

Name: _____ Date: _____ Period: _____

Choosing a Pair O' Dice

Task 1: Predict and Experiment

Make a prediction about which set of dice will come in first, second, third, fourth and fifth.

Dice: $(1 \rightarrow 8) + (1 \rightarrow 4)$ $(1 \rightarrow 4) + (1 \rightarrow 4) + (1 \rightarrow 4)$ $(1 \rightarrow 12)$ $(1 \rightarrow 6) + (1 \rightarrow 6)$ $(1 \rightarrow 20) - (1 \rightarrow 8)$

Place: _____

Your team's dice: _____

Roll your dice 20 times. Record the results here and then combine your sum with TWO other groups.

For each set of dice, record the following.

	$(1 \rightarrow 8) + (1 \rightarrow 4)$	$(1 \rightarrow 4) + (1 \rightarrow 4) + (1 \rightarrow 4)$	$(1 \rightarrow 12)$	$(1 \rightarrow 6) + (1 \rightarrow 6)$	$(1 \rightarrow 20) - (1 \rightarrow 8)$
Sum (of 60 rolls)					
Mean (of 60 rolls)					
Mode (of 20 rolls)					

Task 2: Analysis

1. Which set of dice had the greatest sum? _____

2. Do you think this investigation was *fair*? Did the sets of dice have equally likely outcomes?

3. How could you determine (a) which set of dice *should* have won and (b) the probability of rolling a sum of 12 with each set?

4. How do you think the results would change if the dice were rolled 1,000 times each?



Name: _____ Date: _____ Period: _____

Task 3: Showing Possible Outcomes (Sample Space)

Using a table, tree diagram, or list, show all of the possible outcomes from rolling your dice and another set assigned to your group.

Our Dice: _____

What is the **mean** of the possible outcomes?

What is the **mode** of the possible outcomes?

Assigned Dice: _____

What is the **mean** of the possible outcomes?

What is the **mode** of the possible outcomes?



Task 4: Calculating Probability (Use the Sample Spaces to answer these questions)

1. What is the probability of rolling a sum of 12 for each of the sets of dice?

$$P(12) \text{ when rolling } 1 \rightarrow 8 + 1 \rightarrow 4 =$$

$$P(12) \text{ when rolling } 1 \rightarrow 4 + 1 \rightarrow 4 + 1 \rightarrow 4 =$$

$$P(12) \text{ when rolling } 1 \rightarrow 12 =$$

$$P(12) \text{ when rolling } 1 \rightarrow 6 + 1 \rightarrow 6 =$$

$$P(12) \text{ when rolling } 1 \rightarrow 20 - 1 \rightarrow 8 =$$

2. What is the probability of rolling a sum of 7 for each of the sets of dice?

$$P(7) \text{ when rolling } 1 \rightarrow 8 + 1 \rightarrow 4 =$$

$$P(7) \text{ when rolling } 1 \rightarrow 4 + 1 \rightarrow 4 + 1 \rightarrow 4 =$$

$$P(7) \text{ when rolling } 1 \rightarrow 12 =$$

$$P(7) \text{ when rolling } 1 \rightarrow 6 + 1 \rightarrow 6 =$$

$$P(7) \text{ when rolling } 1 \rightarrow 20 - 1 \rightarrow 8 =$$

3. What is the probability of rolling a sum LESS than 0 for each of the sets of dice?

$$P(\# < 0) \text{ when rolling } 1 \rightarrow 8 + 1 \rightarrow 4 =$$

$$P(\# < 0) \text{ when rolling } 1 \rightarrow 4 + 1 \rightarrow 4 + 1 \rightarrow 4 =$$

$$P(\# < 0) \text{ when rolling } 1 \rightarrow 12 =$$

$$P(\# < 0) \text{ when rolling } 1 \rightarrow 6 + 1 \rightarrow 6 =$$

$$P(\# < 0) \text{ when rolling } 1 \rightarrow 20 - 1 \rightarrow 8 =$$



Name: _____ Date: _____ Period: _____

4. What is the probability of NOT rolling a sum of 12 for each of the sets of dice?

P(NOT 12) when rolling $1 \rightarrow 8 + 1 \rightarrow 4$ =

P(NOT 12) when rolling $1 \rightarrow 4 + 1 \rightarrow 4 + 1 \rightarrow 4$ =

P(NOT 12) when rolling $1 \rightarrow 12$ =

P(NOT 12) when rolling $1 \rightarrow 6 + 1 \rightarrow 6$ =

P(NOT 12) when rolling $1 \rightarrow 20 - 1 \rightarrow 8$ =

5. Which set of dice has the greatest probability of getting a sum of 5? Give a detailed answer, including the probability of getting a 5.



Teacher Directions

Objective

By analyzing sums of different sets of dice that all have a maximum sum of 12, students will understand and find sample spaces to represent all possible outcomes.

Materials

- 6-sided dice – approx. 8
- 4-sided dice (tetrahedral dice) – approx. 16
- 8-sided dice – approx. 8
- 12-sided dice – approx. 4
- 20-sided dice – approx. 4
- Calculators (optional)

Note: Different groups will explore with different sets of dice.

Activity Notes

Introduction

Pass out only the activity sheet. Show the students the top row and model what each set of dice looks like and how they work. First, show an 8-sided die and then show them a 4-sided die as well as how to read this die (the number is read on the bottom of any edge for some). Explain that for this set, they will roll each die and find the sum. Do one sample roll and have them practice finding the sum. They are NOT to record it under $(1 \rightarrow 8) + (1 \rightarrow 4)$; this is for their prediction.

Next, show them what three 4-sided dice would be by rolling and finding the sum. Next, show a 12-sided die and explain that, in this case, they will just get what they roll. Then, show two standard dice (6-sided), roll them and demonstrate how to find the sum. Finally, show them a 20-sided die. Roll that and THEN the 8-sided die and SUBTRACT the number on the 8-sided die FROM the number rolled on the 20-sided die (yes, this means it could be negative, but don't tell them that—let them figure it out!).

Predict and Experiment

Now, give each pair 2 minutes to discuss and then predict the order the dice will finish (from least to greatest value) when each set is rolled 100 times and the sums are added together. Have groups raise their hands to vote for which set will be first, second, etc. Then, let each group pick a set to do the experiment with (or assign them). Ideally, you want 3-4 groups investigating each set of dice and it is nice if they can investigate the one they thought would win.

Once each pair gets their dice, have them roll 20 times and record the sums in the table provided. Then, have each group that had the same set come together to share just the sum of their 20 rolls. Have a volunteer from each group come up to the document camera and record their data on your table (for the sum, mean and mode).

Note: There will be a different mode for each small group, but just one combined sum and mean.

Ask the class which number (sum, mean or mode) tells you which set won. (It should be the mean or the sum if all sets had an equal number of trials).

Analysis Questions

Give the class 5-8 minutes to work on the 4 analysis questions and then discuss them as a class. Your goal here is to get the students to agree that they need to list all possible outcomes to verify if it was



Teacher Directions

“fair” (recall equally likely outcomes conversation from *2 Coins, 3 People*). Your secondary goal is to help students see that more trials means the data will tend to be closer to the theoretical probability.

Showing Possible Outcomes

In order to verify if the experimental winner *should* have been the winner and to be able to calculate probability, we will need to show all possible outcomes for each set of dice (this is called the *sample space*). Have each group first try to do this for the set of dice they rolled and once they are successful, assign them a second set to do this for. Below are some notes to help you guide them:

- All of them can be analyzed with a Tree Diagram
- All of them (except 1 to 12) can be analyzed with a table (which is often easier)
- All of them can be analyzed with a list, but it gets messy and easy to forget.
- The tree is the most likely choice for the three 4-sided dice.
- The three 4-sided dice CAN be analyzed with a table IF you do the first table as comparing one 4-sided with another 4-sided (you’ll end up with 16 outcomes) and then make a second table to compare those 16 with the third 4-sided dice.

Have each pair also answer the two questions in the text box so that you can compare the theoretical probability with the experimental results.

Have a student come up to show their sample space for each set of dice and have any group who used a different method also come up to explain the method.

Task 4: Calculating Probabilities

Before doing this on day 2, make a copy of the sample space from Task 3 for EACH set of dice for EACH pair of students to have. Some have been provided (pages 7 and 8).

Pass out the tables or tree diagrams for each set of dice to each pair. Model 1-2 problems as a class, and then have them complete the remainder of the task.

Answers					
	8-sided and 4-sided	three 4-sided	12-sided	two 6-sided	20-sided and 8-sided
P(12)	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{12}$	$\frac{1}{36}$	$\frac{1}{20}$
P(7)	$\frac{1}{8}$	$\frac{12}{64}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{20}$
P(# < 0)	Impossible	Impossible	Impossible	Impossible	$\frac{7}{40}$
P(NOT 12)	$\frac{31}{32}$	$\frac{63}{64}$	$\frac{11}{12}$	$\frac{35}{36}$	$\frac{19}{20}$



Sample Space

$$(1 \rightarrow 8) + (1 \rightarrow 4)$$

	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12

$$(1 \rightarrow 6) + (1 \rightarrow 6)$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$(1 \rightarrow 20) - (1 \rightarrow 8)$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12

12-Sided Die	1	2	3	4	5	6	7	8	9	10	11	12
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For the three 4-sided dice, use a tree diagram or two tables. There are 64 outcomes.

Table 1: $(1 \rightarrow 4) + (1 \rightarrow 4)$

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8



Sample Space

Table 2: Results from two 4-sided dice from table 1 above with the additional 4-sided die.

	1	2	3	4
2	3	4	5	6
3	4	5	6	7
3	4	5	6	7
4	5	6	7	8
4	5	6	7	8
4	5	6	7	8
5	6	7	8	9
5	6	7	8	9
5	6	7	8	9
5	6	7	8	9
6	7	8	9	10
6	7	8	9	10
6	7	8	9	10
7	8	9	10	11
7	8	9	10	11
8	9	10	10	12

