

Wait-what? Multiplication is more than just equal groups?

Conceptual understanding of multiplication begins before students formally learn to multiply. Let's examine activities to demonstrate the learning progression of multiplication that can be immediately used in your classroom. We will engage in tasks and explore models designed to facilitate students' development of multiplicative reasoning.



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Wait, what?

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Multiplication is more than just equal groups?

NCTM San Antonio, TX 2017



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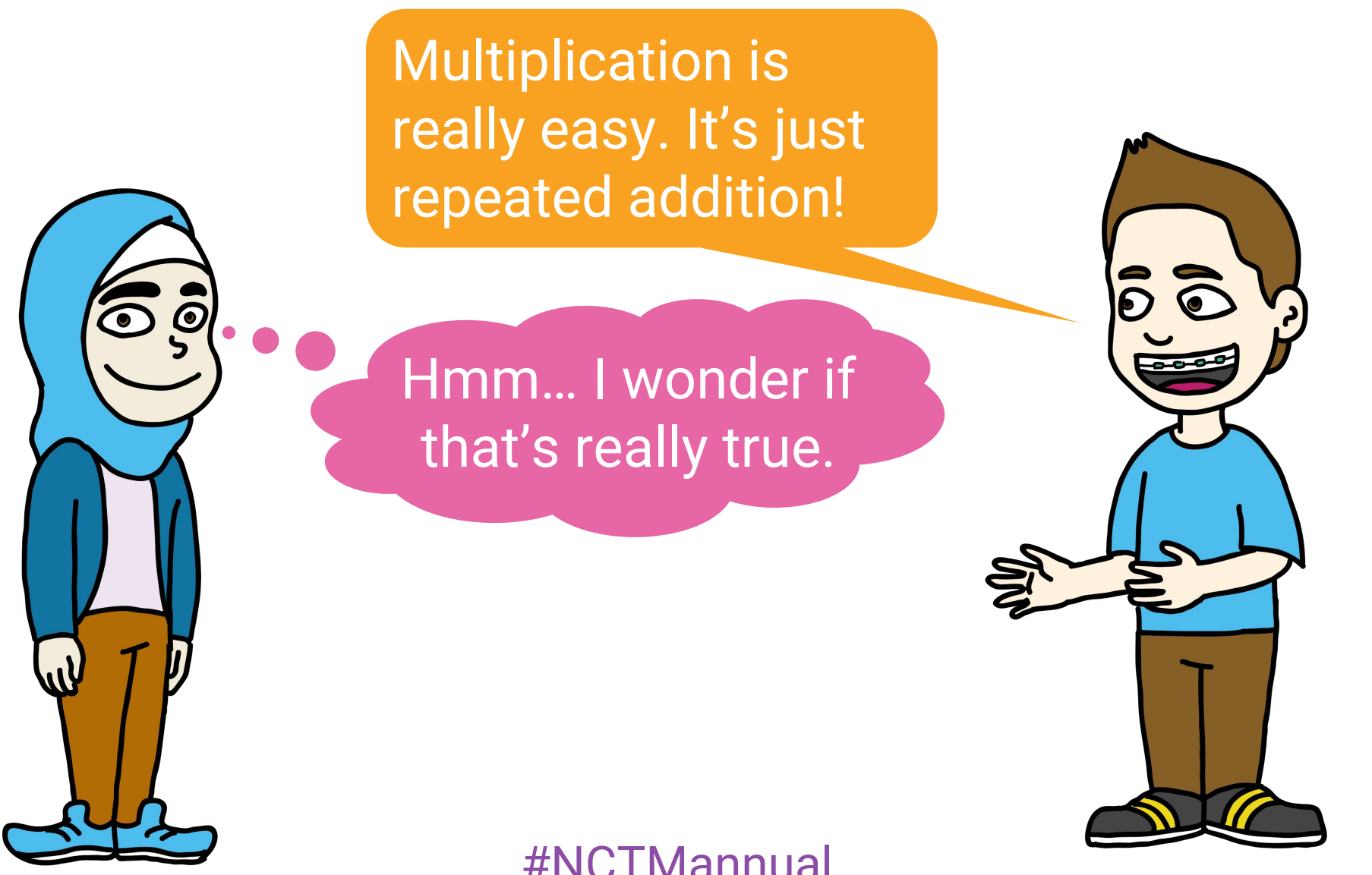
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What do you think about Jacob's explanation?



Multiplication is really easy. It's just repeated addition!

Hmm... I wonder if that's really true.

5.NF.5

Interpret multiplication as scaling (resizing), by:

A. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

B. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

TEK 6.3B Number and operations.

The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:

(B) determine, with and without computation, whether a quantity is increased or decreased when multiplied by a fraction, including values greater than or less than one

3.OA.1

Interpret products of whole numbers

e.g., interpret 5×7 as the total number of objects in five groups of seven objects each.

$$5 \times 7$$

five groups of seven

TEK 3.4E Number and operations.

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to.

(E) represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting

4.OA.1

Interpret a multiplication equation as a comparison

$$35 = 5 \times 7$$

“35 is five times as many as
seven”

and

“35 is seven times as many as five”

TEK 3.5C Algebraic reasoning.

The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:

(C) describe a multiplication expression as a comparison such as 3×24 represents 3 times as much as 24

$$3 \times 24$$

“three times as much as twenty-four”

5.NF.5

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Multiplication of whole numbers

Focus on the **relationship of the size** of the product to the size of the factors **not only repeated addition**

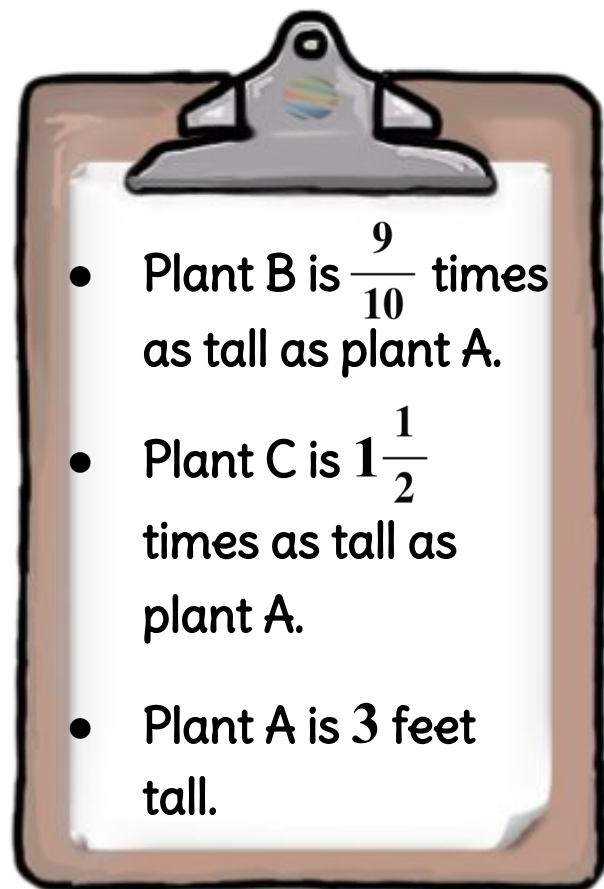
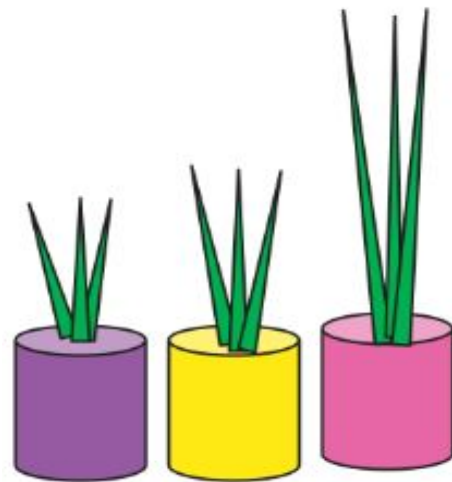
The multiplier expresses the number of times the multiplicand increases or decreases; the multiplier is the **scaling factor**.

$$\text{multiplier} \times \text{multiplicand} = \text{product}$$

Kieran & Ryan did a plant growth experiment, but their labels fell off and now they don't know which plant is which.

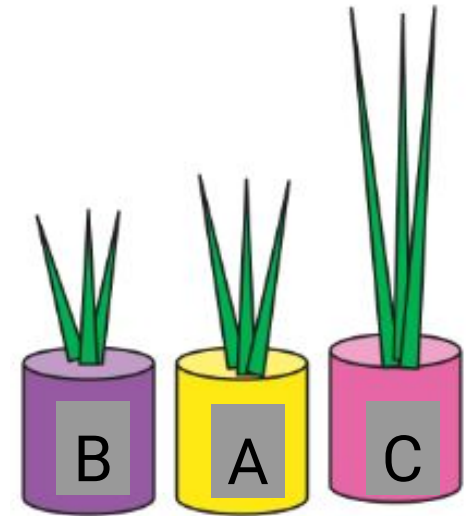
Luckily they had to write a lab report to explain their results.

Write an expression to show the heights of plants B & C. Then list the plants in order from shortest to tallest.



Task Solution

plant	expression
A	
B	$\frac{9}{10} \times 3\text{ft.}$
C	$1\frac{1}{2} \times 3\text{ft.}$



shortest $\longrightarrow$ tallest

B A C

A game of sorts...



- Form a triad.
- Each person takes a labeled colored card.
- Take turns flipping cards from the deck one at a time.
- Find the product of the two factors for each expression on the card you flip.
- Any player may take the card **if** it makes a true equation with the colored card they have.

Turn and Talk

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What did you notice about the factors with products that are less than three? Greater than three? Equal to three?

Based on the patterns you see, what conjectures can you make?

How might you use this activity with your students?

A Number Talk Idea

$$1 \times 16$$

$$\frac{1}{2} \times 16$$

$$\frac{1}{4} \times 16$$

- What do you notice about the factors?
- Why are some of the products smaller than the whole number factor?
- Name a fractional factor that will increase the scale of the product.
- Describe how $16 \times \frac{1}{4}$ is different from $\frac{1}{4} \times 16$.

Problem:

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Amalia is making bows for gifts.



Her original bows each use three feet of ribbon.

She wants to make a bow that is four times that size.

How many feet of ribbon will she need for the larger bow?

$$4 \text{ copies of } 3 \text{ feet} = \underline{\hspace{2cm}} \text{ feet}$$

$$4 \times 3 \text{ feet} = \underline{\hspace{2cm}} \text{ feet}$$

multiplier $\times$ multiplicand = product

Think about it as a compare problem:

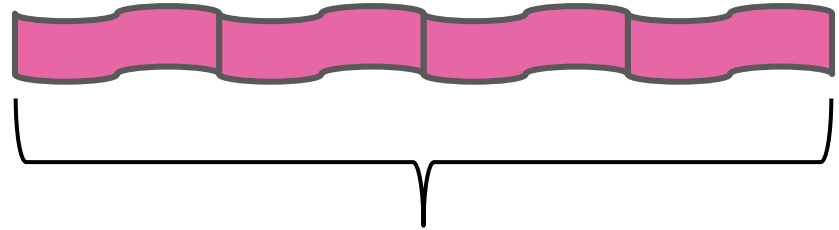
reference set:

3 feet of ribbon



multiplier:

4 times larger



product

Back to Amalia



Next, she is going to make a much smaller bow. This bow will be one-third the size of the three foot bow.

How much ribbon will she need?

$$\frac{1}{3} \times 3 \text{ feet} = \underline{\hspace{1cm}} \text{ feet}$$

(multiplier $\times$ multiplicand = product)

Think about it as a compare problem:

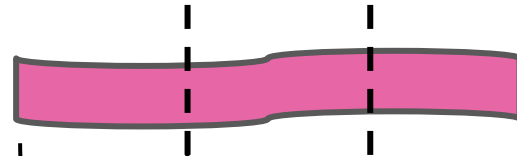
reference set:

3 feet of ribbon



multiplier:

$\frac{1}{3}$ the size



Product

Back to Amalia...

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Now Amalia needs a medium bow.

Looks like the three foot bow will be too large!

This bow will require $\frac{5}{12}$ the length of ribbon that she needs for the three foot bow.

How much ribbon will she need?

Avoid using the algorithm and solve using a visual representation.

Back to Amalia...

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She now needs a huge bow!
That three foot bow will be too small.

This bow requires $2\frac{1}{2}$ times the length of ribbon
that she used for the three foot bow.

How much ribbon will she need?

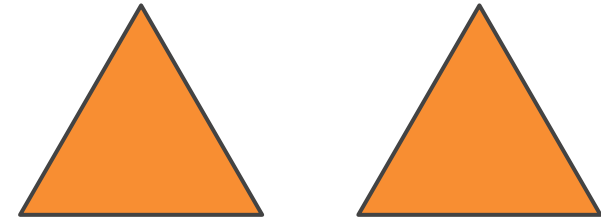
Avoid using the algorithm and solve using a
visual representation.

Think of a copy machine...

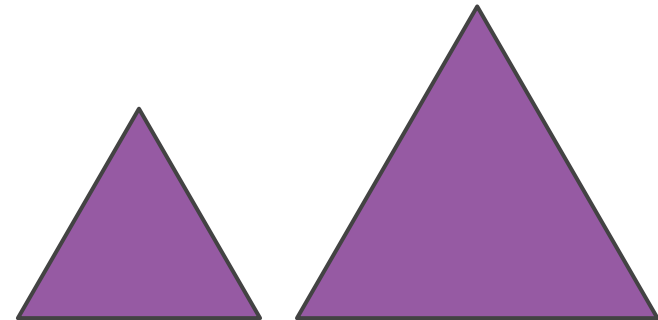
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When copying the same size –
you are not changing the size.
You are multiplying by one

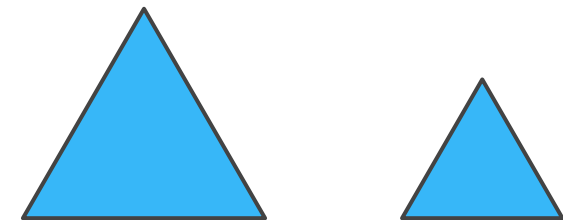
Identity property



When enlarging – the multiplier is
greater than one – the product
will be greater than the
multiplicand.



When shrinking – the multiplier is
a fraction less than one – the
product will be smaller than the
multiplicand.





Unit: Interpreting Multiplying Fractions as Scaling



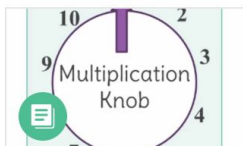
1. Understanding additive and multiplicative comparisons (C)

Lesson objective: Extend understanding that while additive comparisons show "n more than," multiplicative, or relative, comparisons show "n times as many" or "n times as large." Students bring prior knowledge...

Created By: Kim Bobadilla

Standards: 5.NF.B.5a, 5.NF.B.5b, 5.NF.B.5, 5.NF.B.6, MP2, MP4, MP6

Tags: Conceptual, K-8 math curriculum



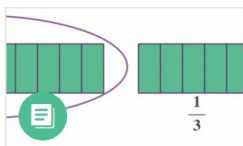
2. Understand that scaling does not always make numbers larger (C)

Lesson objective: Understand that scaling does not always make numbers larger and that if we multiply a number by a fraction less than one the number will scale down. Students bring prior knowledge of scaling by whole numbers and multiplying fractions...

Created By: Kim Bobadilla

Standards: 5.NF.B.5a, 5.NF.B.5b, 5.NF.B.5, 5.NF.B.6, MP2, MP6

Tags: Conceptual, K-8 math curriculum



3. Practice scaling down (FP)

Lesson objective: Fluently predict products using the magnitude of the scaling factor. This lesson helps to build fluency with multiplication of whole numbers and fractions less than one. Tape diagrams and number lines are used here because...

Created By: Kim Bobadilla

Standards: 5.NF.B.5a, MP4

Tags: K-8 math curriculum, Procedural skills



4. Apply scaling down to a real-world situation (A)

Lesson objective: Apply understanding of multiplication of fractions less than one to solve real-world problems. This lesson provides an opportunity for students to apply their knowledge and understanding of multiplying fractions and whole numbers a ...

Created By: Kim Bobadilla

Standards: 5.NF.B.5a, 5.NF.B.5b, 5.NF.B.5, 5.NF.B.6, MP2, MP4, MP6

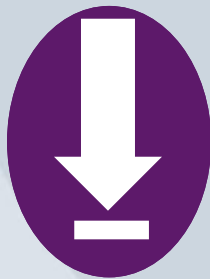


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Thank You!



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