

Ask yourself,



“Does the
game
promote



modeling?”



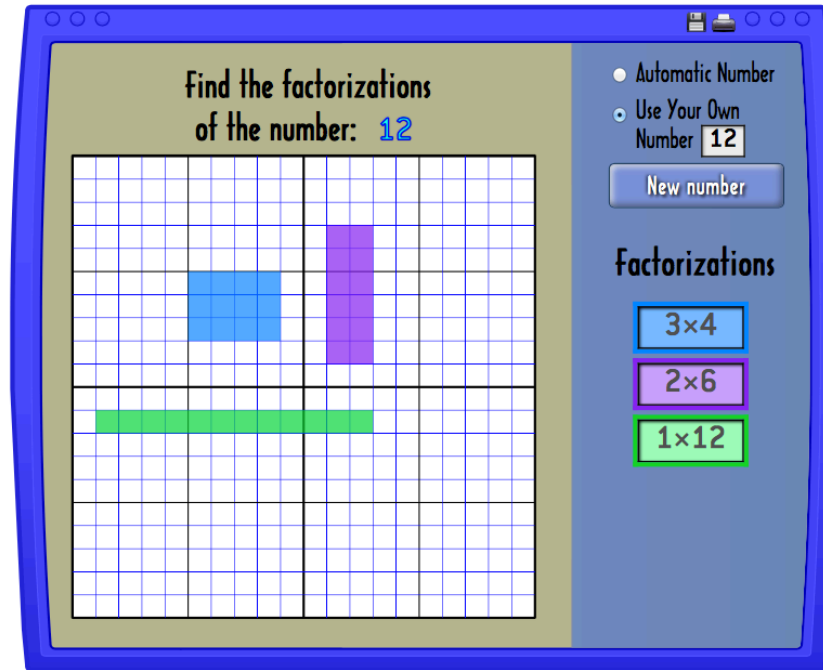
strategies?”



flexibility?”

Choose online games carefully.

Factorize



This game supports students' development of (circle all that apply):

models

informal strategies

counting strategies

reasoning strategies

flexibility

connections

recall

Interactive from [Illuminations.nctm.org](https://illuminations.nctm.org)

Bunny Times

The screenshot shows a user interface for a math game. On the left is a vertical sidebar with four buttons: 'Featured Lesson Plan', 'Featured Mobile Game', 'Featured Interactive' (highlighted in orange), and 'Featured Brain Teaser'. The main area features a farm scene with a barn, a pond, and a field of carrots. A cartoon bunny character is in the bottom right corner, holding a sign that says '0'. A speech bubble from the bunny says: 'Help the bunny eat carrots. How many are there? Tap the digits to enter the number.' In the top right corner of the main area, the text reads: 'Featured Interactive Pre-K-Grade 5; Become confident in facts up to 12×12 using visual models that stress the conceptual aspects of multiplication.' At the bottom right, there is a 'read more +' button.

Interactive from [Illuminations.nctm.org](https://illuminations.nctm.org)

What can we do to develop fluency?

- Patiently help students construct number sense by encouraging them to invent, share, and refine informal strategies.
- Encourage students to look for patterns and relationships and to use these discoveries to construct reasoning strategies.
- Encourage children to build on what they already know.
- Assign practice wisely to discover patterns and relationships and to make reasoning strategies more automatic – not on drilling isolated facts.
- Encourage students to flexibly use a variety of strategies.
- Provide opportunity to engage in meaningful dialog about learnings: share, discuss, compare strategies.

We can choose activities that

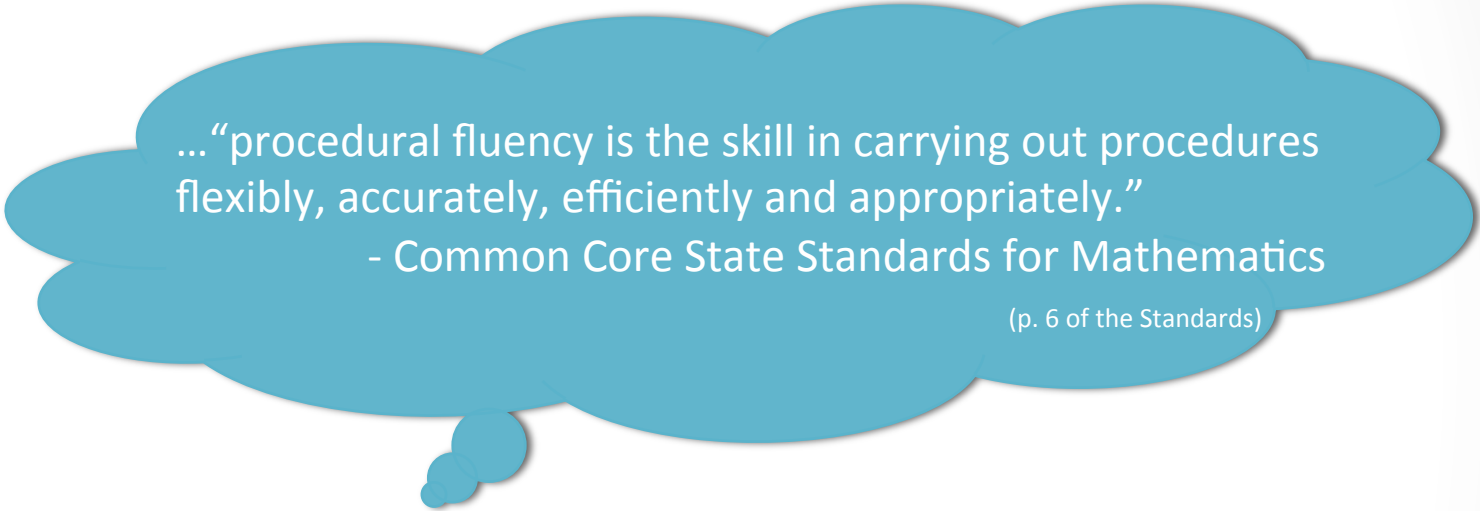
create
understanding,

develop
strategies,

and provide
meaningful and
enjoyable
practice.



In conclusion from the intro to the Standards of Mathematical Practice ...



...“procedural fluency is the skill in carrying out procedures flexibly, accurately, efficiently and appropriately.”

- Common Core State Standards for Mathematics

(p. 6 of the Standards)

Games can support students as they develop problem solving skills and flexibility with numbers and operations!

References

- Baroody, A.J. "Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them." In Teaching Children Mathematics, 13 (1), 22–31. National Council of Teachers of Mathematics, Reston, VA, 2006.
- Bay-Williams, Jenny and Kling, Gina. "Three Steps to Mastering Multiplication Facts." In Teaching Children Mathematics, 21 (9), 548-559. National Council of Teachers of Mathematics, Reston, VA, 2015.
- Boaler, J. "Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts." at youcubed at Stanford University, 2015. <http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf>
- Boaler, J. "Research Suggests Timed Tests Cause Math Anxiety." In Teaching Children Mathematics, 20 (8), 469-474. National Council of Teachers of Mathematics, Reston, VA, 2014.
- Dweck, Carol S. "Even Geniuses Work Hard." In Educational Leadership, 2010.
- National Council of Teachers of Mathematics (2015). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA. National Council of Teachers of Mathematics.
- National Research Council (2001). *Adding It Up*. Washington D.C.: National Academy Press.
- Smith, M. & Stein, M. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA. National Council of Teachers of Mathematics.
- Wagreich, P., Goldberg, H., et alia. (2007, 2004, 2008). *Math Trailblazers: A Mathematical Journey Using Science and Language Arts*. Dubuque, IA: Kendall/Hunt.