ALGORITHMS

PAST SUCCESS / PRESENT FINDINGS/ FUTURE IMPLICATIONS SUPPORTING UNDERSTANDING AND FLUENCY

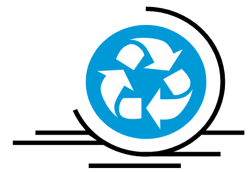
NCTM Annual Meeting & Exposition
San Antonio, Texas
April 6, 2017

Christine Moynihan, Ph.D.
Educational Consultant
moynihan.christine@gmail.com

THE THREE Rs OF ARITHMETIC

REASONING

RELEVANCE



RELATIONSHIPS

(Moynihan)

THE THREE COMPONENTS OF COMPUTATIONAL FLUENCY

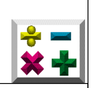
What defines computational fluency?

- ❖ Generate a list of what this looks like.
- ❖ Prepare to share your thinking.

~ EFFICIENCY ~

~ ACCURACY ~

~ FLEXIBILITY ~



(Russell, 2002)

ALGORITHMS

What is an ALGORITHM???????

- ❖ Write your own definition.
- ❖ Prepare to share your thinking.

- a step-by-step operational procedure
- a recipe of sorts used to complete tasks
- a basic technique used to get a job done
- a series of well-defined steps used to solve problems
- a set of rules which will yield a correct answer if done properly

LET'S GET CRACKING

Caitlin was so excited that she found a great deal for Spring Break! Jet Blue was offering a flight to Fort Lauderdale for $396 round trip. She was then planning to split a room with three of her close girlfriends and her share would be $235. What will be the cost of her flight and hotel bill?

ADDITION
U.S. HISTORICALLY-TAUGHT

Explanation: Start by adding the ones and "carry" if necessary. Follow the same procedure with the tens, hundreds, etc.

Example:

1. Add the ones (7 + 9 = 16)
   Put down the 6/carry the 1.

$$\begin{array}{r} {}^{1} \\ 257 \\ +\ \underline{389} \\ 6 \end{array}$$

2. Add the tens (1 + 5 + 8 = 14)
   Put down the 4/carry the 1.

$$\begin{array}{r} {}^{1\,1} \\ 257 \\ +\ \underline{389} \\ 46 \end{array}$$

3. Add the hundreds (1 + 2 + 3 = 6)

$$\begin{array}{r} {}^{1\,1} \\ 257 \\ +\ \underline{389} \\ 646 \end{array}$$

DR. JOHN VAN de WALLE

- Internationally-known mathematics educator
- Author of multiple books/resources
- *Elementary and Middle School Mathematics: Teaching Developmentally;* series often used as text in pre-service classes
- Taught at Virginia Commonwealth University for almost 30 years both pre-service and in-service teachers

It is fun to figure out the puzzle of how children go about making sense of mathematics and then how to help teachers help kids.

TRADTIONAL ALGORITHMS

THE TRADITIONAL ALGORITHMS ARE NOT SERVING US WELL.

The four traditional algorithms are each clever procedures for computation that have been devised to work for all numbers with attention only to basic facts.

Some of the problems with these methods are inherent in the algorithms themselves. Others are related to the difficulties of teaching these algorithms in light of their reduced need in today's society.

(Van de Walle)

IN THE WORDS OF KEITH DEVLIN

All the Mathematical Methods I Learned In My University Math Degree Became Obsolete In My Lifetime

This article is an expanded version of an essay published on Edge.org. January 1, 2017

ALGORITHM ASSESSMENT

Let's Take A Look

ALGORITHM ASSESSMENT

Name of Algorithm: ______________________

Operation: ______________________

Example

	YES	NO
1. Is number-oriented rather than digit-oriented	___	___
2. Is left-handed / starts with most significant numbers	___	___
3. Is something students could generate on their own	___	___
4. Relies on understanding rather than memorization	___	___
5. Can be completed mentally, involving little/no regrouping	___	___
6. Supports flexibility in thinking about numbers	___	___
7. Supports use of precise vocabulary	___	___
8. Is relatively efficient	___	___
9. Is visually clear, easy to follow w/ few chances for error	___	___
10. Supports development of number sense	___	___

ADDITION – U.S. HISTORICALLY-TAUGHT ASSESSMENT

	YES	NO
1. Is number-oriented rather than digit-oriented.		✔
2. Is left-handed/starts w/ most significant numbers.		✔
3. Is something students could generate on their own.		✔
4. Relies on understanding rather than memorization.		✔
5. Can be completed mentally, involving little or no regrouping.		✔

ADDITION – U.S. HISTORICALLY-TAUGHT ASSESSMENT

	YES	NO
6. Supports flexibility in thinking about numbers.		✔
7. Supports use of precise vocabulary.		✔
8. Is relatively efficient.	✔	
9. Is visually clear, easy to follow w/ few chances for error.		✔
10. Supports development of number sense.		✔

ADDITION
LANDMARK / FRIENDLY NUMBERS

Explanation: Change number(s) to "landmark" number(s) – "friendly" number(s) in our number system that are easy to work with; often multiples of 10

Example:	<u>498 + 236</u>
1. Change 498 to 500.	498 + 2 = 500
2. Add the two addends.	500 + 236 = 736
3. Compensate for the +2 by subtracting 2.	736 - 2 = 734

ADDITION
EQUAL / OPPOSITE CHANGE

Explanation: Change one number to a landmark number. Equalize that change by performing the opposite operation on the other addend.

Example:	<u>34 + 48</u>
1. Change 48 to 50.	48 + 2 = 50
2. Apply an equal but opposite change to 34.	34 – 2 = 32
3. Add the two changed addends.	50 + 32 = 82

ADDITION
PARTIAL SUMS (LEFT-TO-RIGHT)

Explanation: Add the components of the addends by place value. Add the partial sums to find the total sum.

Example:	364 <u>+ 168</u>
1. Add the hundreds: (300 + 100).	400
2. Add the tens: (60 + 60).	120
3. Add the ones: (8 + 4).	12
4. Add partial sums: (400 + 120 + 12).	532

ADDITION ALGORITHMS

- Landmark / Friendly Numbers
- Equal and Opposite Change
- Partial Sums: Left-to-Right
- Partial Sums: Right-to-Left
- U.S. Historically Taught

NUMBER SENSE

To the person without number sense, arithmetic is a bewildering territory in which any deviation from the known path may rapidly lead to being totally lost.

(Dowker)

SUBTRACTION
PARTIAL DIFFERENCES (LEFT-TO-RIGHT)

Explanation: Subtract components of the numbers by place value. Subtract the partial differences to find total difference.

Example:

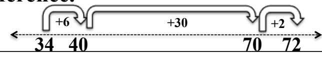
	534
	- 256
1. Subtract hundreds: (500 - 200).	300
2. Subtract the tens: (30 - 50).	-20
3. Subtract the ones: (4 - 6).	-2
4. Combine partials: (300 - 20 - 2).	278

SUBTRACTION
ADDING ON

Explanation: Place numbers on the number line (mentally or pictorially). Find the difference by adding on to subtrahend until reaching minuend.

Example: 72 - 34

1. Place numbers on number line.	
2. Add 6 to 34 to get to 40.	+ 6
3. Add 30 to 40 to get to 70.	+ 30
4. Add 2 to 70 to get to 72.	+ 2
5. Combine all moves for total difference.	6 + 30 + 2 = 38



SUBTRACTION
SAME CHANGE

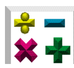
Explanation: Change the subtrahend to the nearest multiple of 10, 100, etc. Apply the same change to the minuend and find the difference.

Example: 836 - 378

1. Change 378 to 400.	378 + 22 = 400
2. Apply the same change to the minuend.	836 + 22 = 858
3. Find the difference.	858 - 400 = 458

SUBTRACTION ALGORITHMS

- ADDING ON
- SAME CHANGE
- PARTIAL DIFFERENCES: LEFT-TO-RIGHT
- PARTIAL DIFFERENCES: RIGHT-TO-LEFT
- LEFT-TO-RIGHT
- U.S. HISTORICALLY-TAUGHT



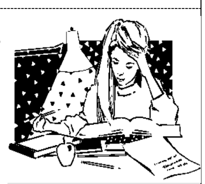
MULTIPLE WAYS TO CALCULATE

Being able to calculate in multiple ways means that one has transcended the formality of the algorithm and reached the essence of numerical operation - the underlying mathematical ideas and principles.

(Liping Ma)

MEMORIZING WITHOUT UNDERSTANDING

Growing evidence suggests that once students have memorized & practiced procedures without understanding …



… they have difficulty learning to bring meaning to their work.

(Hiebert)

MULTIPLICATION PARTIAL PRODUCTS

Explanation: Think of each factor in expanded form. Multiply components of each factor by components of other factor. Add the partial products to obtain the total product.

	34
Example:	x 26
1. Multiply ones by ones (6 x 4).	24
2. Multiply ones by tens (6 x 30).	180
3. Multiply tens by ones (20 x 4).	80
4. Multiply tens by tens (20 x 30).	600

5. Add partial products to obtain the total product. 600 + 180 + 80 + 24 = 884

MULTIPLICATION ALGORITHMS

- CLUSTER / FRIENDLY
- PARTIAL PRODUCTS (BOX)
- LATTICE
- U.S. HISTORICALLY-TAUGHT

DIVISION
PARTIAL QUOTIENTS

Explanation: Multiply the divisor by familiar numbers (begin w/ 10 or multiples of 10 if possible) and subtract that product from the dividend. Continue until remainder < divisor.

Example: 200 ÷ 9

1. 10 x 9 = 90
2. 200 - 90 = 110
3. 10 x 9 = 90
4. 110 – 90 = 20
5. 2 x 9 = 18
6. 20 - 18 = 2
7. Add partial quotients/account for remainder: 22 R2

DIVISION ALGORITHMS

- CLUSTER / FRIENDLY
- PARTIAL QUOTIENTS
- U.S. HISTORICALLY-TAUGHT

CHILDREN'S FIRST METHODS

Children's first methods are admittedly inefficient.

However …
if they are free to do their own thinking,
they invent increasingly efficient procedures just as our ancestors did.

By trying to bypass the constructive process,
we prevent them from making sense of arithmetic.

(Kamii & Livingston)

IF I TEACH MATHEMATICS …

If I teach mathematics

ONLY

in the way I understand it, …

then only children who understand it
in the same way

will learn it.

(Lee Stiff)

LOOKING AHEAD FOR OUR STUDENTS

We can no longer afford the time required for the teaching, reteaching, and remediation of outmoded skills [traditional algorithms].

Good workers in almost every field will need number sense. Today's curriculum shortchanges students in the necessary skills while wasting time teaching those skills that are obsolete in the work place.

(Van de Walle)

A WORD TO ALL

… whereas the focus used to be on mastering the skills with the goal of carrying out the procedures accurately- something that, thanks to the learning capacity of the human brain, could be achieved <u>without</u> deep [or] conceptual understanding - the focus today is on that conceptual understanding.

That is a very different goal, and quite frankly a much more difficult one to reach.

(Devlin 2017)

A FAVORITE

Learning is not attained by chance.

It must be sought for with ardor
and
attended to with diligence.

~ Abigail Adams ~