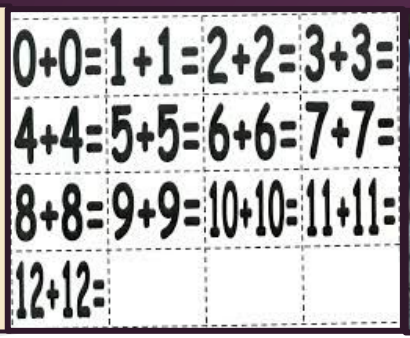
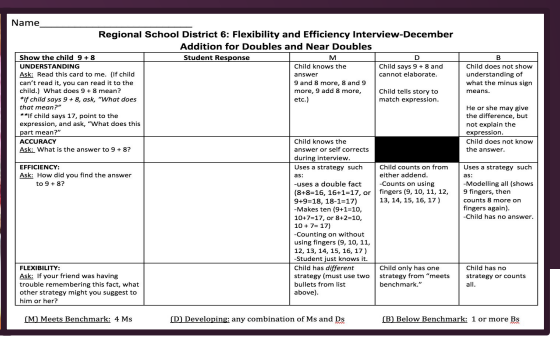
Making Fact Fluency Assessments Meaningful





Name_____

Regional School District 6: Flexibility and Efficiency Interview-December
Addition for Doubles and Near Doubles

Show the child 9 + 8	Student Response	M	D	B
UNDERSTANDING Ask: Read this card to me. (If child can't read it, you can read it to the child.) What does 9 + 8 mean? **If child says 9 + 8, ask, "What does that mean?"* **If child says 17, point to the expression, and ask, "What does this part mean?"		Child knows the answer 9 and 8 more, 8 and 9 more, 9 add 8 more, etc.)	Child says 9 + 8 and cannot elaborate. Child tells story to match expression.	Child does not show understanding of what the minus sign means. He or she may give the difference, but not explain the expression.
ACCURACY Ask: What is the answer to 9 + 8?		Child knows the answer or self corrects during interview.	[redacted]	Child does not know the answer.
EFFICIENCY: Ask: How did you find the answer to 9 + 8?		Uses a strategy such as: -uses a double fact (8+8=16, 1&+1=17, or 9+9=18, 18-1=17) -Makes ten (9+1=10, 10+7=17, or 8+2=10, 10 + 7= 17) -Counting on without using fingers (9, 10, 11, 12, 13, 14, 15, 16, 17) -Student just knows it.	Child counts on from either addend: -Counts on using fingers (9, 10, 11, 12, 13, 14, 15, 16, 17)	Uses a strategy such as: -Modelling all (shows 9 fingers, then counts 8 more on fingers again). -Child has no answer.
FLEXIBILITY: Ask: If your friend was having trouble remembering this fact, what other strategy might you suggest to him or her?		Child has *different* strategy (must use two bullets from list above).	Child only has one strategy from "meets benchmark."	Child has no strategy or counts all.

(M) Meets Benchmark: 4 Ms (D) Developing: any combination of Ms and Ds (B) Below Benchmark: 1 or more Bs

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Learning Targets

To broaden participants' knowledge about how meaningful fact fluency assessments can drive instruction and improve learning.

To motivate participants to become change agents in their schools and think differently about student's fluency.


SALLY FORTH by Greg Howard
HILARY'S FACT FLUENCY TEST IS TODAY, TED. CAN YOU GIVE HER ONE LAST RUN-THROUGH?
SURE.
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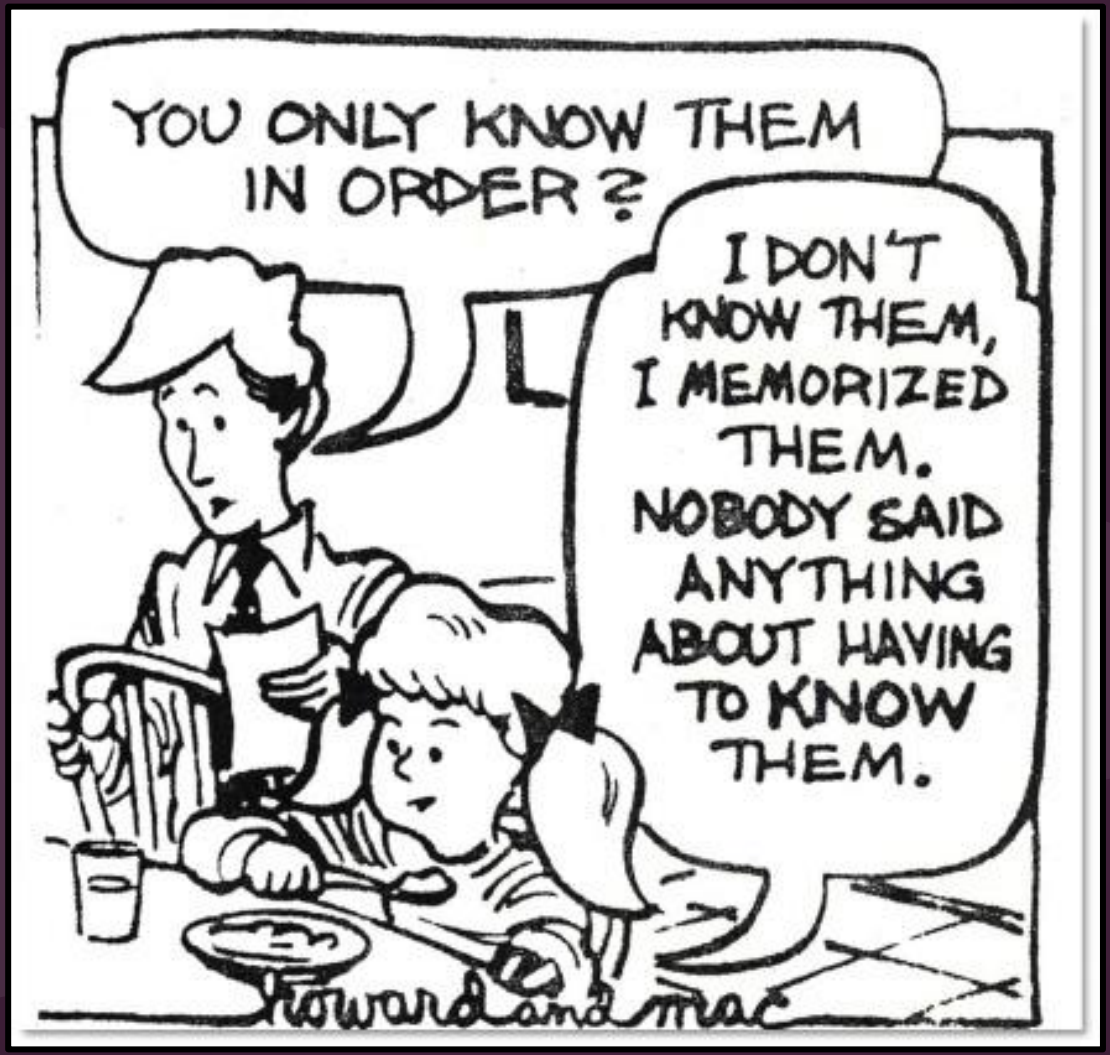
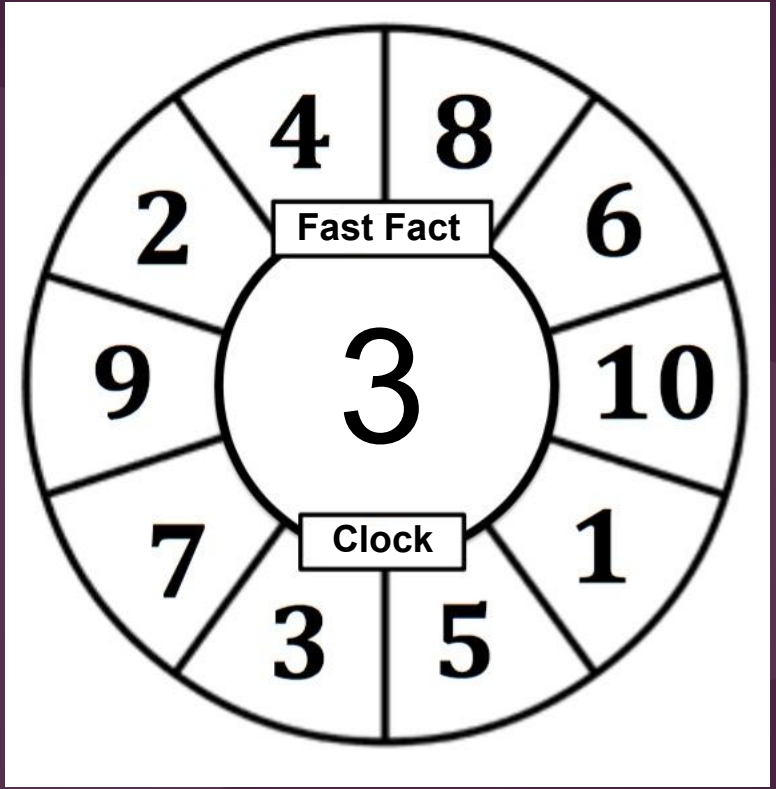
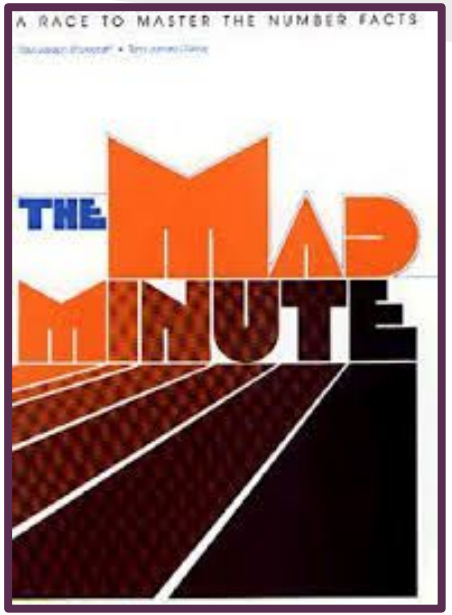
READY?
8 x 4
THAT'S NOT THE FIRST FACT, DAD. HOW CAN I SOLVE THEM IF YOU JUMBLE THEM UP?

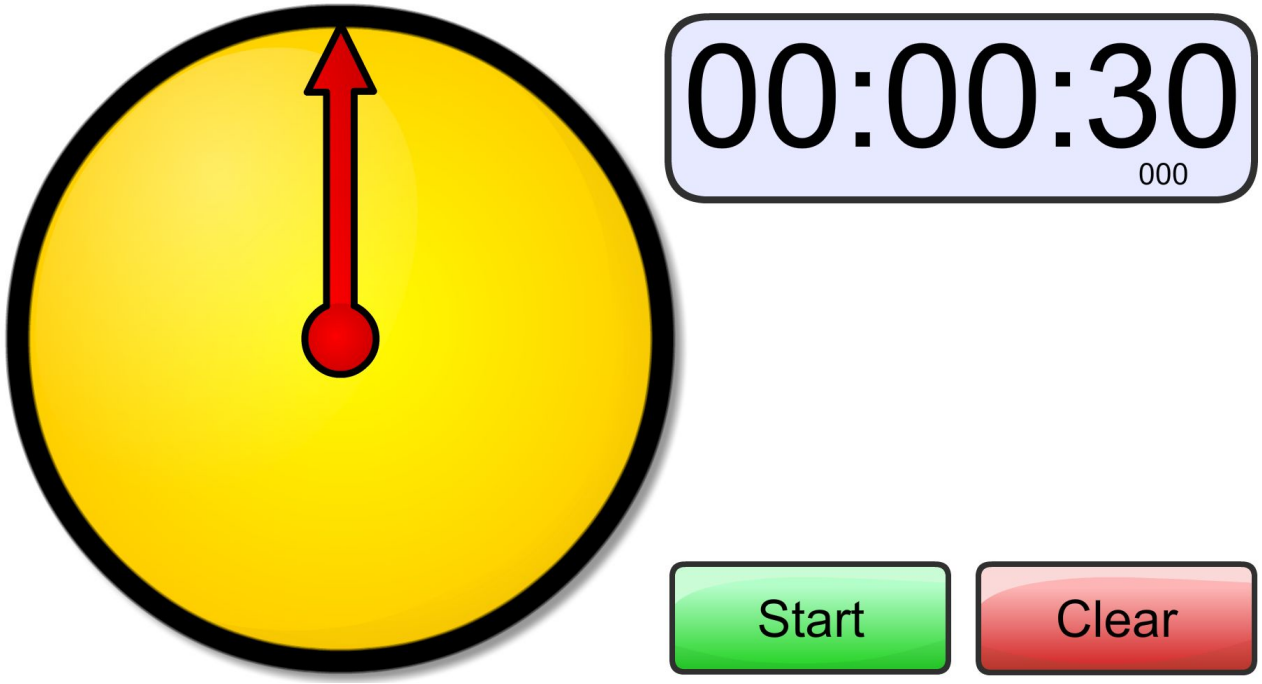

YOU ONLY KNOW THEM IN ORDER?
I DON'T KNOW THEM, I MEMORIZED THEM. NOBODY SAID ANYTHING ABOUT HAVING TO KNOW THEM.
howard and mac


4
8
2
Fast Fact
6
3
9
10
Clock
7
1
3
5

The Way it Was…

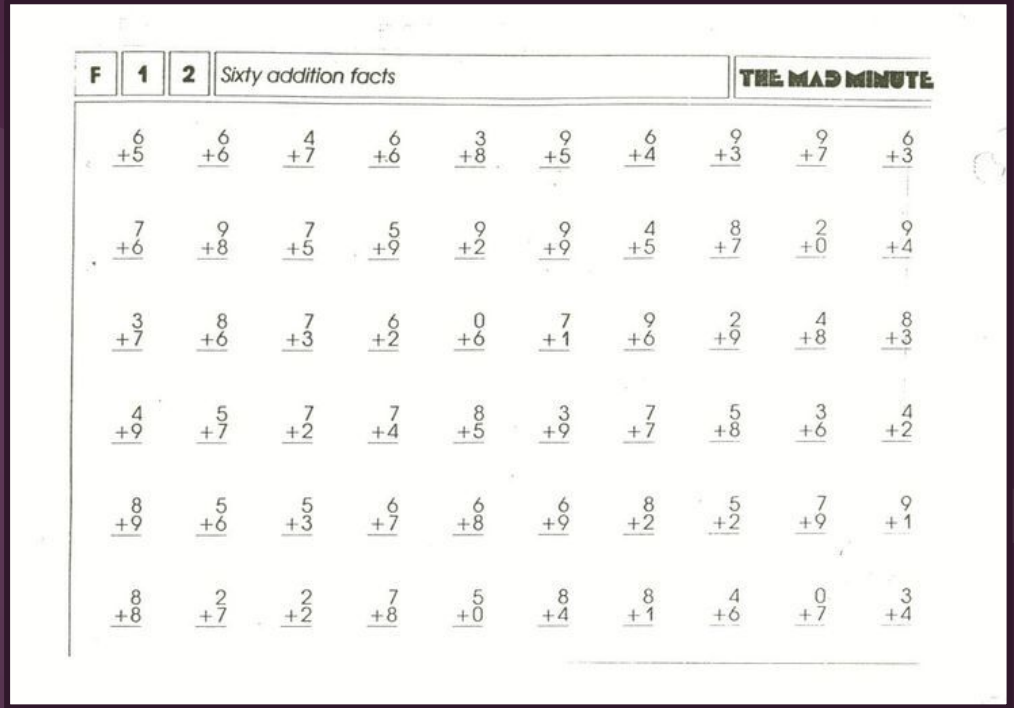


Test Time!



Switch assessments and correct.

What does this assessment tell you about your partner's understanding of mathematics.

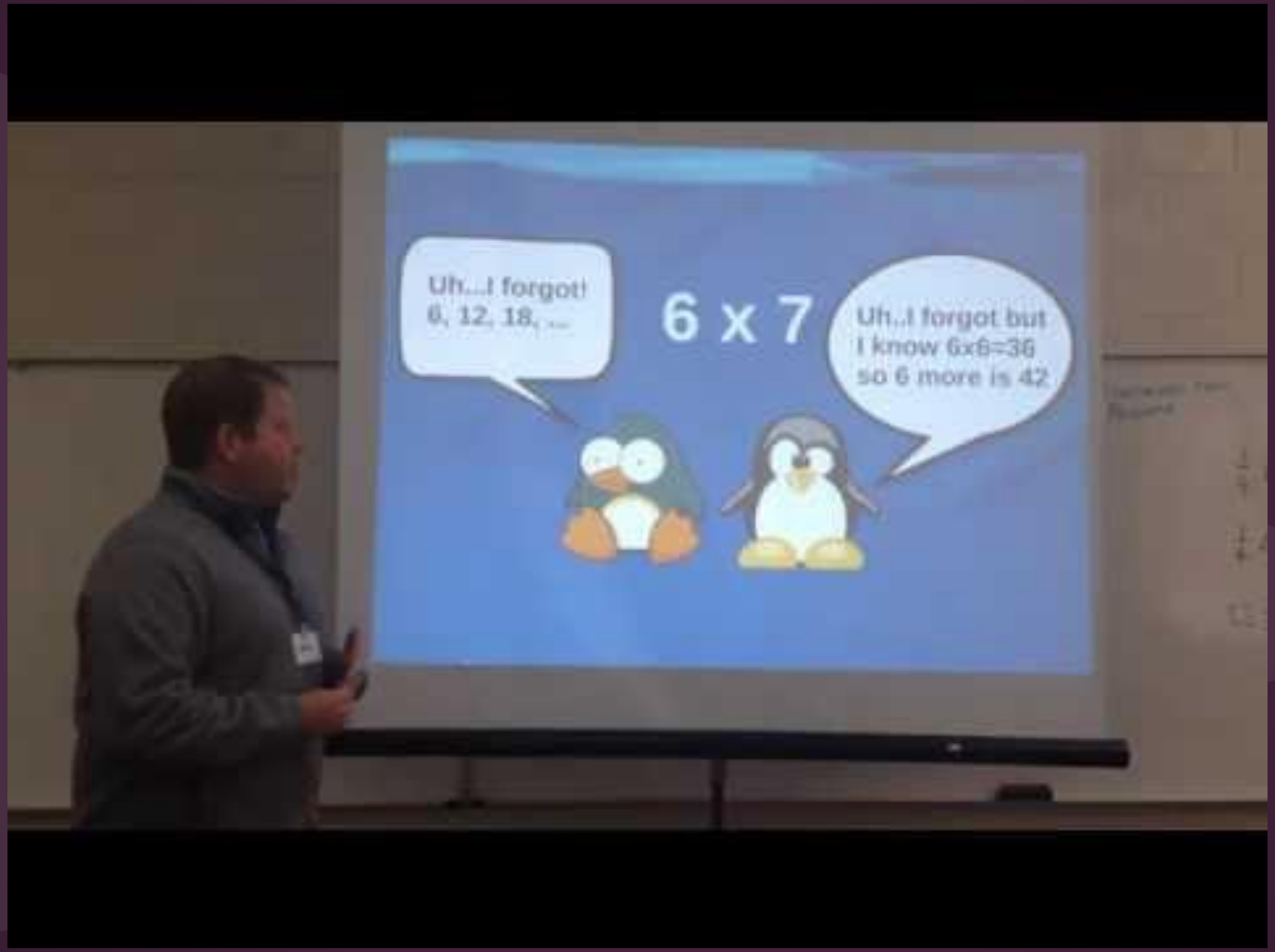


F | 1 | 2 | Sixty addition facts | THE MAD MINUTE

6 +5	6 +6	4 +7	6 +6	3 +8	9 +5	6 +4	9 +3	9 +7	6 +3
7 +6	9 +8	7 +5	5 +9	9 +2	9 +9	4 +5	8 +7	2 +0	9 +4
3 +7	8 +6	7 +3	6 +2	0 +6	7 +1	9 +6	2 +9	4 +8	8 +3
4 +9	5 +7	7 +2	7 +4	8 +5	3 +9	7 +7	5 +8	3 +6	4 +2
8 +9	5 +6	5 +3	6 +7	6 +8	6 +9	8 +2	5 +2	7 +9	9 +1
8 +8	2 +7	2 +2	7 +8	5 +0	8 +4	8 +1	4 +6	0 +7	3 +4

Think-Pair-Share

If you've memorized basic facts,have you learned them? Why or why not?



Fletcher, 2014

First Steps in Number, 2006

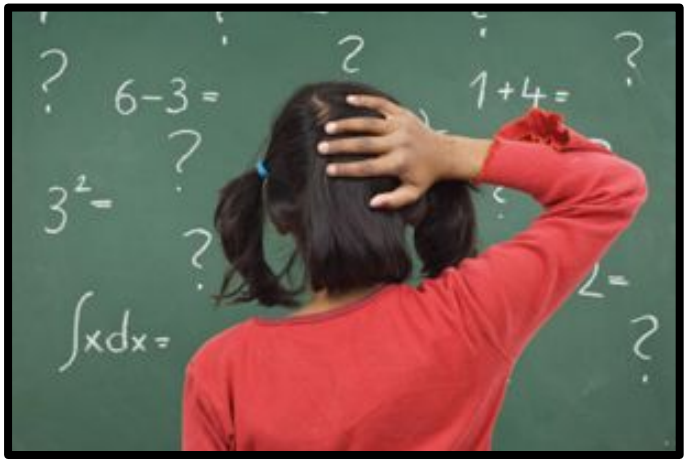
Fluency of Basic Facts

Efficient, appropriate and flexible application of calculation skills and is an essential aspect of mathematical proficiency (Baroody, 2006).

Fluently means noticing relationships and using strategies.

Fluency is “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (CCSSI 2010, p.6).

From memory does not mean *“memorized”*



What is *Flexibility, Efficiency, & Accuracy?*

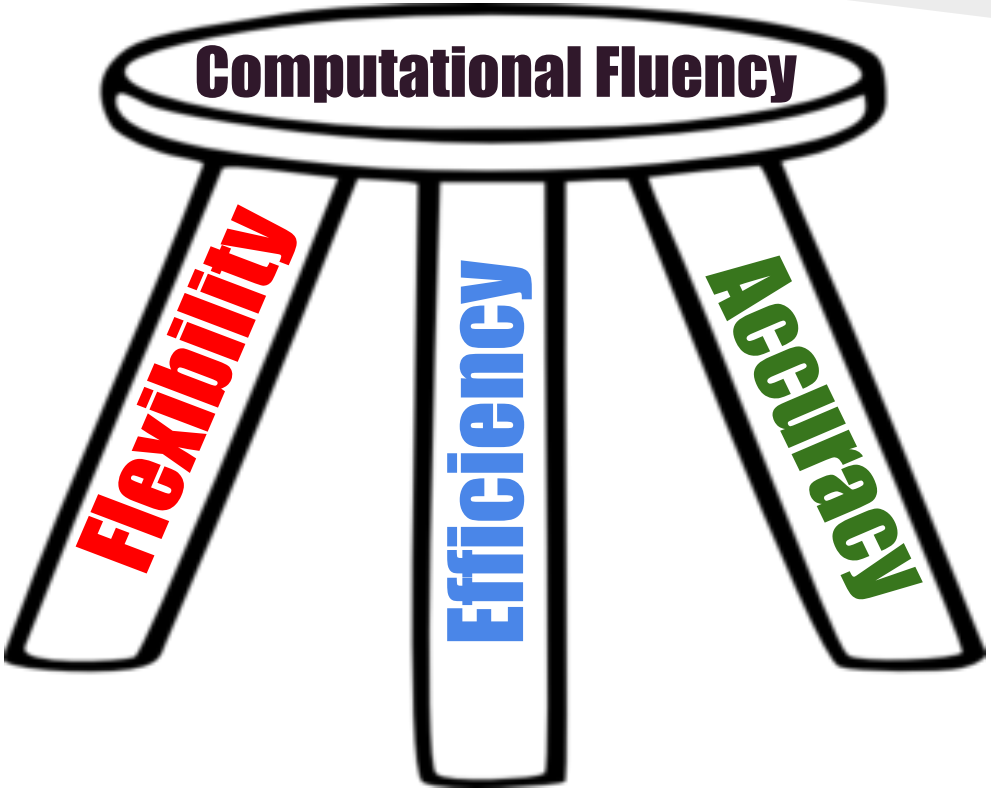
Flexibility means the ability to use number relationships with ease in computation.

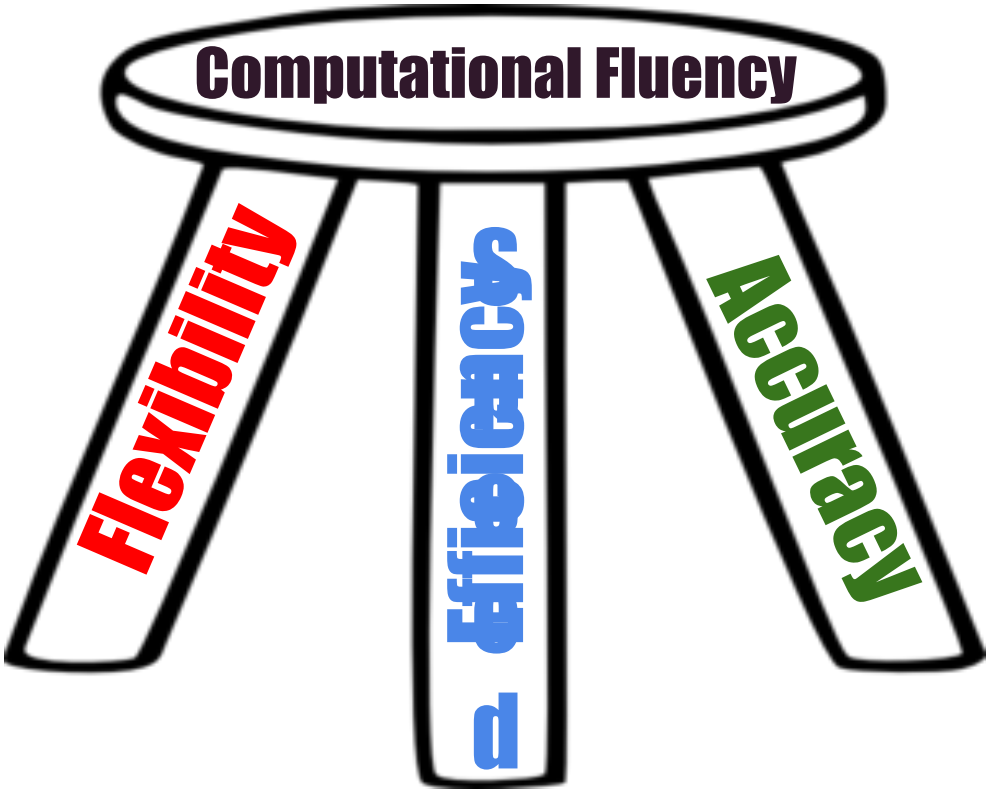
Efficiency refers to the ability to choose an appropriate, expedient strategy for a specific computation problem.

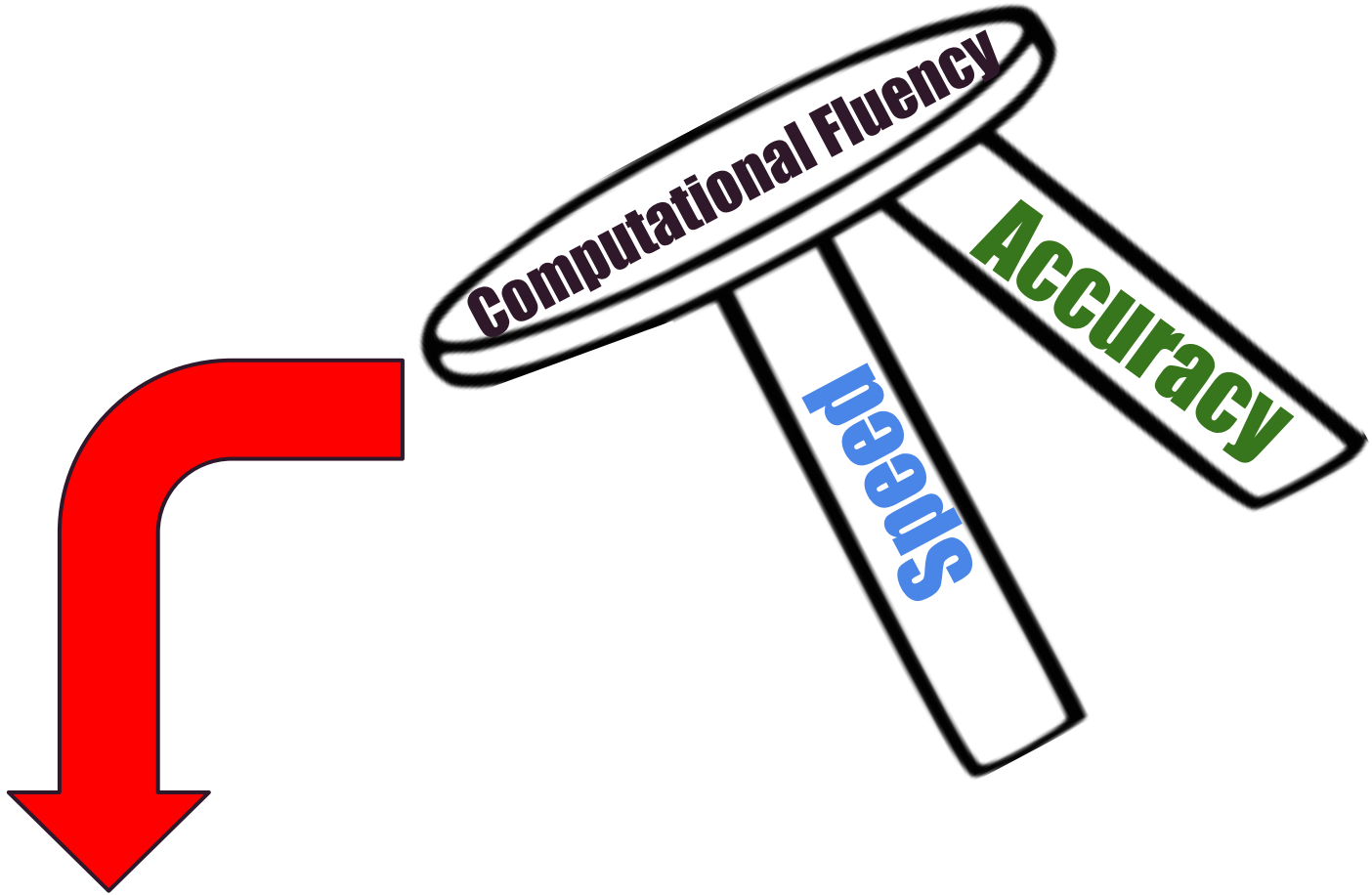
Accuracy denotes the ability to produce a correct answer.

(Parish, 2010)

Components of Fact Fluency




Computational Fluency
Flexibility
Efficiency
Accuracy


Computational Fluency
Accuracy
Speed

Creating the Vision

- Create a sense of urgency
- Develop the team
- Research, research, research
- Dialogue
- Meet on Common Ground
- Agree to Disagree
- Create Assessment

Fluency Mission Statement

The teachers of Region 6 believe that all students can develop single and multi-digit computational fluency (+, -, x, ÷ of whole number, fractions and decimals) through mathematics instruction that balances and connects ***conceptual understanding and procedural fluency .***

To achieve computational fluency, students must integrate:

the meaning of operations and their relationships to each other;
number relationships; and
the Base-Ten Number system (Russell, 2000, p. 154-155)

Computational fluency demands more of students than memorizing a single procedure or basic facts. It is the ability to solve single-digit and multi-digit computation with *flexibility, efficiency* and *accuracy*.

Flexibility means the ability to use number relationships with ease in computation.
Efficiency refers to the ability to choose an appropriate, expedient strategy for a specific computation problem.
Accuracy denotes the ability to produce a correct answer. (Parish, 2010, p. 5)

Computationally fluent students can compute using a variety of tools including manipulatives, representations, mental math, paper and pencil, calculators or other technology, and can wisely and comfortably choose which strategy is appropriate for a given situation. Regardless of the particular method used, students should be able to explain their method.

Fluency Mission Statement cont.

Instant recall of basic facts

The teachers of Region 6 believes that instant recall of basic facts, as a component of computational fluency, can be helpful as this allows students to solve complex mathematical tasks more efficiently in later grades.

Committing facts to memory is a *process* where students begin by refining and extending their natural strategies for solving simpler problems. Embracing multiple strategies promotes deep understanding, which then connects to fact knowledge. This helps students develop methods for mental and multi-digit computation. Gradually students master more and more efficient strategies and commit more facts to memory. (Isaacs & Carroll, 1999, p 509)

By developing students' deep conceptual understanding through flexible strategies (procedural fluency) for addition, subtraction, multiplication and division, they will be able to figure out a solution If a student rotely memorizes his or her facts without these opportunities, he or she will have no way of figuring out a solution if the fact is forgotten or unknown. In essence, the students will spend more time trying to retrieve the fact, rather than applying a known strategy to solve the fact.

Research shows that when properly instructed, the basic facts offer excellent opportunities for students to reason mathematically. (Isaacs & Carroll, 1999, p 509)

What changed? Not much!

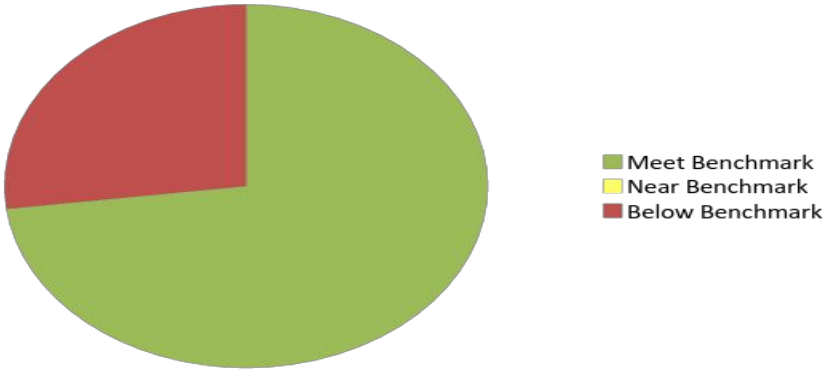
Scoring Rubric

(based on 45 facts in 4 minutes/5 seconds per fact)

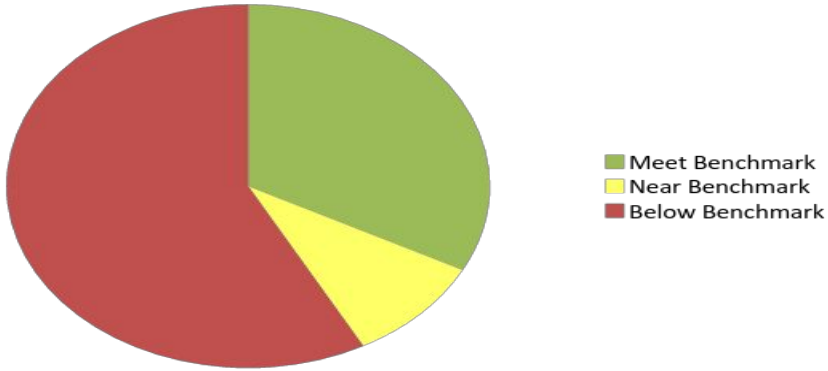
Expectation	Number Correct	
	Addition	**Subtraction**
Exceeds	49-54	49-56
Meets	45-48	45-48
Near	40-44	40-44
Below	0-39	0-39

The Data-Grade 1 Results

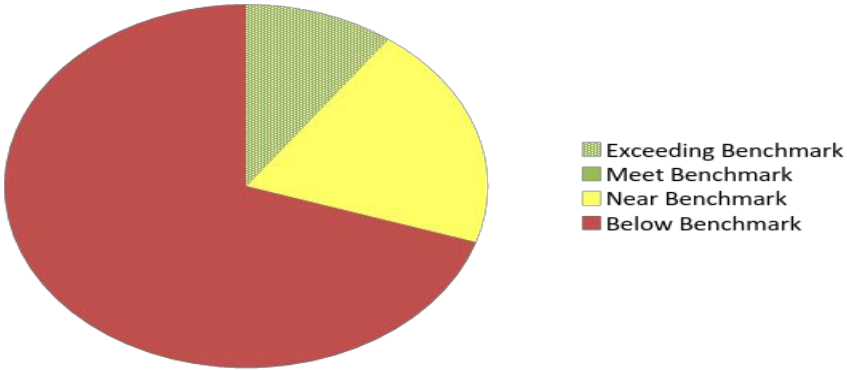
Addition to 5



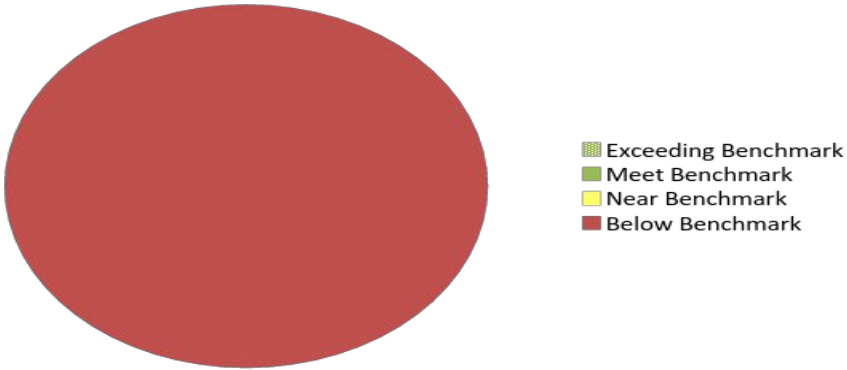
Subtraction to 5



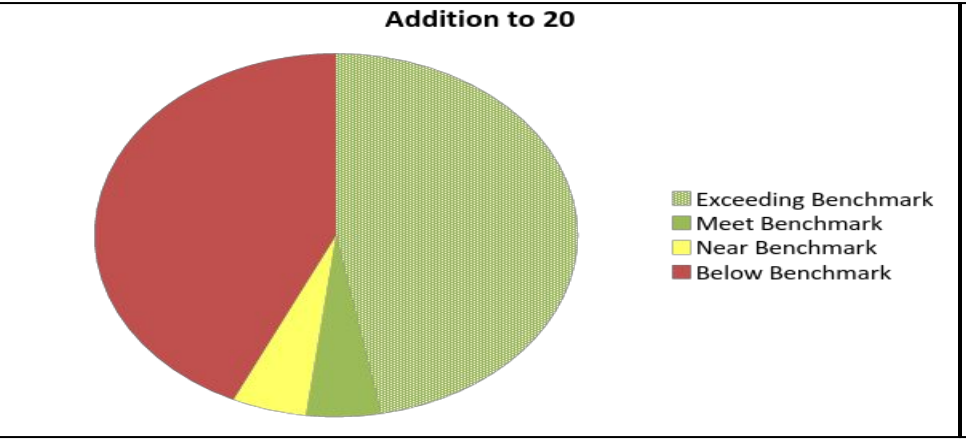
Addition to 10

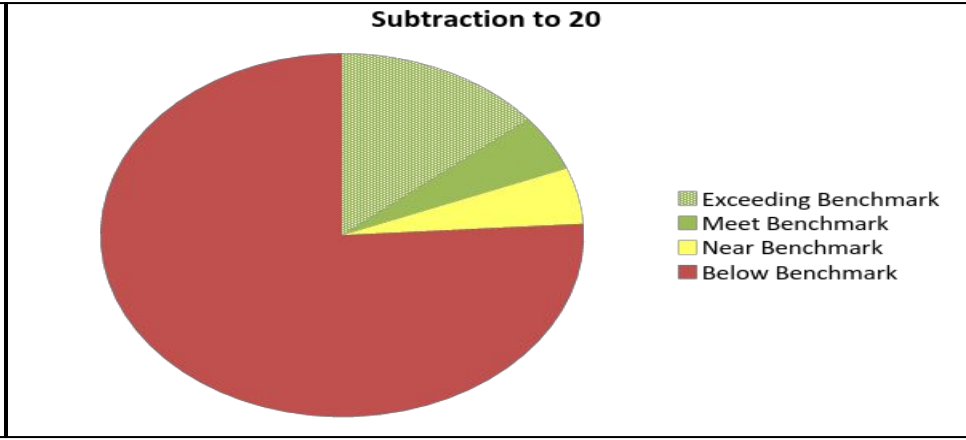


Subtraction to 10



The Data-3 Grade Results

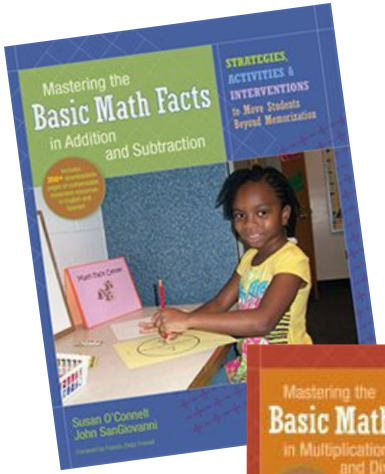


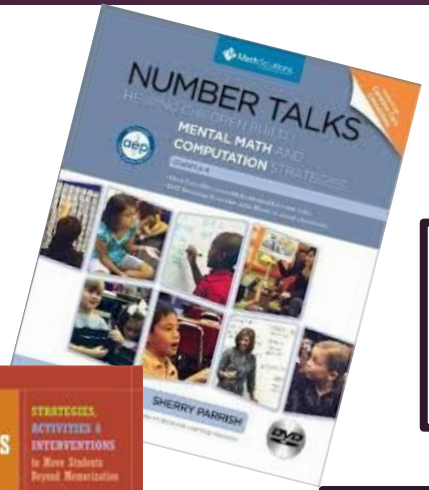


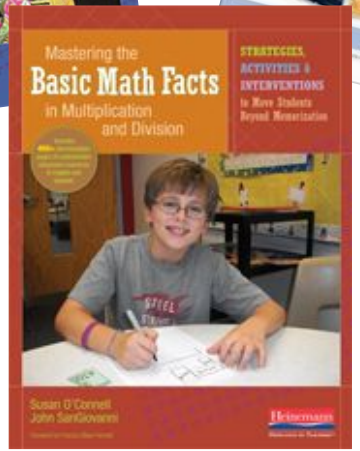
Grades 4-6 Data

	Grade 4 (10 x 10)			Grade 5 (12 x 12)			6 (12 x 12)	
	Multiply	Divide		Multiply	Divide		Multiply	Divide
W	0/12	0/12	W			W	0/15	0/15
	0%	0%					0%	0%
M	0/17	0/17	M	4/27	2/27	M	3/27	1/27
	0%	0%		15%	7%		11%	4%
G	12/23	6/23	G	1/29	2/29	G	2/38	1/38
	52%	26%		3%	6%		5%	2%
D	12/52	6/52	D	5/56	4/56	D	5/80	2/80
	23%	11.5%		9%	7%		6%	3%

Back to the Drawing Board…







K, Counting and Cardinality; K–5, Operations and Algebraic Thinking

ACHIEVE THE CORE

teaching children mathematics

Phases of Basic Fact Mastery

Traditional approaches to learning facts (flashcards, drill, and timed testing) attempt to move students from counting all directly to mastery.

This approach is ineffective—many students do not retain what they memorized in the long term, moving to grade 4 and beyond still not knowing their facts.

Even if students remember facts, they are unlikely to be fluent as defined above, as they will not have learned to flexibly apply strategies to find the answer to a addition and subtraction facts or more complex computation. (Baroody 2006)

Methods for solving single-digit addition and subtraction problems

Level 1

Direct Modeling by Counting All or Taking Away **-** Represent situational or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the total or addend.

Level 2

Counting on*-Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.*

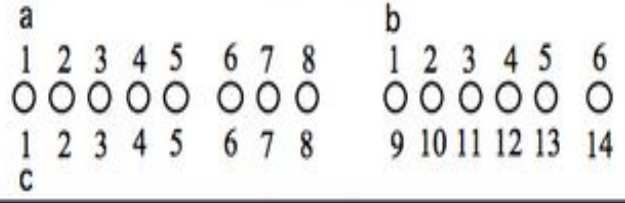
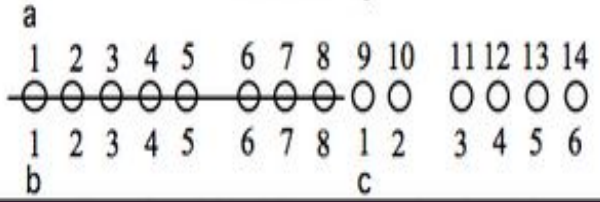
Level 3

Convert to an easier problem **-**Decompose an addend to compose a part with another addend.

Progressions for the Common Core State Standards in Mathematics (Draft). 2011.

Level 1

Direct Modeling by Counting All or Taking From **-** Represent situational or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the total or addend.

Levels	8 + 6 = 14	14 – 8 = 6
Level 1: Count all	Count All 	Take Away 

Progressions for the Common Core State Standards in Mathematics (Draft). 2011.

Level 2

Counting on-*Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.*

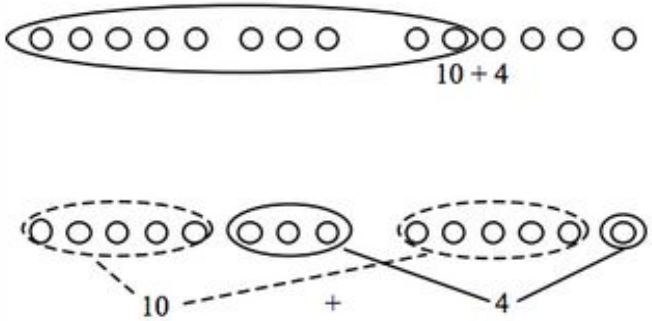
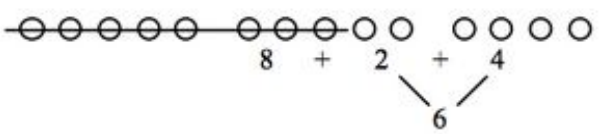
Levels	8 + 6 = 14	14 – 8 = 6
Level 2: Count on	Count On  	To solve 14 – 8 I count on 8 + ? = 14  8 to 14 is 6 so 14 – 8 = 6

Progressions for the Common Core State Standards in Mathematics (Draft). 2011.

Methods for solving single-digit addition and subtraction problems

Level 3

Convert to an easier problem **-**Decompose an addend to compose a part with another addend.

Levels	8 + 6 = 14	14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Recompose: Make a Ten  	14 – 8: I make a ten for 8 + ? = 14  8 + 6 = 14
Doubles ± *n*	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Progressions for the Common Core State Standards in Mathematics (Draft). 211.

Explicitly Teaching Strategies….

DOES NOT MEAN

teaching a specific strategy and then asking students to use it. This approach ***removes*** the reasoning component and adds to what the student is being asked to ***memorize.***

MEANS

supporting thinking, including asking students which strategies they might use in a given situation

helping students *see* the possibilities and letting them *choose* strategies that help them arrive at a solution.

***It can take 2-4 lessons before students will internalize the reasoning strategies discussed in class* (Steinbery, 1985).**

Number Sense

Teachers should help students develop math facts, not by emphasizing facts for the sake of facts or using ‘timed tests’ but by encouraging students to use, work with and explore numbers.

As students work on meaningful number activities they will commit math facts “to heart” at the same time as understanding numbers and math. **They will enjoy and learn important mathematics rather than memorize, dread and fear mathematics.**

Number Sense not Math Anxiety

Number sense, **critically important to students' mathematical development**, is inhibited by over-emphasis on the memorization of math facts in classrooms and homes. The more we emphasize memorization to students the less willing they become to think about numbers and their relations and to use and develop number sense (Boaler, 2009)

Boaler, Jo. Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts (2014)
http://tinyurl.com/pjqwnjp

Games = Formative Assessment

NCTM Assessment Principle states "*Assessment should support the learning of important mathematics and furnish useful information*"(NCTM, 2000)

Monitor progress through

- observations
- interviews
- math journals (Kling and Bay-Williams, 2014)

Data is more useful, as "efficiency and accuracy can be negatively influenced by timed testing" (Henry and Brown 2012) and timed testing has a negative impact on students (Boaler 2012).

+ & - Fluency Progression

Grade Level	Skill	How It is Assessed
Kindergarten	Within 5: • Conceptual understanding and accuracy	• Interviews with problems in context
Grade 1	Within 10: • Understanding, efficiency, flexibility, accuracy	• Strategy Checklists • Interviews • End of Year Assessments (untimed)
Grade 2	Within 20: • Understanding, efficiency, flexibility, accuracy	• Strategy Checklists • Interviews • End of Year Assessments (untimed)
Grade 3	Within 20: • Automaticity and accuracy	• End of Year Assessments (timed)

Kindergarten Interview

"Students act out addition and subtraction situations by representing quantities in the situation with objects, their fingers, and math drawings (MP 5, K.OA.1). To do this, students must mathematize a real world situations (MP 4) focusing on their quantities and their relationships rather than non-mathematical aspects of the situation."

"Students solve addition and subtraction equations for numbers within 5 (2+1 = __ or 3-1 = ___) while still connecting these equations to situations verbally or with drawings. Experience with decompositions of numbers and with Add To or Take From situations enables students to begin to fluently add and subtract within 5."

From the Progressions for the Common Core State Standards (2011)

Kindergarten Interview

Regional School District #6 Kindergarten June Benchmark - Facts to 5
Student Sheet

1. 4 bunnies sat on the grass. 5 more bunnies hopped there. How many bunnies are there now? *(Add to-Result Unknown)*

2. 10 apples were on the table. I ate 6 apples. How many apples are on the table now? (*Take From-Result Unknown)*

3. There are 4 yellow pencils and 3 red pencil on the table. How many pencils are on the table? (Put Together)

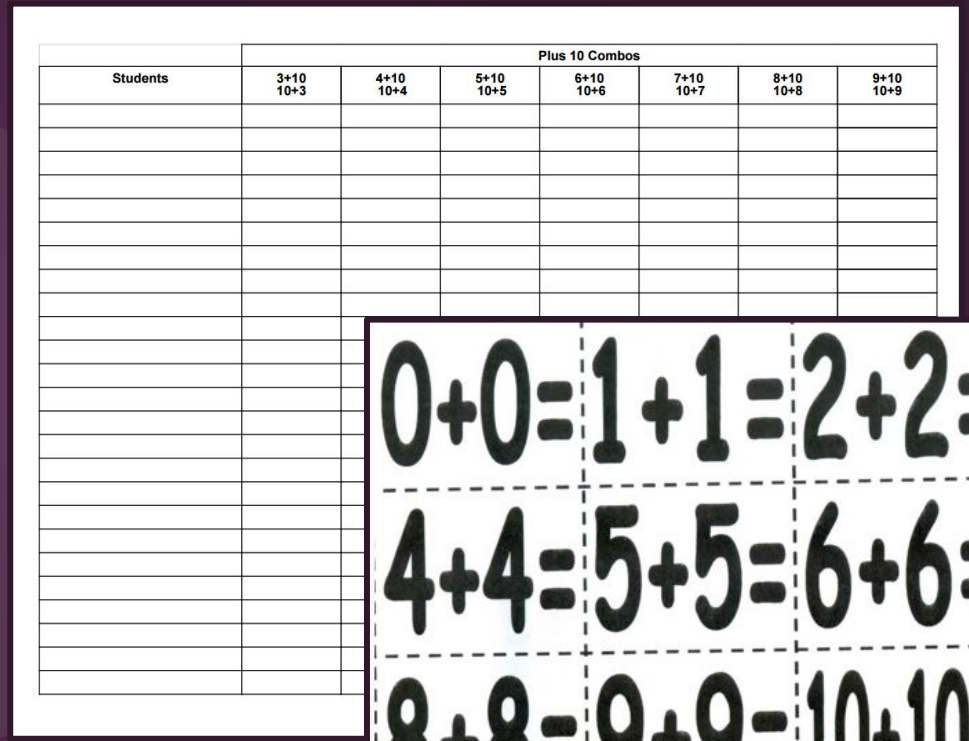
Regional School District #6 Rubric Question 1-June

Question 1: 4 bunnies sat on the grass. 2 more bunnies hopped there. How many bunnies are there now? *(Add to-Result Unknown)*

What child says	What child does (check all that apply)	Rubric (check one)
	□ role plays with the bunnies, acting out the situation. □ just grabs 4 bunnies and 5 bunnies without role playing. □ role plays with non-realistic items, acting out the situation. □ just grabs 4, then 5 non-realistic items without role playing. □ represents problem using paper and pencil to draw □ uses numbers along with drawing of representation □ uses symbols and equations to show along with drawing of representation □ uses symbols and equations and no drawing □ solves mentally □ uses symbols □ correct operation (adds) □ incorrect operation (subtracts) □ appears to guess □ cannot solve situation	**_____Meets End of Year Target** Student solves Add To Result Unknown problem using the following strategy • uses tools such as tiles, linking cubes, ten frame, counters. • uses paper and pencil to draw realistic or non realistic representation using shapes, etc., and counts all or counts on. • paper, pencil representation using numbers and symbols • solves mentally and can explain using number words or symbols, either verbally or with paper and pencil. **____Progressing Toward End of Year Target** Student solves Add To Result Unknown problem using any of the following strategies • correct operation, realistic objects and role playing **____Below Benchmark for End of Year Target** Incorrect answer or student is not able to solve problem.
	Other:	

Grade 2 Checklists

Time Frame	Facts To Assess
September	+1, +2, and Make Ten
October	-1, -2, Subtraction Make Ten
November	Plus 10 Combos, Subtraction Plus 10
December	Doubles and Near Doubles Addition
January-February	Doubles and Near Doubles Subtraction
March	Plus 9 Combos and Remaining
April	Subtraction 9 Combos and Remaining
May-June	Maintenance



Students	Plus 10 Combos						
	3+10 10+3	4+10 10+4	5+10 10+5	6+10 10+6	7+10 10+7	8+10 10+8	9+10 10+9

Flexibility and Efficiency Interview

Name_____

Regional School District 6: Flexibility and Efficiency Interview-December
Addition for Doubles and Near Doubles

Show the child 9 + 8	Student Response	M	D	B
UNDERSTANDING Ask: Read this card to me. (If child can't read it, you can read it to the child.) What does 9 + 8 mean? **If child says 9 + 8, ask, "What does that mean?"* **If child says 17, point to the expression, and ask, "What does this part mean?"		Child knows the answer 9 and 8 more, 8 and 9 more, 9 add 8 more, etc.)	Child says 9 + 8 and cannot elaborate. Child tells story to match expression.	Child does not show understanding of what the minus sign means. He or she may give the difference, but not explain the expression.
ACCURACY Ask: What is the answer to 9 + 8?		Child knows the answer or self corrects during interview.		Child does not know the answer.
EFFICIENCY: Ask: How did you find the answer to 9 + 8?		Uses a strategy such as: -uses a double fact (8+8=16, 16+1=17, or 9+9=18, 18-1=17) -Makes ten (9+1=10, 10+7=17, or 8+2=10, 10 + 7= 17) -Counting on without using fingers (9, 10, 11, 12, 13, 14, 15, 16, 17) -Student just knows it.	Child counts on from either addend. -Counts on using fingers (9, 10, 11, 12, 13, 14, 15, 16, 17)	Uses a strategy such as: -Modelling all (shows 9 fingers, then counts 8 more on fingers again). -Child has no answer.
FLEXIBILITY: Ask: If your friend was having trouble remembering this fact, what other strategy might you suggest to him or her?		Child has *different* strategy (must use two bullets from list above).	Child only has one strategy from "meets benchmark."	Child has no strategy or counts all.

(M) Meets Benchmark: 4 Ms (D) Developing: any combination of Ms and Ds (B) Below Benchmark: 1 or more Bs

Name_____

Regional School District 6: Flexibility and Efficiency Interview-December

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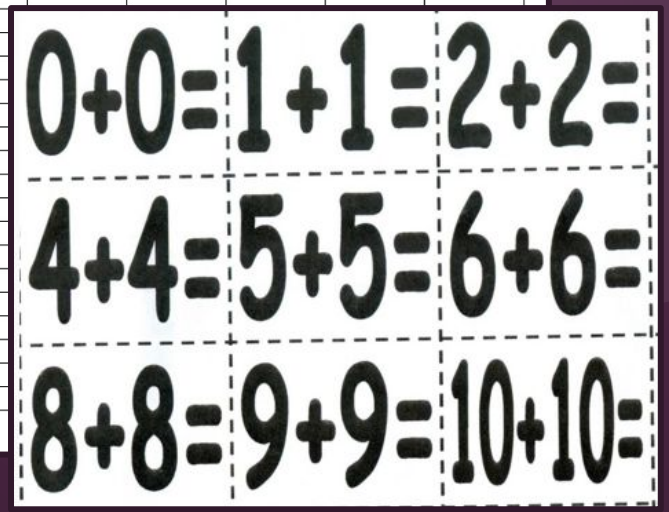
ACCURACY Ask: What is the answer				
EFFICIENCY: Ask: How did you find th to 9 + 8?				
FLEXIBILITY: Ask: If your friend was h trouble remembering thi other strategy might you him or her?				

(M) Meets Benchma

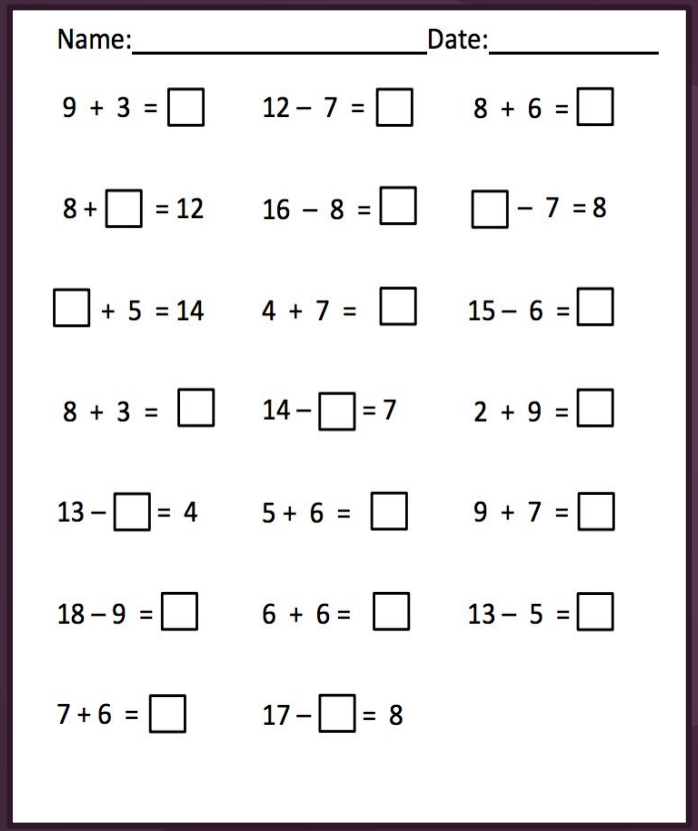
Students	Plus 10 Combos						
	3+10 10+3	4+10 10+4	5+10 10+5	6+10 10+6	7+10 10+7	8+10 10+8	9+10 10+9



What do you think about this approach?

What would you expect to learn about your students by combining the checklist with the interview?

End of Year (Mini) Assessment


Name:_____ Date:_____

9 + 3 = ☐ 12 – 7 = ☐ 8 + 6 = ☐

8 + ☐ = 12 16 – 8 = ☐ ☐ – 7 = 8

☐ + 5 = 14 4 + 7 = ☐ 15 – 6 = ☐

8 + 3 = ☐ 14 – ☐ = 7 2 + 9 = ☐

13 – ☐ = 4 5 + 6 = ☐ 9 + 7 = ☐

18 – 9 = ☐ 6 + 6 = ☐ 13 – 5 = ☐

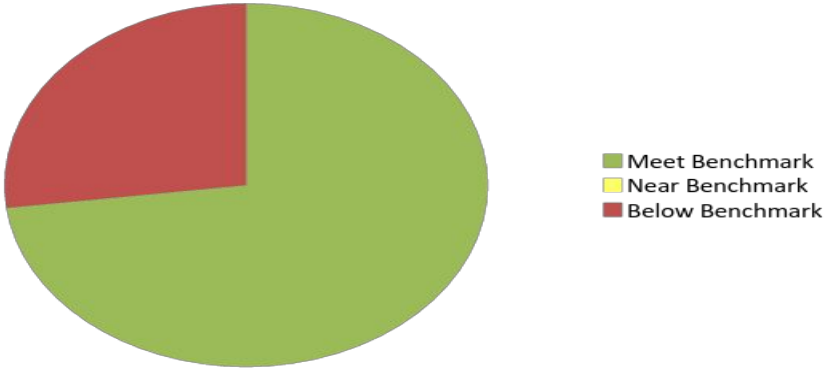
7 + 6 = ☐ 17 – ☐ = 8

- Administered 3-5x per year (dependent on grade level.
- Growth Mindset – students are timed up rather than back.
- Application of facts, built on conceptual to procedural foundation – RIGOR
- Focuses on the relationship between the operations and their properties.

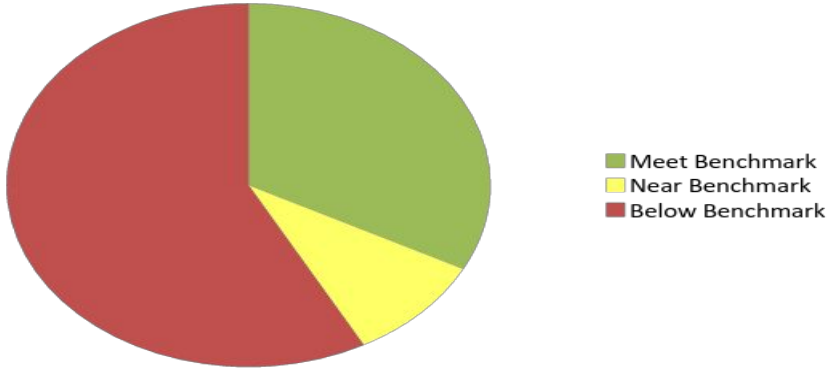
The Way It Was-

Grade 1 Results, 2014

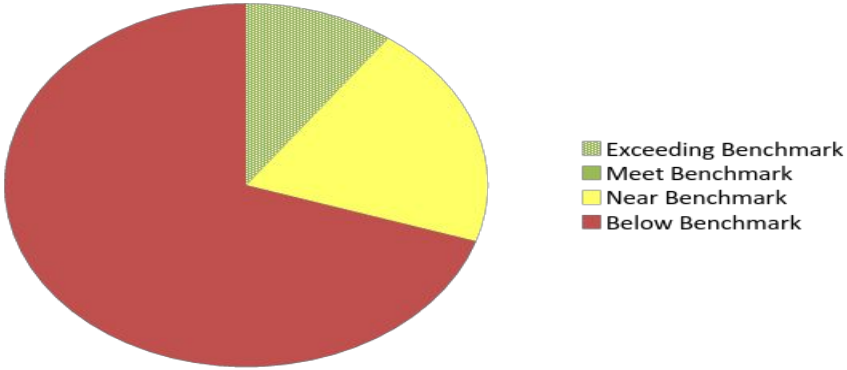
Addition to 5



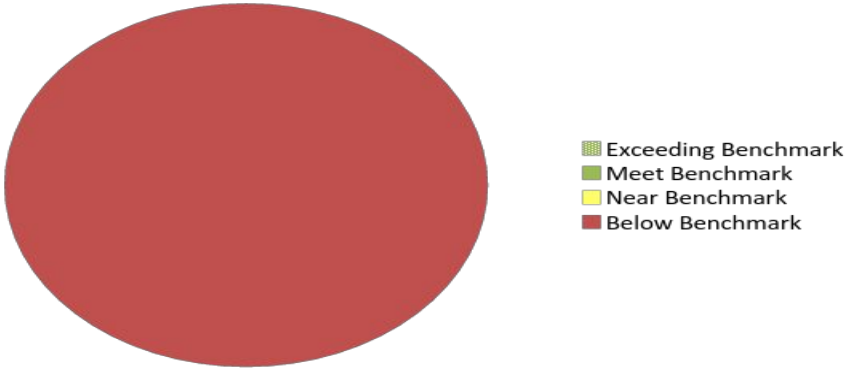
Subtraction to 5



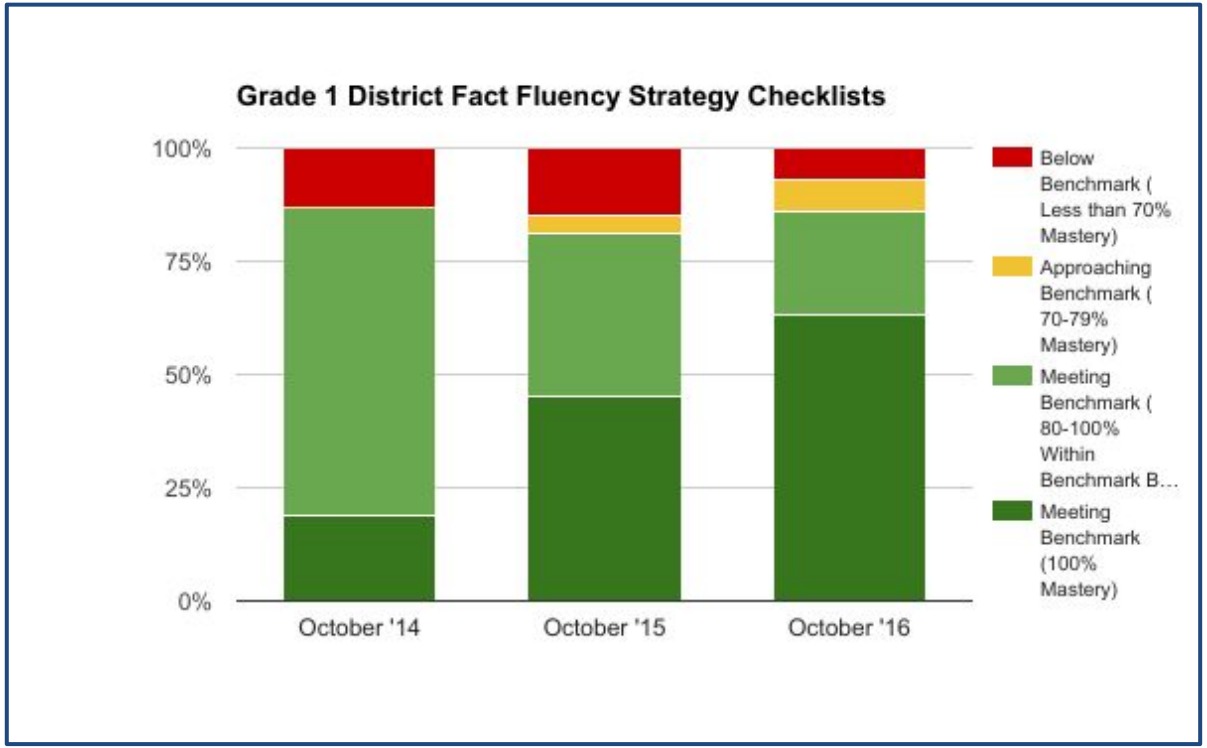
Addition to 10



Subtraction to 10

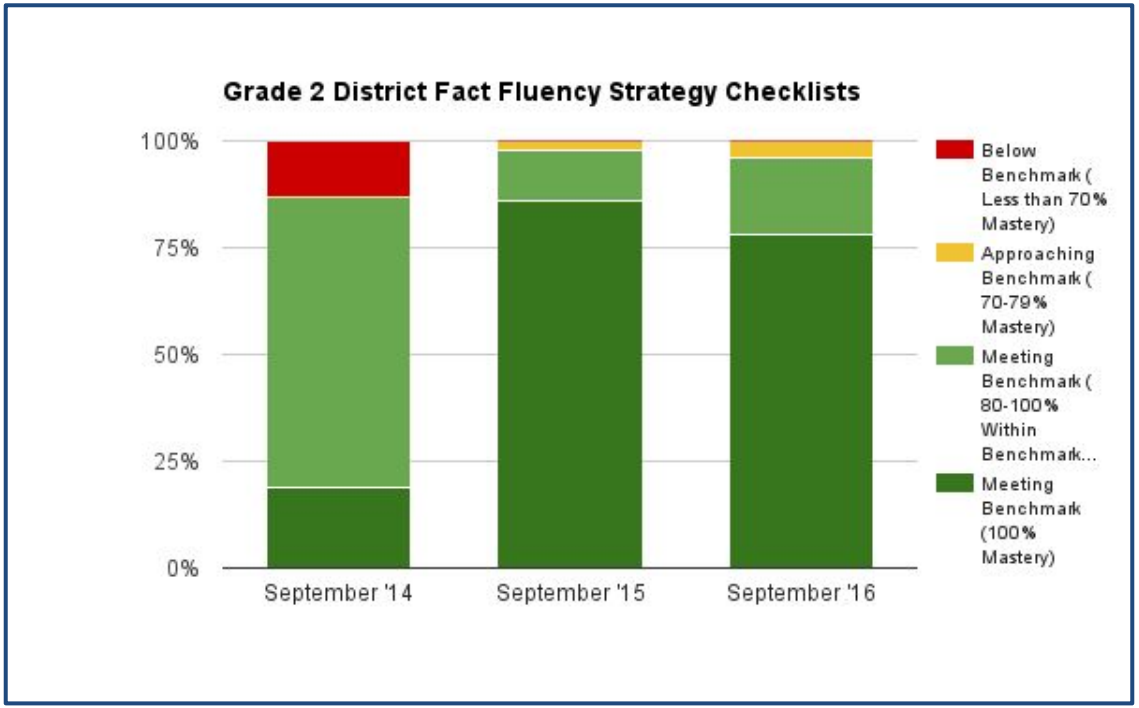


The Way It Is-Grade 1 Results



- The number of students reaching 100% mastery has greatly improved since implementation

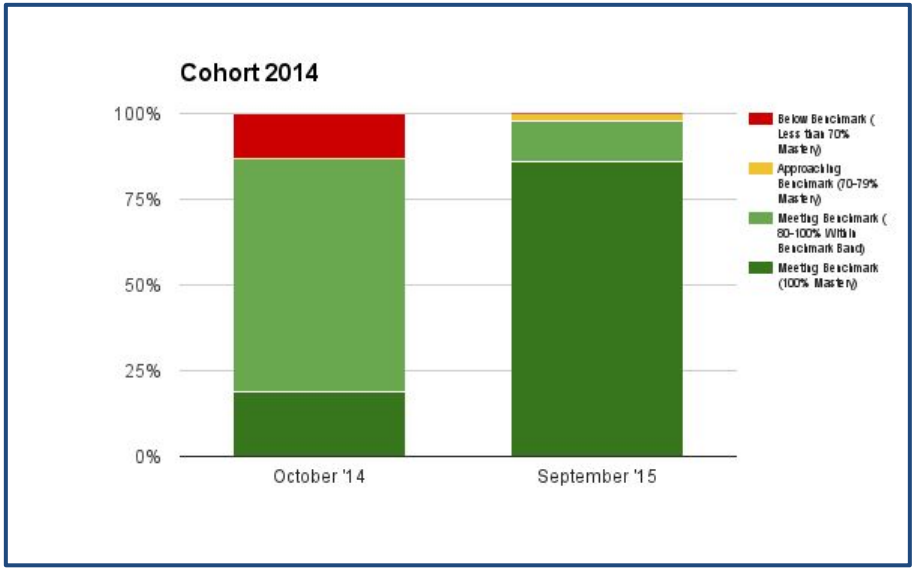
The Way It Is-Grade 2 Results

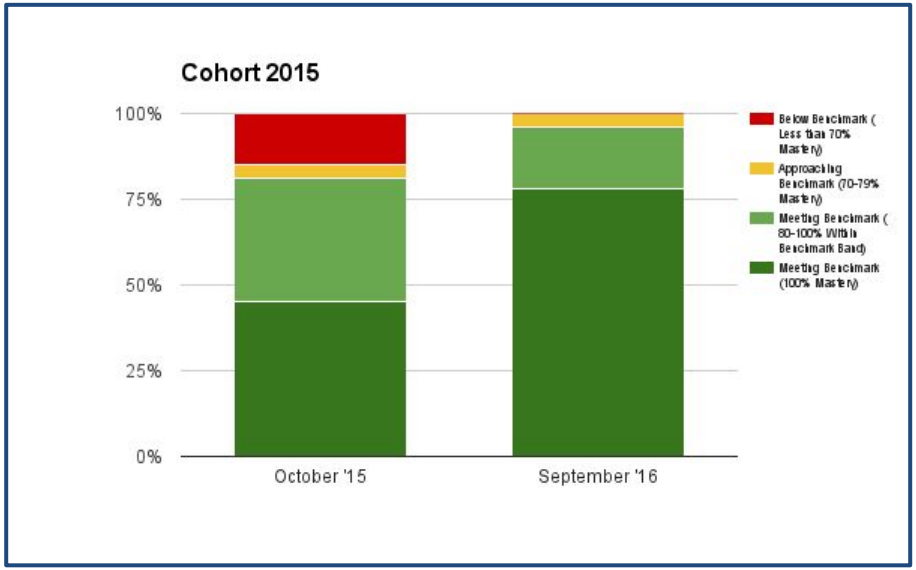


- The number of students meeting benchmark and reaching 100% mastery has greatly improved since implementation

The Way It Is-Year to Year Growth

Grade 1 > Grade 2



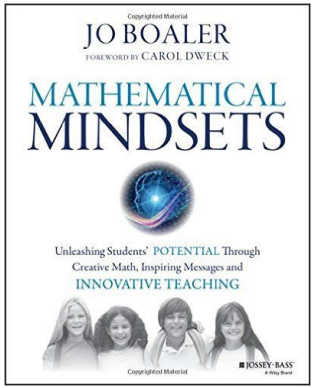


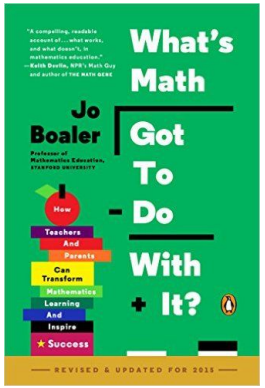
- More students met benchmark in 2nd year
- No students were in need of support in 2nd year
- More students had 100% mastery in 2nd year

To Sum it all up.....

Fluency comes about when students develop number sense, when they are mathematically confident because they understand numbers (Boaler, 2015).

youcubed at Stanford University





Jo-Boaler's Youcubed Links

- Youcubed.org
- Aligning Assessment to Brain Science
- Depth, not Speed
- Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts
- Speed and Time Pressure Blocks Working Memory
- Think It Up! Mistakes Grow Your Brain

A Call to Action…

What is one component of this assessment practice you will take back to your classroom or school?

Resources

- "Addition and Subtraction within 20 Mini-Assessment." *Achieve the Core*. Student Achievement Partners, 13 Oct. 2013. Web.
- Baroody, Arthur J. 2006. "Why children have difficulty mastering the basic number combinations and how to help them." *Teaching Children Mathematics* 13 (August): 22-31.
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- Henry, Valerie J., and Richard S. Brown. 2008. "First-Grade basic facts: An investigation into teaching and learning of an accelerated, high-demand memorization standard." *Journal for Research in Mathematics Education* 39 (March):153-83

Resources

- Kling, Gina and Jennifer M. Bay-Williams. 2014. "Assessing Basic Fact Fluency." *Teaching Children Mathematics* 20 (April): 490-96.
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