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HANDS-ON GEOMETRY FOR DEEPER UNDERSTANDING FOR ALL LEARNERS IN GRADES 3-5

Margie Mason – mmmaso@wm.edu
Sam Rhodes - srrhod@wm.edu

The Tidewater Team
The College of William and Mary

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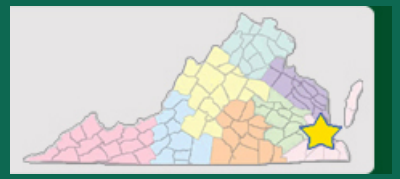
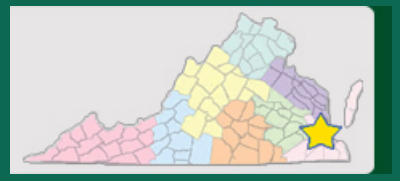
Please sketch a right triangle and set it aside.



The van Hiele Levels of Geometric Understanding

- Described by Dutch educators P.M. van Hiele and Dina van Hiele-Geldof in the 1950's
- Learning is a discontinuous process with jumps that suggest "levels."
- Originally 5 levels, but Clements and Battista hypothesized an additional level.



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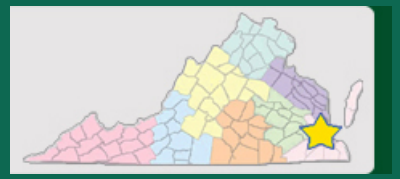
Level 0: Pre-recognition

Students at this level notice only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between figures. For example, they may distinguish between triangles and quadrilaterals, but may not be able to distinguish between a rhombus and a parallelogram.



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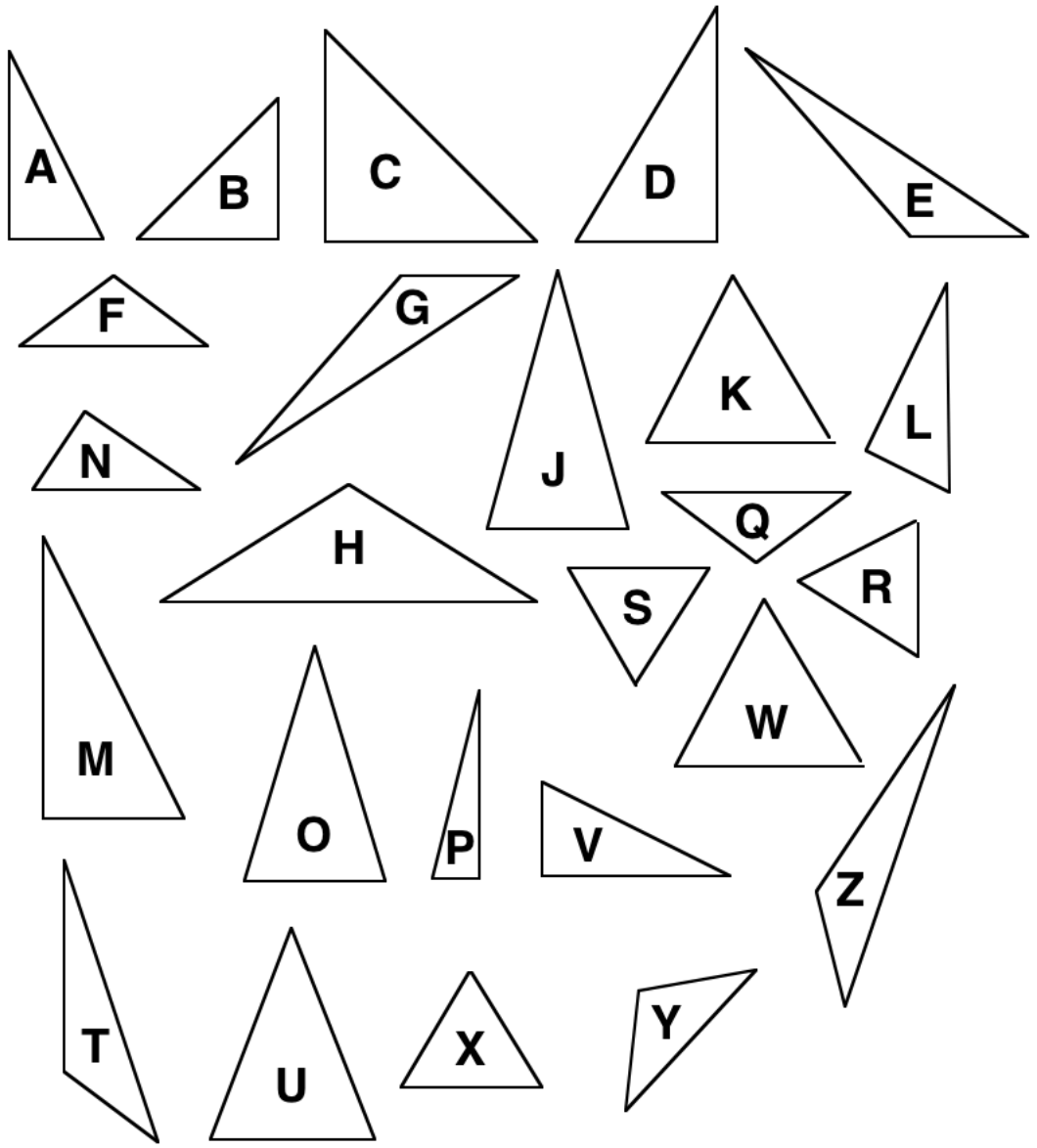
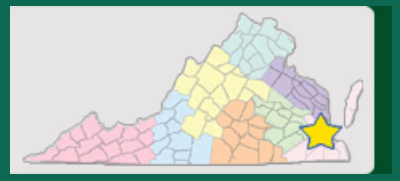
Triangle Sorting Pieces (p.11)





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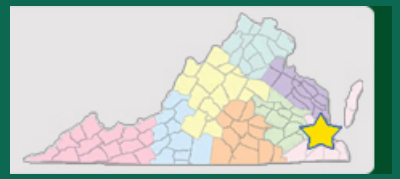
Level 0: Pre-recognition



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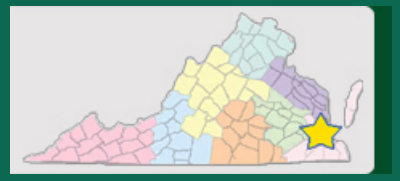
Level 1: Visualization

- Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of the figure. A student should recognize & name figures and distinguish a given figure from others that look somewhat the same.

- "I know it's a rectangle because it looks like a door and I know that the door is a rectangle."



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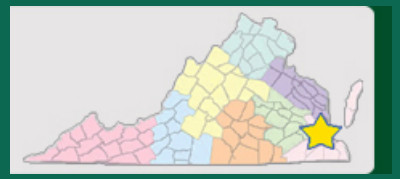
Level 1: Visualization



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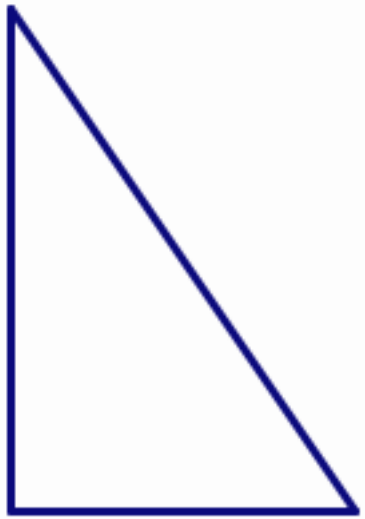
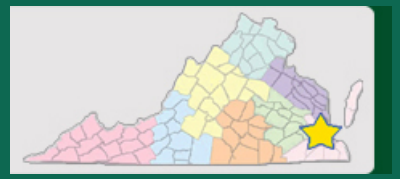
Right Triangles

Does your right triangle look like this?





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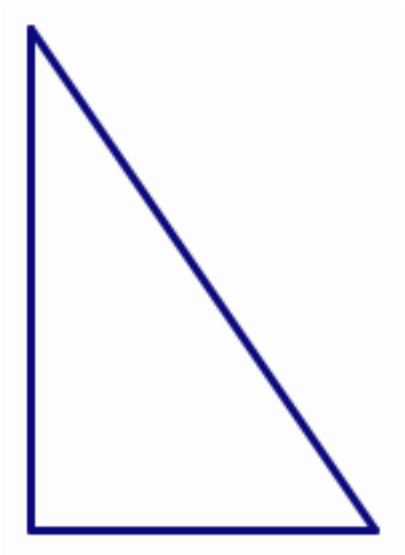
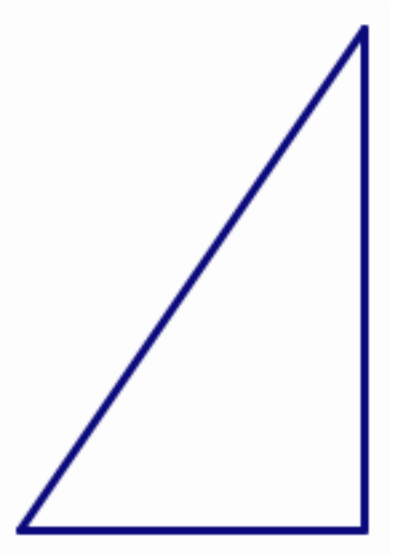
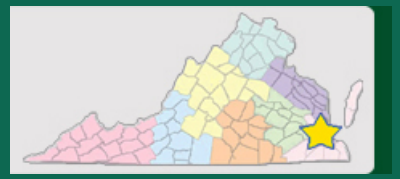
Right Triangles

If this is a right triangle, what is this figure?

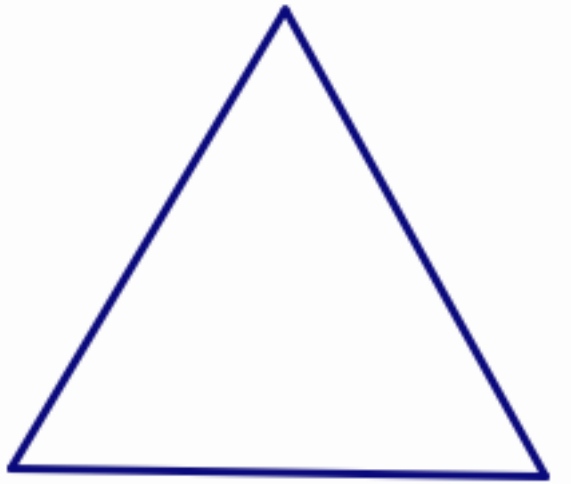


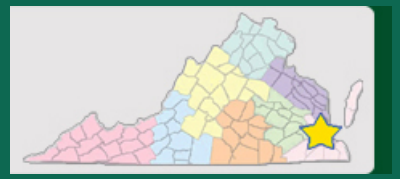




What is This Figure?

If this is a right triangle, what is this figure?





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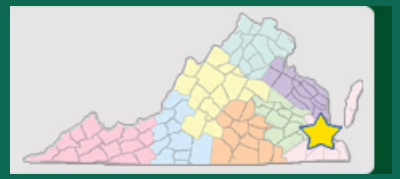
Right and Left Triangles



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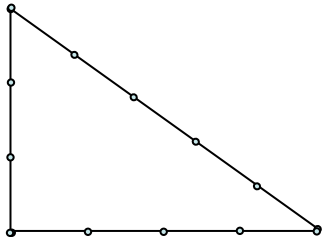
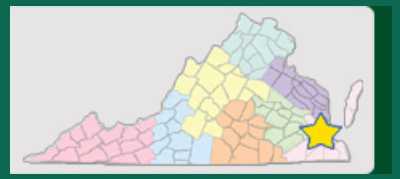
Thirteen Knots in a Rope

- Make 13 equally spaced knots on a rope.
- Connect knot #1 to knot #13.
- Make a triangle using knots 1, 4, and 8 as the vertices.



- Pull tightly at the vertices. What kind of triangle does this make?
- The Egyptians were called “rope stretchers.” How might the Egyptians have used such a rope?



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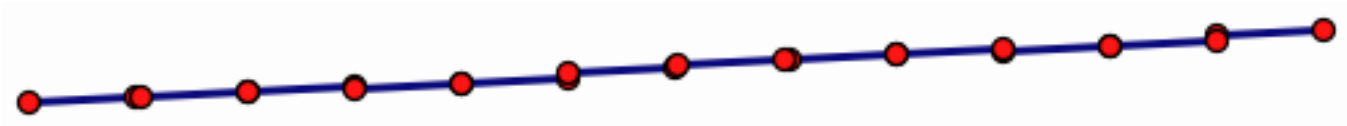
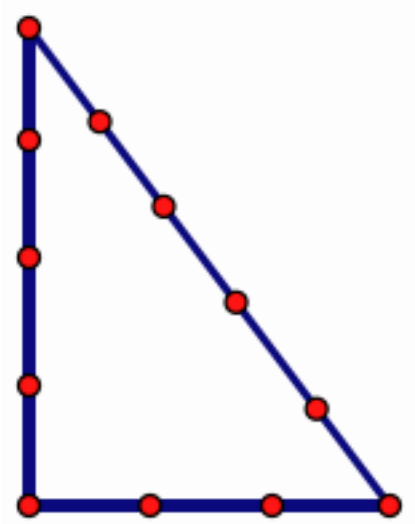
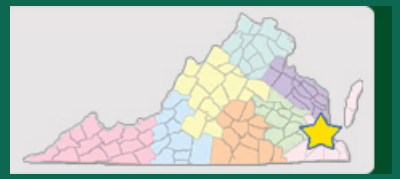
Egyptian Rope Stretchers







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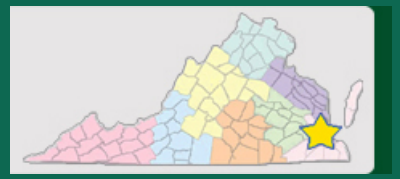
Level 2: Analysis

- Properties are perceived, but are isolated and unrelated. A student should recognize and name properties of geometric figures.

- "I know it's a rectangle because it is closed, it has 4 sides and 4 right angles, opposite sides are parallel, opposite sides are congruent, diagonals bisect each other, adjacent sides are perpendicular, ..."



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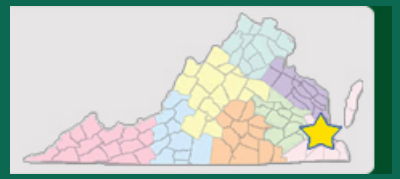
Level 2: Analysis



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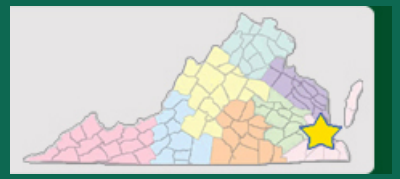
Level 3: Abstraction

- Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood.
- "I know its a rectangle because it's a parallelogram with right angles."



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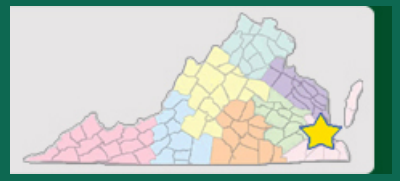
Level 4: Deduction

The student can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. A student should be able to supply reasons for steps in a proof.



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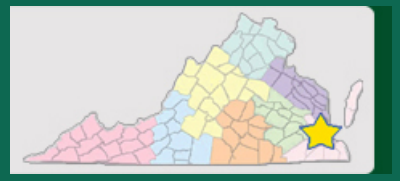
Level 5: Rigor

The standards of rigor and abstraction represented by modern geometries characterize level 5. Symbols without referents can be manipulated according to the laws of formal logic. A student should understand the role and necessity of indirect proof and proof by contrapositive.



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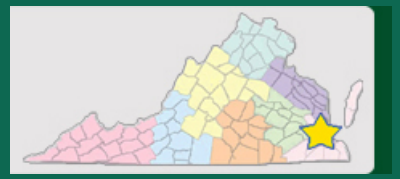
Additional Points

1. The learner can not achieve one level without passing through the previous levels.

2. Progress from one level to another is more dependent on educational experience than on age or maturation.

3. Certain types of experiences can facilitate or impede progress within a level or to a higher level.



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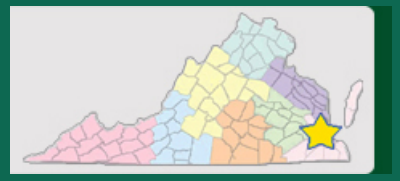
Textbooks

- Most textbooks provide activities requiring only Level 1 thinking up through sixth grade.

- Teachers must provide different types of tasks to facilitate the development of the higher levels of thought.



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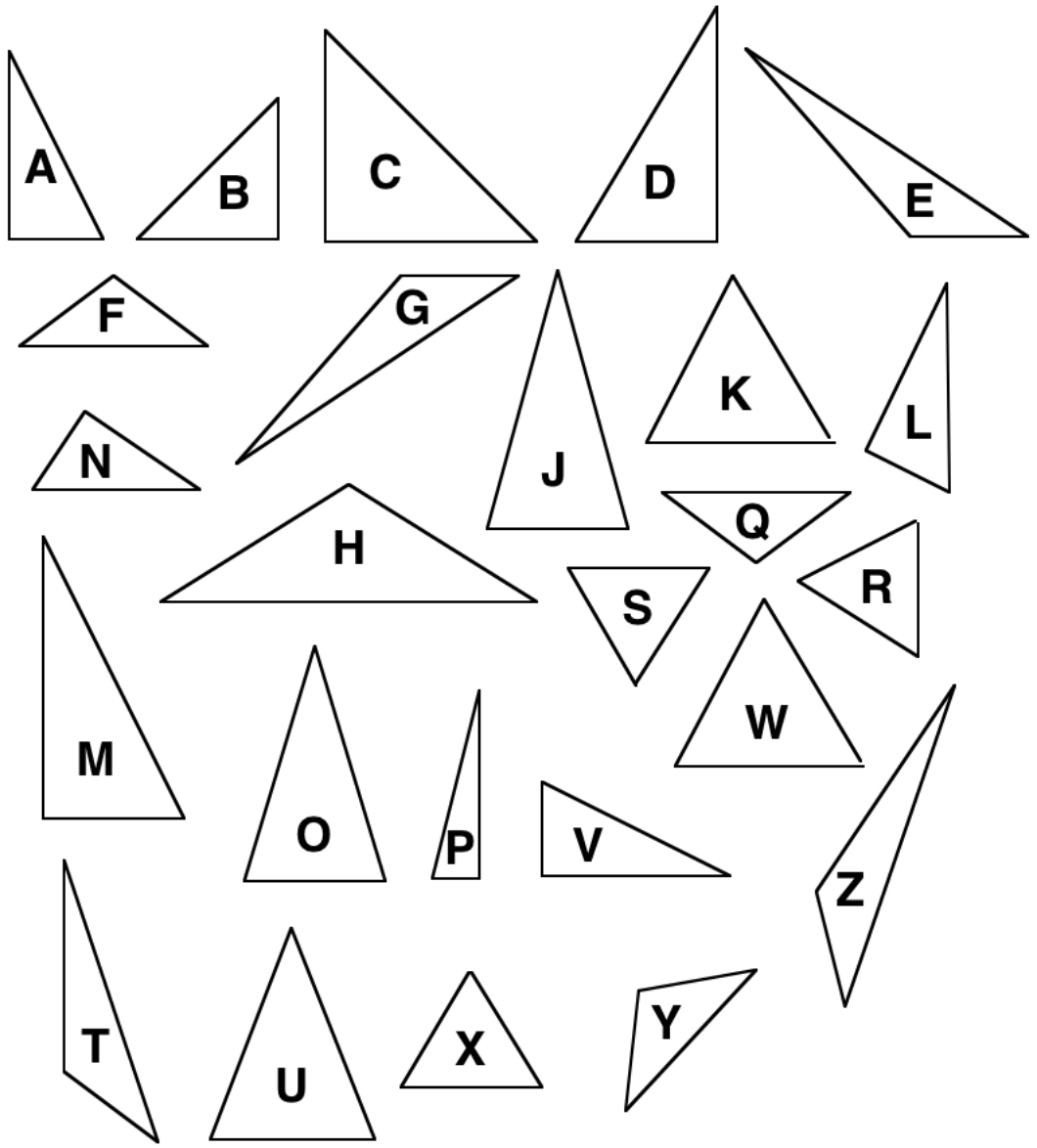
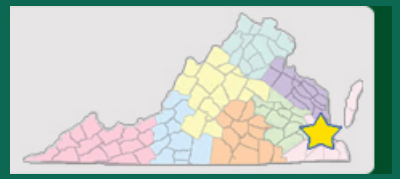
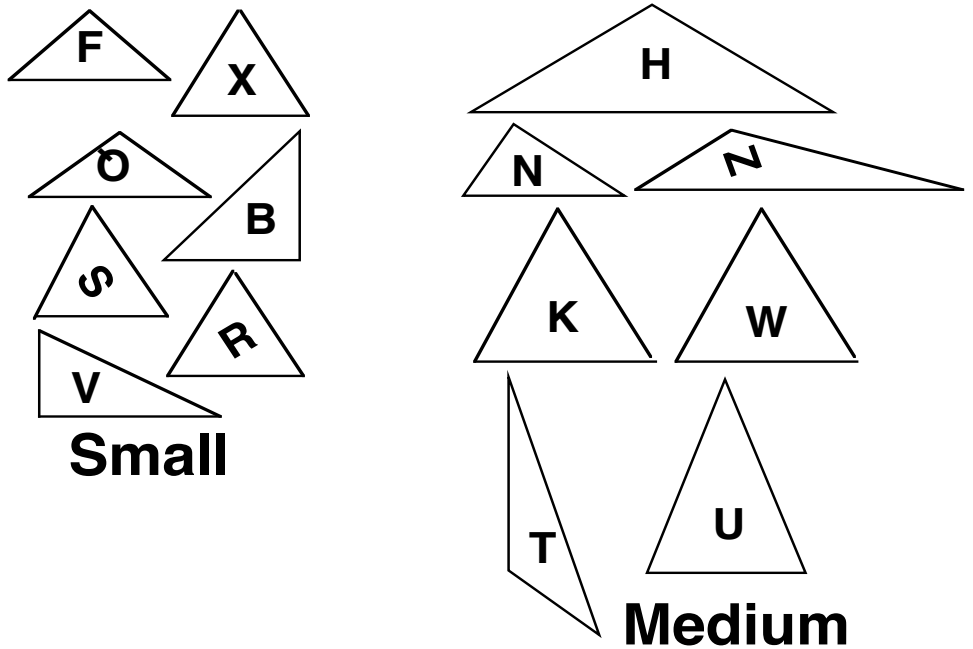
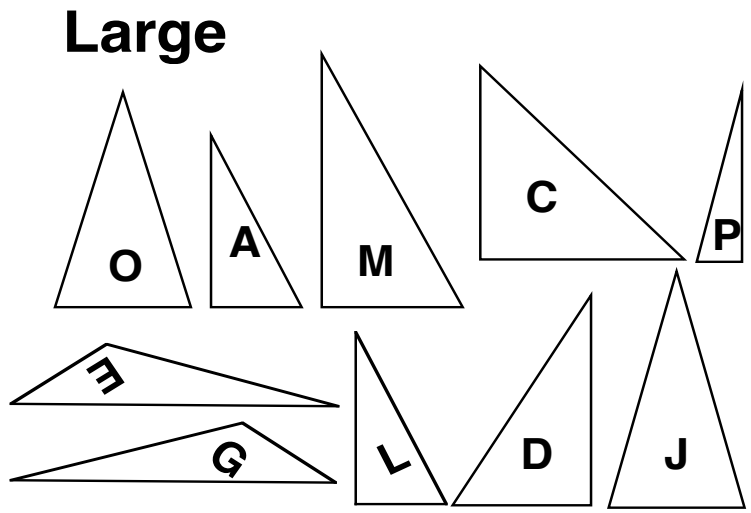
Triangle Sorting Pieces (p.11)

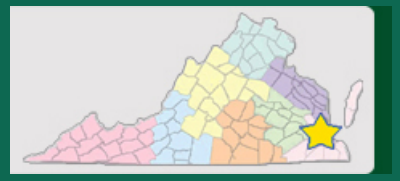




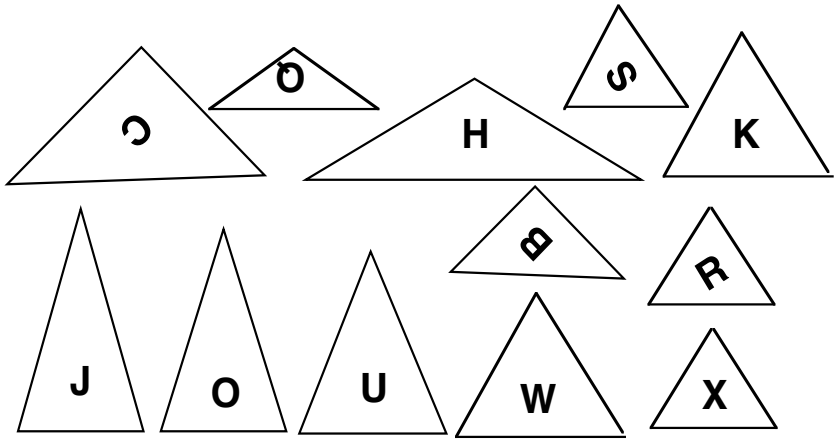
Sample Student Sort 1



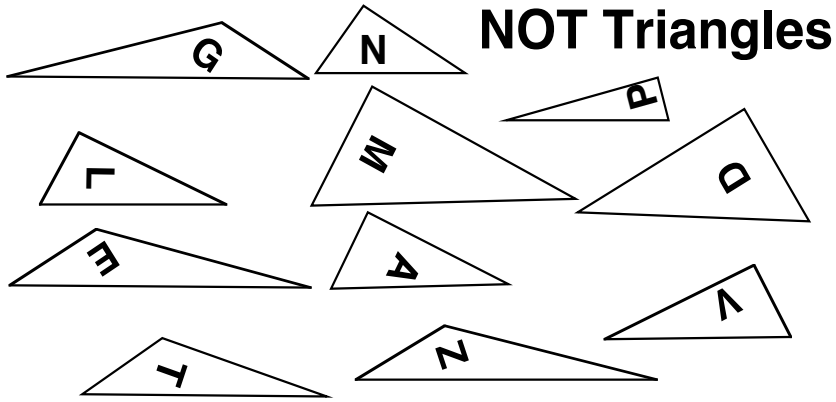




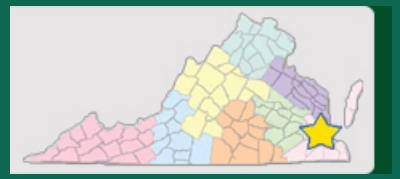
Sample Student Sort 2



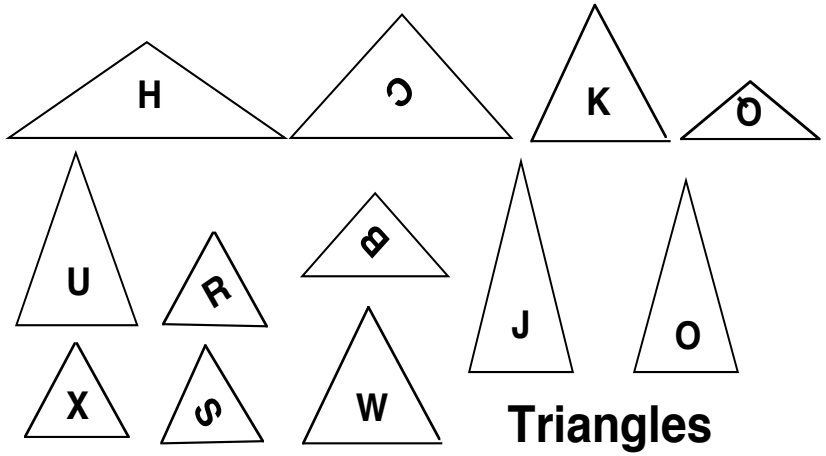
Triangles



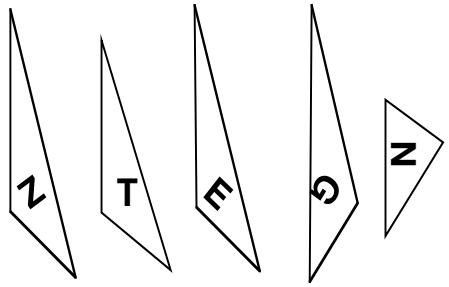
NOT Triangles



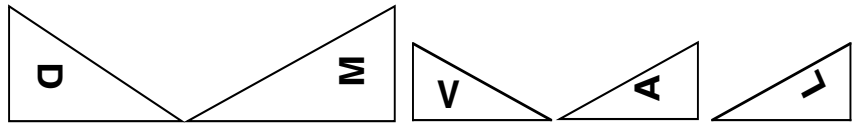
Sample Student Sort 3



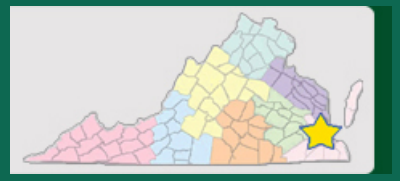
Triangles



Look alike...
N is smaller...
NOT triangles



look like ramps... NOT triangles



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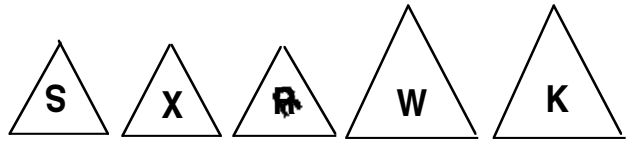
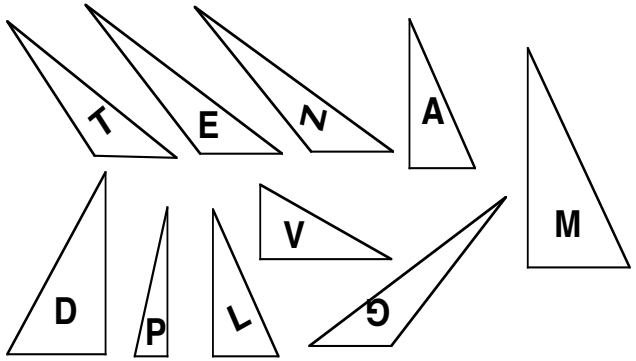
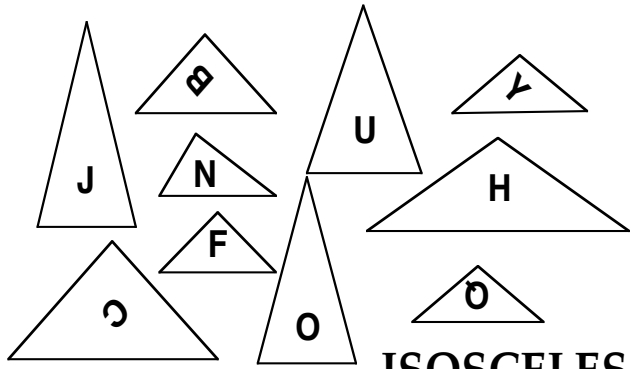


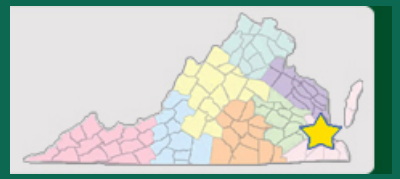
EQUILATERAL



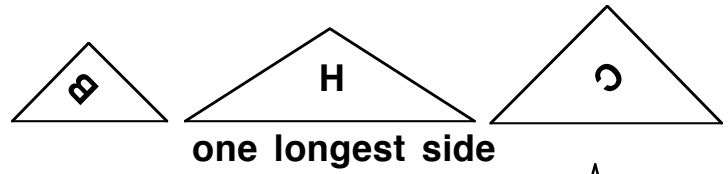
SCALENE



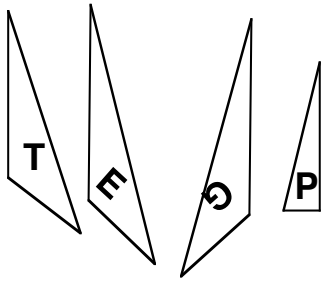
ISOSCELES



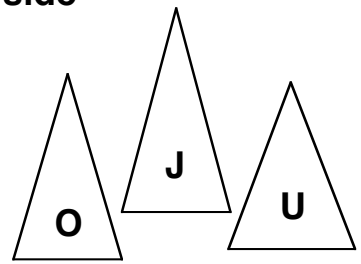
Sample Student Sort 5



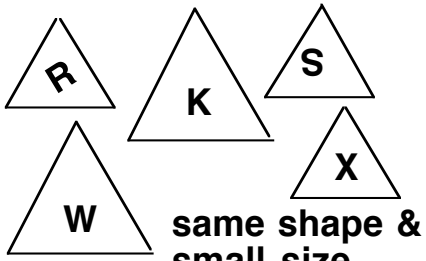
one longest side



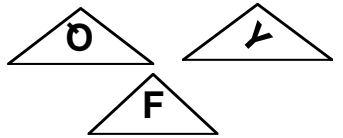
irregular & very narrow



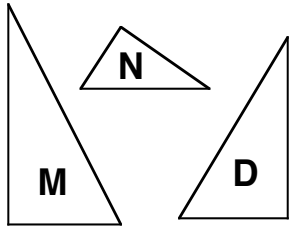
2 sides are similar, one is shorter



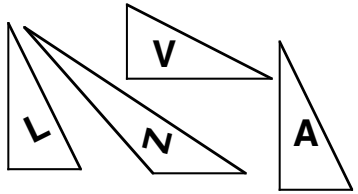
same shape & small size



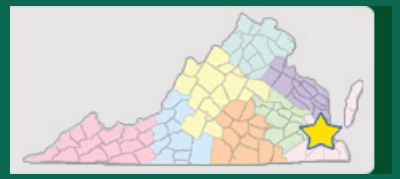
2 sides are similar, one is longer



irregular sides



3 uneven sides



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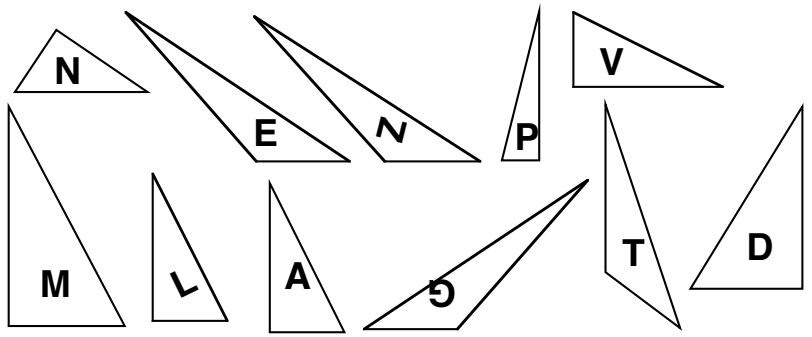
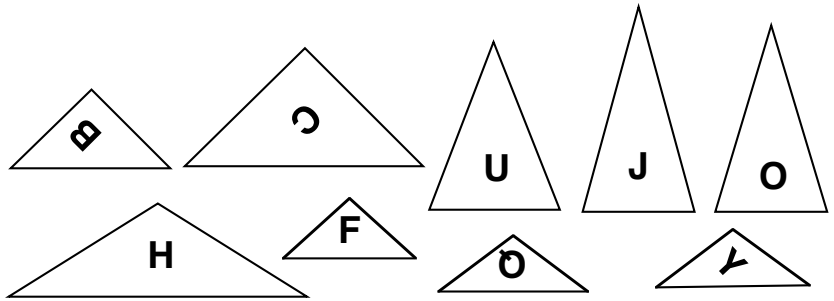
Sample Student Sort 6

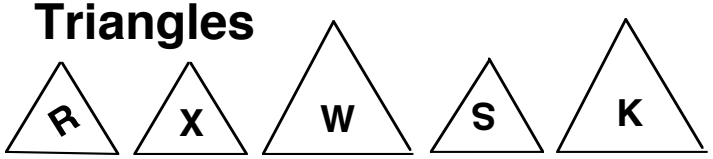


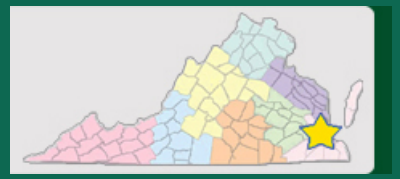
Every side has a different size



2 sides =, 3rd side smaller or larger

Perfect Triangles





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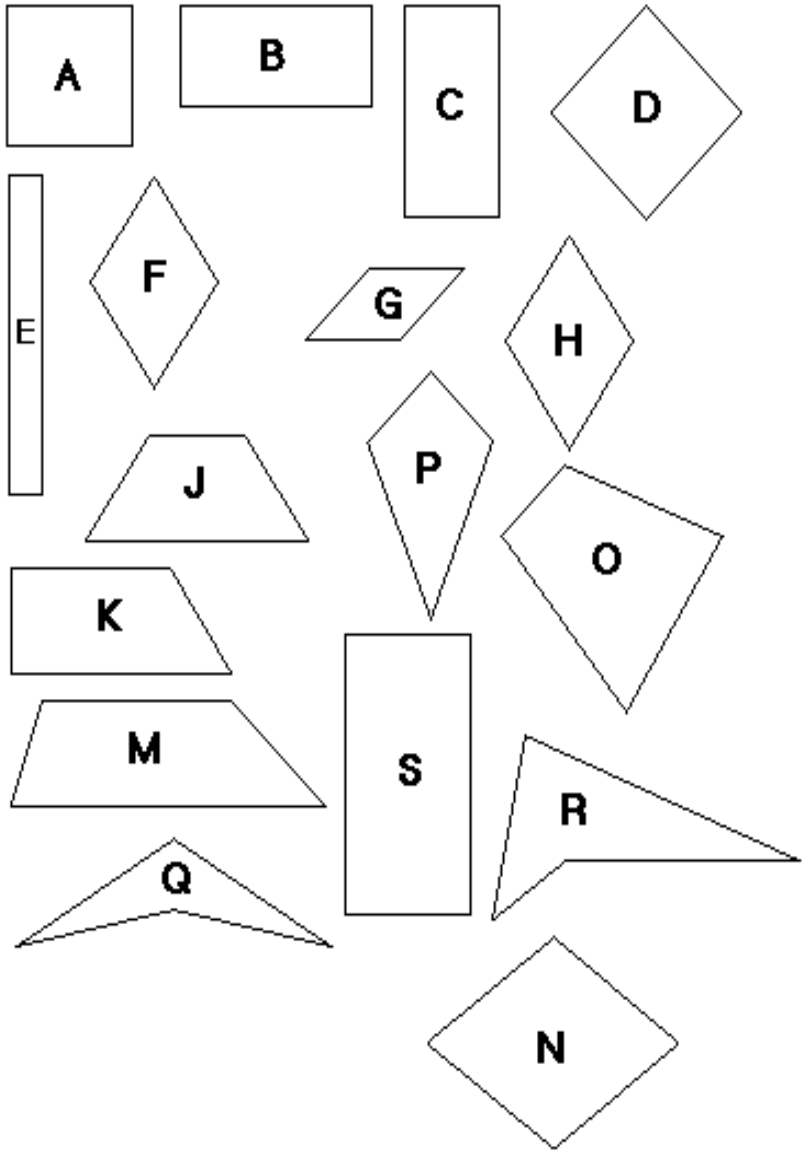
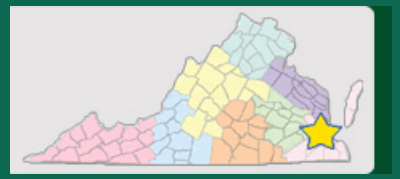
Quadrilateral Sorting Pieces





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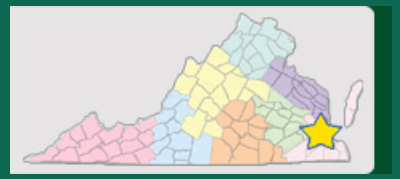
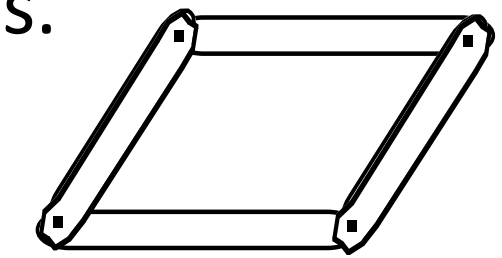
1. What's My Rule?

- The sorter uses a written down "secret rule" to sort the quadrilateral pieces into two or more piles.
- Other players guess the rule. A correct identification is worth 5 points. A correct answer, but not the written one, is worth 1 point. Each incorrect guess results in a 2-point penalty. The winner is the first one to accumulate 10 points.

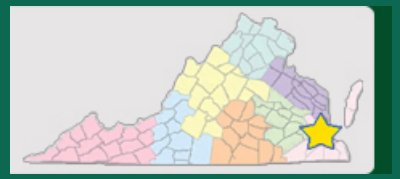


2. Quadrilateral Properties Laboratory

1. Divide yourselves into groups of three or four.
2. Pick two pairs of congruent segments and connect them as shown below. Have them flex the figure to different positions.



3. What stays the same? What changes?
   What do you notice about the opposite sides of this quadrilateral?



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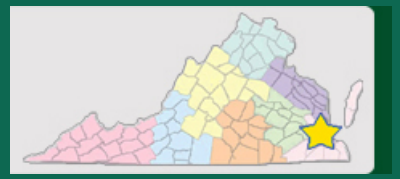
Quadrilateral Properties Laboratory

- Make one of the angles a right angle (You can use the square corner of a piece of paper or a file card to check your accuracy.)
- What happens to the other angles?
- Will this always be true when you make one angle of a parallelogram a right angle? How do you know?
- Is it still a parallelogram? a quadrilateral? a polygon?
- What other name, besides polygon, quadrilateral, and parallelogram, can be given to it now?



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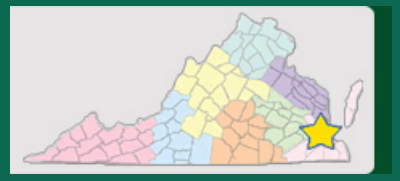
3. Quadrilateral Sorting Laboratory

1. Complete the Table by listing the letters of all the quadrilaterals that meet the given criteria.
2. Which category is the largest? What name can be used to describe this category?
3. Which lists which are the same? What name can be used to describe quadrilaterals with these properties?
4. Are there any lists that are proper subsets of another list? If so, which ones?
5. Are there any lists that aren't subsets of one another that have some but not all members in common? If so, which ones?
6. Which lists have no members in common?



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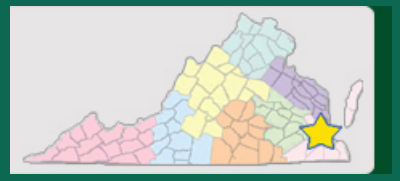
Quadrilateral Sorting Laboratory

1. Label each of the categories in the Table with the most specific name possible using the labels kite, quadrilateral, parallelogram, rectangle, rhombus, square, and trapezoid. For example, #1 - a quadrilateral that has 4 right angles is a rectangle. (Having 4 right angles isn't enough to make it a square; it would need 4 congruent sides as well.)
2. Compare your results to that of the other Lab Groups. Then fill out the family tree by inserting the names kites, rectangles, squares, and trapezoids into the appropriate places on the diagram.



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Quadrilateral Table

1. has 4 right angles

---

2. has exactly one pair of parallel sides

---

3. has two pair of opposite sides congruent

---

4. has 4 congruent sides

---

5. has two pair of opposite sides parallel

---

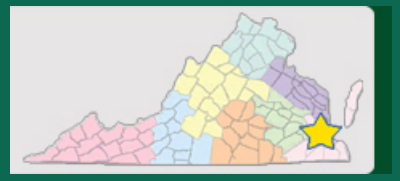
6. has no sides congruent

---

7. is convex with two pair of adjacent sides congruent, but not all sides congruent

---

8. has perpendicular diagonals



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Quadrilateral Table

9. has opposite angles congruent

---

10. is concave

---

11. is convex

---

12. its diagonals bisect one another

---

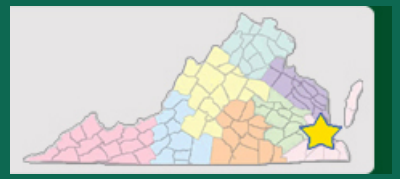
13. has four sides

---

14. has four congruent angles

---

15. has four congruent sides and four congruent angles



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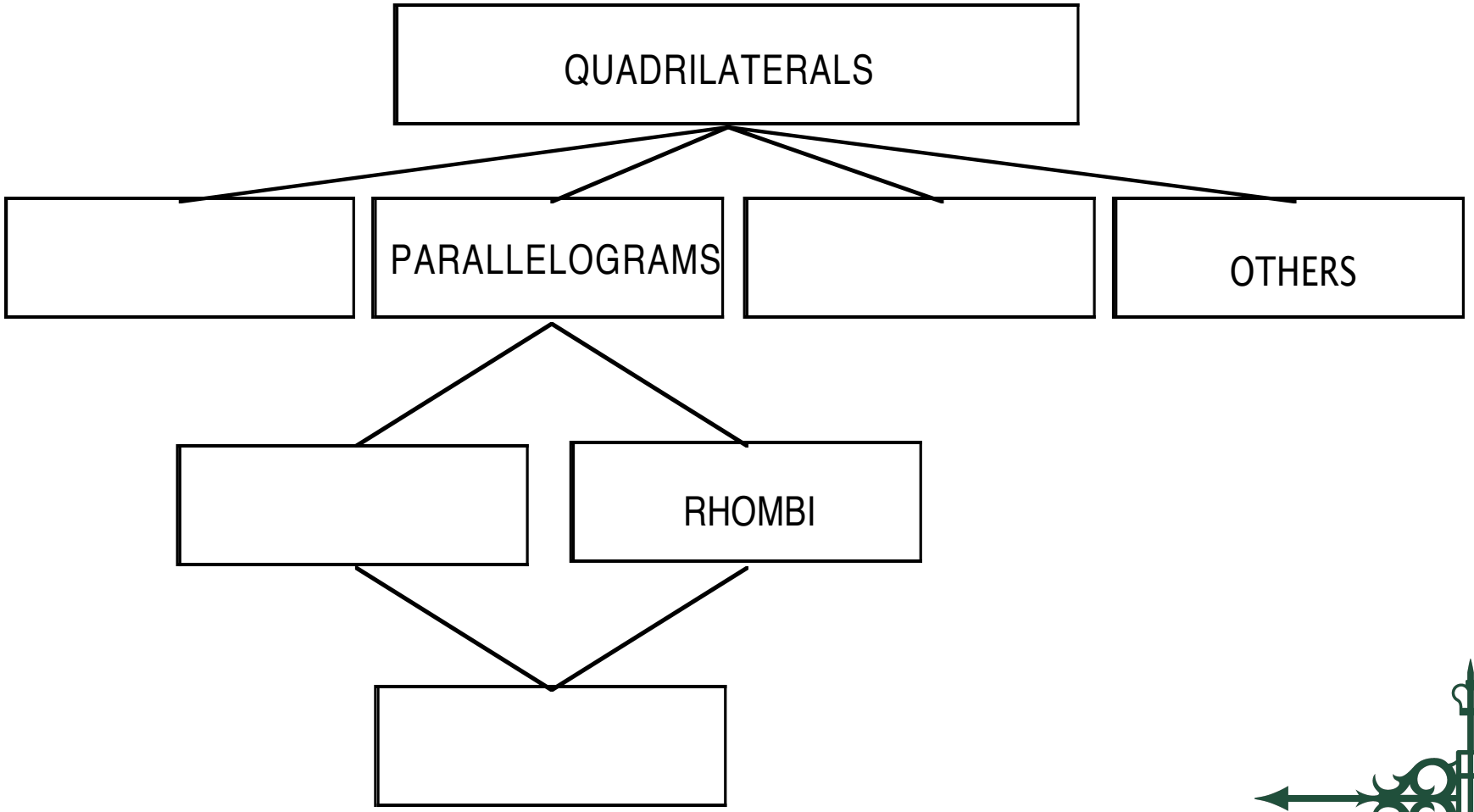
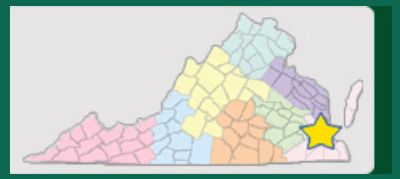
Quadrilateral Table





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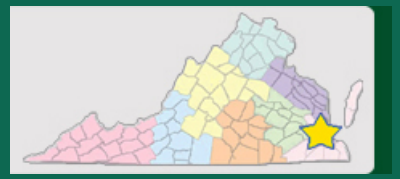
5. More Knots in a Rope

- Make a polygon with your rope.
- Make a parallelogram with four sides of the same length.
- Make a quadrilateral with opposite sides two units in length and four units in length.
- Make a parallelogram with four sides of the same length.



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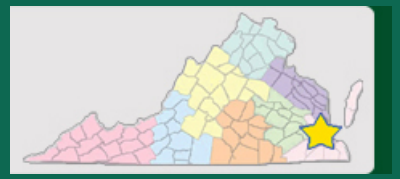
5. More Knots in a Rope

- Make a quadrilateral whose sides are 1-1-5-5 units.
- Make another whose sides are 2-2-4-4.
- Can you make a trapezoid with your rope?



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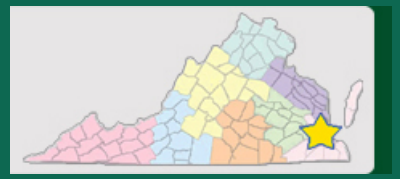
5. Other Possible Rope Topics

- Triangle types (equilateral, isosceles, scalene, obtuse, acute)
- Make all the hexagons you can
- Pythagorean triples
- The Triangle Inequality Theorem



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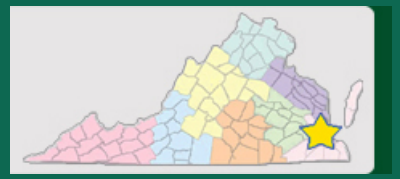
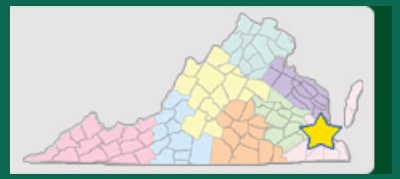
6. The perimeter is 24. What's the area?

- Working in groups of two or three, cut off a strip of adding machine paper that is at least 24 " long.
- Glue it together so that it forms a paper collar that is about 24.1 inches around.
- Count out 38 one-inch cubes.
- Using as many of these cubes as needed, construct **rectangular** arrays that fill the paper collar you just made.
- Record the dimensions of each array and record its area and perimeter. Note any patterns.



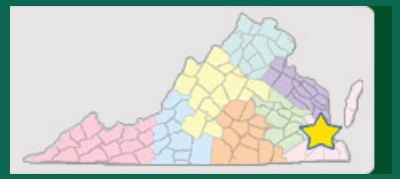
6. The perimeter is 24. What's the area?

Length	Width	Perimeter	Area
1	11	24	11
2	10	24	20
3	9	24	27
4	8	24	32
5	7	24	35
6	6	24	36



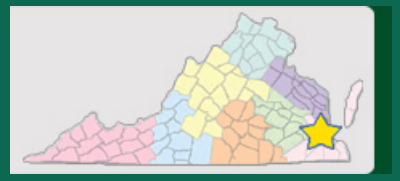
6. The perimeter is 24. What's the area?

- Working in groups of two or three, count out 24 one-inch cubes.
- Form these 24 cubes into a **rectangular** array, using all 24 cubes.
- Record the dimensions of each array and record its area and perimeter. Note any patterns you find.

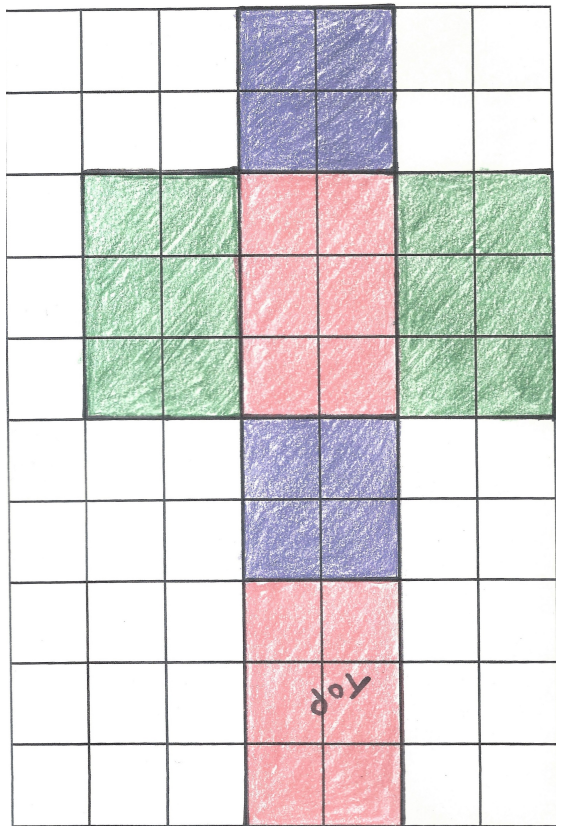


6. The perimeter is 24. What's the area?

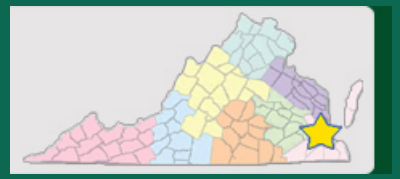
Length	Width	Perimeter	Area
1	24	50	24
2	12	28	24
3	8	22	24
4	6	20	24



7. Volume and Surface Area (p. 15)



Scale Factor	Length	Width	Height	S.A.
Starting Prism				
Doubling				
Tripling				
Quadriu-ling				



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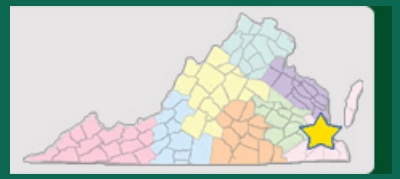
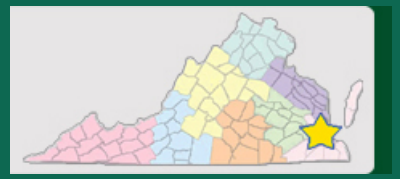
7. Volume and Surface Area (p. 15)

Scale Factor	Length	Width	Height	Surface Area	Volumne
Starting Prism	2	3	1	Counted 32 squares	6 cubes
Doubling the Width	2	6	2	12 + 12 + 4 + 4 +12 + 12 = 56 squares	24 cubes
Tripling the Width	2	9	2	2(lw + lh + wh) 2(2*9 + 2*2 + 9*2) = 80 squares	36 cubes
Doubling the Height	2	3	4	6 + 6 + 8 + 8 + 12 + 12 = 52 squares	24 cubes
Tripling the Height	2	3	6	2(lw + lh + wh) 2(2*3 + 2*6 + 3*6) = 72 squares	36 cubes



7. Volume and Surface Area (p. 15)

- When we doubled or tripled one measurement the volume was doubled or tripled. So if we know the volume of the first prism we can multiply its volume by whatever scale factor we are increasing one measurement by to find out how much the volume of a new prism is.
- There is a direct relationship to changing one measured attribute to its change in Volume.
- What if we change more than one dimension?



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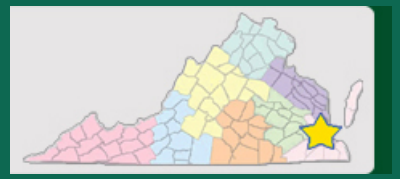
7. Volume and Surface Area (p. 15)

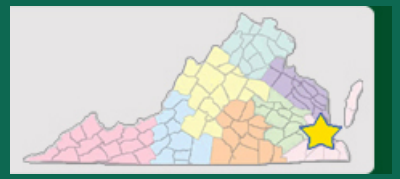
- When we doubled or tripled one measurement, it increases the total surface area, but we did not see any constant change.

- There is no direct relationship to changing one measured attribute to its changing surface area.



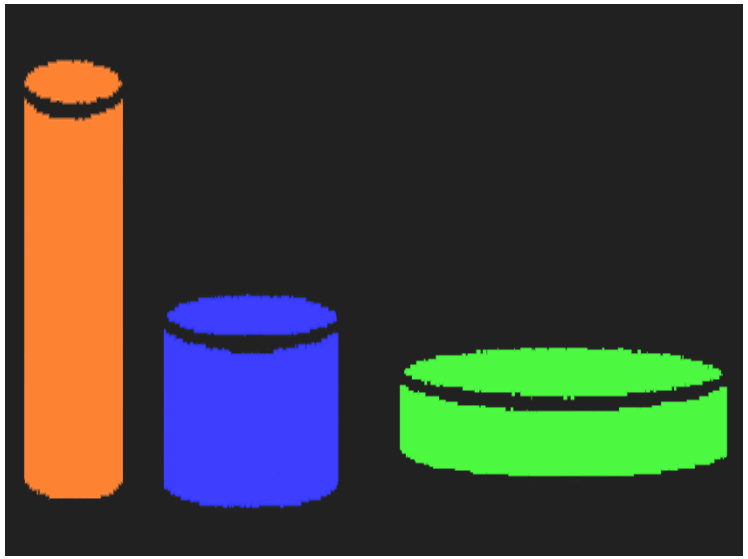
8. Tubes and Rice (p. 16)

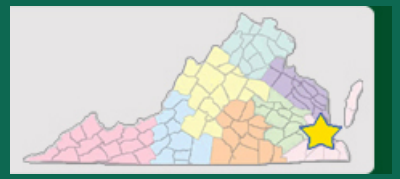
- Do 3 tubes with the same surface area have the same volume?
- Take one transparency & tape sides to make a tube (first tube).
- Take another transparency, cut into two equal pieces & tape sides to make a second tube.
- Take a third transparency, cut into three equal pieces & tape sides to make a third tube.
- Which tube has the greatest volume or do they all hold the same amount?



8. Tubes and Rice (p. 16)

- Fill the tall tube with plastic markers or rice.
- Put the second tube around the first.
- Pull up the first to see how the plastic markers "fit" the second tube.
- What do you think would happen with the third tube?





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QUESTIONS/CONTACT

Thank you!

Please feel free to contact us at:

Dr. Margie Mason – mmmaso@wm.edu

Sam Rhodes – srrhod@wm.edu

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