Questions, Discourse, and Productive Struggle: Integrating Three Effective Teaching Practices

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Today's Action-Packed Agenda

- Focusing on Three Effective Mathematics Teaching Practices
- Solve the Bike & Truck Task
- **Questioning**: Pay It Forward and The Cases of Vanessa Culver & Steven Taylor
- **Discourse**: Bike & Truck and The Case of Shalunda Shackelford
- **Productive Struggle**: The Lifeguard Task
- Connecting productive struggle, questioning, and discourse

Effective Mathematics Teaching Practices

A research-based consensus on what constitutes good teaching

Changing Instruction Means Changing Beliefs

Many parents and educators believe that students should be taught as they were taught, through **memorizing facts, formulas, and procedures and then practicing skills** over and over again (e.g., Sam and Ernest 2000).

This view perpetuates the **traditional lesson paradigm** that features **review, demonstration, and practice** and is still pervasive in many classrooms (Banilower et al. 2006; Weiss and Pasley 2004).

Teachers, as well as parents, are **often not convinced** that straying from these established beliefs and practices will be more effective for student learning (Barkatsas and Malone 2005; Wilken 2008).

Effective Mathematics Teaching Practices

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2. Implement tasks that promote reasoning and p
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4. Facilitate meaningful math
5. Pose purposeful
6. Build
7.
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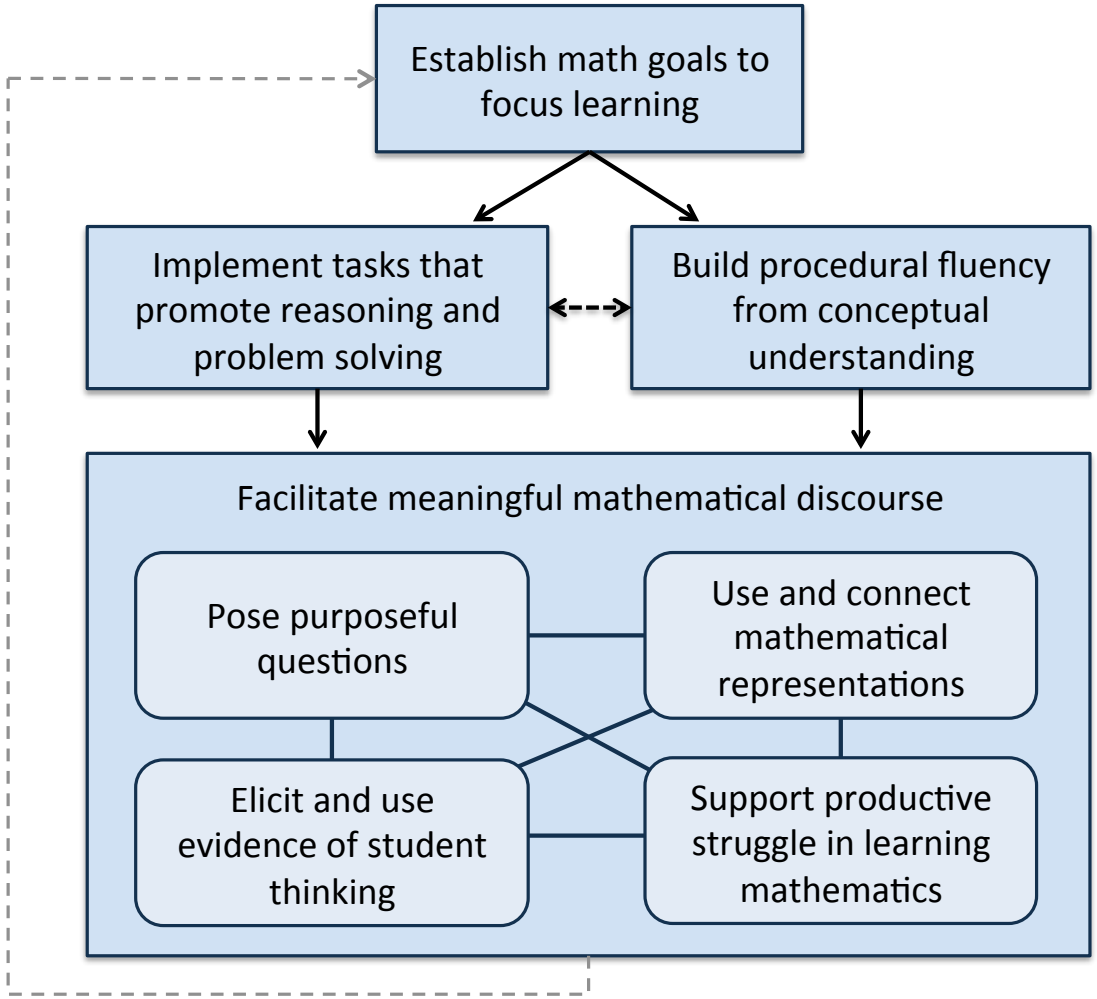
A research-based consensus on what constitutes good teaching

Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. **Facilitate meaningful mathematical discourse.**
5. **Pose purposeful questions.**
6. Build procedural fluency from conceptual understanding.
7. **Support productive struggle in learning mathematics.**
8. Elicit and use evidence of student thinking.

Effective Mathematics Teaching Practices



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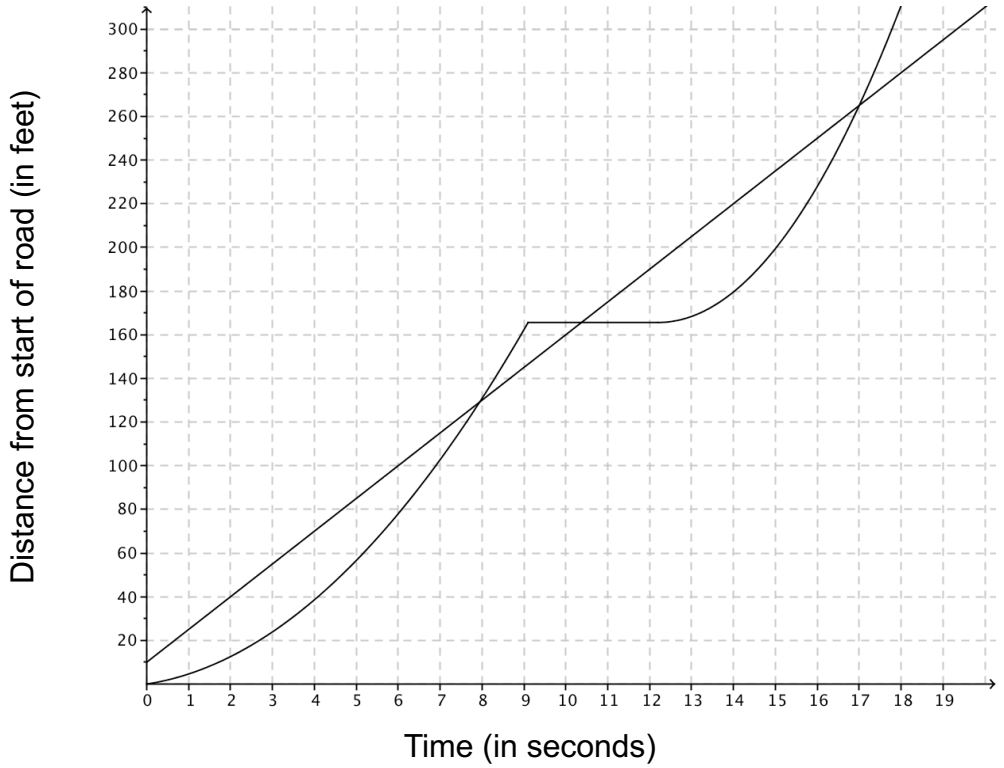
Solving and Discussing the Bike and Truck Task

A task of high cognitive demand

Bike and Truck Task

A bicycle traveling at a steady rate and a truck are moving along a road in the same direction. The graph below shows their positions as a function of time. Let B(t) represent the bicycle's distance and K(t) represent the truck's distance.



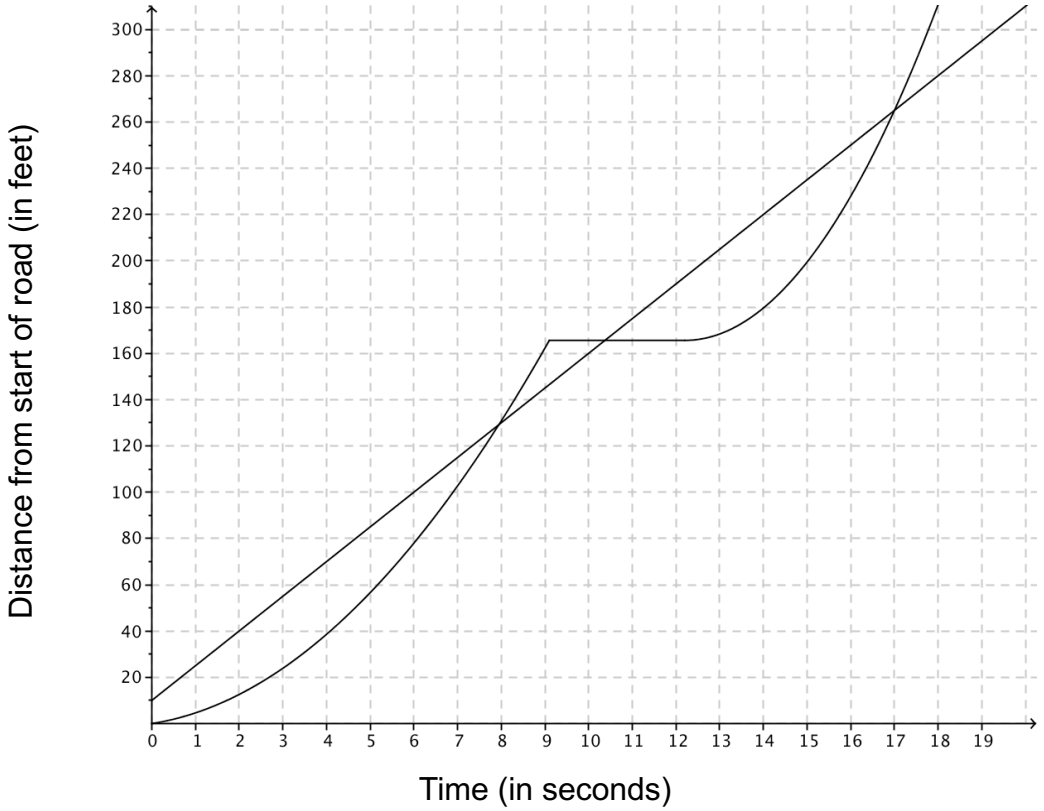
Bike and Truck Task

1. Label the graphs appropriately with B(t) and K(t). Explain how you made your decision.
2. Describe the movement of the truck. Explain how you used the values of B(t) and K(t) to make decisions about your description.
3. Which vehicle was first to reach 300 feet from the start of the road? How can you use the domain and/or range to determine which vehicle was the first to reach 300 feet? Explain your reasoning in words.
4. Jack claims that the average rate of change for both the bicycle and the truck was the same in the first 17 seconds of travel. Explain why you agree or disagree with Jack and why.

Bike and Truck Task

Discussion question introduced by the teacher:

- Between what two seconds did the truck drive the fastest? How do you know?



Questioning

The Cases of Vanessa Culver and Steven Taylor

The Pay It Forward Task

In the movie “Pay It Forward,” a student, Trevor, comes up with an idea that he thought could change the world. He decides to do a good deed for three people and then each of the three people would do a good deed for three more people and so on. He believed that before long there would be good things happening to billions of people. At stage 1 of the process, Trevor completes three good deeds.

How does the number of good deeds grow from stage to stage?

How many good deeds would be completed at stage 5?

Describe a function that would model the Pay It Forward process at *any* stage.

The Cases of Vanessa Culver and Steven Taylor

Read the Case of Vanessa Culver and The Case of Steven Taylor, both teachers using the Pay It Forward Task with their classes.

Consider the following:

- What did each teacher learn about students' thinking?
- How are the types of questions asked by Ms. Culver and Mr. Taylor the same and how are they different?
- Do the differences in the types of questions matter?

Be prepared to cite line numbers as evidence for your claims.

Pose Purposeful Questions

Effective Questions should:

- Reveal students' current understandings;
- Encourage students to explain, elaborate, or clarify their thinking; and
- Make the mathematics more visible and accessible for student examination and discussion.

Teachers' questions are crucial in helping students make connections and learn important mathematics and science concepts. Teachers need to know how students typically think about particular concepts, how to determine what a particular student or group of students thinks about those ideas, and how to help students deepen their understanding.

(Weiss & Pasley, 2004)

Characteristics of Questions That Support Students' Exploration

Assessing

- Based closely on the work the student has produced
- Clarify what the stud… what tha…
- …nation to the teacher about what the student understands

Stay & listen

Advancing

- Use what students have produced as a b…is for making progress toward the
- Mo… thinking by pre… what they know to a new …
- Press students to think about something they are not currently thinking about

Walk away

Discourse

The Case of Shalunda Shackelford

Bike and Truck Task: Video Context

School: Tyner Academy

Principal: Carol Goss

Teacher: Shalunda Shackelford

Class: Algebra 1

Curriculum: IFL Lessons Creating and Interpreting Functions

Class Size: 26

At the time the video was filmed, Shalunda Shackelford was a teacher at Tyner Academy in the Hamilton County School District. The lesson occurred in April in an Algebra 1 class.

The teacher is working on promoting classroom discourse.

Ms. Shackelford's Mathematics Learning Goals

Students will understand that:

- The language of change and rate of change (increasing, decreasing, constant, relative maximum or minimum) can be used to describe how two quantities vary together over a range of possible values.
- Context is important for interpreting key features of a graph portraying the relationship between time and distance.
- The average rate of change is the ratio of the change in the dependent variable to the change in the independent variable for a specified interval in the domain.

Connections to the Common Core Standards

Interpreting Functions F–IF

Interpret functions that arise in applications in terms of the context.

F-IF.B4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

★Indicates that the standard is also a Mathematical Modeling standard.

Connections to the Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. Reason abstractly and quantitatively.
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Bike and Truck Task: Context of the Video Clip

Prior to the lesson:

- The teacher has been working on facilitating mathematical discussions and targeting the Common Core standards for content and mathematical practice in very deliberate ways.

Video Clip:

- In the video, you will see students discussing their work on the Bike and Truck task and defending different mathematical ideas and conceptions.

Lens for Watching Video Clip 1

As you watch the first clip, pay attention to the ways in which the teacher facilitates discourse. Specifically:

- How does the teacher support students to share and defend their own ideas?
- How does the teacher provide students with the opportunity to clarify understandings?
- How does the teacher provide students with the opportunity to develop convincing arguments?

Facilitate Meaningful Mathematical Discourse

Mathematical Discourse should:

- Build on and honor students' thinking;
- Provide students with the opportunity to share ideas, clarify understandings, and develop convincing arguments; and
- Advance the mathematical learning of the whole class.

Discussions that focus on cognitively challenging mathematical tasks…are a primary mechanism for promoting conceptual understanding of mathematics (Hatano & Inagaki 1991; Michaels, O'Connor & Resnick 2008).

(Smith, Hughes, Engle and Stein 2009, p. 549)

Facilitate Meaningful Mathematics Discourse

In effective discourse teachers:

- Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations;
- Select and sequence student approaches and solution strategies for whole-class analysis and discussion;
- Facilitate discourse among students by positioning them as authors of ideas, who explain and defund their approaches; and
- Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.

Facilitate Meaningful Mathematical Discourse

5 Practices for Orchestrating Productive Discussions

- Anticipating
- Monitoring
- Selecting
- Sequencing
- Connecting

Smith & Stein (2011)

Teacher Discourse Moves

- Waiting
- Inviting student participation
- Revoicing
- Asking students to revoice
- Probing a student's thinking
- Creating opportunities to engage with another's reasoning

Herbel-Eisenmann, Steele, & Cirillo (2013)

Productive Struggle

The Lifeguard Task

What is productive struggle?

Turn and talk to the people near you.

- What does productive struggle look like and sound like?
  - For students?
  - For teachers?
- What does unproductive struggle look like and sound like?
- What can teachers do to:
  - Support productive struggle?
  - Convert unproductive struggle to productive struggle?

Support Productive Struggle in Learning Mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.

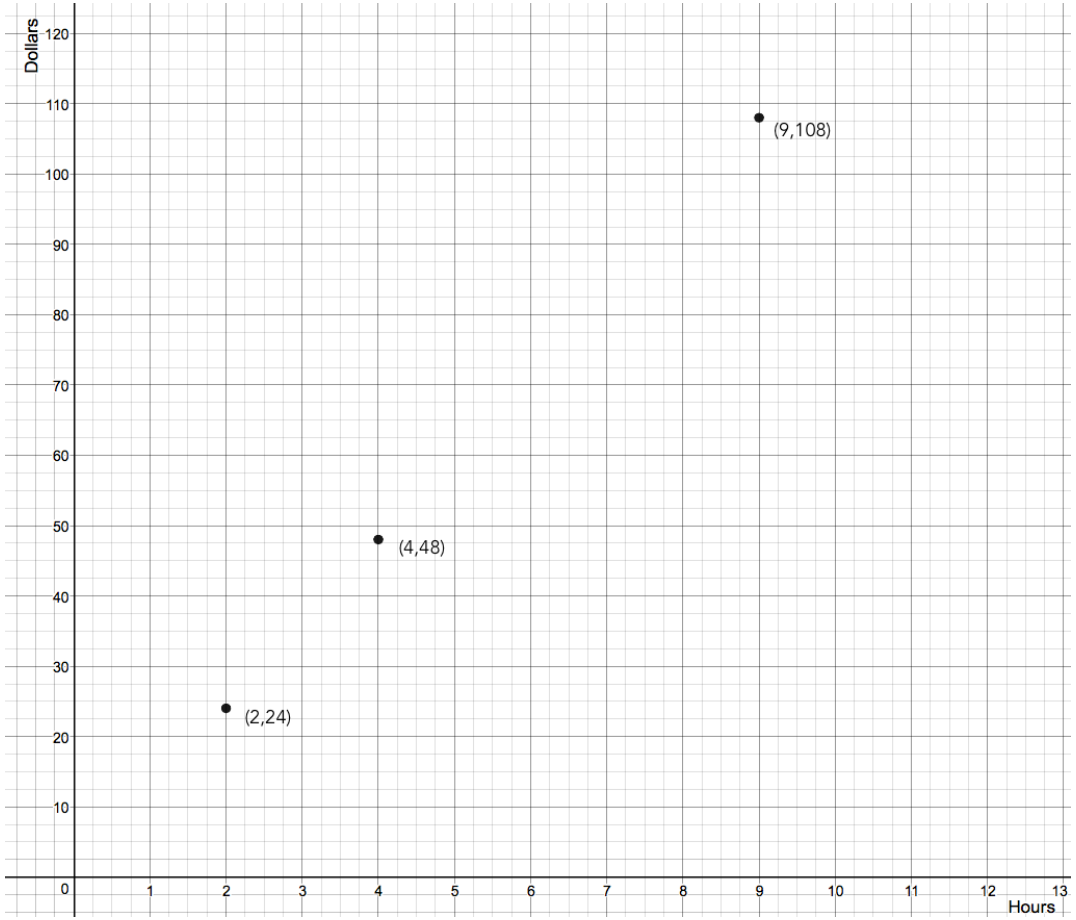
By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated.

Hiebert & Grouws, 2007, pp. 387-388

The Lifeguard Task

Benjamin's friend, Maleeka, is taking a job as a lifeguard at the city pool. She will be paid a constant rate per hour. The graph represents Maleeka's potential earnings for three different numbers of hours of summer work.

1. Maleeka's short-term goal is to earn more than $80 for new Xbox One™ games. Describe the number of hours that she must work in order to make enough money for the games. Show all work and represent your answer using words, inequality notation, and a number line.
2. Maleeka's long-term summer goal is to make more than $750. Determine the number of hours that she must work in order to meet this goal. Show all work and represent your answer using words, inequality notation, and a number line.



Adapted from Institute for Learning (2015a). Lesson guides and student workbooks available at ifl.pitt.edu.

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Read the mini-dialogues, then:

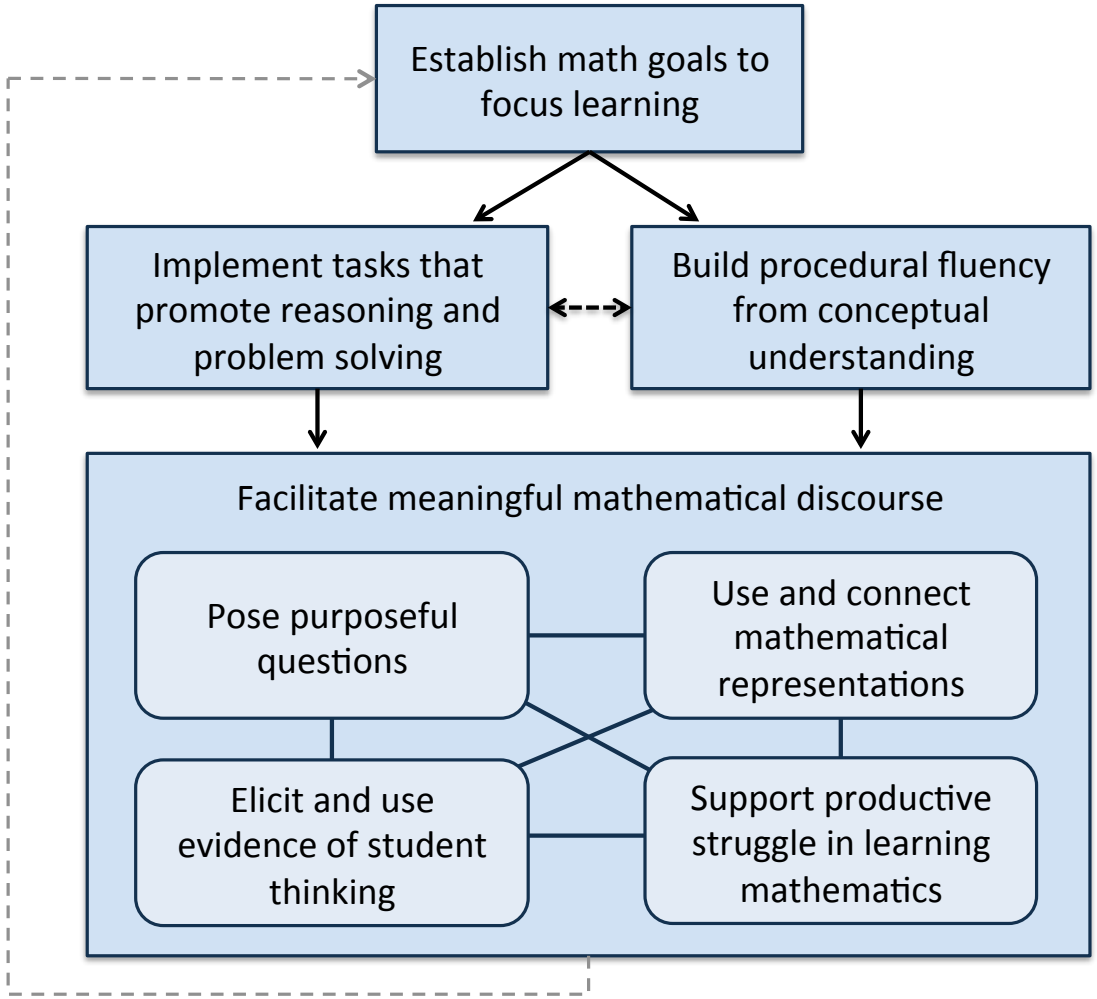
- Discuss the nature of each student's struggle.
- Identify what the teacher does to help students move beyond the impasse they had reached.
- Determine whether or not the teacher supported students' productive struggle.

Five Teacher Responses to Productive Struggle

- Telling (4)
  supplying students with information that removes the struggle
- Directed guidance (1)
  redirecting students to another strategy consistent with the teacher's thinking
- Probing guidance (3)
  determining what the student is thinking, encouraging self-reflection, offering ideas based on student thinking
- Affordance (2)
  asking the student to articulate what they have done, encouraging continued effort with limited intervention, allowing students time to work
- Unfocused or vague (5)
  provides a suggestion that is too general to be supportive

Adapted from Warshauer, 2015 and Smith, Steele, & Raith, 2017

Effective Mathematics Teaching Practices



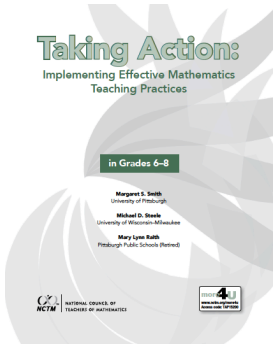
Next Steps

- How can you integrate opportunities for productive struggle in your classroom through the tasks you choose and goals you set?
- How will you promote productive struggle through questioning and discourse?
- How will you share with your students and colleagues the importance of productive struggle in learning mathematics?

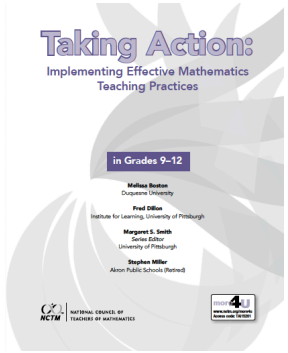
Taking Action Series

Coming Soon!

Huinker, DeAnn and Victoria Bill. Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5. Reston, VA: National Council of Teachers of Mathematics, 2017.



Smith, Margaret, Michael D. Steele, and Mary Lynn Raith. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8.* Reston, VA: National Council of Teachers of Mathematics, 2017.



Boston, Melissa, Fredrick Dillon, Margaret Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12.* Reston, VA: National Council of Teachers of Mathematics, 2017.

Taking Action Sessions At NCTM

Taking Action in **Middle** School: Implementing Effective Mathematics Teaching Practices

- Who: Smith and Steele
- When: Friday, April 7
- Time: 11:00 am - 12:00 pm
- Where: Hemisfair 1

Taking Action in **High** School: Implementing Effective Mathematics Teaching Practices

- Who: Boston and Dillon
- When: Friday, April 7
- Time: 2:00 – 3:00 pm
- Where: Hemisfair 1

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Thank you!

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