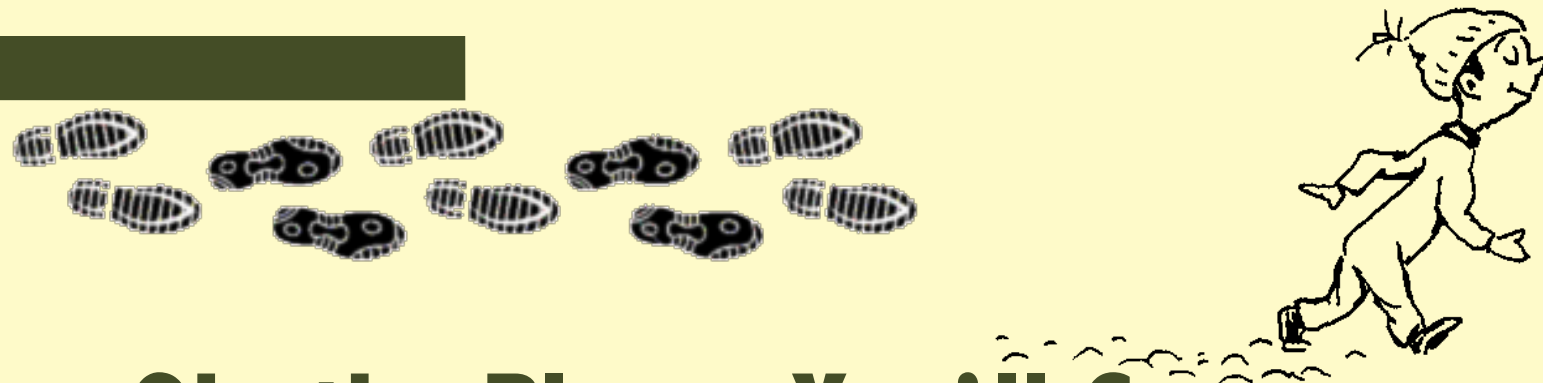




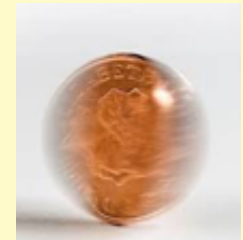
Oh, the Places You'll Go... Or Will You?

NCTM Annual Meeting & Exposition

April 6, 2017

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Going Places?

Congratulations!
Today is your day.
You're off to Great Places!
You're off and away!

You have brains in your head.
You have feet in your shoes.
You can steer yourself
any direction you choose.





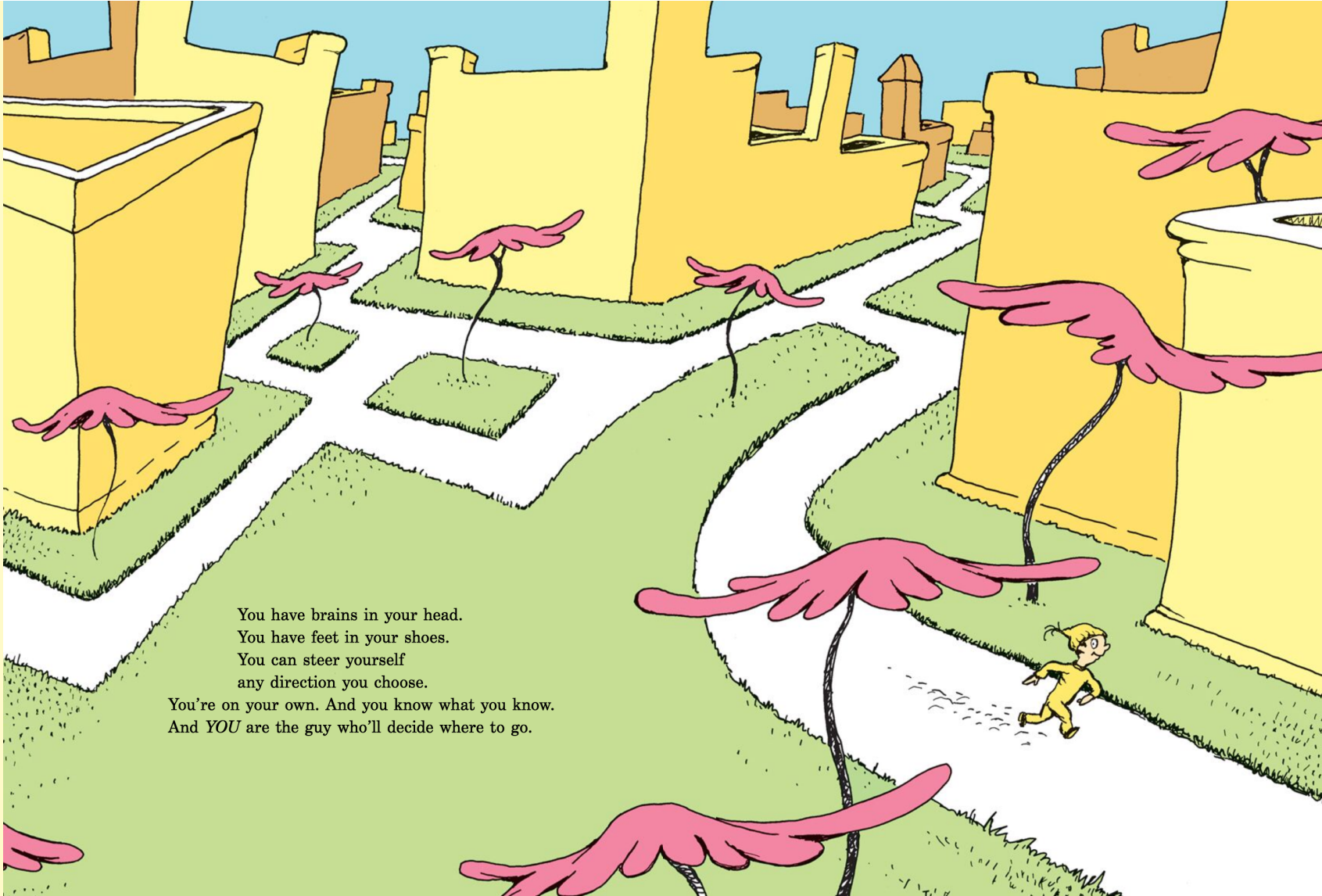
Going Places

You have **a** brain in your head.

You have **a coin** in your **hand**.

**It will steer you
in any direction it does choose.**

You're on your own. **Do you know what you know?**



You have brains in your head.
You have feet in your shoes.
You can steer yourself
any direction you choose.
You're on your own. And you know what you know.
And *YOU* are the guy who'll decide where to go.

Going Places

And will you succeed?

Yes! You will, indeed!

(98 and 3/4 percent guaranteed.)

Be W.I.S.E.

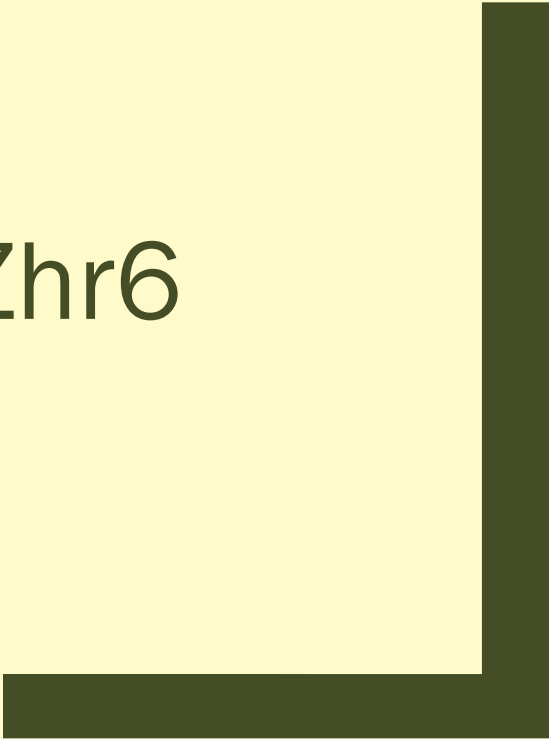
- Wonder
- Investigate
- Study
- Extend

Going Places

You'll get mixed up, of course,
as you already know.

You'll get mixed up
with many strange birds as you go.





<https://goo.gl/NrZhr6>



NATIONAL MUSEUM OF MATHEMATICS

THE 2017 ROSENTHAL PRIZE

for Innovation and Inspiration
in Math Teaching



**Do you have a classroom activity that is: Innovative? Inspiring? Engaging?
Hands-on? Original? Replicable? Designed for students in grades 4-8?**

In this lesson, students use coin flip results to take a simultaneous random walk on a number line. Using their own motions as a probability simulation, students investigate, develop, and analyze a probability model for compound events.

Students should be familiar with finding probabilities of events like coin flips. Students should also have experience with plotting numerical data in plots on a number line or histogram.

Lesson Goals for Grade 7

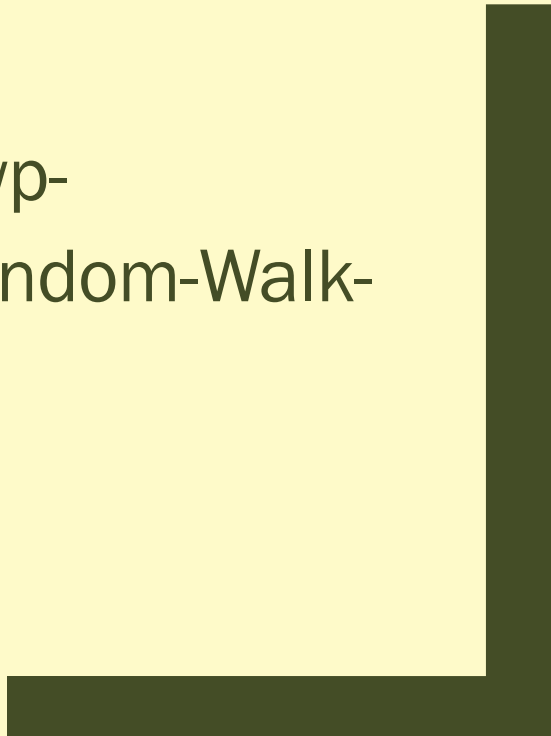
Students will:

- use a simulation to generate frequencies for compound events
- draw conclusions about probability models by examining frequencies in data
- find probabilities of compound events by creating representations of sample spaces
- use a probability distribution to generate and answer questions about probability

Lesson Day 1

Overview

A student takes a random walk on a number line determined by flips of a fair coin: one step forward with a flip of a heads, and one step back with a flip of tails. After students propose questions about an individual's random walk, the whole class, joined by other students if possible, conducts a simultaneous random walk. Students arrive in various locations on the number line and then walk into columns to create a human bar graph that records their final location. Students then investigate the theoretical probabilities involved in the random walk.



<http://momath.org/wp-content/uploads/2016/11/Random-Walk-lesson-9.12.15.pdf>

Mathematical Habits

- Notice, observe, wonder, speculate.
- Seek connections, and structures.
- Plan ahead.
- Ask questions and discuss your thinking with others.
- Tinker, try different strategies, and keep an open mind.
- Make mistakes and revise your thinking.
- Record your ideas and reasoning.
- Check your work and ask if it makes sense.
- Organize and memorize.
- Take on a new challenge!
- Believe in the power of your own effort.

Investigate chance processes and develop, use, and evaluate probability models.

- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

Investigate chance processes and develop, use, and evaluate probability models.

- Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Investigate chance processes and develop, use, and evaluate probability models.

- Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Investigate chance processes and develop, use, and evaluate probability models.

- Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

Investigate chance processes and develop, use, and evaluate probability models.

- Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

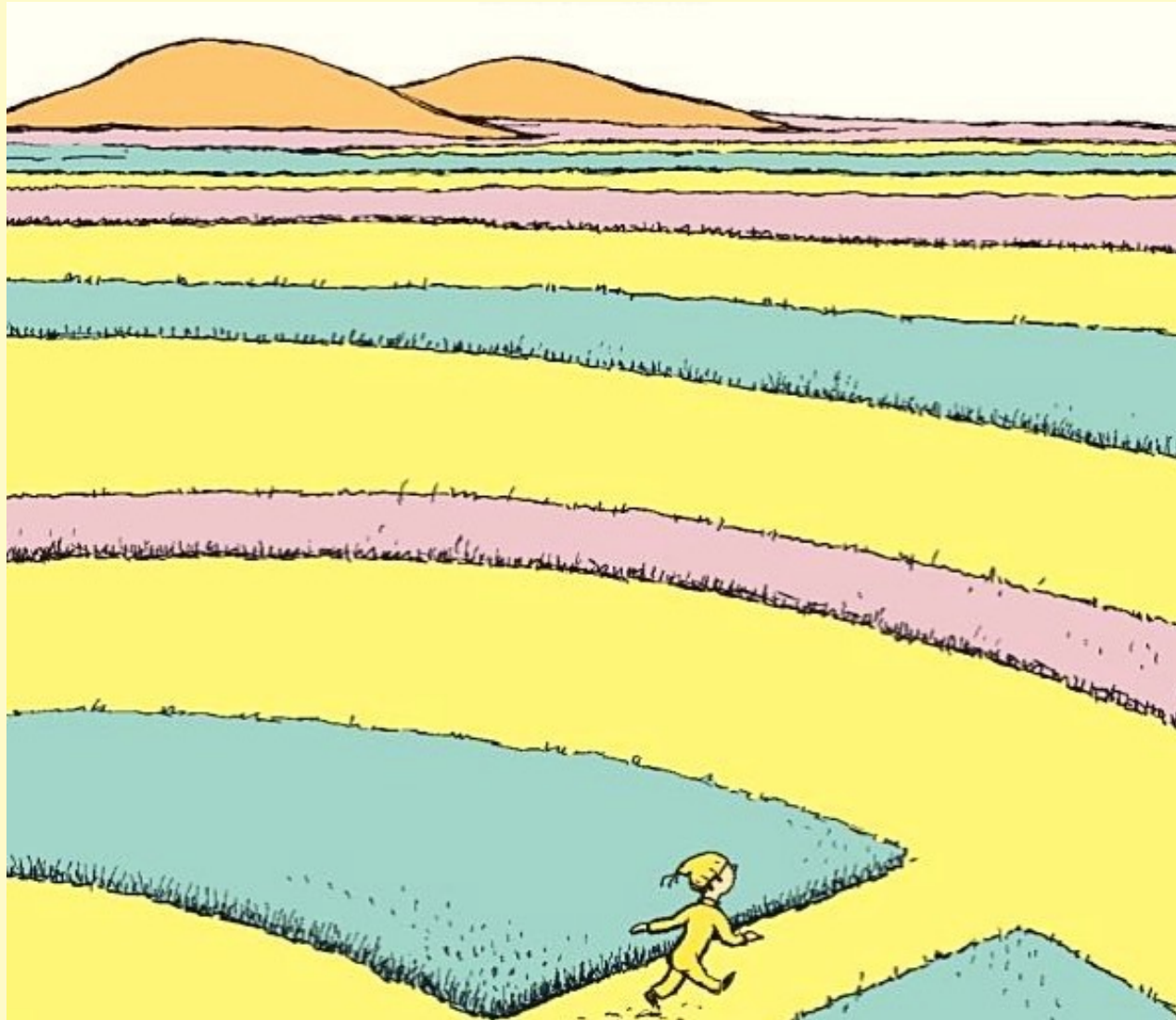
Investigate chance processes and develop, use, and evaluate probability models.

- (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

Going Places

It's opener there
in the wide open air.

Out there things can happen
and frequently do
to people as brainy
and footsy as you.

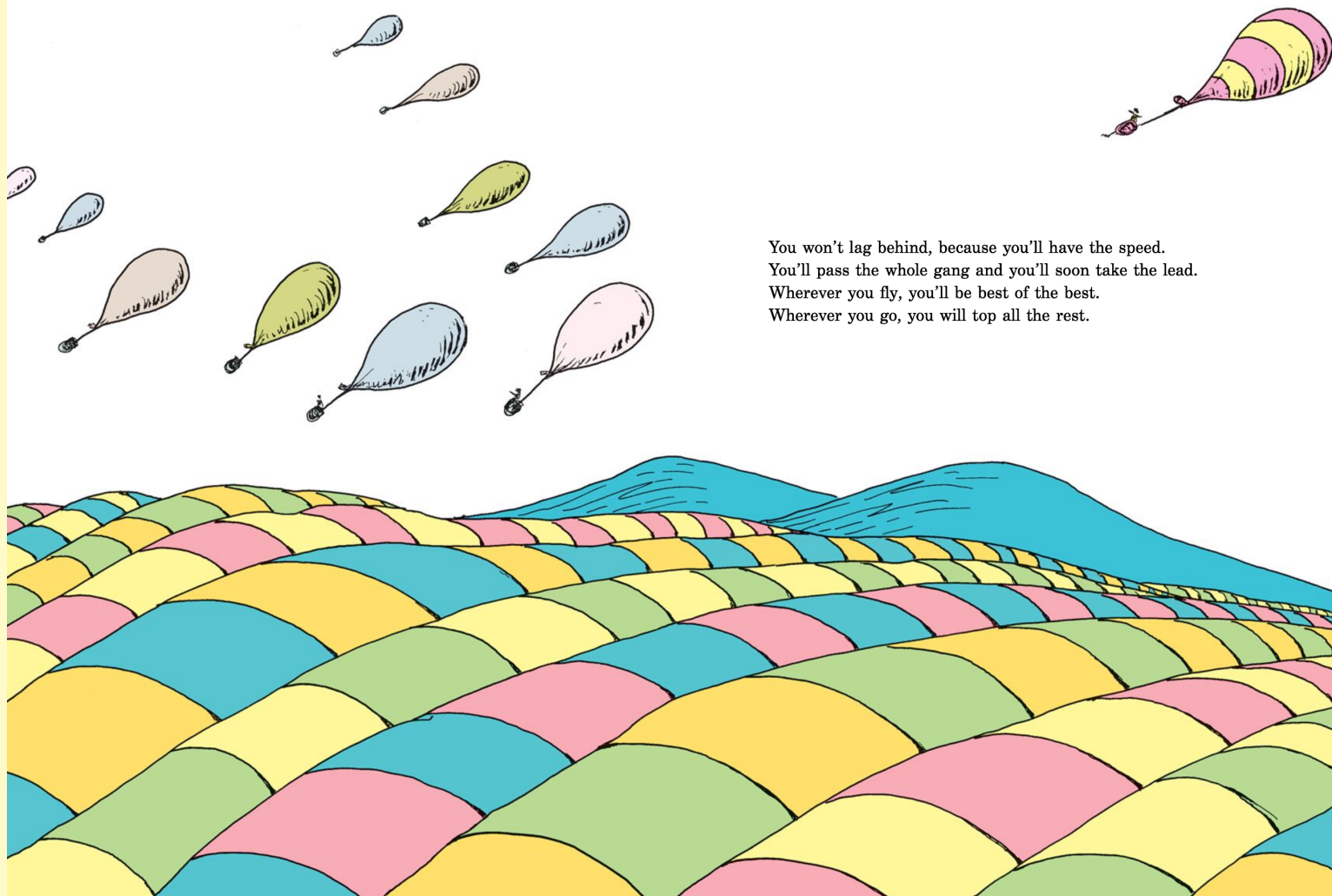


Anyone Have Any Questions?

Anyone Have Any Questions?

- Some students think this is a bad thing!

Ask A Question!



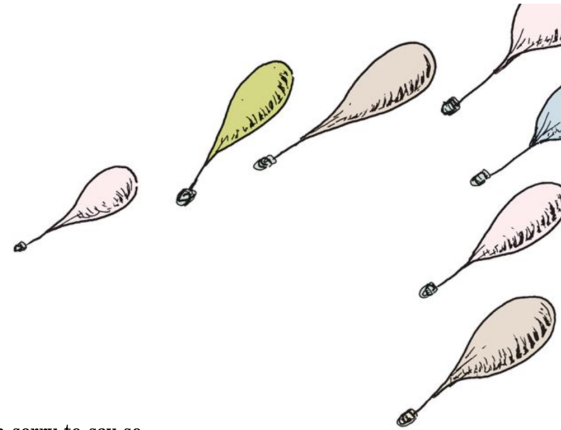
You won't lag behind, because you'll have the speed.
You'll pass the whole gang and you'll soon take the lead.
Wherever you fly, you'll be best of the best.
Wherever you go, you will top all the rest.

Except when you *don't*.
Because, sometimes, you *won't*.



I'm sorry to say so
but, sadly, it's true
that Bang-ups
and Hang-ups
can happen to you.

You can get all hung up
in a prickly perch.
And your gang will fly on.
You'll be left in a Lurch.



Investigations

- Questions such as “*How often will I get heads or tails?*” can be answered using the experimental data and the theoretical probability models. An individual may flip heads more often than tails, but a large sample will produce approximately equal percentages of heads and tails.

Investigations

- Students may notice that the number of total possible outcomes when flipping a coin N times doubles as the number of flips grows by 1. Drawing a tree diagram may explain why this is the case.
- Students may be curious about the probability of everyone in the class flipping heads at the same time and moving in the same direction. What are the chances of that event?

Investigations

- Ask students to respond to the prompt, “*Where will I arrive after a certain number of flips?*” in their own words. Examine students’ different responses to the question as they record them and ask students to share their statements with others. The statement, “There is a $\frac{3}{8}$ chance I will arrive back at zero after 4 flips,” for example, may be surprising to some students.
- What is the average distance a person will be from zero after 4 flips? Introduce students to the idea of expected value.

Investigations

- Return to students' experience with the 6-Flip Trip. *Do you think the chance that a person will return to zero is greater than or smaller than $3/8$?* List all the possible ways to arrive at zero after 6 flips and determine the probability. Students may be interested in exploring how to find the total number of unique sequences of M heads within N coin flips.

Investigations

- By listing the number of ways to arrive at each location in Flip Trips of different length, students may notice that the number of ways to arrive at a location in an N -Flip Trip is the sum of the ways of arriving at the two adjacent locations in an $(N-1)$ -Flip Trip. Why is this so? (For example, there are 6 ways to arrive at 0 in a 4-Flip Trip, and 4 ways of arriving at 2. There will be 10 ways of arriving at 1 in a 5-Trip Flip. Why?) Students can begin to investigate some of the properties of Pascal's Triangle.

Investigations

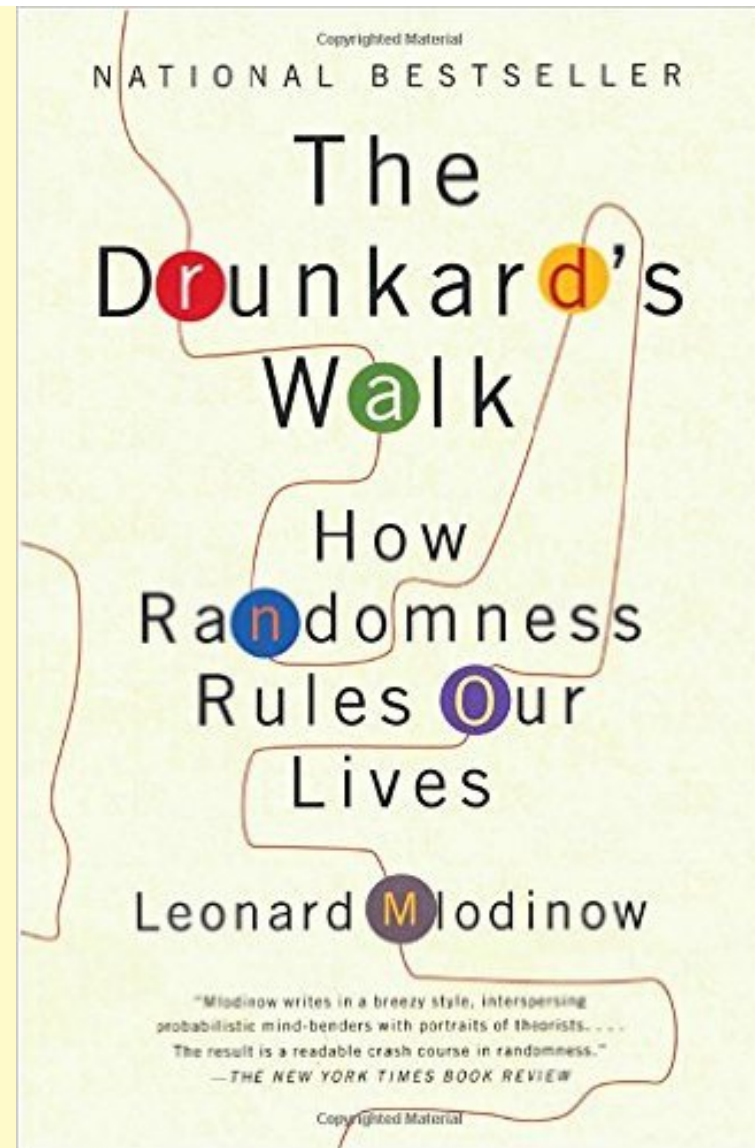
- What if we used dice and made a walk with a $\frac{2}{3}$ probability of moving forward and $\frac{1}{3}$ probability of moving back? How would the probabilities of arriving at the different final positions change?
- Pizza!

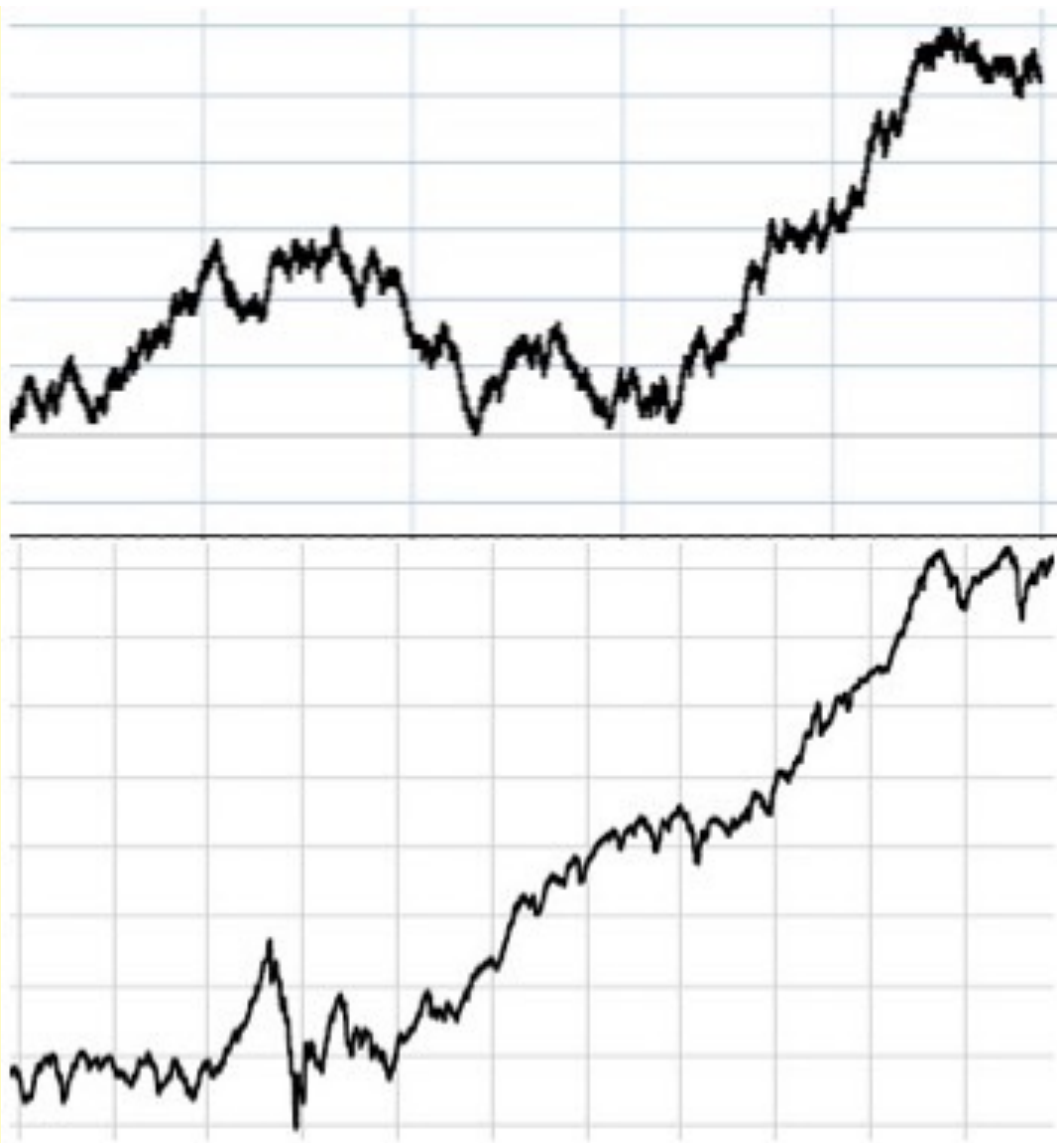
Investigations

- By showing us the true nature of chance and revealing the psychological illusions that cause us to misjudge the world around us, Mlodinow gives us the tools we need to make more informed decisions. From the classroom to the courtroom and from financial markets to supermarkets, Mlodinow's intriguing and illuminating look at how randomness, chance, and probability affect our daily lives will intrigue, awe, and inspire.

Leonard Mlodinow vividly demonstrates how our lives are profoundly informed by chance and randomness and how everything from wine ratings and corporate success to school grades and political polls are less reliable than we believe.

By showing us the true nature of chance and revealing the psychological illusions that cause us to misjudge the world around us, Mlodinow gives us the tools we need to make more informed decisions. From the classroom to the courtroom and from financial markets to supermarkets, Mlodinow's intriguing and illuminating look at how randomness, chance, and probability affect our daily lives will intrigue, awe, and inspire.





Curious Behavior

You'll get mixed up, of course,
as you already know.
You'll get mixed up
with many strange birds as you go.

How many times will a random walk cross a boundary line if permitted to continue walking forever? A simple random walk will cross every point an infinite number of times. And it will take literally forever.

Curious Behavior

The law of long leads, more properly known as the arcsine law, says that in a coin-tossing game, a surprisingly large fraction of sample paths leave one player in the lead almost all the time, and in very few cases will the lead change sides and fluctuate in the manner that is naively expected of a well-behaved coin.

Curious Behavior

Since we are tossing a fair coin to determine the steps of the walker, you might easily think that the random walk should be positive half of the time and negative half of the time, and that it should return to 0 frequently. But in fact, the arcsine law implies that with probability $1/2$, there will be *no* return to 0 during the second half of the walk, from time $n+1$ to $2n$, regardless of n , and it is not uncommon for the walk to stay positive (or negative) during the entire time from 1 to $2n$.

Curious Behavior

Player A is said to be in the lead at time n if the random walk is above the t -axis at that time. One can ask what is the most probable number of times that player A is in the lead, in a game of length $2m$. Most people will say that the answer to this question is m . Actually, m is the least likely number of times that player A is in the lead, and the most likely number of times in the lead is 0 or $2m$.

Curious Behavior

The probability of being at the origin goes to 0, but there is probability 1 of eventually returning to the origin.

Although the walk will return infinitely often, the expected time between returns is infinite!

Thank you!

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