Unleash the Mathematician Within

Crafting Rich and Interesting Problems

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CALIFORNIA STATE UNIVERSITY FULLERTON™

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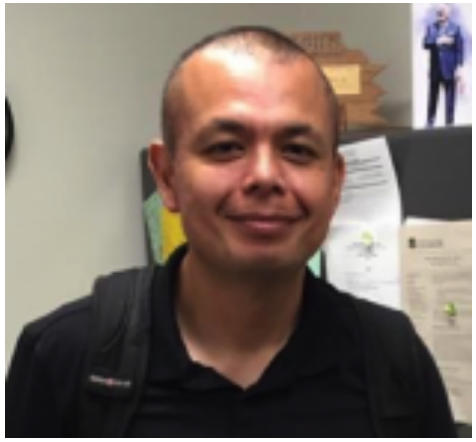
Introduction

Roberto Soto, former high school teacher, district program specialist, now assistant professor of mathematics at California State University, Fullerton



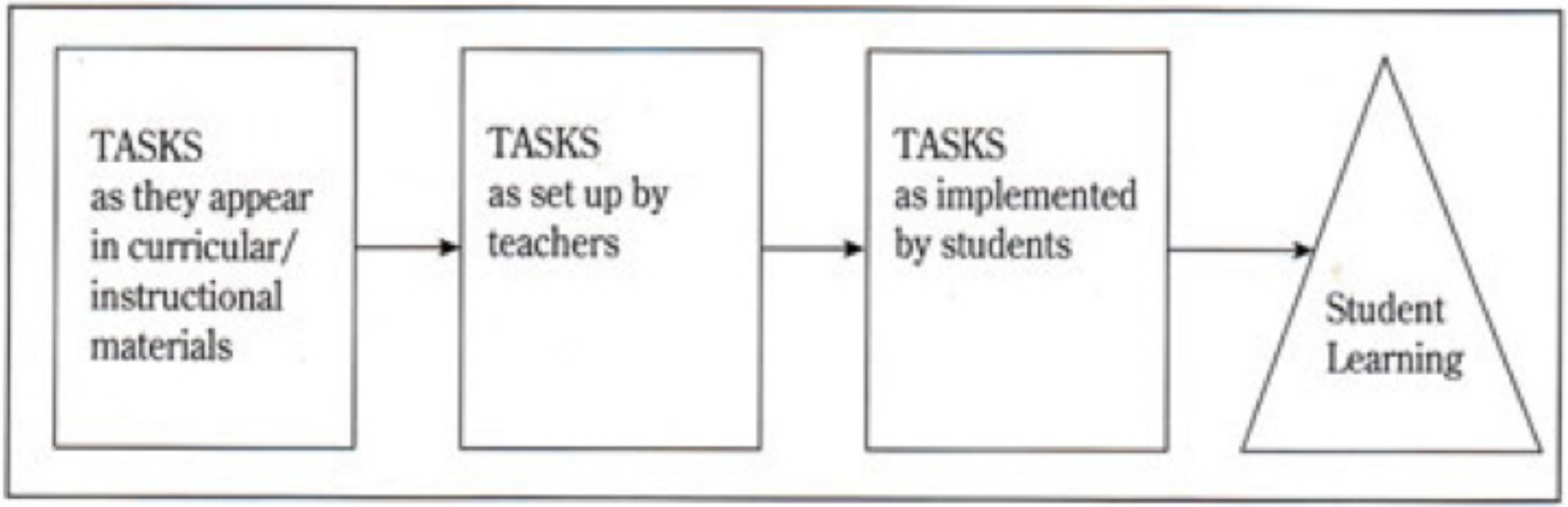
Bridget Druken, assistant professor of mathematics education at California State University, Fullerton, teaches math content courses and supervises single subject teacher candidates

Goals

1. Learn how to **unpack a textbook exercise** and **create tasks that are more open** and **offer a richer problem solving experience**.
2. Better understand how new problems **elicit the CCSS Standards of Mathematical Practice** (SMPs).

"Not all tasks are created **equal**,
and **different tasks**
will provoke
different levels and **kinds of student thinking**."

Stein, Smith, Henningsen, & Silver, 2000



Stein, Smith, Henningsen, & Silver, 2000

What is a problem?

A **problem** is a task where the solver understands the situation and has tools to solve it, yet does not know exactly where to begin process of providing a solution.

- Prior knowledge is important
- Learning trajectory is important (sequence)
- Problem solving motivates needing to know procedures

What is an exercise?

Practicing a skill to maintain a level of proficiency

- Practice doing vs. problem solving

Levels of Cognitive Demand

From the QUASAR Project

Low-Level Cognitive Demand Tasks
– memorization
– procedures without connections to meaning

High-Level Cognitive Demand Tasks
– procedures with connections to meaning
– doing mathematics

Cognitive Demand: type and amount of mental resources required to engage with task

Stein, Smith, Henningsen, & Silver, 2000

Levels of Cognitive Demand

Low-Level Tasks
– memorization
– procedures without connections to meaning

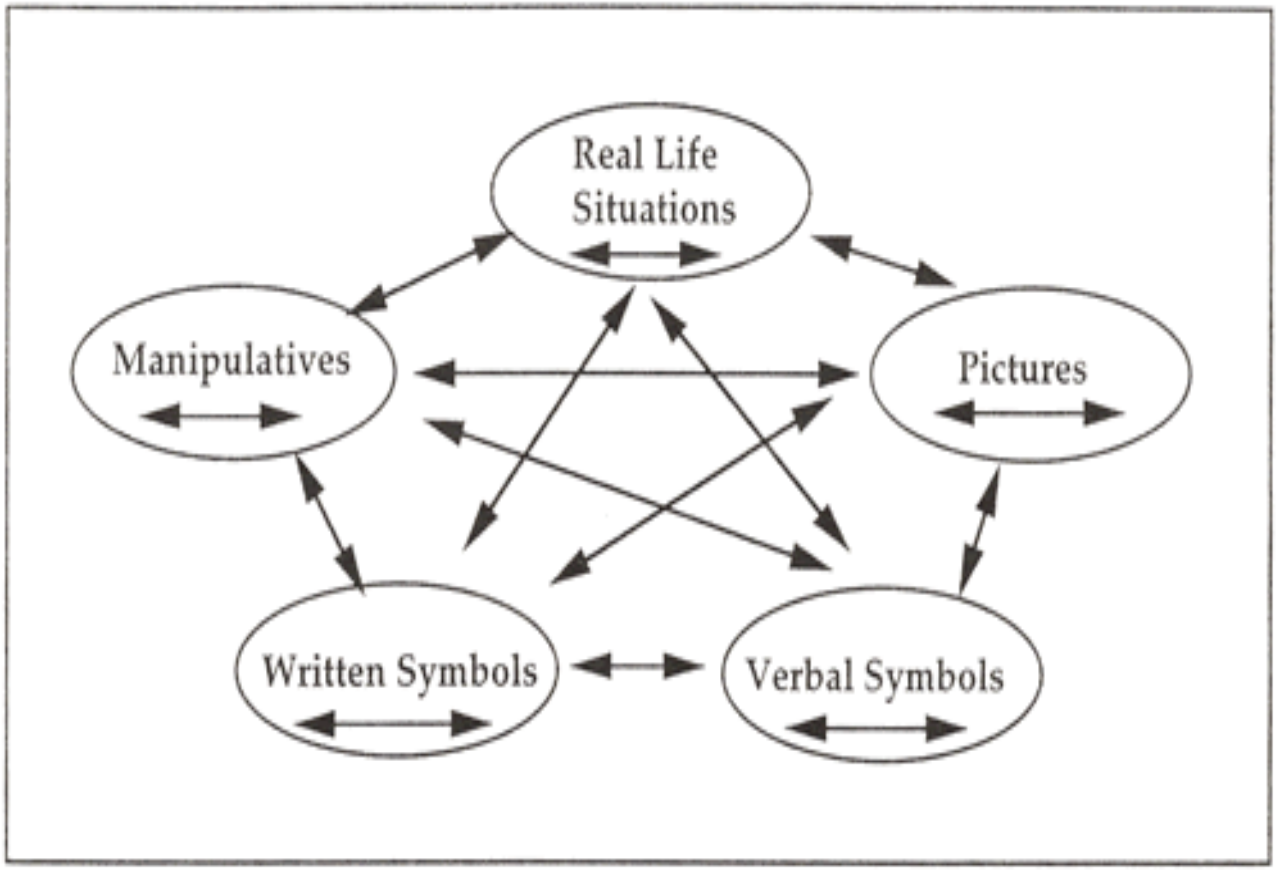
Contributes to decline in cognitive demand:
- sets of procedures without connection to underlying concepts, meaning and understanding;
- an unsystematic exploration;
- a lack of mathematical activity.

High-Level Tasks
– procedures with connections to meaning
– doing mathematics

Supports high-level tasks:
- extent task builds on prior student knowledge;
- scaffolding of the task;
- appropriate time spent solving the task;
- modeling of high-level performance;
- maintained pressure for explanation and meaning.

Stein, Smith, Henningsen, & Silver, 2000

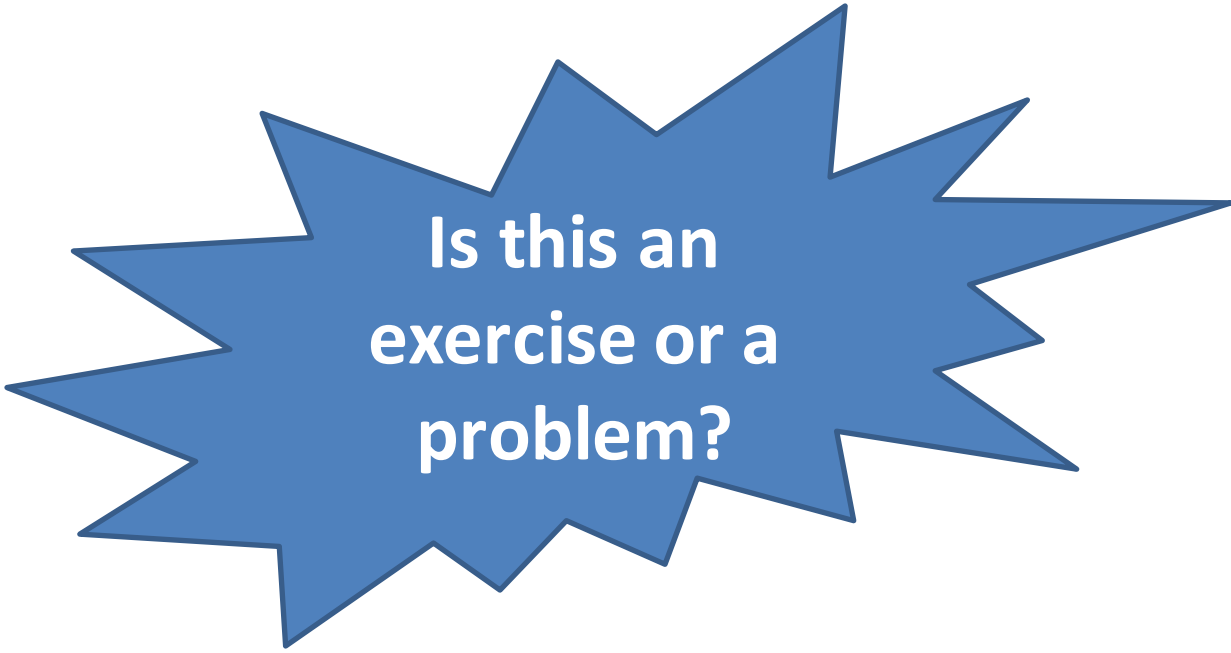
Mathematical Representations



Lesh, Post, & Pehr, 1987

Melissa's Wrapping Paper

Melissa bought wrapping paper. It was 3 foot by 8 foot. What is the area of the paper?



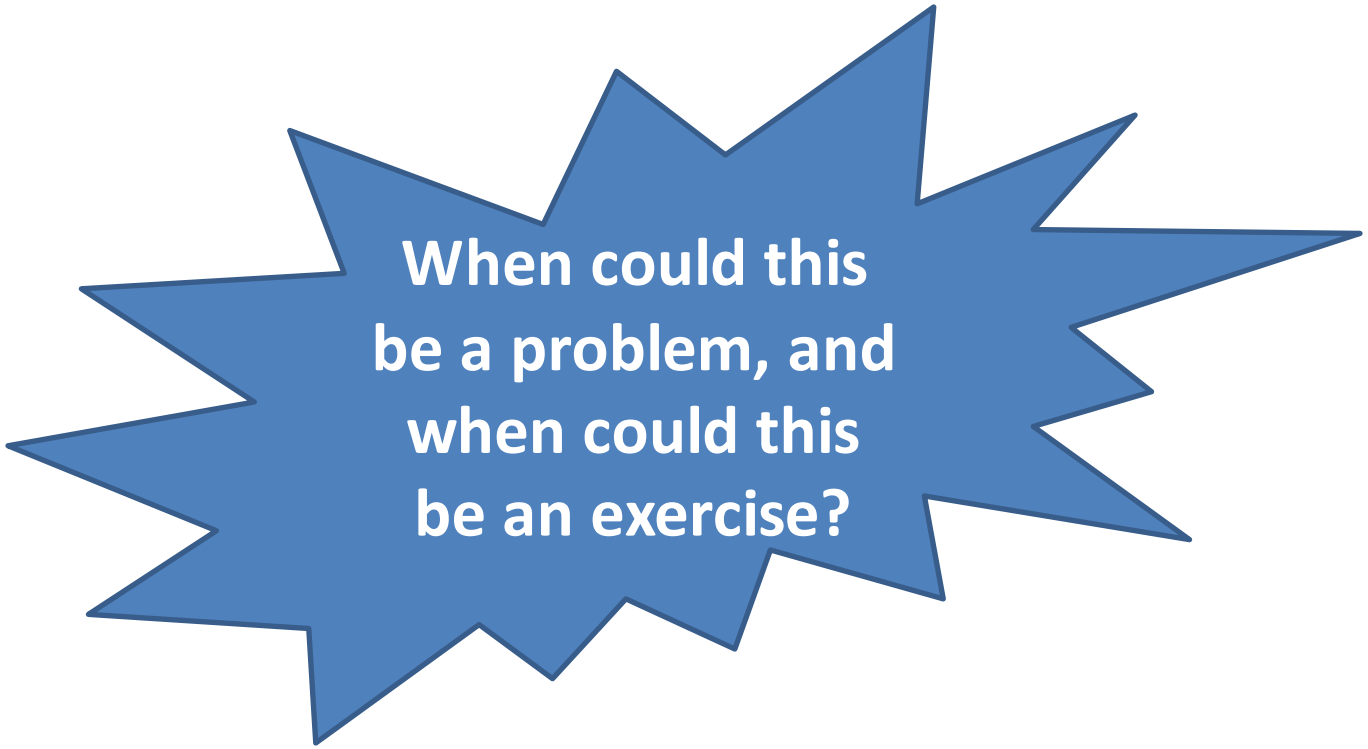
Can an exercise be a problem?

It depends.

If exercise is in the **zone of proximal development** (ZPD) and scaffolded by instructor, then it could be considered a problem for the student.

Melissa's Wrapping Paper

Melissa bought wrapping paper. It was 3 foot by 8 foot. What is the area of the paper?



Melissa's Wrapping Paper 2.0

Melissa bought wrapping paper. It was 3 foot by 8 foot. What is the area of the paper? Solve in more than one way.

Melissa's Wrapping Paper 3.0

Melissa bought wrapping paper. It was 3 foot by 8 foot. ~~What is the area of the paper? Solve in more than one way.~~

Roberto bought wrapping paper, too. It was 6 foot by 4 foot. Who can wrap more objects?

Melissa's Wrapping Paper 4.0

Melissa's wrapping paper is 24 square feet. Identify a possible length and width for her rectangle.

Melissa's Wrapping Project

Melissa needs to wrap these gifts.

How much wrapping paper does she need to buy?

Melissa's Wrapping Project

Melissa needs to wrap these gifts.

How much wrapping paper does she need to buy?

Will students engage in any math practices while solving this task?

Melissa's Wrapping Project

Melissa needs to wrap these gifts.

How much wrapping paper does she need to buy?

MP 1, 2, 3, 4, 6, 8

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Activity: From **Exercise** to **Problem**

On next slide, you will see four different tasks commonly found in textbooks.

Directions:

- In a group of 4, select one task and problematize it. **How could you modify task to create a more open and richer problem solving experience?**
- Create one poster with ideas and possible solution sketches
- After 20 minutes, we will do Gallery Walk to view each other's ideas

From **Exercise** to **Problem**

(3rd grade): Which windows will use the same number of square feet of glass: 6' by 4', 3' by 5', 3' x 7', 3' x 8'?

(4th grade): A rectangle has a perimeter of 36 inches. One side measures 6 inches. What are the measures of the other three sides?

(5th grade): The Sotos' pool measures 10 meters long, 8 meters wide, and 3 meters deep. How many cubic meters of water will the pool hold?

-- or -- Choose an exercise from your own curriculum!

Features of Problems

- start initially unknown
- need to ask some questions before problem solving
- many decision points & multiple steps
- have tools, but solution isn't immediately evident
- explanation required
- Multiple acceptable solutions
- Variety of mathematical tools/representations used (e.g. manipulatives, graphs, tables, diagram)
- Real-world context

Features contd from audience

Comments, Questions, Concerns

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Take aways

1. Think about **unpacking a textbook exercise** and **creating tasks that are more open** and **offer a richer problem solving experience**.
2. Understand that problems **elicit the CCSS Standards of Mathematical Practice** (SMPs).

Resources & References

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education,* 28 (5), 524-549.

Lesh, Richard, Tom Post, and Merlyn Behr. "Representations and Translations among Representations in Mathematics Learning and Problem Solving," In Problems of Representation in the Teaching and Learning of Mathematics, edited by Claude Janvier, pp. 33–40. Hillsdale, NJ: Lawrence Erlbaum Associates, 1987.

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Dan Meyers

http://threeacts.mrmeyer.com/watertank/

Robert Kaplinsky

http://robertkaplinsky.com/work/sink-hole/

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