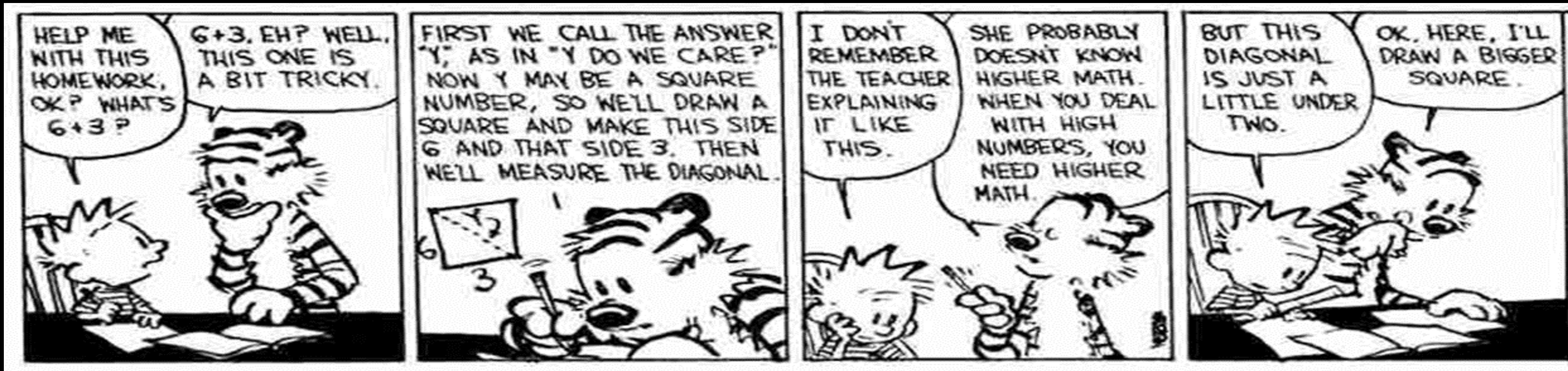
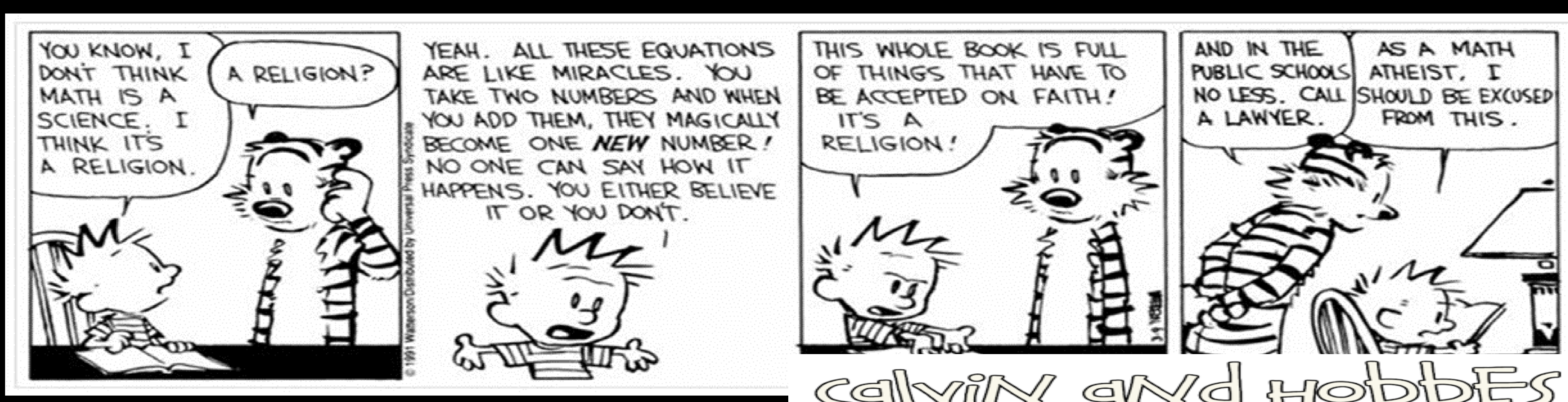


Help!! My students are counting on their fingers!

The truth about fluency and how to achieve it.



NCTM 2017

Alison J. Mello

Foxborough Public Schools

Math Consulting Professionals

@alisonmellomath

Getting to know each other

I'll go first…………

- Alison Mello
- Educator for 20+ years
- Foxboro, Massachusetts
- Grades 4-8, specialist, curriculum director
- Graduate instructor
- Math consultant
- Doctoral student
- Board member: ATMIM and MassMATE
- Mom of 16 year old twin girls ☺

Your Turn…

[Visit kahoot.it](kahoot.it)

Is this familiar?

What is fluency anyway?

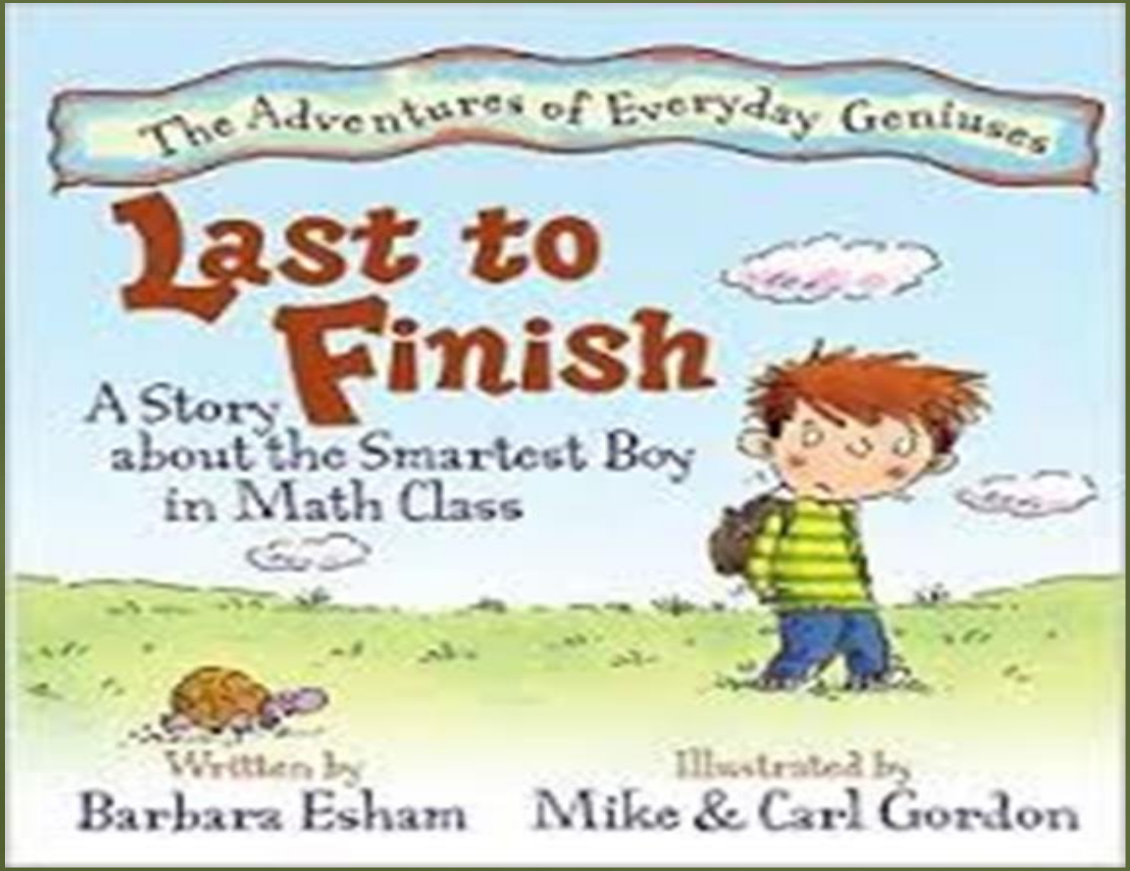
Fluency includes three ideas:

- ***Efficiency*** means that the student can carry out the strategy easily.
- ***Accuracy*** depends on precise recording, knowledge of number relationships, and checking results.
- ***Flexibility*** requires the knowledge of more than one approach to solve the problem and to check the results.

"Developing Computational Fluency with Whole Numbers in the Elementary Grades" – Susan Jo Russell

One indication of fluency in mathematics is when students are able to explain a strategy used to solve basic addition facts problem and do not solely rely on recall. In the Henry and Brown study, students explained their strategies for solving a set of addition problems during one-on-one interviews and small group sessions. Evidence in the Henry and Brown study indicated that successful arithmetic skills are often stemmed from many techniques, not relying on counting, memorization, or the use of strategies alone.

Simply Speed??? NOPE!!



To count or not to count?

- From the neurocognitive perspective, finger counting provides multisensory input, which conveys both cardinal and ordinal aspects of numbers.
- Recent data indicate that children with good finger-based numerical representations show better arithmetic skills and that training finger gnosis, or "finger sense," enhances mathematical skills.
- Therefore neurocognitive researchers conclude that elaborate finger-based numerical representations are beneficial for later numerical development.

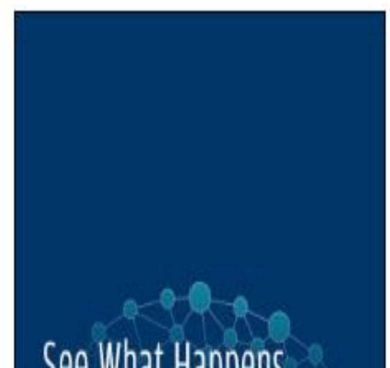
Moeller, Martignon, Wessolowski, Engel, & Nuerk (2011)

Philippe Lissac / Godong / Corbis

Why Kids Should Use Their Fingers in Math Class

Evidence from brain science suggests that far from being "babyish," the technique is essential for mathematical achievement.

JO BOALER AND LANG CHEN | APR 13, 2016 | EDUCATION



Ok, but the story doesn't end there…

- Research in mathematics education recommends **fostering mentally based numerical representations so as to induce children to abandon finger counting at the end of first or beginning of second grade.**

- More precisely, mathematics education recommends first using finger counting, then concrete structured representations and, finally, mental representations of numbers to perform numerical operations.

Moeller, Martignon, Wessolowski, Engel, & Nuerk (2011)

So who's right, the neuroscientists or the math researchers?

To answer that, reflect on what you see in your classrooms

Do you see what I see?

- Students using fingers for multi-digit computations?
- Students who miss that their answers are unreasonable?
- A lack of perseverance when involved in calculations?
- A heavy reliance on pencil and paper or calculators?

If so, what's the plan?

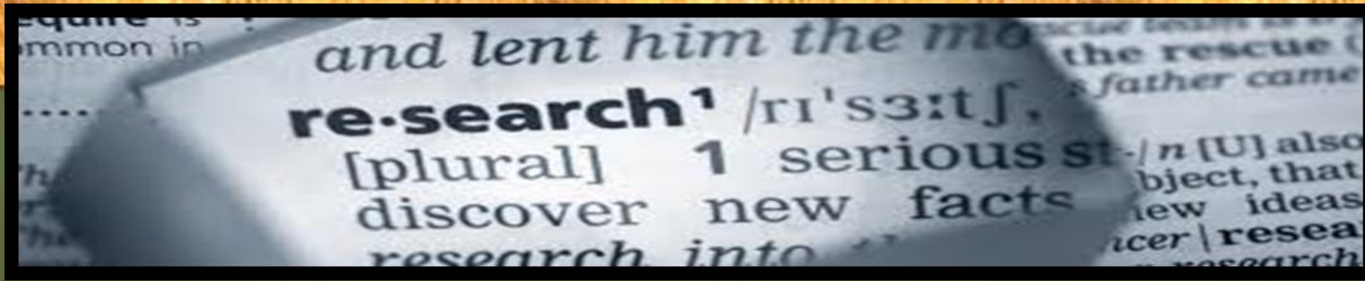
- When 3rd graders cannot add fluently, do you push them on to multiplication?
- What happens when 4th graders can't subtract?
- How about in 5th grade when lack of fluency
- with multiplication impedes fraction instruction?

Familiar "solutions"

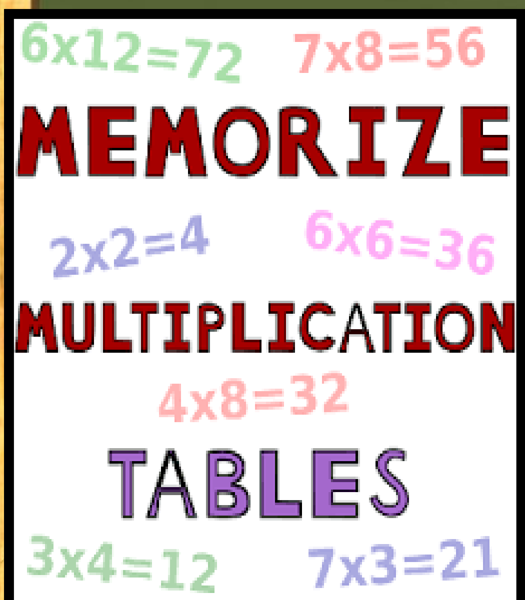
Rote Drill

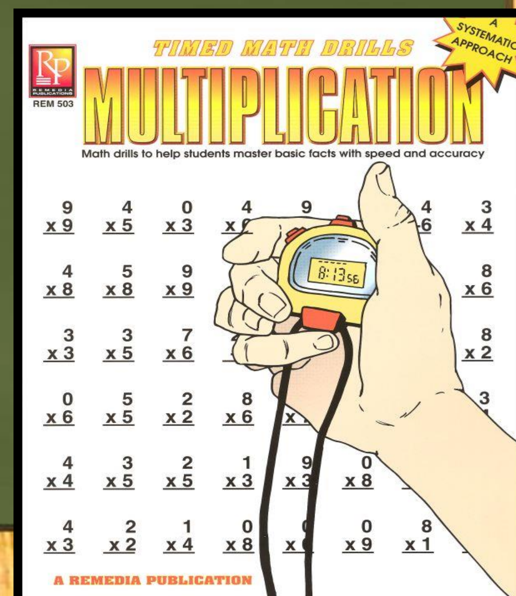
Timed Tests

Memorization



The research is clear about why traditional solutions don't work.





Rote Drill

"Do not subject any student to fact drills unless the student has developed efficient strategies for the facts being practiced. . . . Short-term gains are almost certain to be lost over time. Practice prior to development of efficient methods is simply a waste of precious instruction time."

(**Van de Walle**, 2001, p. 144)

Timed Tests

Teachers who use timed tests believe that the tests help children learn basic facts. This perspective makes no instructional sense. . . . Children who have difficulty with skills or who work more slowly run the risk of reinforcing wrong practices under pressure. Also, they can become fearful about, and negative toward, their mathematics learning."
(**Marilyn Burns**, 1995, p. 408)

Memorization

Memorization of basic facts usually refers to committing the results of unrelated operations to memory so that thinking through a computation is unnecessary. Isolated additions and subtractions are practiced one after another as if there were no relationships among them; the emphasis is on recalling the answers.

Teaching facts for automaticity, in contrast, relies on thinking. Answers to facts must be automatic, produced in only a few seconds; counting each time to obtain an answer is not acceptable. But thinking about the relationships among the facts is critical. A child who thinks of 9 + 6 as 10 + 5 produces the answer of 15 quickly, but **thinking rather than memorization** is the focus (although over time these facts are eventually remembered).

Cathy Fosnot

So how do students get stuck in the

FINGER ZONE?

"Every child's journey in math begins with counting. But it is the next step – moving from counting to adding – that is critical. Unfortunately, many kids never make the transition from counting numbers one at a time to thinking abstractly and efficiently in groups. No wonder they find math difficult!" ***Greg Tang***

To move students away from immature counting strategies and fingers…

The fix is to identify methods that are more…

Efficient

Effective

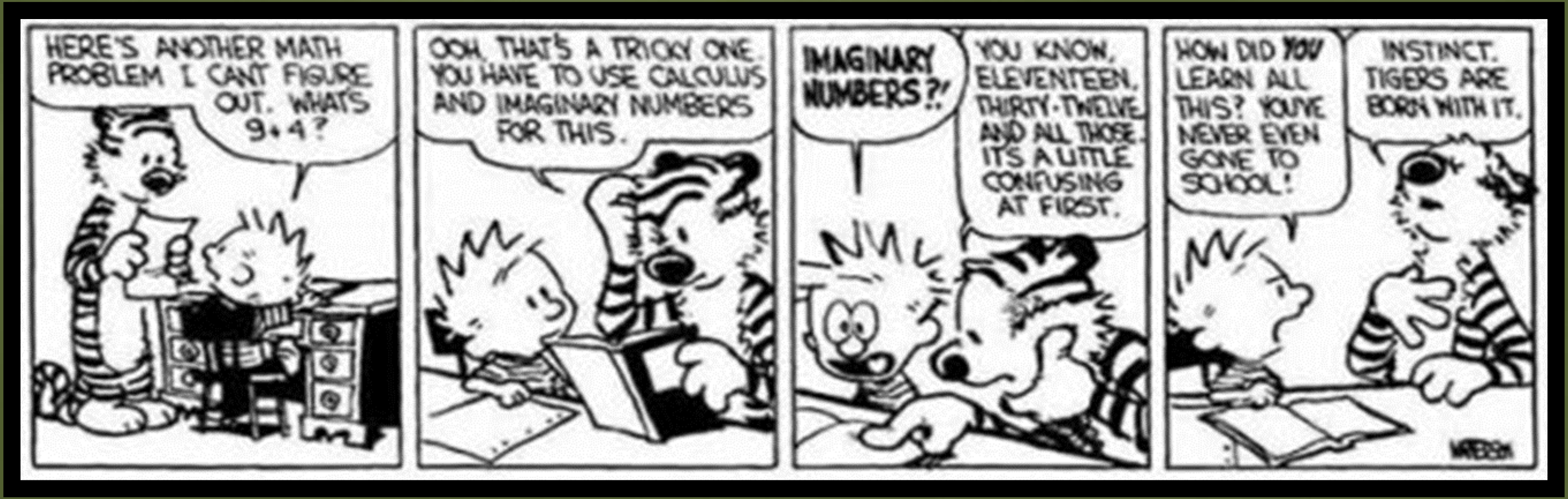
Easy

Math must be more than rote calculations. It must make sense!

Charlie: "My math teacher is crazy".

Mother: "Why?"

Charlie: "Yesterday she told us that five is 4+1; today she is telling us that five is 3 +2."



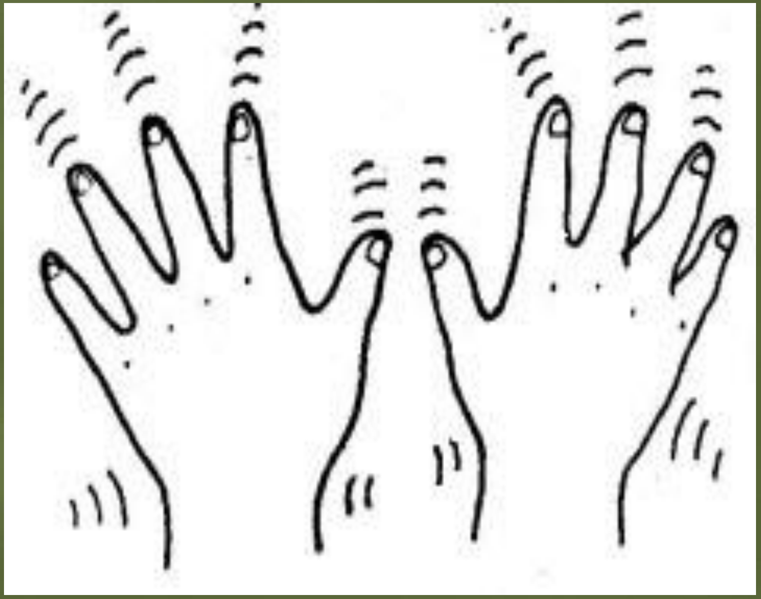
We cannot resort to tricks or shortcuts!!

WHY???????????????????????????????????

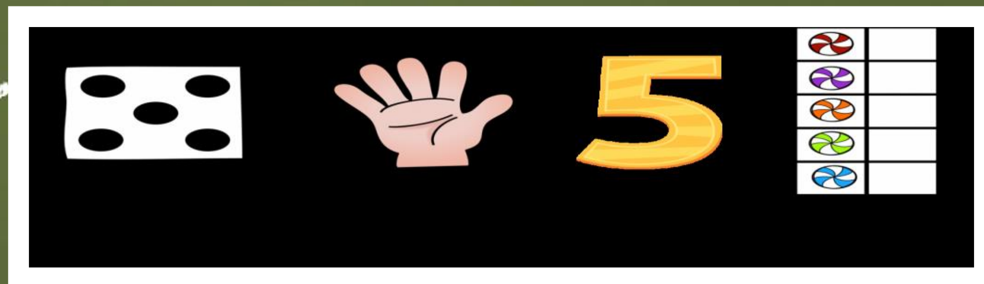
Really???????

What????

Hall of Shame



Prior to fingers and counting



Subitizing

There are two components of subitizing:

- Perceptual is the instant visual recognition of a pattern such as the dots on a die.
- Conceptual means recognizing smaller groups and adding them together, such as two dots plus two dots equals four dots.

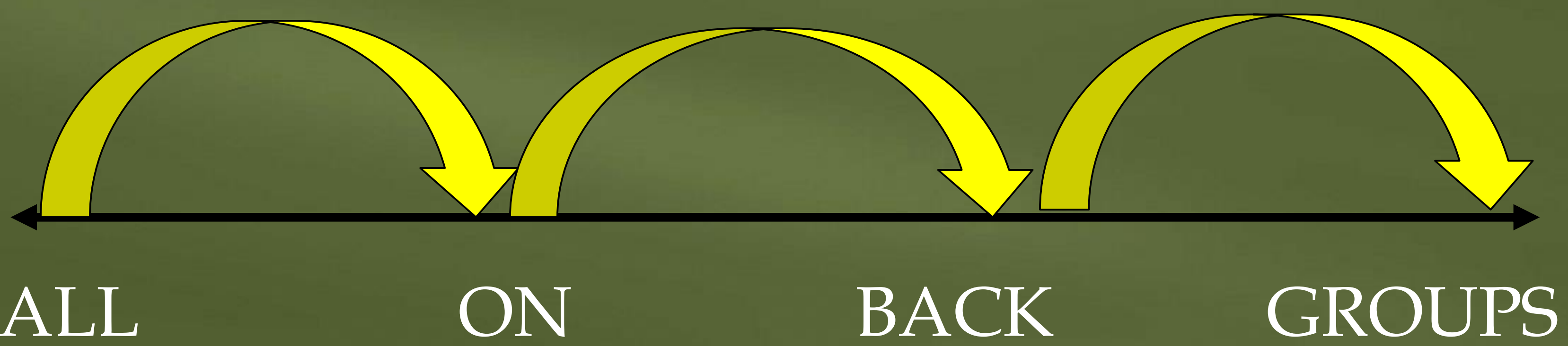
Phase 1 – Counting & Constructing Meaning

1:1 correspondence

Cardinality-last number represents quantity

Sequence----number string

Counting



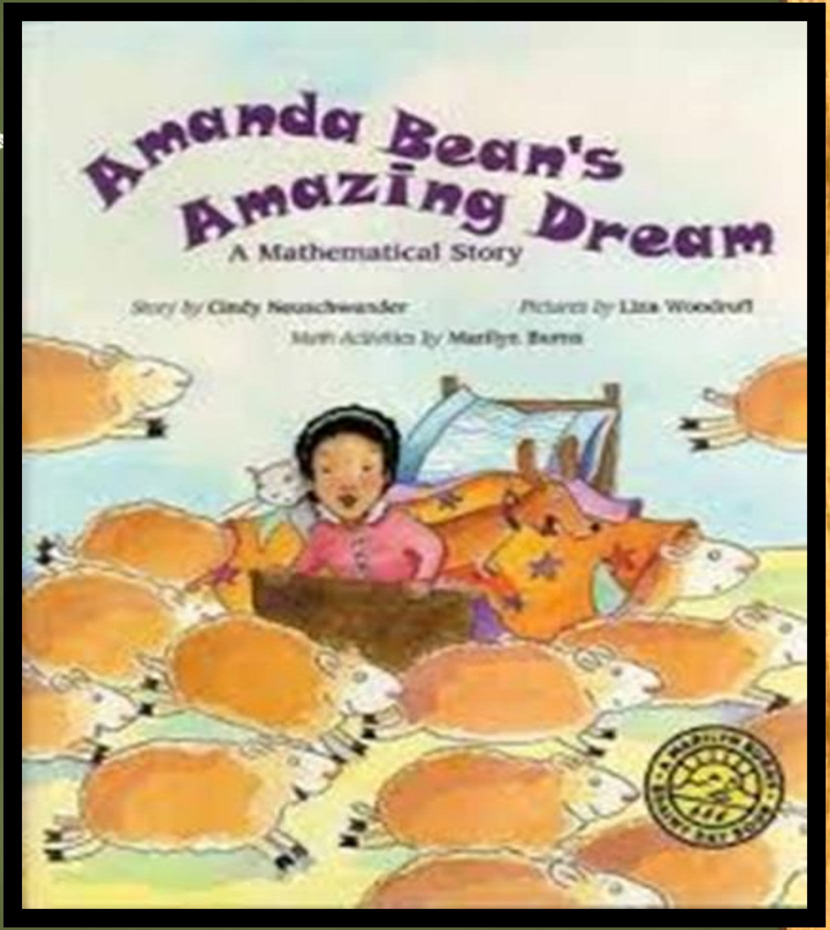
During this phase…

Focus on…

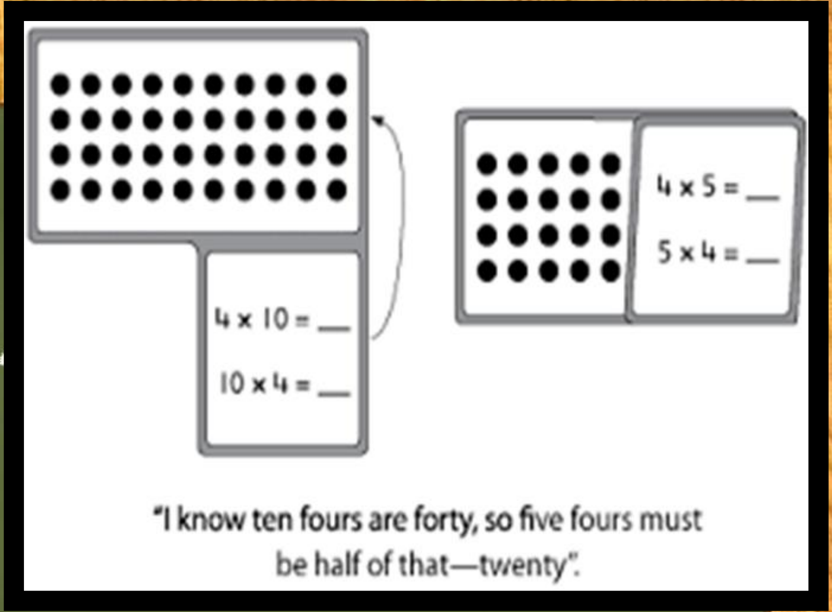
- Patterns
- Efficiency
- Flexibility

Routines to try…

- Count around the room
- Start with…get to



Phase 2 – Reasoning Strategies



- Use known facts to derive unknown facts
- Discover relationships between numbers

7 x 2 **double**

7 x 4 -- same as 7 x 2 x 2 **double double**

7 x 8 -- same as 7 x 2 x 2 x 2 **double double double**

Double-Double-Double

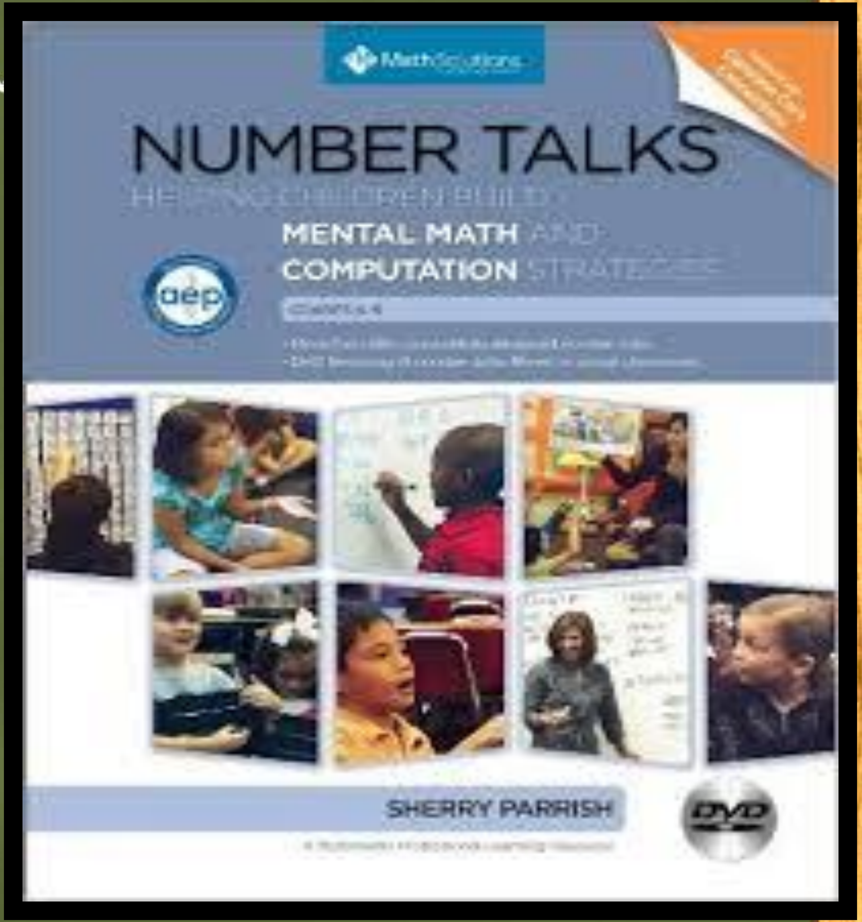
During this phase…

Focus on…

- Strategies
- Properties
- Flexibility

Routines to try…

- Number Talks
- Number of the Day
- True or False

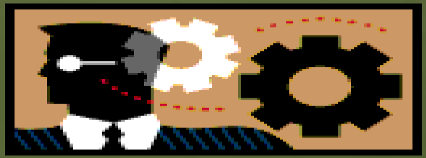


True / False

[True/False]

a versatile routine for all grades

Warm-Ups are great too!



Which of the following problems has the largest product? Try to figure it out by solving as few problems as possible. How did you choose which problems to do or not to do?

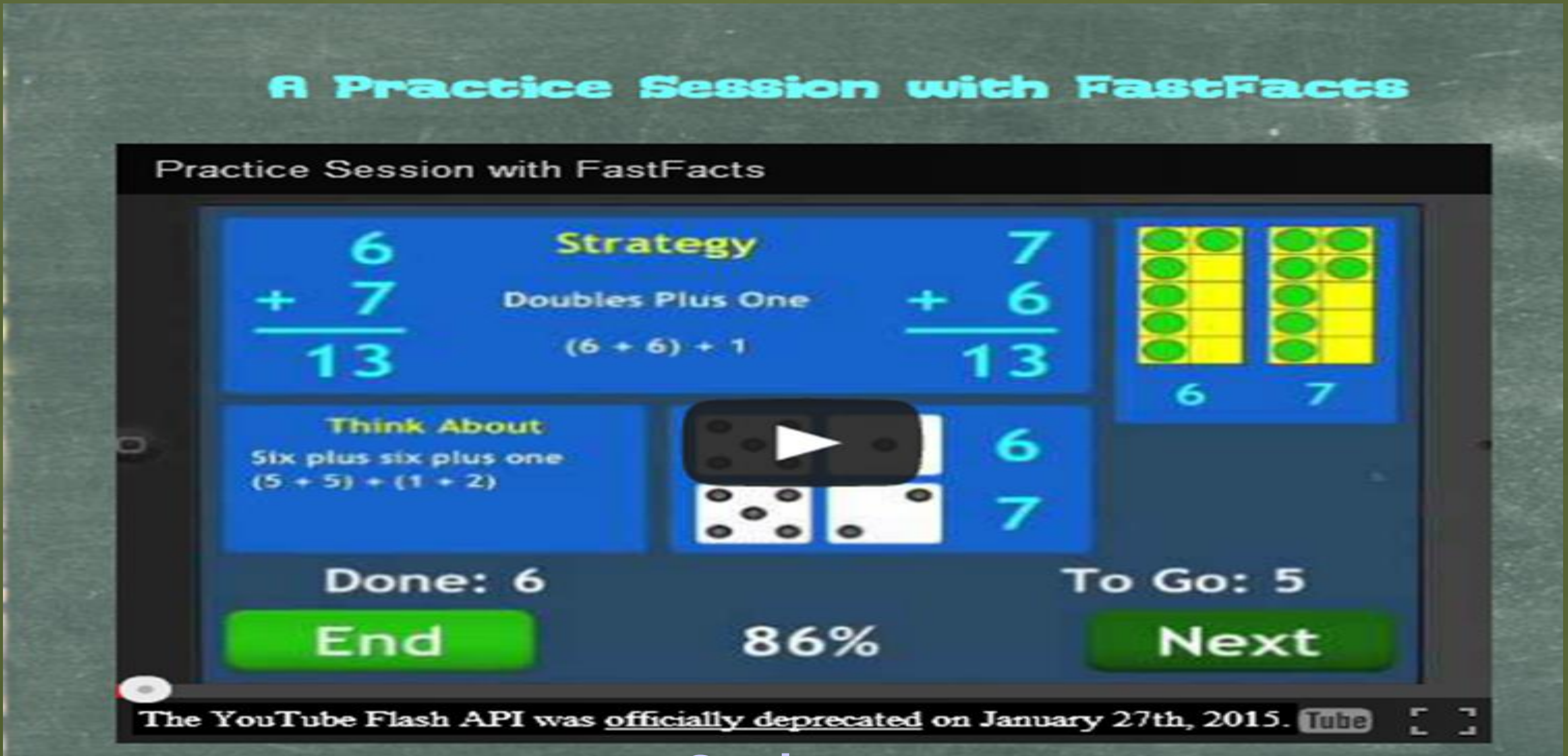
42 x 17	24 x 12	52 x 11
40 x 20	50 x 24	43 x 16
36 x 36	12 x 14	42 x 42

The importance of these phases cannot be understated

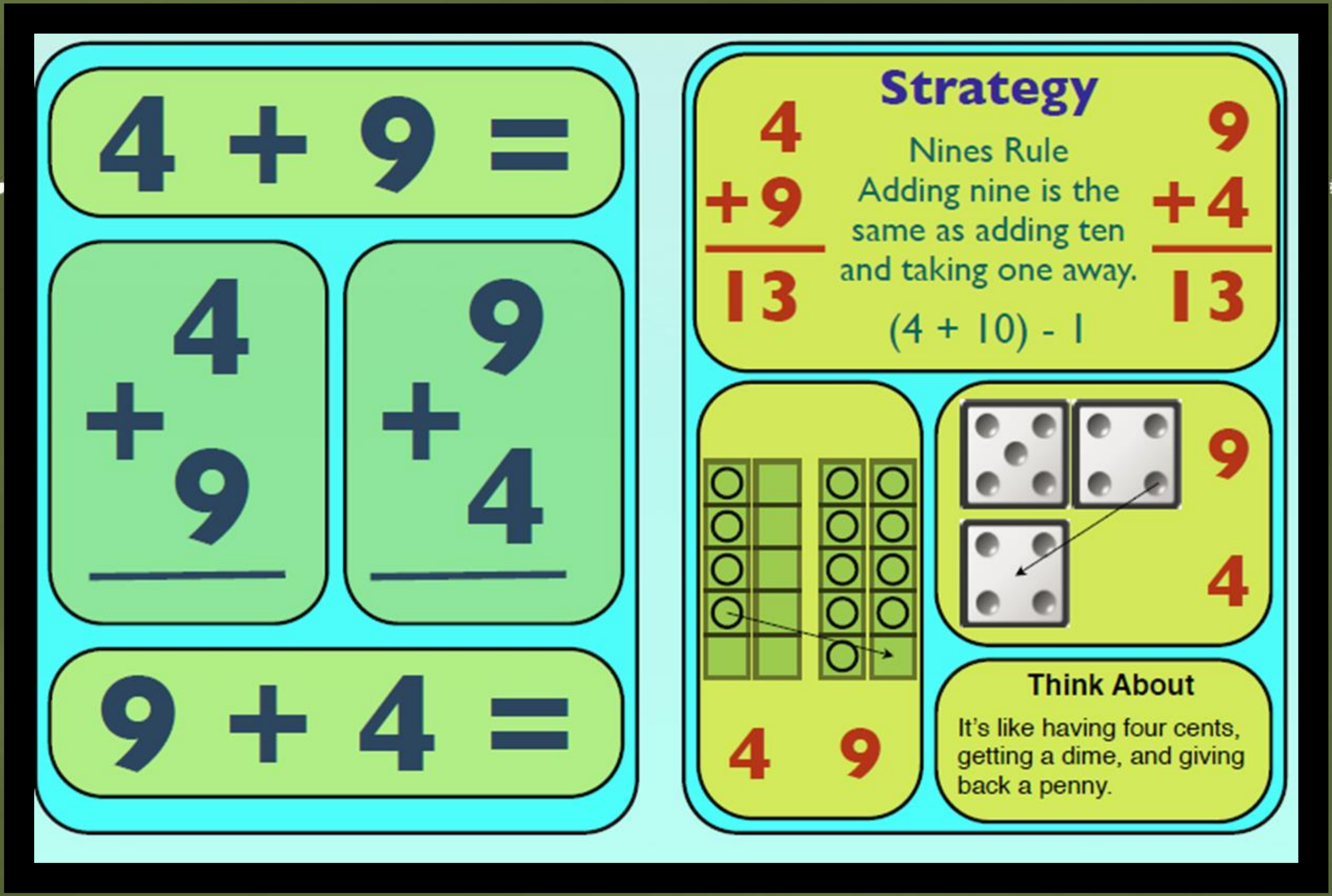
When focusing on the acquisition of basic math facts **Phases I and II are by far the most critical**. Phase II involves the student in seeking efficient strategies for the solving of basic number combinations. In so doing, students acquire the ability to look at numbers and operations in many equivalent ways. “Number relationships provide the foundation for strategies that help students remember basic facts.”

Van De Walle, 2007, 2013 p. 171

During this phase, provide rich representations of basic facts



Studysmart



Strategy Cards

How can images like this foster strategic reasoning?

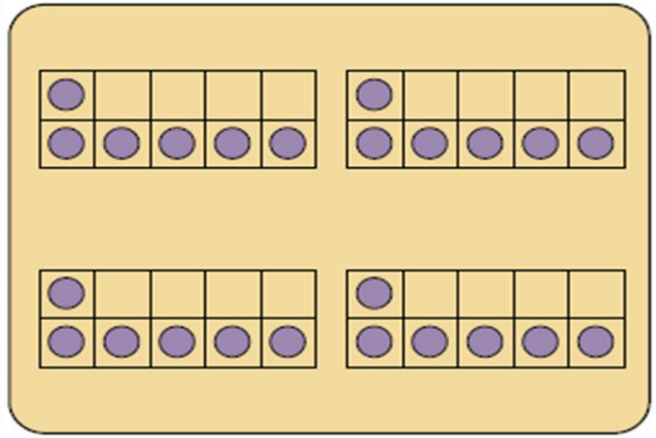
___ + ___ = ___

$4 \times 6 =$

$$\begin{array}{r} 6 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ \times\ 6 \\ \hline \end{array}$$

$6 \times 4 =$

$$\begin{array}{r} 6 \\ \times\ 4 \\ \hline 24 \end{array} \qquad \begin{array}{r} 4 \\ \times\ 6 \\ \hline 24 \end{array}$$



How many groups of juice?
How many cans in each group?

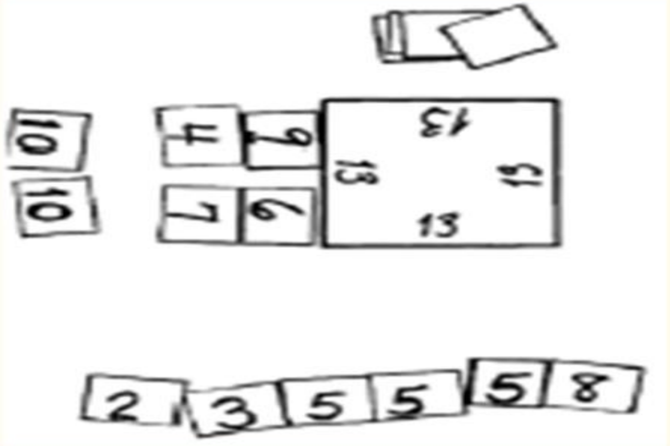
____ x ____ = ____

Figure 2

Examples of a composition-decomposition activity (based on Baroody, Lai, and Mix [in press])

The Number Goal game

Two to six children can play this game. A large, square center card is placed in the middle with a number, such as 13, printed on it. From a pile of small squares, all facing down and having a number from 1 to 10, each player draws six squares. The players turn over their squares. Taking turns, each player tries to combine two or more of his or her squares to yield a sum equal to the number on the center card.

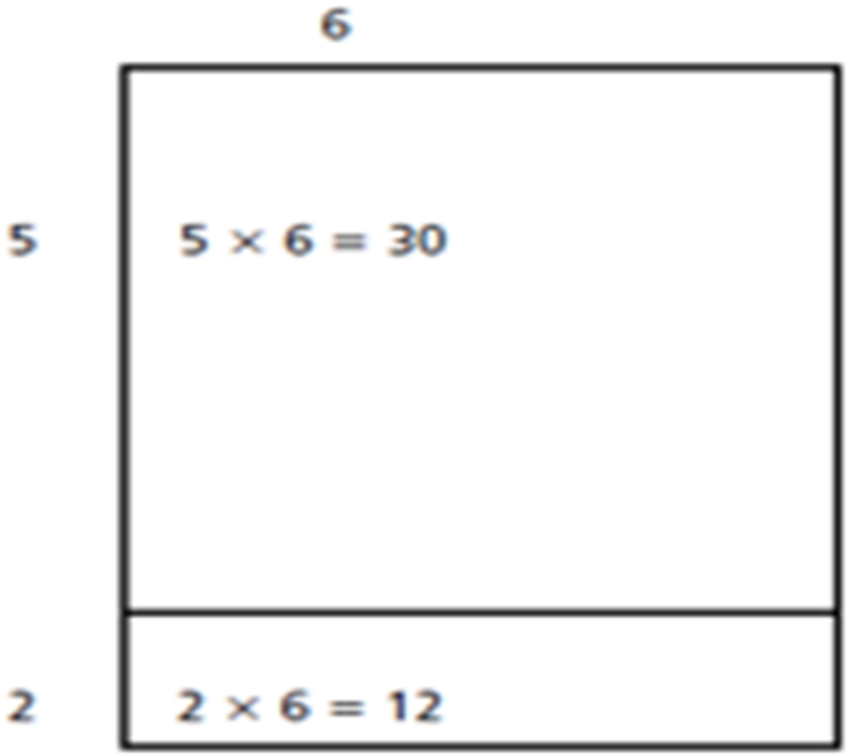


If a player had squares 2, 3, 5, 5, 5, and 8, she could combine 5 and 8 and also combine 3, 5, and 5 to make 13. Because each solution would be worth 1 point, the player would get 2 points for the round. If the player had chosen to combine 2 + 3 + 8, no other possible combinations of 13 would be left, and the player would have scored only 1 point for the round. An alternative way of playing (scoring) the game is to award points for both the number of parts used to compose the target number (e.g., the play 3 + 5 + 5 and 5 + 8 would be scored as 5 points, whereas the play 2 + 3 + 8 would be scored as 3 points).

Number Goal Tic-Tac-Toe (or Three in a Row)

This game is similar to the Number Goal game. Two children can play this game. From a pile of small squares, all facing down and having a number from 1 to 10, each player draws six squares. The players turn over their squares. Taking turns, each player tries to combine two or more of his or her squares to create a sum equal to one of the numbers in the 3 x 3 grid. If a player can do so, she or he places her or his marker on that sum in the 3 x 3 grid, discards the squares used, and draws replacement squares. The goal is the same as that for tic-tac-toe—that is, to get three markers in a row.

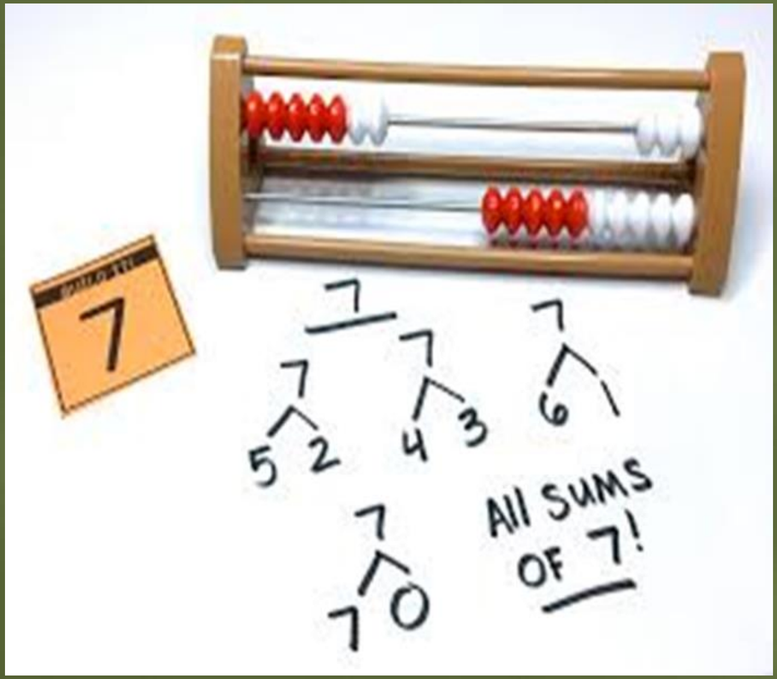
(c) Using an area representation to decompose the fact 7×6

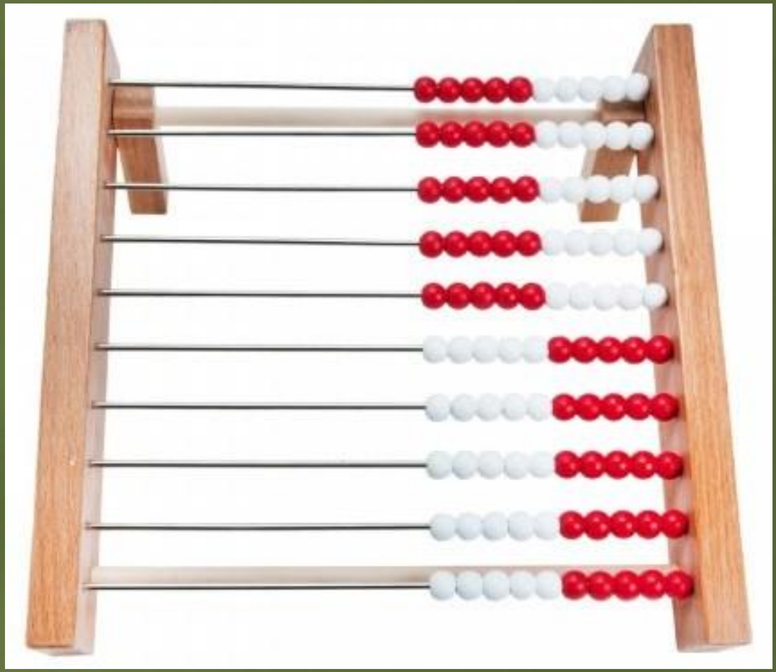


$$\begin{aligned} 7 \times 6 &= 5 \times 6 + 2 \times 6 \\ &= 30 + 12 \\ &= 42 \end{aligned}$$

TOOLS…Your secret weapon!!!

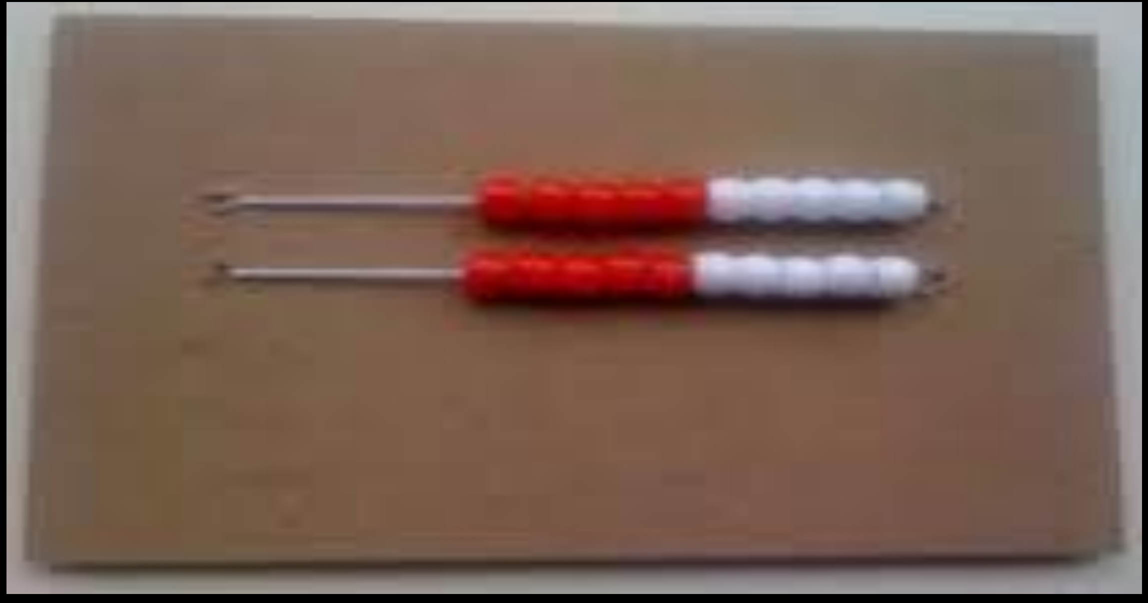
Fill student's toolboxes with alternatives to fingers……..



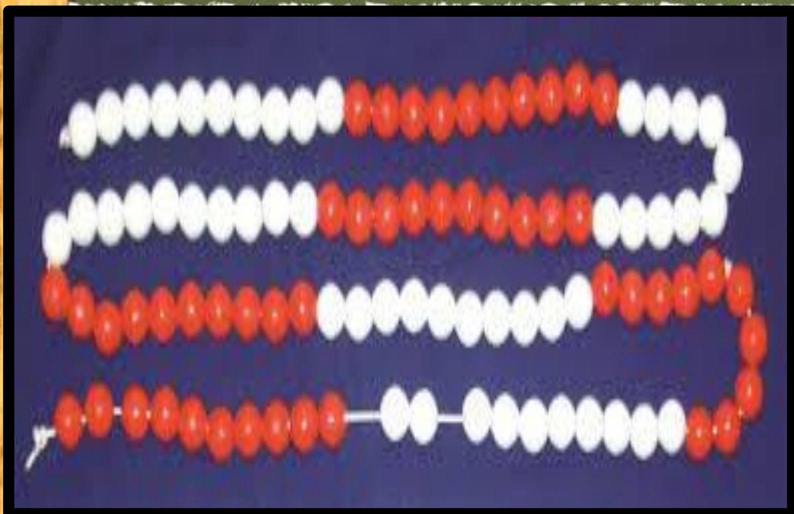


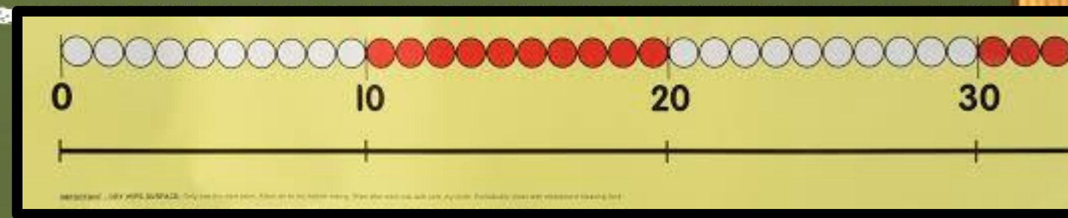
Rekenreks (20 and 100 bead)

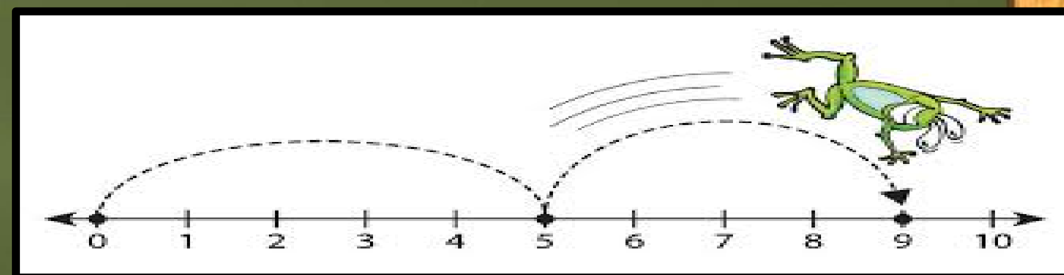
Make your own Rekenrek



Beaded Number Line. BEST. TOOL. EVER!!

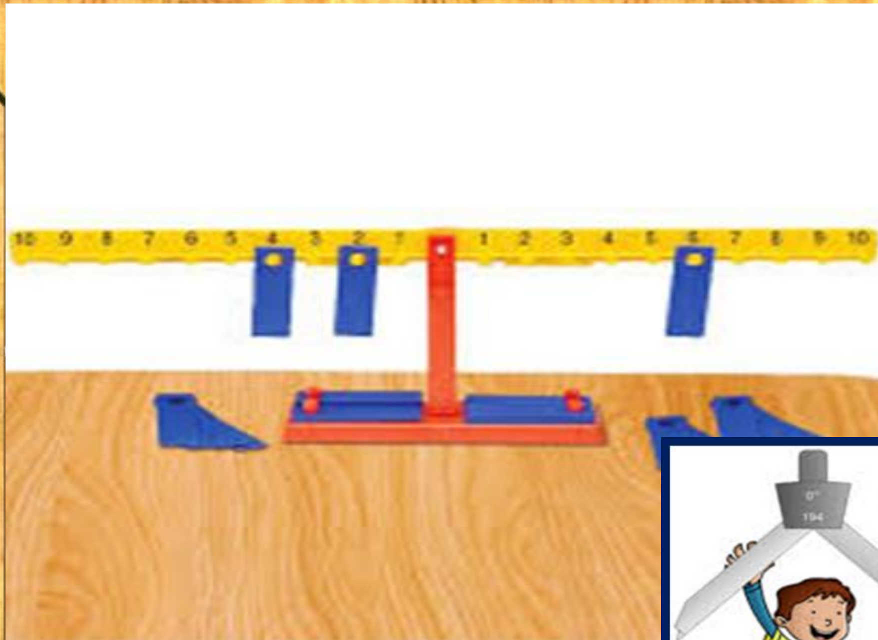


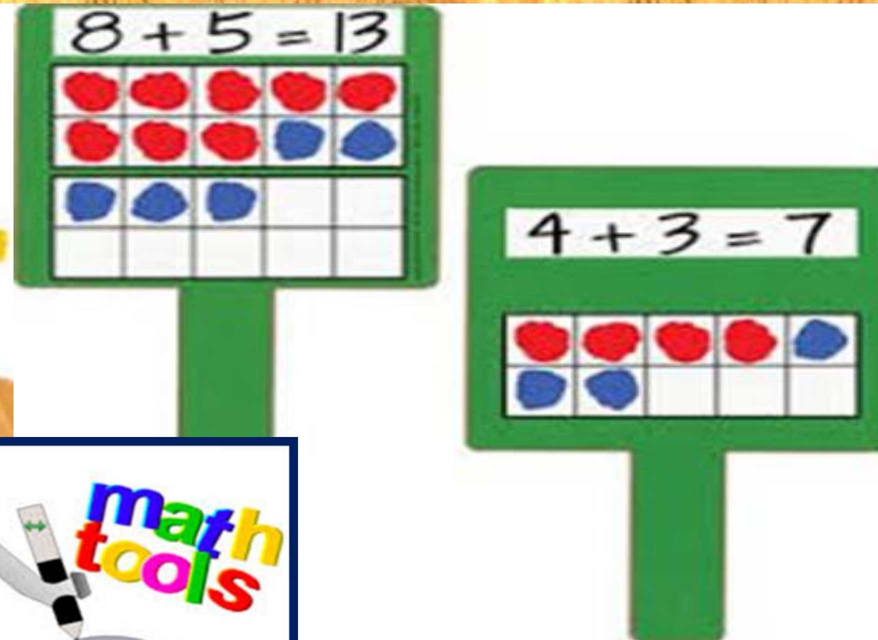


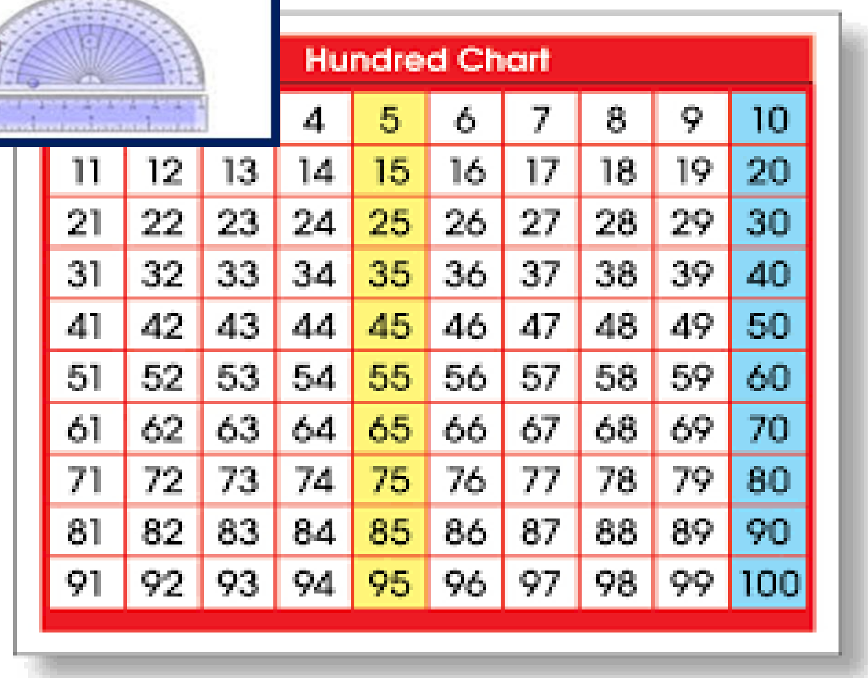


Beaded number line

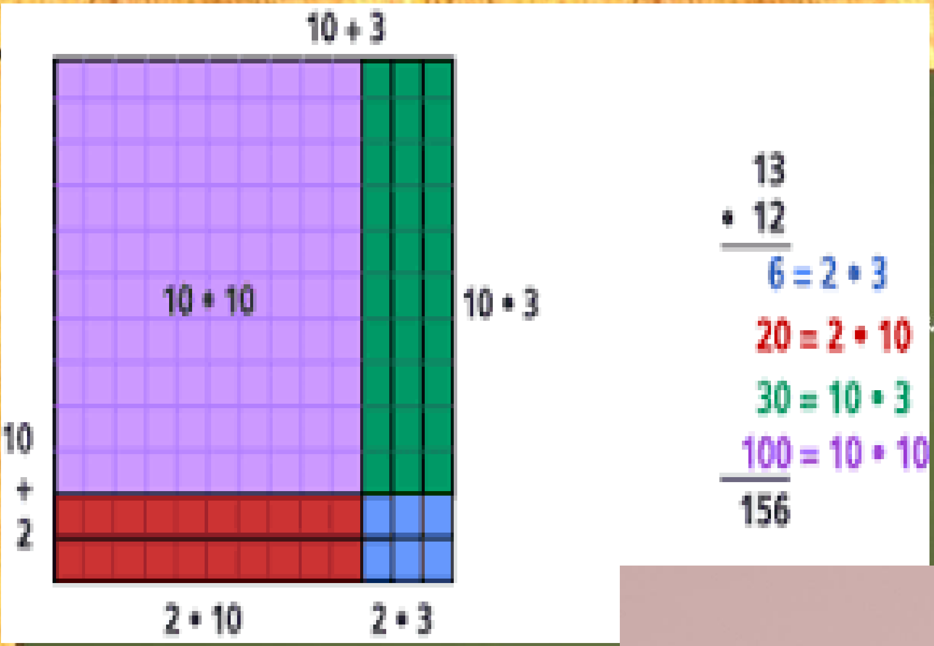
comparing...skip counting...adding...subtracting...rounding.... place value...decimals..

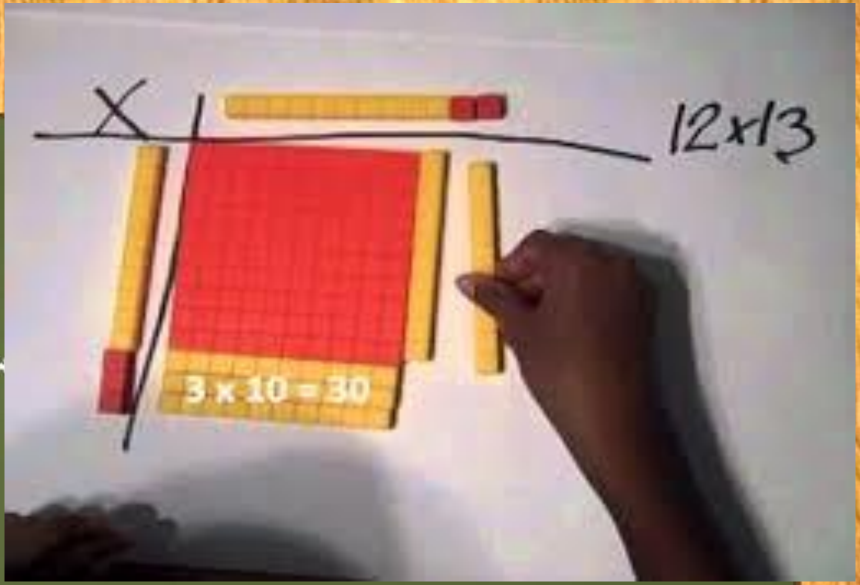


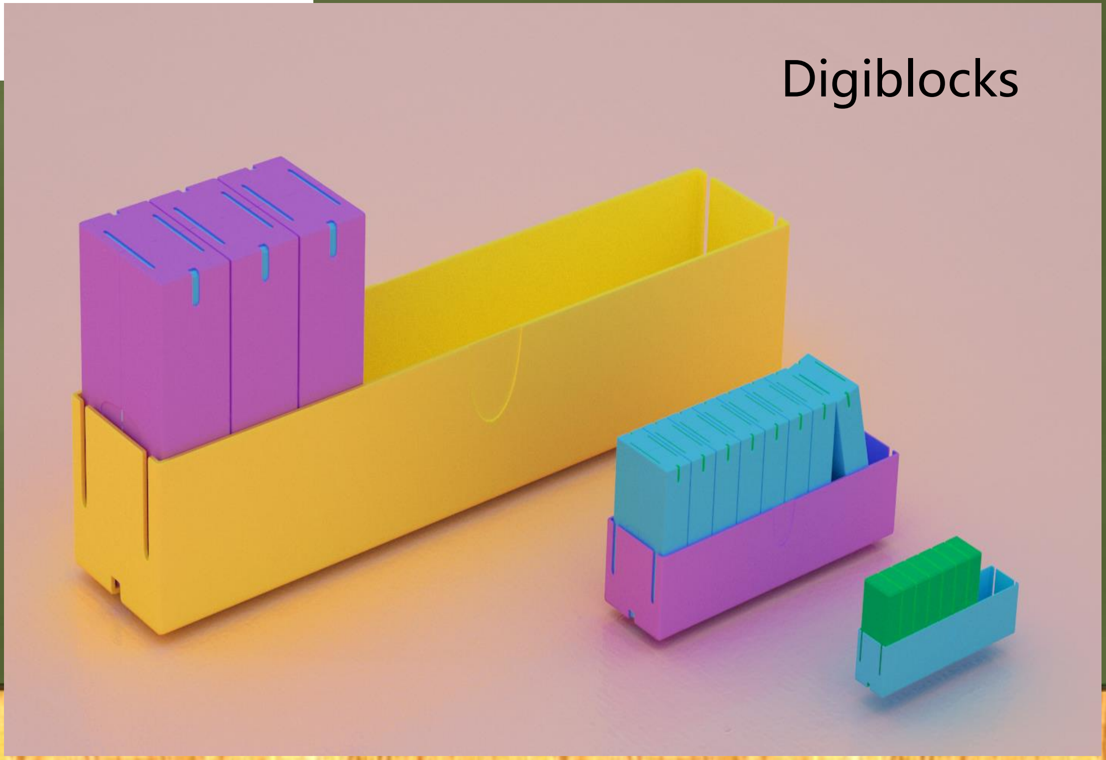




			4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100


10 + 3
10 • 10
10 • 3
10
+
2
2 • 10
2 • 3
13
• 12
6 = 2 • 3
20 = 2 • 10
30 = 10 • 3
100 = 10 • 10
156


X
12x13
3 x 10 = 30

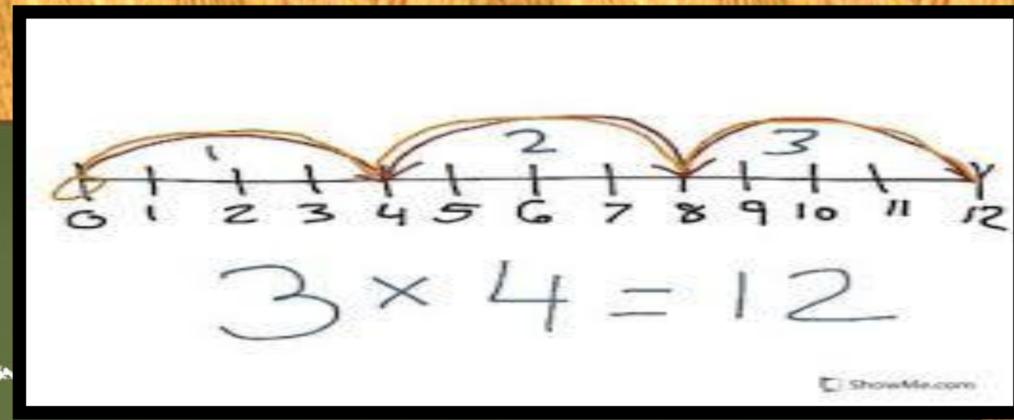

Digiblocks

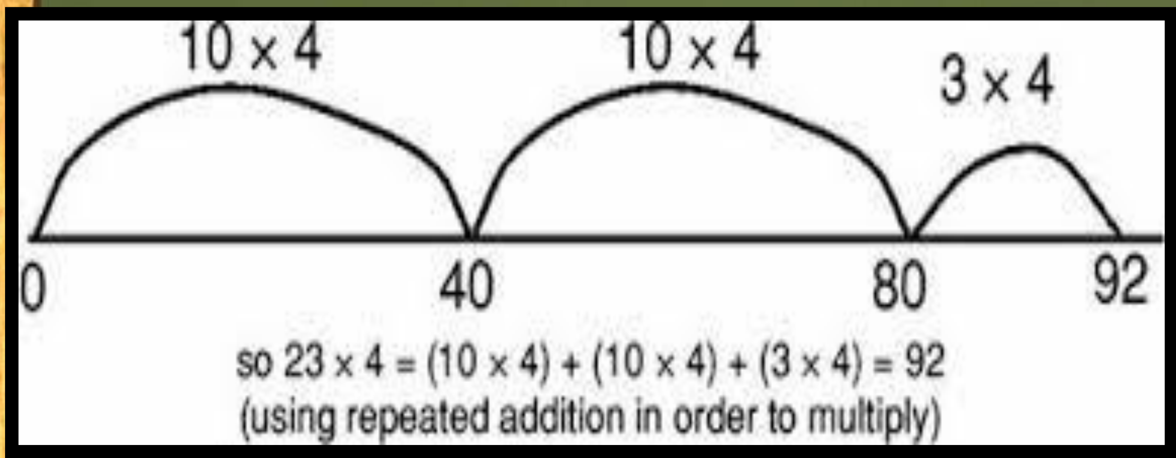
From concrete to visual

Number Lines…..
Start with beaded or Rekenrod

Move to closed and then open
Supports flexibility and reasoning

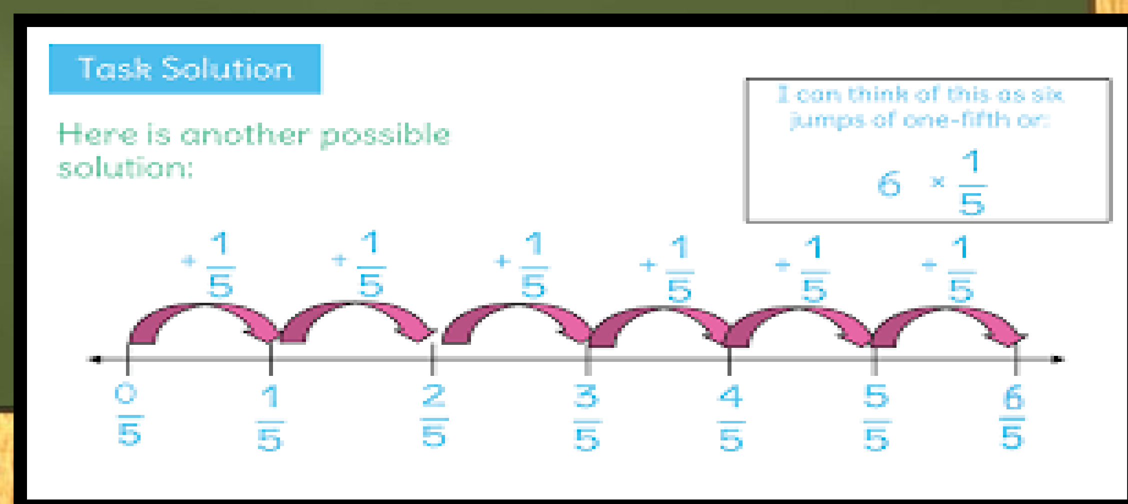
Building computational fluency



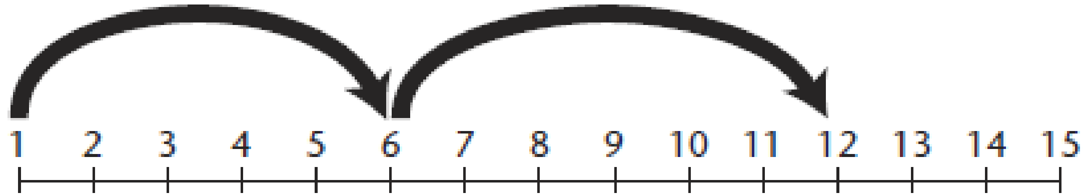


Developing strategic reasoning

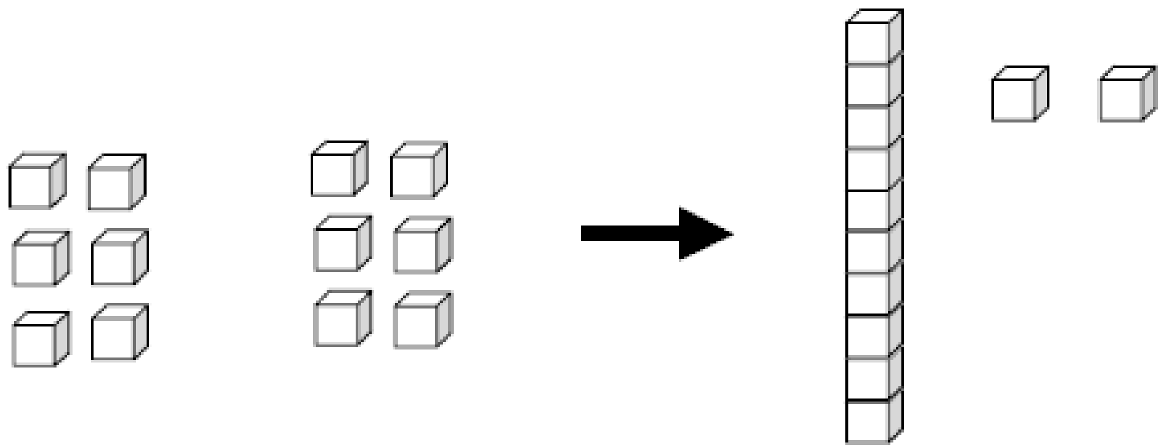
Focusing on conceptual understanding



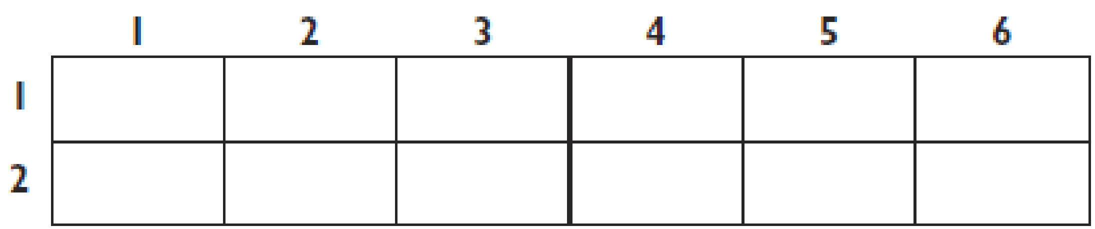
Number line:



Base ten blocks:



Arrays:



2 rows x 6 columns

BETTER THAN FINGERS!!

What do all of the tools and models have in common?

They highlight parts and wholes, help make connections, and build

FLEXIBILITY!!!

Phase 3 –
Working Toward Quick Recall

Quick recall generally accepted to be 3 seconds or less (without resorting to inefficient methods like counting).

Includes student using a known fact to quickly derive an unknown one.

Phase III is simply about increasing the quick recall of facts.

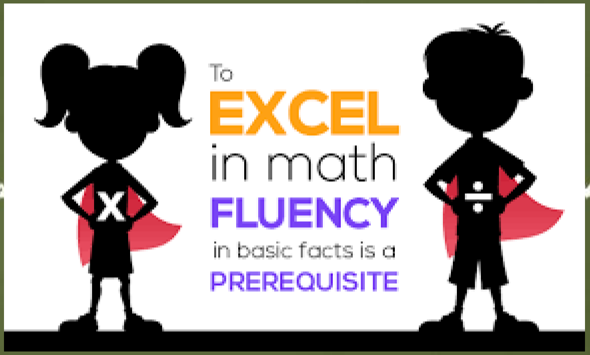
This involves:

- Increasing the speed in which the student selects and applies a strategy for solving
- Using highly organized and purposeful practice to help students attain automatic retreiva;

Phase III may include practice with specific groups of facts through fact triangle cards, games, paper/pencil practice, and the use of software specifically designed for this purpose.

***Van de Walle**, 2001*

During this phase…



Focus on…

- Automatic recall
- Strategic reasoning for sticky facts
- Frequent/engaging practice (think games & routines)

Routines to try…

- I Love Math
- Power Towers
- Musical Cards

Often we skip too quickly to Phase III!!

Successfully addressing Phases I and II will significantly decrease the need for prolonged work in Phase III. Often this phase focuses on particularly stubborn facts for which fewer relationships exist. Once students acquire the ability to quickly recall the facts, Phase III focuses on maintenance of the facts. ***Van de Walle**, 2001*

Sounds great, but who has time?

(Are we ok with just admiring the problem, or do we actually want to solve it?)

Consider a workshop model

- Fluency station
- Fact Wallets
- Apps
- Games

Create an opportunity with homework

- Personalized plans with students tracking their own progress

[Resources]()

[More resources]()

The missing piece…ASSESSMENT

- Ditch timed tests!!!!!!!!!!!
- Consider oral assessments
- Ask questions about strategies
- Math Running Records
  https://mathrunningrecords.wordpress.com/

Questions??

Follow me on Twitter

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@mrsmellomathsci

Contact Me!

alisonmellomathpd@gmail.com

Thank You!

Resources Technology

- http://catalog.mathlearningcenter.org/apps