Adopting New Math Books?

Start by Selecting an Effective Textbook Analysis Toolkit to Inform Your Work!

NCTM
Annual Meeting
& Exhibition

April 2017
San Antonio, TX

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A knowledgeable eye for critical features of instructional materials in order to assess the degree to which a series will support each and every students' learning of college and career-readiness standards.

Mills & Briars, April 2017

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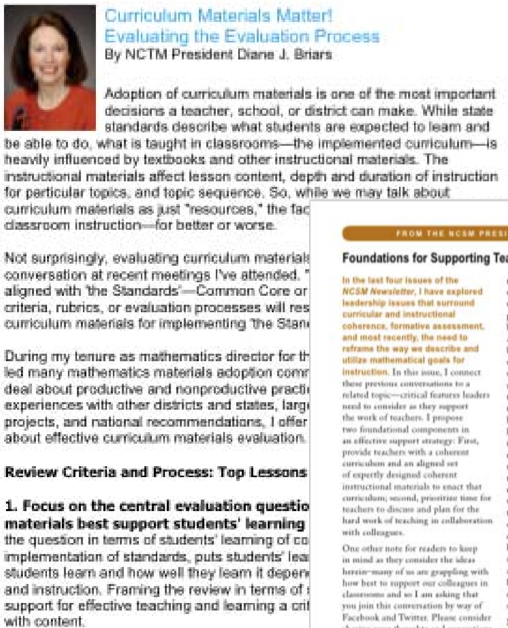
Agenda

- Introduction to the Curriculum Analysis Toolkit
- *Look Fors* - Resources to Support the Content Reviews
- *Look Fors* - Resources to Support the Practices Review

Partner Discussion

Discuss your current thinking about features of CCSSM or your state's college and career readiness standards that need to be reflected in instructional materials and make a few notes.

Presidents' Messages



Curriculum Materials Matter!
Evaluating the Evaluation Process
By NCTM President Diane J. Briars

Adoption of curriculum materials is one of the most important decisions a teacher, school, or district can make. While state standards describe what students are expected to learn and be able to do, what is taught in classrooms—the implemented curriculum—is heavily influenced by textbooks and other instructional materials. The instructional materials affect lesson content, depth and duration of instruction for particular topics, and topic sequence. So, while we may talk about curriculum materials as just "resources," the fac
classroom instruction—for better or worse.

Not surprisingly, evaluating curriculum material
conversation at recent meetings I've attended. "
aligned with 'the Standards'—Common Core or
criteria, rubrics, or evaluation processes will res
curriculum materials for implementing 'the Stan

During my tenure as mathematics director for th
led many mathematics materials adoption comm
deal about productive and nonproductive practi
experiences with other districts and states, larg
projects, and national recommendations, I offer
about effective curriculum materials evaluation.

Review Criteria and Process: Top Lessons

1. Focus on the central evaluation questio
materials best support students' learning
the question in terms of students' learning of co
implementation of standards, puts students' lea
students learn and how well they learn it depen
and instruction. Framing the review in terms of
support for effective teaching and learning a cri
with content.

FROM THE NCSM PRESIDENT—By Valerie L. Mills

Foundations for Supporting Teachers and the Work of Teaching

Valerie L. Mills

In the last four issues of the *NCSM Newsletter*, I have explored leadership issues that surround curricular and instructional coherence, formative assessment, and most recently, the need to reframe the way we describe and utilize mathematical goals for instruction. In this issue, I connect these previous conversations to a related topic—critical features leaders need to consider as they support the work of teachers. I propose two foundational components in an effective support strategy: First, provide teachers with a coherent curriculum and an aligned set of expertly designed coherent instructional materials to enact that curriculum; second, prioritize time for teachers to discuss and plan for the hard work of teaching in collaboration with colleagues.

One other note for readers to keep in mind as they consider the ideas herein–many of us are grappling with how best to support our colleagues in classrooms and so I am asking that you join this conversation by way of Facebook and Twitter. Please consider sharing your thoughts and suggestions for strategies you believe are foundational in supporting teachers so that we can all benefit from our collective wisdom and experiences.

First, provide teachers with a coherent curriculum and an aligned set of expertly designed coherent instructional materials to enact that curriculum.
NCTM published *Curriculum and Evaluation Standards for School Mathematics* in 1989, and by the mid-90s it had prompted the publication of "supplemental books" with rich mathematical tasks that could be used to bring problem solving and discrete mathematics topics into classrooms where they were using traditional textbooks and wanted to more closely align their practice with the new standards. Also by the mid-90s, the new, so-called *Standards-based* textbooks were becoming available. These materials were developed in an entirely new way, as research projects, by teams of university faculty working together to design, pilot, revise, and field test carefully sequenced sets of lessons. These textbooks produced lessons that not only stood the "Is it in the book?" test for a list of required content standards, but far more importantly and for the first time, they helped teachers build mathematical understanding and skills with meticulously structured lessons that worked as a coordinated sequence of challenges. These materials were designed to develop mathematical knowledge and reasoning in far more sophisticated and complex ways than a collated collection of stand alone lessons and their use has now been demonstrated to improve mathematics success for all students.

Much like 25 years ago we find ourselves today in an era of new mathematics standards, and like those times, supplemental materials are widely available to help teachers align their practice to these new standards. Now instead of buying them, you can Google them. They are generally free, and certainly plentiful. Many administrators are looking at their shrinking budgets and once again asking teachers to pull together their own instructional materials using these free resources. The question to be considered, both 25 years ago and today, is: What might you get drawing on these now electronically available lessons in comparison with a research-based, standards-based textbook?

To help answer that question, it is worth reminding ourselves what goes into the development of a *coherent* mathematics textbook series using a research-based approach. These author teams structure lessons to develop a mathematically related constellation of ideas rather than a single discrete skill. The lessons are sequenced beginning with concrete contexts and representations and they move gradually toward greater abstraction and mathematical complexity. This is true for the design of a unit of study, the set of units that compose a textbook, and across a series of textbooks.

Supporting this progression toward greater mathematical sophistication, mathematical representations (drawings, words, tables, graphs, symbols) are intentionally selected and sequenced, lesson and student assignments are composed to encourage the construction of mathematical connections among topics and representations, teacher notes suggest ways to improve the nature of the classroom discourse and planning for possible student misconceptions, and mathematical tools are strategically and appropriately introduced. In addition, great care is given to the tasks in lessons, assignments, and assessments. These tasks are designed to be open enough to provide access to a range of students using a variety of approaches, and scaffolded to support the learning trajectory. They utilize engaging contexts, include an appropriate balance and sequence of items that are cognitively more and less sophisticated, and require students to reason mathematically and to synthesize related concepts

... CONTINUED ON NEXT PAGE

Form a group of three.

- Each person read one:
  A. Briars #1 & #2
  B. Briars #1 & #3 - #5
  C. Mills Section 1: Coherent Curriculum
- As you read your selection, note aspects of CCSSM and instructional materials that are particularly critical for any discussion of textbook selection.
- Share key ideas from your reading with others in your group.

Core Premises

- The central review question is "To what extent do the materials support students' learning of the standards?"
- Final selection must be based on the standards as presented in your state's standards in their entirety.
- Review must focus on the nature and organization of the mathematics learning experiences that students will engage in every day. Therefore, focus of review should be on the content of the student and teacher editions of the materials.
- No materials are perfect. There will be flaws. Important to distinguish between flaws that can easily be corrected and ones that cannot.

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Brookhill Institute of Mathematics

CCSSO Council of Chief State School Officers

Texas Instruments

Analysis Tool Components

Executive Summary
User's Guide
Tool 1: Content Analysis
Tool 2: Mathematical Practices Analysis
Tool 3: Overarching Considerations

- Equity
- Assessment
- Technology

Professional Development Facilitator Guide
PowerPoint Slides

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[Mathedleadership.Org]()

- [CCSS]()
- [Overview]()
  - [*CCCSS Curriculum Analysis Tools*]()

[www.mathedleadership.org/ccss/materials.html]()

Implementing the Common Core State Standards for Mathematics: The CCSS Curriculum Materials Analysis Tools

This webinar provides an overview of Curriculum Materials Analysis Tools recently developed by a committee led by Bill Bush at the University of Louisville. The set of three tools can assist textbook selection committees, school administrators, and K-12 teachers in the selection of curriculum materials that support implementation of the Common Core State Standards in Mathematics. The tools are designed to provide educators with objective measures and information to guide their selection of mathematics curriculum materials based on evidence of the materials' alignment with the CCSSM including the Standards for Mathematical Practice, grade level content, equity, technology, and assessment.

Links:
November 8, 2011 [Webinar Replay]() (length: 56 minutes)
[Webinar Slides]()

COMMON CORESTATE STANDARDS

Related Links

[CCSS]() (Common Core State Standards)
[CCSSO]() (Council of Chief State School Officers)
[Inside Mathematics]()
[Noyce Foundation]()
[Shell Center/MARS]()

CCSS Curriculum Analysis Tool and Professional Development Materials

Led by Bill Bush, University of Louisville, and initiated at the request of Council of Chief State School Officers (CCSSO), this project is developing tools for assessing the potential of curriculum materials to support students' attainment of the CCSS, including the Standards for Mathematical Practice. The tools, and supporting professional development materials, will be disseminated by CCSSO and NCSM.

Links:
[Executive Summary Curriculum Materials Analysis Tools]()
[Professional Development for CCSSM Curriculum Analysis Reviewers]()
[Common Core State Standards (CCSS) Mathematics Curriculum Materials Analysis Project]()

Analysis Tool Components

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Tool 1: Content Analysis

- Determine the extent to which the CCSS are addressed in the materials
- Determine the extent to which CCSS are sequenced appropriately in the materials
- Determine the extent to which the materials provide a balanced treatment of the CCSS in terms of conceptual development and procedural fluency

Tool 1: Content Analysis (K-8)

CCSSM Curriculum Analysis Tool 1— Ratios and Proportional Relationships for Grades 6-8

Name of Reviewer ______________________ **School/District** ____**Date** _______________

Name of Curriculum Materials____ **Publication Date** _____________________**Grade Level(s)** _____________

Content Coverage Rubric (Cont):
Not Found (N) -The mathematics content was not found.
Low (L) - Major gaps in the mathematics content were found.
Marginal (M) - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled.
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Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):
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CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
6.RP Ratios and Proportional Relationships	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.RP Ratios and Proportional Relationships**	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.EE Expressions and Equations**	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Understand ratio concepts and use ratio reasoning to solve problems.				**Analyze proportional relationships and use them to solve real-world and mathematical problems.**				**Understand connections between proportional relationships, lines, and linear equations.**			
1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak."				1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.				5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.			

Tool 1: Content Analysis (HS)

CCSSM Curriculum Analysis Tool 1—Interpreting Functions in Grades 9-12
Name of Reviewer ______________ School/District ______________ Date ______________
Name of Curriculum Materials ______________ Publication Date ______________ Course(s) ______________

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CCSSM Standards Grades 9-12	Chapter pages	Cont N-L-M-A-H	Bal N-L-M-A-H	Notes/Explanation
Interpreting Functions (F-IF)				
Understand the concept of a function and use function notation				
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.				
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.				
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.				
Interpret functions that arise in applications in terms of the context				
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*				

Content Coverage Rubric

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Content Summary Discussion

Standards Alignment:

1. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation?
2. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard?
3. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?

+ Content Summary Discussion

Balance between Mathematical Understanding and Procedural Skills

4. Do the curriculum materials support the development of students ‘mathematical understanding?
5. Do the curriculum materials support the development of students ‘ proficiency with procedural skills?
6. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills?
7. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills?
8. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding & procedural skills?

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Content Summary Discussion

Overall Impressions:

9. What are your overall impressions of the curriculum materials examined?
10. What are the strengths and weaknesses of the materials you examined?

Content Coverage Rubric

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Making Sense of Content Rubrics:

- What does a “gap” in the content look like… is it just skipping some standards? Or is it more?
- What does it mean to develop deep conceptual understanding?
- What does it mean to balance concepts and skills?
- What will it look like to make connections?
- What kinds of tasks will support understanding, balance and connections?

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Tool 1a: Content Coverage Rubric and *Look Fors*

1A. Content Coverage/Treatment Rubric:	Key Evidence and Where to Find It!	Look Fors:
In the rubric below, "gap" refers to IF, WHERE, and HOW content is treated in the materials. **Not Found (N)** - The mathematics content was not found. **Low (L)** - Major gaps in the mathematics content were found. **Marginal (M)** - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled. **Acceptable (A)** - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled. **High (H)** - The content was fully formed as described in the standards	• Base this analysis on lessons as presented in the student and teachers' editions, since these determine students' core instructional experiences. • This analysis addresses IF, WHERE, and HOW content is treated in the materials. Examining whether content is included is insufficient to determine whether students will have the opportunity to learn content as specified in CCSSM. • This analysis must be done not only within grades, but across grades to determine whether the materials adequately address and connect the mathematical ideas as they develop within and across grades, as described in the standards. (The complete the *CCSS Curriculum Materials Analysis Toolkit* contains grade-band analysis sheets for specific CCSS content domains.) • For High School – in addition reviewers will need to explore and understand the author's rationale for distributing content into and cross the three HS courses. Noting particularly *focus* - extensive course level experiences without re-teaching, and *coherence* - building on prior knowledge from within and across courses.	Content development is focused, coherent, and rigorous: 1. **CCSS Content:** CCSS Content Standards for the grade range are thoroughly developed 2. **Focus:** Content present respects the foci and learning progressions built into CCSS grade level standards, so that the content present outside this is limited to: connecting to prior knowledge without re-teaching, and previewing future content without expecting proficiency. 3. **Mathematical Range:** In major topics, lessons pursue conceptual understanding, procedural skill, and fluency, and application 4. **Representations:** Types and range of representations, sequence of representations, and the use of critical representations as identified in the CCSSM 5. **Connections:** Degree to which lessons support students in making connections among related mathematical concepts and algorithms as described in CCSSM. (E.g., Content cluster heads that begin with "Extend and apply . . . .")

Summary Questions—Content Coverage/Treatment

1. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation?
2. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard?
3. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?

+ *Understanding in CCSS*

4.NBT

- Generalize place value ***understanding*** for multi-digit whole numbers.
- Use place value ***understanding*** and properties of operations to perform multi-digit arithmetic.

4.NF

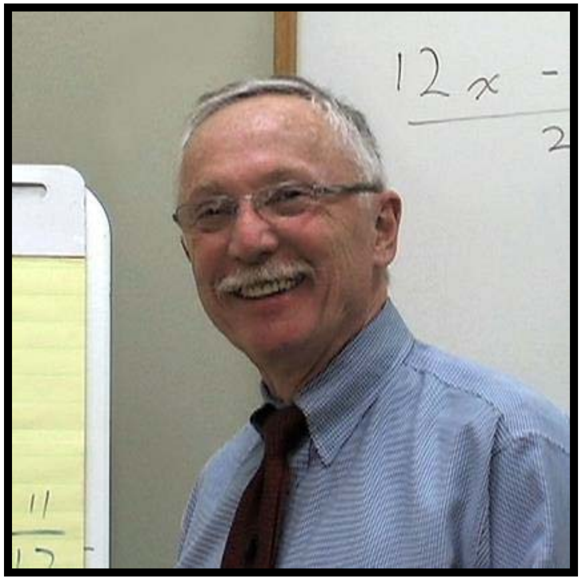
- Extend ***understanding*** of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous ***understandings*** of operations on whole numbers.
- ***Understand*** decimal notation for fractions and compare decimal fractions.

4.MD

- Geometric measurement: ***understand*** concepts of angle and measure angles.

Evaluating *Understanding*

"Understand" is intended to mean that students can explain the concept with mathematical reasoning including concrete illustrations, mathematical representations, and example applications.



Phil Daro, Author CCSS

+ Making Sense of *Understanding*

Students who ***understand*** a concept can:

A. Use it to make sense of and explain quantitative situations (Model with Mathematics)

B. Incorporate it into their own arguments and use it to evaluate the arguments of others (Construct viable arguments and critique the reasoning of others)

C. Bring it to bear on the solutions to problems (Make sense of problems and persevere in solving them)

D. Make connections between it and related concepts

- Phil Daro, CC writing team ppt. NCSM

Tool 1b: Content Balance Rubric and *Look Fors*

1B. Balance of Mathematical Understanding & Procedural Skills Rubric	Key Evidence and Where to Find It!	Look Fors:
Not Found (N) - The content was not found. **Low (L)** - The content was not developed or developed superficially. **Marginal (M)** - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills. **Acceptable (A)** - The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed. **High (H)**-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.	***Conceptual Understanding* –** comprehension of mathematical concepts, operations, and relations. **"*Understand*"** means that students can explain the concept with mathematical reasoning including concrete illustrations, mathematical representations, and example applications. ***Procedural Fluency* –** skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.	1. **Procedures from Concepts:** Activities designed to develop conceptual understanding are leveraged and explicitly connected to the development of related procedures and algorithms 2. **Task Range:** Tasks are designed and sequenced so that students are ask to work across the full range of cognitive demand levels Opportunities for students to: 3. **Model:** Use concepts to make sense of and explain quantitative situations ("Model with mathematics") 4. **Reason:** Incorporate concepts into their own arguments and use them to evaluate the arguments of others (see "Construct viable arguments and critique the reasoning of others") 5. **Problem Solve:** Bring them to bear on the solutions to problems (see "Make sense of problems and persevere in solving them") 6. **Connect:** Make connections between related concepts

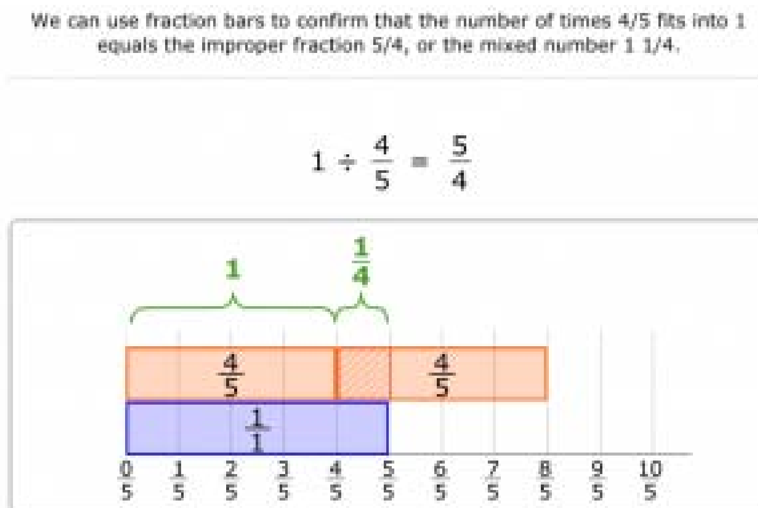
Summary Questions: Balance between Mathematical Understanding and Procedural Skills:

1. Do the curriculum materials support the development of students' mathematical understanding?
2. Do the curriculum materials support the development of students' proficiency with procedural skills?
3. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills?
4. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills?
5. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

+ Illustrations ≟ Understanding

Look closely at the **tasks** in each of the opening and closing lessons then compare and contrast using these questions:

- What do you notice about the development of mathematical understanding in the opening lessons? (Understanding)
- How is the conceptual understanding related/not related to the development of the algorithm in summarizing the lesson? (Connections)
- Locate the places in the lesson where the mathematics is generalized.



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Dividing Fractions - Book 1

Initial Lesson

Investigation 4

Dividing With Fractions

In earlier investigations of this unit, you learned to use addition, subtraction, and multiplication of fractions in a variety of situations. There are times when you also need to divide fractions. To develop ideas about when and how to divide fractions, let's review the meaning of division in problems involving only whole numbers.

Getting Ready for Problem 4.1

Students at Lakeside Middle School raise funds to take a field trip each spring. In each of the following fundraising examples, explain how you recognize what operation(s) to use. Then write a number sentence to show the required calculations.

- The 24 members of the school swim team get dollar-per-mile pledges for a swim marathon they enter. The team goal is to swim 120 miles. How many miles should each swimmer swim?

- There are 360 students going on the field trip. Each school bus carries 30 students. How many buses are needed?
- The school band plans to sell 600 boxes of cookies. There are 20 members in the band. How many boxes should each member sell to reach the goal if each sells the same number of boxes?

Compare your number sentences and reasoning about these problems with classmates. Decide which are correct and why.

4.1 Preparing Food

There are times when the amounts given in a division situation are not whole numbers but fractions. First, you need to understand what division of fractions means. Then you can learn how to calculate quotients when the divisor or the dividend, or both, is a fraction.

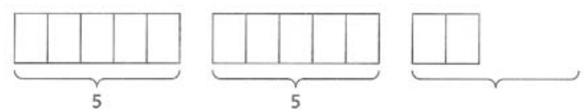
When you do the division $12 \div 5$, what does the answer mean?

The answer should tell you how many fives are in 12 wholes. Because there is not a whole number of fives in 12, you might write:

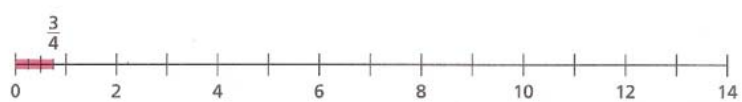
$$12 \div 5 = 2\tfrac{2}{5}$$

Now the question is, what does the *fractional part* of the answer mean?

The answer means you can make 2 fives and $\frac{2}{5}$ *of another five*.



Suppose you ask, "How many $\frac{3}{4}$'s are in 14?" You can write this as a division problem, $14 \div \frac{3}{4}$.



Can you make a whole number of $\frac{3}{4}$'s out of 14 wholes?

If not, what does the fractional part of the answer mean?

As you work through the problems in this investigation, keep these two questions in mind.

What does the answer to a division problem mean?

What does the fractional part of the answer to a division problem mean?

Dividing Fractions - Book 1

Initial Lesson

Problem 4.1 Dividing a Whole Number by a Fraction

Use written explanations or diagrams to show your reasoning for each part. Write a number sentence showing your calculation(s).

A. Naylah plans to make small cheese pizzas to sell at a school fundraiser. She has nine bars of cheese. How many pizzas can she make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{1}{4}$ bar **3.** $\frac{1}{5}$ bar

4. $\frac{1}{6}$ bar **5.** $\frac{1}{7}$ bar **6.** $\frac{1}{8}$ bar

B. Frank also has nine bars of cheese. How many pizzas can he make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{2}{3}$ bar **3.** $\frac{3}{3}$ bar **4.** $\frac{4}{3}$ bar

5. The answer to part (2) is a mixed number. What does the fractional part of the answer mean?

C. Use what you learned from Questions A and B to complete the following calculations.

1. $12 \div \frac{1}{3}$ **2.** $12 \div \frac{2}{3}$ **3.** $12 \div \frac{5}{3}$

4. $12 \div \frac{1}{6}$ **5.** $12 \div \frac{5}{6}$ **6.** $12 \div \frac{7}{6}$

7. The answer to part (3) is a mixed number. What does the fractional part of the answer mean in the context of cheese pizzas?

D. 1. Explain why $8 \div \frac{1}{3} = 24$ and $8 \div \frac{2}{3} = 12$.

2. Why is the answer to $8 \div \frac{2}{3}$ exactly half the answer to $8 \div \frac{1}{3}$?

E. Write an algorithm that seems to make sense for dividing any whole number by any fraction.

F. Write a story problem that can be solved using $12 \div \frac{2}{3}$. Explain why the calculation matches the story.

Dividing Fractions - Book 1

Initial Lesson--Homework

Problem 4.1 Homework

1. The Easy Baking Company makes muffins. Some are small and some are huge. There are 20 cups of flour in the packages of flour they buy. How many muffins can be made from a package of flour if each takes the following amounts of flour?

   a. $\frac{1}{4}$ cup **b.** $\frac{2}{4}$ cup **c.** $\frac{3}{4}$ cup

   d. $\frac{1}{10}$ cup **e.** $\frac{2}{10}$ cup **f.** $\frac{7}{10}$ cup

   g. $\frac{1}{7}$ cup **h.** $\frac{2}{7}$ cup **i.** $\frac{6}{7}$ cup

   j. Explain how the answers for $20 \div \frac{1}{7}$, $20 \div \frac{2}{7}$, and $20 \div \frac{6}{7}$ are related. Show why this makes sense.

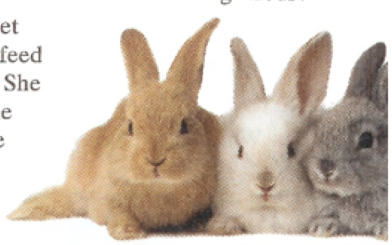
2. Find each quotient.

   a. $6 \div \frac{3}{5}$ **b.** $5 \div \frac{2}{9}$ **c.** $3 \div \frac{1}{4}$ **d.** $4 \div \frac{5}{8}$

3. For parts (a)–(c), do the following steps:
   - Draw pictures or write number sentences to show why your answer is correct.
   - If there is a remainder, tell what the remainder means for the situation.

   a. Bill is making 22 small pizzas for a party. He has 16 cups of flour. Each pizza crust takes $\frac{3}{4}$ cup of flour. Does he have enough flour?

   b. There are 12 baby rabbits at the pet store. The manager lets Gabriella feed vegetables to the rabbits as treats. She has $5\frac{1}{4}$ ounces of parsley today. She wants to give each rabbit the same amount. How much parsley does each rabbit get?



   c. It takes $18\frac{3}{8}$ inches of wood to make a frame for a small snapshot. Ms. Jones has 3 yards of wood. How many frames can she make?

Problem 4.1 Homework, cont.

4. Find each quotient. Describe any patterns that you see.

   a. $5 \div \frac{1}{4}$ **b.** $5 \div \frac{1}{8}$ **c.** $5 \div \frac{1}{16}$

24. Mr. Delgado jogs $2\frac{2}{5}$ km on a trail and then sits down to wait for his friend Mr. Prem. Mr. Prem has jogged $1\frac{1}{2}$ km down the trail. How much farther will Mr. Prem have to jog to reach Mr. Delgado?

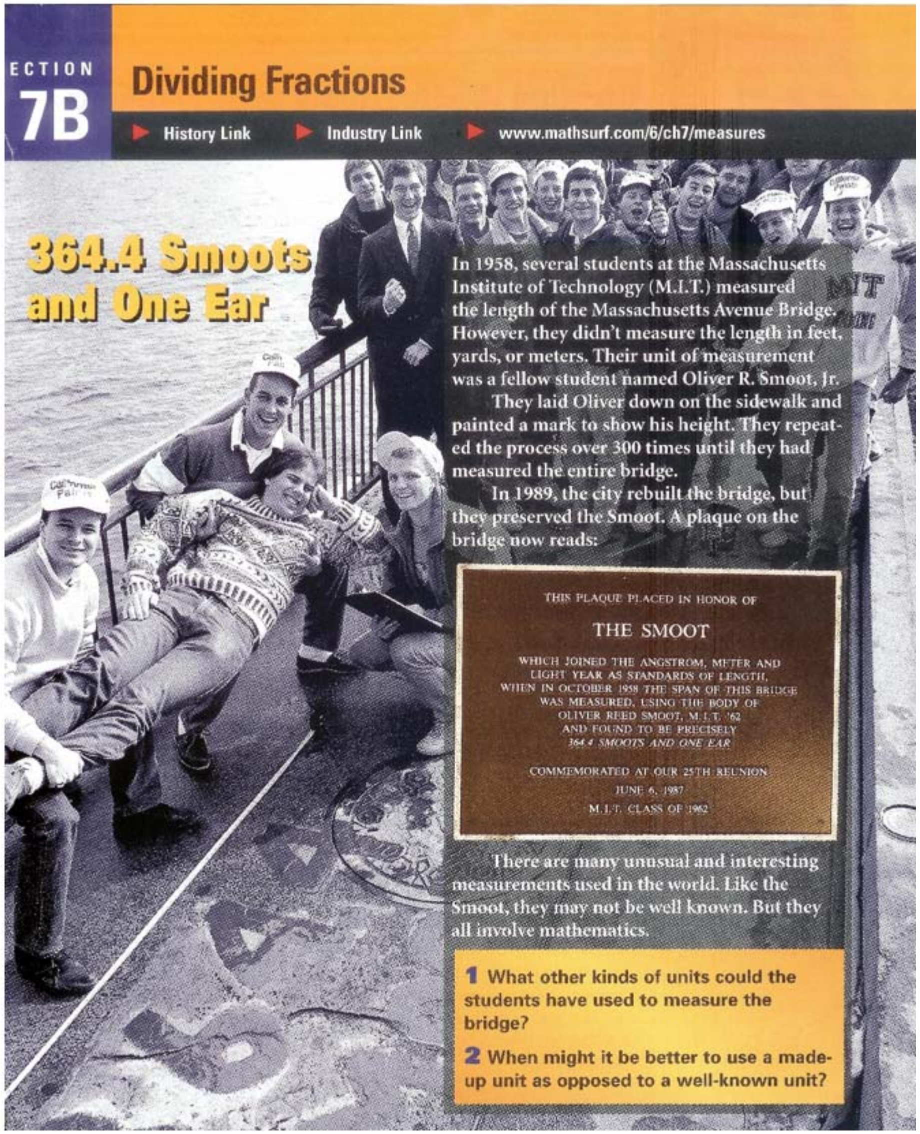
25. Toshi has to work at the car wash for 3 hours. So far, he has worked $1\frac{3}{4}$ hours. How many more hours before he can leave work?

For Exercises 26–29, find each sum or difference. Then, give another fraction that is equivalent to the answer.

26. $\frac{9}{10} + \frac{1}{5}$ **27.** $\frac{5}{6} + \frac{7}{8}$ **28.** $\frac{2}{3} + 1\frac{1}{3}$ **29.** $12\frac{5}{6} - 8\frac{1}{4}$

Dividing Fractions - Book 2

Initial Lesson



ECTION 7B

Dividing Fractions

History Link · Industry Link · www.mathsurf.com/6/ch7/measures

364.4 Smoots and One Ear

In 1958, several students at the Massachusetts Institute of Technology (M.I.T.) measured the length of the Massachusetts Avenue Bridge. However, they didn't measure the length in feet, yards, or meters. Their unit of measurement was a fellow student named Oliver R. Smoot, Jr.

They laid Oliver down on the sidewalk and painted a mark to show his height. They repeated the process over 300 times until they had measured the entire bridge.

In 1989, the city rebuilt the bridge, but they preserved the Smoot. A plaque on the bridge now reads:

THIS PLAQUE PLACED IN HONOR OF

THE SMOOT

WHICH JOINED THE ANGSTROM, METER AND LIGHT YEAR AS STANDARDS OF LENGTH, WHEN IN OCTOBER 1958 THE SPAN OF THIS BRIDGE WAS MEASURED, USING THE BODY OF OLIVER REED SMOOT, M.I.T. '62 AND FOUND TO BE PRECISELY *364.4 SMOOTS AND ONE EAR*

COMMEMORATED AT OUR 25TH REUNION

JUNE 6, 1987

M.I.T. CLASS OF 1962

There are many unusual and interesting measurements used in the world. Like the Smoot, they may not be well known. But they all involve mathematics.

1 What other kinds of units could the students have used to measure the bridge?

2 When might it be better to use a made-up unit as opposed to a well-known unit?

Mills & Briars, April 2017

Dividing Fractions - Book 2

Initial Lesson

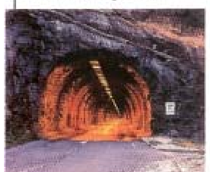
7-4 Dividing Whole Numbers by Fractions

You'll Learn ...

- to divide a whole number by a fraction

... How It's Used

Structural engineers divide whole numbers by fractions when building tunnels.



Vocabulary

reciprocal

▶ Lesson Link In the last section, you learned to multiply whole numbers by fractions. Now you'll divide whole numbers by fractions. ◀

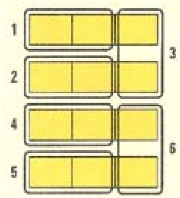
Explore Dividing Whole Numbers by Fractions

Circles and Strips Forever

Dividing a Whole Number by a Fraction

- Draw a number of strips equal to the whole number.
- Divide the strips into equal pieces. The number of pieces in each strip should be equal to the fraction denominator.
- Circle groups of equal pieces. The number of pieces in each circled group should equal the numerator.
- Describe the number of groups circled.

$4 \div \frac{2}{3} = 6$



1. Model these problems.

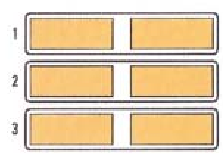
   a. $6 \div \frac{2}{3}$ b. $7 \div \frac{1}{2}$ c. $5 \div \frac{5}{6}$ d. $4 \div \frac{3}{6}$ e. $2 \div \frac{2}{7}$

2. When you divide a whole number by a fraction less than 1, is the quotient larger or smaller than the original whole number? Why?

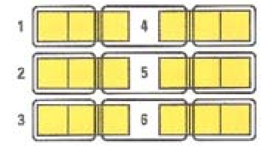
3. Will $3 \div \frac{2}{5}$ have a whole-number answer? Explain.

Learn Dividing Whole Numbers by Fractions

You can think of division as taking a given amount and breaking it down into groups of a certain size. For example, $6 \div 2$ can be modeled as 6 loaves of bread divided into groups of 2. The quotient, 3, is the number of groups you have.



You can think of dividing by fractions in the same way. For example, $6 \div \frac{2}{3}$ is the same as 6 loaves of bread divided into groups of $\frac{2}{3}$. The number of groups you have, 9, is the quotient.



Notice that to find the answer, you first found the number of thirds by multiplying the number of loaves, 6, by the denominator, 3. Then, you divided the number of thirds by the numerator, 2.

$6 \div \frac{2}{3} = 6 \times 3 \div 2 = 9$

Dividing by a fraction is the same as multiplying by its **reciprocal**. Reciprocals are numbers whose numerators and denominators have been switched. When two numbers are reciprocals, their product is 1.

Dividing	Multiplying by reciprocal
$6 \div \frac{2}{3} = 9$	$6 \times \frac{3}{2} = \frac{6}{1} \times \frac{3}{2}$
	$= \frac{18}{2}$
	$= 9$

Remember

The numerator is the number on top of a fraction. The denominator is the number on the bottom. [Page 287]

Examples

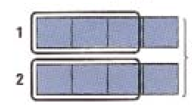
1 Divide: $2 \div \frac{3}{4}$

$2 \div \frac{3}{4} = \frac{2}{1} \times \frac{4}{3}$ Multiply by the reciprocal of the fraction.

$= \frac{2 \times 4}{1 \times 3}$

$= \frac{8}{3}$ or $2\frac{2}{3}$

$2 \div \frac{3}{4} = 2\frac{2}{3}$



2 1 *nail* = $\frac{9}{4}$ in. of cloth. Find the length of 5 in. of cloth in nails.

$5 \div \frac{9}{4} = \frac{5}{1} \times \frac{4}{9}$ Multiply by the reciprocal.

$= \frac{20}{9}$ or $2\frac{2}{9}$ Simplify.

A 5-inch piece of cloth is $2\frac{2}{9}$ nails long.

Try It

Divide. a. $4 \div \frac{3}{5}$ b. $1 \div \frac{4}{7}$ c. $10 \div \frac{17}{4}$ d. $3 \div \frac{3}{5}$

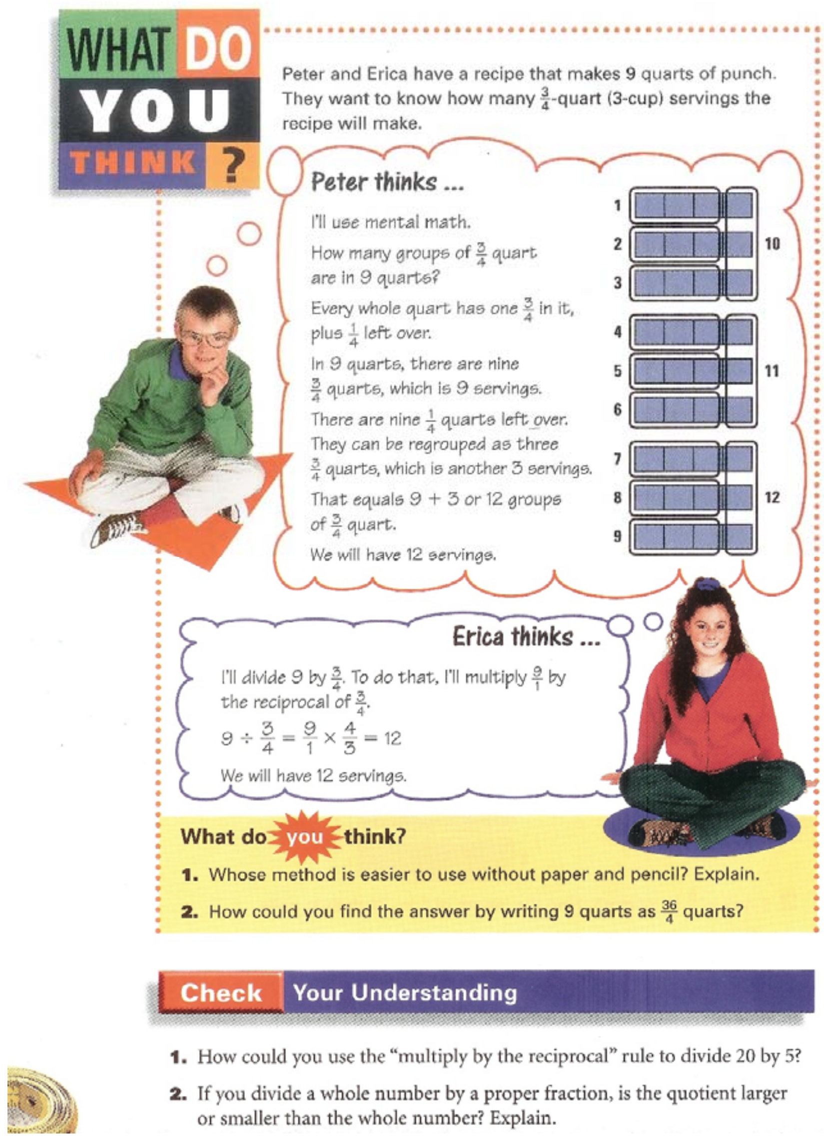
DID YOU KNOW?

Three measurements used primarily for cloth include the *nail*, the *finger*, and the *span*. A *finger* is equal to $4\frac{1}{2}$ inches. A *span* is equal to 9 inches.

Mills & Briars, April 2017

Dividing Fractions - Book 2

Initial Lesson



WHAT DO YOU THINK?

Peter and Erica have a recipe that makes 9 quarts of punch. They want to know how many $\frac{3}{4}$-quart (3-cup) servings the recipe will make.

Peter thinks ...

I'll use mental math.

How many groups of $\frac{3}{4}$ quart are in 9 quarts?

Every whole quart has one $\frac{3}{4}$ in it, plus $\frac{1}{4}$ left over.

In 9 quarts, there are nine $\frac{3}{4}$ quarts, which is 9 servings.

There are nine $\frac{1}{4}$ quarts left over. They can be regrouped as three $\frac{3}{4}$ quarts, which is another 3 servings.

That equals 9 + 3 or 12 groups of $\frac{3}{4}$ quart.

We will have 12 servings.

1 2 3 10 4 5 6 11 7 8 9 12

Erica thinks ...

I'll divide 9 by $\frac{3}{4}$. To do that, I'll multiply $\frac{9}{1}$ by the reciprocal of $\frac{3}{4}$.

$9 \div \frac{3}{4} = \frac{9}{1} \times \frac{4}{3} = 12$

We will have 12 servings.

What do you think?

1. Whose method is easier to use without paper and pencil? Explain.
2. How could you find the answer by writing 9 quarts as $\frac{36}{4}$ quarts?

Check Your Understanding

1. How could you use the "multiply by the reciprocal" rule to divide 20 by 5?
2. If you divide a whole number by a proper fraction, is the quotient larger or smaller than the whole number? Explain.

Dividing Fractions - Book 2

Initial Lesson--Homework

7-4 Exercises and Applications

Practice and Apply

Getting Started State the reciprocal.

1. $\frac{5}{7}$ **2.** $\frac{1}{2}$ **3.** $\frac{2}{9}$ **4.** $\frac{10}{14}$ **5.** $\frac{1}{4}$ **6.** $\frac{4}{5}$

Simplify.

7. $6 \div \frac{2}{3}$ **8.** $2 \div \frac{3}{5}$ **9.** $3 \div \frac{6}{7}$ **10.** $1 \div 1\frac{1}{2}$

11. $9 \div \frac{4}{5}$ **12.** $7 \div \frac{6}{5}$ **13.** $4 \div 3\frac{5}{8}$ **14.** $5 \div \frac{1}{4}$

15. $10 \div 7\frac{2}{3}$ **16.** $8 \div 8\frac{7}{8}$ **17.** $3 \div \frac{10}{11}$ **18.** $5 \div \frac{9}{2}$

19. $16 \div \frac{2}{5}$ **20.** $7 \div 6\frac{3}{4}$ **21.** $8 \div 2\frac{1}{6}$ **22.** $2 \div 4\frac{2}{7}$

23. $1 \div 3\frac{5}{9}$ **24.** $4 \div 1\frac{1}{2}$ **25.** $9 \div \frac{6}{7}$ **26.** $6 \div \frac{8}{12}$

27. $11 \div \frac{13}{2}$ **28.** $10 \div 9\frac{8}{9}$ **29.** $3 \div 11\frac{1}{3}$ **30.** $7 \div 2\frac{3}{8}$

31. **Test Prep** Which two expressions have the same quotient as $6 \div 1\frac{3}{4}$?

I. $\frac{6}{1} \div \frac{7}{4}$ II. $\frac{6}{1} \times \frac{7}{4}$ III. $\frac{6}{1} \div \frac{4}{7}$ IV. $\frac{6}{1} \times \frac{4}{7}$

Ⓐ I and II Ⓑ I and IV

Ⓒ III and II Ⓓ III and IV

32. **Science** $\frac{4}{5}$ of a cubic foot of copper weighs 440 pounds. What is the weight of 1 cubic foot of copper?

33. **Social Studies** As a result of the 1990 census, Pennsylvania has 21 seats in the House of Representatives. This is $\frac{7}{10}$ as many seats as Texas has. How many seats does Texas have?

Problem Solving and Reasoning

34. Journal Explain how you can tell if two numbers are reciprocals of each other.

35. **Critical Thinking** This recipe makes 1 batch of cookies. About how many batches can you make if you change the recipe to include the following? Explain your answers.

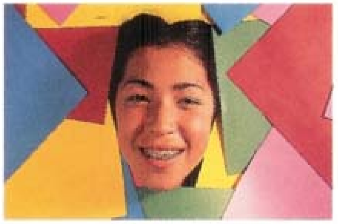
a. A 2-pound bag of flour? (1 cup = $\frac{1}{4}$ pound)

b. A pound of margarine? (1 cup = $\frac{1}{2}$ pound)

c. A 4-pound bag of white sugar? (1 cup = $\frac{1}{2}$ pound)

36. **Communicate** Is $4 \div \frac{2}{5}$ the same as $\frac{2}{5} \div 4$? Explain your reasoning.

37. **Critical Thinking** A *ream* of paper is 500 sheets. A *quire* of paper is $\frac{1}{20}$ of a ream. Monique wanted to know how many sheets of paper were in a quire. She calculated $500 \div \frac{1}{20} = 10{,}000$, and decided a quire of paper was 10,000 sheets. Is her answer reasonable? Explain.



Mixed Review

Convert. *[Lesson 4-3]*

38. 144 ounces = ☐ pounds **39.** 56 pounds = ☐ ounces **40.** 80 ounces = ☐ pounds

41. 100 gallons = ☐ quarts **42.** 64 quarts = ☐ gallons **43.** 40 gallons = ☐ quarts

For each fraction, draw a model. *[Lesson 5-4]*

44. $\frac{1}{4}$ **45.** $\frac{7}{8}$ **46.** $\frac{4}{7}$ **47.** $\frac{80}{100}$ **48.** $\frac{9}{15}$ **49.** $\frac{5}{7}$

Project Progress

Choose 10 of the items from your list. Make a chart detailing how much each item cost when your senior citizen was your age, and how much it costs today. Estimate the fraction or mixed number you would need to multiply the old price by to get the current price.

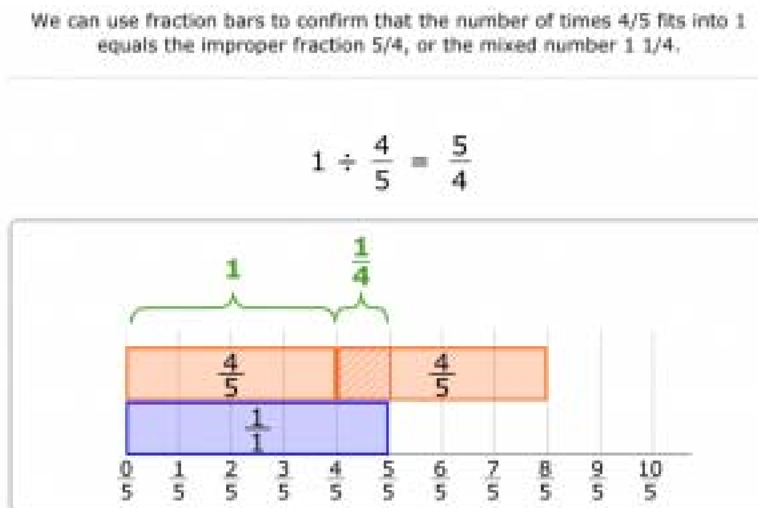
Problem Solving

Understand
Plan
Solve
Look Back

+ Illustrations $\stackrel{?}{=}$ Understanding

Look closely at the **tasks** in each of the opening and closing lessons then compare and contrast using these questions:

- What do you notice about the development of mathematical understanding in the opening lessons? (Understanding)
- How is the conceptual understanding related/not related to the development of the algorithm in summarizing the lesson? (Connections)
- Locate the places in the lesson where the mathematics is generalized.



Mills & Briars, April 2017

Dividing Fractions - Book 1

Initial Lesson

Problem 4.1 Dividing a Whole Number by a Fraction

Use written explanations or diagrams to show your reasoning for each part. Write a number sentence showing your calculation(s).

A. Naylah plans to make small cheese pizzas to sell at a school fundraiser. She has nine bars of cheese. How many pizzas can she make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{1}{4}$ bar **3.** $\frac{1}{5}$ bar

4. $\frac{1}{6}$ bar **5.** $\frac{1}{7}$ bar **6.** $\frac{1}{8}$ bar

B. Frank also has nine bars of cheese. How many pizzas can he make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{2}{3}$ bar **3.** $\frac{3}{3}$ bar **4.** $\frac{4}{3}$ bar

5. The answer to part (2) is a mixed number. What does the fractional part of the answer mean?

C. Use what you learned from Questions A and B to complete the following calculations.

1. $12 \div \frac{1}{3}$ **2.** $12 \div \frac{2}{3}$ **3.** $12 \div \frac{5}{3}$

4. $12 \div \frac{1}{6}$ **5.** $12 \div \frac{5}{6}$ **6.** $12 \div \frac{7}{6}$

7. The answer to part (3) is a mixed number. What does the fractional part of the answer mean in the context of cheese pizzas?

D. 1. Explain why $8 \div \frac{1}{3} = 24$ and $8 \div \frac{2}{3} = 12$.

2. Why is the answer to $8 \div \frac{2}{3}$ exactly half the answer to $8 \div \frac{1}{3}$?

E. Write an algorithm that seems to make sense for dividing any whole number by any fraction.

F. Write a story problem that can be solved using $12 \div \frac{2}{3}$. Explain why the calculation matches the story.

+ Dividing Fractions - Book 2

Initial Lesson

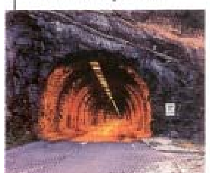
7-4 Dividing Whole Numbers by Fractions

You'll Learn ...

- to divide a whole number by a fraction

... How It's Used

Structural engineers divide whole numbers by fractions when building tunnels.



Vocabulary

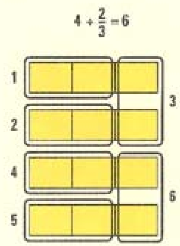
reciprocal

▶ Lesson Link In the last section, you learned to multiply whole numbers by fractions. Now you'll divide whole numbers by fractions. ◀

Explore — Dividing Whole Numbers by Fractions

Circles and Strips Forever

Dividing a Whole Number by a Fraction



- Draw a number of strips equal to the whole number.
- Divide the strips into equal pieces. The number of pieces in each strip should be equal to the fraction denominator.
- Circle groups of equal pieces. The number of pieces in each circled group should equal the numerator.
- Describe the number of groups circled.

1. Model these problems.

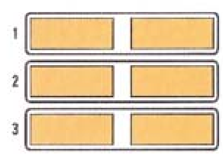
a. $6 \div \frac{2}{3}$ **b.** $7 \div \frac{1}{2}$ **c.** $5 \div \frac{5}{6}$ **d.** $4 \div \frac{3}{6}$ **e.** $2 \div \frac{2}{7}$

2. When you divide a whole number by a fraction less than 1, is the quotient larger or smaller than the original whole number? Why?

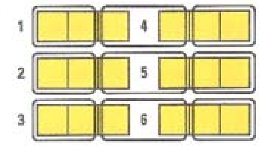
3. Will $3 \div \frac{2}{5}$ have a whole-number answer? Explain.

Learn — Dividing Whole Numbers by Fractions

You can think of division as taking a given amount and breaking it down into groups of a certain size. For example, $6 \div 2$ can be modeled as 6 loaves of bread divided into groups of 2. The quotient, 3, is the number of groups you have.



You can think of dividing by fractions in the same way. For example, $6 \div \frac{2}{3}$ is the same as 6 loaves of bread divided into groups of $\frac{2}{3}$. The number of groups you have, 9, is the quotient.



Notice that to find the answer, you first found the number of thirds by multiplying the number of loaves, 6, by the denominator, 3. Then, you divided the number of thirds by the numerator, 2.

$6 \div \frac{2}{3} = 6 \times 3 \div 2 = 9$

Dividing by a fraction is the same as multiplying by its **reciprocal**. Reciprocals are numbers whose numerators and denominators have been switched. When two numbers are reciprocals, their product is 1.

Dividing	Multiplying by reciprocal
$6 \div \frac{2}{3} = 9$	$6 \times \frac{3}{2} = \frac{6}{1} \times \frac{3}{2}$
	$= \frac{18}{2}$
	$= 9$

Remember

The numerator is the number on top of a fraction. The denominator is the number on the bottom. [Page 287]

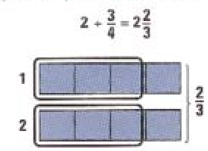
Examples

1 Divide: $2 \div \frac{3}{4}$

$2 \div \frac{3}{4} = \frac{2}{1} \times \frac{4}{3}$ Multiply by the reciprocal of the fraction.

$= \frac{2 \times 4}{1 \times 3}$

$= \frac{8}{3}$ or $2\frac{2}{3}$



2 1 *nail* = $\frac{9}{4}$ in. of cloth. Find the length of 5 in. of cloth in nails.

$5 \div \frac{9}{4} = \frac{5}{1} \times \frac{4}{9}$ Multiply by the reciprocal.

$= \frac{20}{9}$ or $2\frac{2}{9}$ Simplify.

A 5-inch piece of cloth is $2\frac{2}{9}$ nails long.

Try It

Divide. **a.** $4 \div \frac{3}{5}$ **b.** $1 \div \frac{4}{7}$ **c.** $10 \div \frac{17}{4}$ **d.** $3 \div \frac{3}{5}$

DID YOU KNOW?

Three measurements used primarily for cloth include the *nail*, the *finger*, and the *span*. A *finger* is equal to $4\frac{1}{2}$ inches. A *span* is equal to 9 inches.

+ Dividing Fractions - Standard Algorithm

Book 1

4.4 Writing a Division Algorithm

You are ready now to develop an algorithm for dividing fractions. To get started, you will break division problems into categories and write steps for each kind of problem. Then you can see whether there is one "big" algorithm that will solve them all.

A. 1. Find the quotients in each group below.

Group 1	Group 2	Group 3	Group 4
$\frac{1}{3} \div 9$	$12 \div \frac{1}{6}$	$\frac{5}{6} \div \frac{1}{12}$	$5 \div 1\frac{1}{2}$
$\frac{1}{6} \div 12$	$5 \div \frac{2}{3}$	$\frac{3}{4} \div \frac{3}{4}$	$\frac{1}{2} \div 3\frac{2}{3}$
$\frac{3}{5} \div 6$	$3 \div \frac{2}{5}$	$\frac{9}{5} \div \frac{1}{2}$	$3\frac{1}{3} \div \frac{2}{3}$

2. Describe what the problems in each group have in common.

3. Make up one new problem that fits in each group.

4. Write an algorithm that works for dividing *any* two fractions, including mixed numbers. Test your algorithm on the problems in the table. If necessary, change your algorithm until you think it will work all the time.

B. Use your algorithm to divide.

1. $9 \div \frac{4}{5}$ **2.** $1\frac{7}{8} \div 3$ **3.** $1\frac{2}{3} \div \frac{1}{5}$ **4.** $2\frac{5}{6} \div 1\frac{1}{3}$

C. Here is a multiplication-division fact family for whole numbers:

$5 \times 8 = 40$ $8 \times 5 = 40$ $40 \div 5 = 8$ $40 \div 8 = 5$

1. Complete this multiplication-division fact family for fractions.

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

2. Check the division answers by using your algorithm.

D. For each number sentence, find a value for N that makes the sentence true. If needed, use fact families.

1. $\frac{2}{3} \div \frac{4}{5} = N$ **2.** $\frac{3}{4} \div N = \frac{7}{8}$ **3.** $N \div \frac{1}{4} = 3$

Book 2

Dividing Fractions by Fractions

▶ Lesson Link In the last lesson, you learned to divide whole numbers by fractions. Now you'll divide fractions by fractions. ◀

Explore Dividing Fractions by Fractions

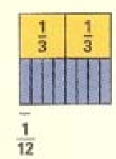
Wish Upon a Bar

Materials: Fraction Bars®

Dividing a Fraction by a Fraction

- Using a Fraction Bar®, draw and label the first fraction.
- Under that, use a Fraction Bar® to draw as many diagrams of the second fraction as will fit.
- Describe the number of diagrams below the first fraction.

$\frac{2}{3} \div \frac{1}{12} = 8$



1. Model each problem.

a. $\frac{3}{6} \div \frac{1}{12}$ **b.** $\frac{1}{2} \div \frac{1}{4}$ **c.** $\frac{2}{3} \div \frac{1}{6}$ **d.** $\frac{2}{4} \div \frac{2}{12}$

2. When you divide a fraction by a fraction less than 1, why is the answer bigger than the fraction you started with?

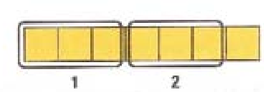
3. How is dividing a fraction by a fraction similar to dividing a whole number by a fraction?

4. Can you use Fraction Bars® to divide $\frac{1}{2} \div \frac{1}{5}$? Explain.

Learn Dividing Fractions by Fractions

When you divide a whole number by a fraction, you get the same result as if you had multiplied the whole number by the fraction's reciprocal. This is also true when you divide a fraction by a fraction.

Dividing	Multiplying by Reciprocal
$\frac{6}{7} \div \frac{3}{7} = 2$	$\frac{6}{7} \times \frac{7}{3} = \frac{42}{21}$ or 2



Mills & Briars, April 2017

Tool 1b: Content Balance Rubric and *Look Fors*

1B. Balance of Mathematical Understanding & Procedural Skills Rubric	Key Evidence and Where to Find It!	Look Fors:
Not Found (N) - The content was not found. **Low (L)** - The content was not developed or developed superficially. **Marginal (M)** - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills. **Acceptable (A)** - The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed. **High (H)**-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.	***Conceptual Understanding* –** comprehension of mathematical concepts, operations, and relations. "***Understand"*** means that students can explain the concept with mathematical reasoning including concrete illustrations, mathematical representations, and example applications. ***Procedural Fluency* –** skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.	1. **Procedures from Concepts:** Activities designed to develop conceptual understanding are leveraged and explicitly connected to the development of related procedures and algorithms 2. **Task Range:** Tasks are designed and sequenced so that students are ask to work across the full range of cognitive demand levels Opportunities for students to: 3. **Model:** Use concepts to make sense of and explain quantitative situations ("Model with mathematics") 4. **Reason:** Incorporate concepts into their own arguments and use them to evaluate the arguments of others (see "Construct viable arguments and critique the reasoning of others") 5. **Problem Solve:** Bring them to bear on the solutions to problems (see "Make sense of problems and persevere in solving them") 6. **Connect:** Make connections between related concepts

Summary Questions: Balance between Mathematical Understanding and Procedural Skills:

1. Do the curriculum materials support the development of students' mathematical understanding?
2. Do the curriculum materials support the development of students' proficiency with procedural skills?
3. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills?
4. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills?
5. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

Tool 1a: Content Coverage Rubric and *Look Fors*

1A. Content Coverage/Treatment Rubric:	Key Evidence and Where to Find It!	Look Fors:
In the rubric below, "gap" refers to IF, WHERE, and HOW content is treated in the materials. **Not Found (N)** - The mathematics content was not found. **Low (L)** - Major gaps in the mathematics content were found. **Marginal (M)** - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled. **Acceptable (A)** - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled. **High (H)** - The content was fully formed as described in the standards	• Base this analysis on lessons as presented in the student and teachers' editions, since these determine students' core instructional experiences. • This analysis addresses IF, WHERE, and HOW content is treated in the materials. Examining whether content is included is insufficient to determine whether students will have the opportunity to learn content as specified in CCSSM. • This analysis must be done not only within grades, but across grades to determine whether the materials adequately address and connect the mathematical ideas as they develop within and across grades, as described in the standards. (The complete the *CCSS Curriculum Materials Analysis Toolkit* contains grade-band analysis sheets for specific CCSS content domains.) • For High School – in addition reviewers will need to explore and understand the author's rationale for distributing content into and cross the three HS courses. Noting particularly *focus* - extensive course level experiences without re-teaching, and *coherence* - building on prior knowledge from within and across courses.	Content development is focused, coherent, and rigorous: 1. **CCSS Content:** CCSS Content Standards for the grade range are thoroughly developed 2. **Focus:** Content present respects the foci and learning progressions built into CCSS grade level standards, so that the content present outside this is limited to: connecting to prior knowledge without re-teaching, and previewing future content without expecting proficiency. 3. **Mathematical Range:** In major topics, lessons pursue conceptual understanding, procedural skill, and fluency, and application 4. **Representations:** Types and range of representations, sequence of representations, and the use of critical representations as identified in the CCSSM 5. **Connections:** Degree to which lessons support students in making connections among related mathematical concepts and algorithms as described in CCSSM. (E.g., Content cluster heads that begin with "Extend and apply . . . . ")

Summary Questions—Content Coverage/Treatment

1. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation?
2. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard?
3. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?

Standards for Mathematical Practice

"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important 'processes and proficiencies' with longstanding importance in mathematics education."

(CCSS, 2010)

Standards for Mathematical Practice

Mathematical facts and procedures, the Content part of what we teach, are the results of the application of mathematical habits of mind reflected in the Practices. ***For that reason, fidelity to the way mathematics is made and used, a big part of the intent of the Mathematical Practices, requires that the Content be taught through the Practices.*** *That way, the connections are real, integrated rather than interspersed.*

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Mills & Briars, April 2017

http://thinkmath.edc.org/index.php/Differences_between%2C_and_connections_between%2C_Content_and_Practice_standards

Tool 2: Standards for Mathematical Practice

CCSSM Mathematical Practices Analysis Tool 2	Page 1

Name of Reviewer ______________________ School/District ______________________ Date ________

Name of Curriculum Materials ______________________ Publication Date ________ Grade Level(s) ________

Tool 1 Domain Considered ______________________

Opportunities to Engage in the Standards for Mathematical Practices
Found Across the Content Standards

Overarching Habits of Mind	1. Make sense of problems and persevere in solving them.	6. Attend to precision.
Evidence of how the Standards for Mathematics Practice were addressed (with page numbers)		
Reasoning and Explaining	**2. Reason abstractly and quantitatively.**	**3. Construct viable arguments and critique the reasoning of others.**
Evidence of how the Standards for Mathematics Practice were addressed (with page numbers)		

Synthesis of Standards for Mathematical Practice	Page 3
(Mathematical Practices → Content) To what extent do the materials demand that students engage in the Standards for Mathematical Practice as the primary vehicle for learning the Content Standards?	
(Content → Mathematical Practices) To what extent do the materials provide opportunities for students to develop the Standards for Mathematical Practice as "habits of mind" (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards?	
To what extent do accompanying assessments of student learning (such as homework, observation checklists, portfolio recommendations, extended tasks, tests, and quizzes) provide evidence regarding students' proficiency with respect to the Standards for Mathematical Practice?	
What is the quality of the instructional support for students' development of the Standards for Mathematical Practice as habits of mind?	

Summative Assessment	Explanation for score
(Low) – The Standards for Mathematical Practice are not addressed or are addressed superficially. (Marginal) The Standards for Mathematical Practice are addressed, but not consistently in a way that is embedded in the development of the Content Standards. (Acceptable) – Attention to the Standards for Mathematical Practice is embedded throughout the curriculum materials in ways that may help students to develop them as habits of mind.	

Mathematical Practices Analysis Tool Rubric

Mathematical Practices → Content

- To what extent do the materials demand that students engage in the Standards for Mathematical Practice as the primary vehicle for learning the Content Standards?

Content → Mathematical Practices

- To what extent do the materials provide opportunities for students to develop the Standards for Mathematical Practice as "habits of mind" (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards?

+ Mathematical Practices Analysis Tool Rubric

Assessment of SMP

- To what extent do accompanying assessments of student learning (such as homework, observation checklists, portfolio recommendations, extended tasks, tests, and quizzes) provide evidence regarding students' proficiency with respect to the Standards for Mathematical Practice?

Teacher Support

- What is the quality of the instructional support for students' development of the Standards for Mathematical Practice as habits of mind?

Tool 2: Standards for Mathematical Practice

2. The Practices	Key Evidence and Where to Find It!	Look Fors:
Low – The Standards for Mathematical Practice are not addressed or are addressed superficially. **Marginal** - The Standards for Mathematical Practice are addressed, but not consistently in a way that is embedded in the development of the Content Standards. **Acceptable** – Attention to the Standards for Mathematical Practice is embedded throughout the curriculum materials in ways that may help students to develop them as habits of mind.	Content standards that explicitly refer to "understand" or "understanding" are especially good opportunities to connect the practices to the content. (CCSS, p. 8) ***Instructional Tasks:*** Examine the extent to which lessons consistently are built around tasks that promote problem solving, reasoning, and engagement in standards for mathematical practice. SMPs should be treated in two ways: 1. Students should engage in the SMPs as they work on tasks to learn specific content; and 2. Developing proficiency in the SMPs should be the explicit goal of some lessons. Occasional opportunities—once a week; a few times a chapter—for students to engage in the SMPs are not sufficient. Explicitly labeling lessons or tasks with particular mathematical practices ("call-outs") is irrelevant. ***Formative Assessment:*** Formal and informal assessments should provide evidence about students' proficiency with the SMPs as well as the content standards. ***Resources:*** • The "Elaborations" on the Standards for Mathematical Practice for Grades K-5 and Grades 6-8 (Illustrative Mathematics) provide additional interpretation of the SMPs for these grade levels. — Grades K-5: http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards/ — Grades 6-8: ommoncoretools.me/2014/05/04/6-8-elaborations-of-the-practice-standards/ • "Model" and "modeling" are used in a variety of ways in mathematics education. See the NCTM-SIAM Committee on Modeling Across the Curriculum's "How to Identify Tasks that Engage Students in Mathematical Modeling" for clarification of SMP 4. Modeling with mathematics	Opportunities for students to: 1. **Mathematical Practices → Content:** To what extent do the materials demand that students engage in the Standards for Mathematical Practice as the primary vehicle for learning the Content Standards? 2. **Content → Mathematical Practices:** To what extent do the materials provide opportunities for students to develop the Standards for Mathematical Practice as "habits of mind" (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards? 3. **Opportunities to Elicit Evidence of Student Thinking:** To what extent do accompanying assessments of student learning (such as homework, observation checklists, portfolio recommendations, extended tasks, tests, and quizzes) provide evidence regarding students' proficiency with respect to the Standards for Mathematical Practice? 4. **Teacher Support:** What is the quality of the instructional support for students' development of the Standards for Mathematical Practice as habits of mind?

Research on Teaching and Tasks
Principles to Action

Research on the use of mathematical tasks over the last two decades has yielded three major findings:

1. Not all tasks provide the same opportunities for student thinking and learning. (Hiebert et al. 1997; Stein et al. 2009)
2. Student learning is *greatest* in classrooms where the tasks consistently encourage high-level student thinking and reasoning and *least* in classrooms where the tasks are routinely procedural in nature. (Boaler and Staples 2008; Hiebert and Wearne 1993; Stein and Lane 1996)
3. Tasks with high cognitive demands are the most difficult to implement well and are often transformed into less demanding tasks during instruction. (Stein, Grover, and Henningsen 1996; Stigler and Hiebert 2004)

Implement Tasks that Promote Reasoning and Problem Solving

Mathematical tasks should:

- Provide opportunities for students to engage in exploration or encourage students to use procedures in ways that are connected to concepts and understanding;
- Build on students' current understanding; and
- Have multiple entry points.

Task Analysis Guide (pp 7-8)

<table>
<tr><th>Lower-Level Demands</th><th>Higher-Level Demands</th></tr>
<tr><td><u>Memorization</u>

• involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory.

• cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.

• are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated.

• have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.</td><td><u>Procedures With Connections</u>

• focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.

• suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.

• usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.

• require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.</td></tr>
<tr><td><u>Procedures Without Connections</u>

• are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or` placement of the task.

• require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.

• have no connection to the concepts or meaning that underlie the procedure being used.

• are focused on producing correct answers rather than developing mathematical understanding.

• require no explanations or explanations that focuses solely on describing the procedure that was used.</td><td><u>Doing Mathematics</u>

• require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).

• require students to explore and understand the nature of mathematical concepts, processes, or relationships.

• demand self-monitoring or self-regulation of one's own cognitive processes.

• require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

• require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.

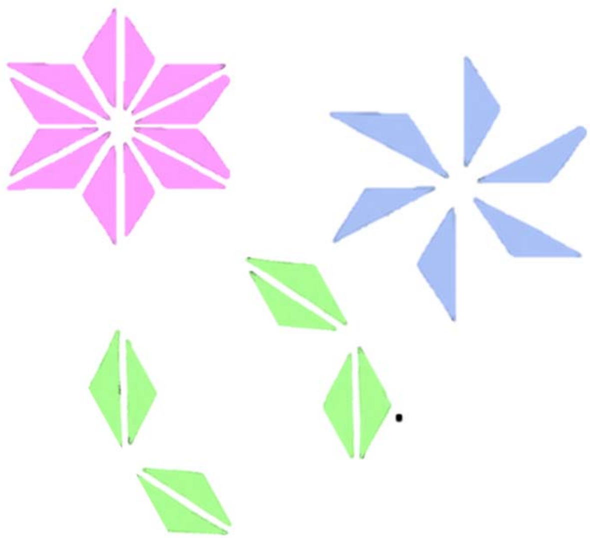
• require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.</td></tr>
</table>

+ Connecting and Exploring: SMPs, Task Demand, and Content Development

For Lesson 1 in the two sample lessons:

1. Agree on the Level of Demand citing one or more of the bullet characteristics.
2. Agree on which, if any, of the SMPs (including specific bullets) students would likely use while working on the task.
3. Agree on a brief description of the mathematical content students would have the opportunity to learn working on the task.

Keep your eye open for patterns and relationships among task demand, SMP and content development.



Mills & Briars, April 2017

Dividing Fractions - Book 1

Initial Lesson

Problem 4.1 Dividing a Whole Number by a Fraction

Use written explanations or diagrams to show your reasoning for each part. Write a number sentence showing your calculation(s).

A. Naylah plans to make small cheese pizzas to sell at a school fundraiser. She has nine bars of cheese. How many pizzas can she make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{1}{4}$ bar **3.** $\frac{1}{5}$ bar

4. $\frac{1}{6}$ bar **5.** $\frac{1}{7}$ bar **6.** $\frac{1}{8}$ bar

B. Frank also has nine bars of cheese. How many pizzas can he make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{2}{3}$ bar **3.** $\frac{3}{3}$ bar **4.** $\frac{4}{3}$ bar

5. The answer to part (2) is a mixed number. What does the fractional part of the answer mean?

C. Use what you learned from Questions A and B to complete the following calculations.

1. $12 \div \frac{1}{3}$ **2.** $12 \div \frac{2}{3}$ **3.** $12 \div \frac{5}{3}$

4. $12 \div \frac{1}{6}$ **5.** $12 \div \frac{5}{6}$ **6.** $12 \div \frac{7}{6}$

7. The answer to part (3) is a mixed number. What does the fractional part of the answer mean in the context of cheese pizzas?

D. **1.** Explain why $8 \div \frac{1}{3} = 24$ and $8 \div \frac{2}{3} = 12$.

2. Why is the answer to $8 \div \frac{2}{3}$ exactly half the answer to $8 \div \frac{1}{3}$?

E. Write an algorithm that seems to make sense for dividing any whole number by any fraction.

F. Write a story problem that can be solved using $12 \div \frac{2}{3}$. Explain why the calculation matches the story.

Dividing Fractions - Book 2

Initial Lesson

7-4

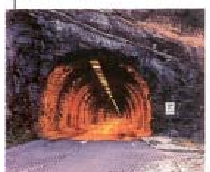
Dividing Whole Numbers by Fractions

You'll Learn ...

- to divide a whole number by a fraction

... How It's Used

Structural engineers divide whole numbers by fractions when building tunnels.



Vocabulary

reciprocal

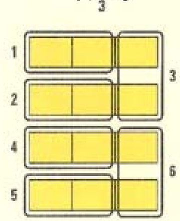
Lesson Link In the last section, you learned to multiply whole numbers by fractions. Now you'll divide whole numbers by fractions.

Explore Dividing Whole Numbers by Fractions

Circles and Strips Forever

Dividing a Whole Number by a Fraction

- Draw a number of strips equal to the whole number.
- Divide the strips into equal pieces. The number of pieces in each strip should be equal to the fraction denominator.
- Circle groups of equal pieces. The number of pieces in each circled group should equal the numerator.
- Describe the number of groups circled.



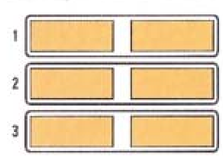
1. Model these problems.

   a. $6 \div \frac{2}{3}$ **b.** $7 \div \frac{1}{2}$ **c.** $5 \div \frac{5}{6}$ **d.** $4 \div \frac{3}{6}$ **e.** $2 \div \frac{2}{7}$

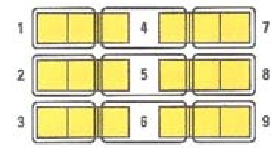
2. When you divide a whole number by a fraction less than 1, is the quotient larger or smaller than the original whole number? Why?
3. Will $3 \div \frac{2}{5}$ have a whole-number answer? Explain.

Learn Dividing Whole Numbers by Fractions

You can think of division as taking a given amount and breaking it down into groups of a certain size. For example, $6 \div 2$ can be modeled as 6 loaves of bread divided into groups of 2. The quotient, 3, is the number of groups you have.



You can think of dividing by fractions in the same way. For example, $6 \div \frac{2}{3}$ is the same as 6 loaves of bread divided into groups of $\frac{2}{3}$. The number of groups you have, 9, is the quotient.



Notice that to find the answer, you first found the number of thirds by multiplying the number of loaves, 6, by the denominator, 3. Then, you divided the number of thirds by the numerator, 2.

$6 \div \frac{2}{3} = 6 \times 3 \div 2 = 9$

Dividing by a fraction is the same as multiplying by its **reciprocal**. Reciprocals are numbers whose numerators and denominators have been switched. When two numbers are reciprocals, their product is 1.

Dividing	Multiplying by reciprocal
$6 \div \frac{2}{3} = 9$	$6 \times \frac{3}{2} = \frac{6}{1} \times \frac{3}{2}$
	$= \frac{18}{2}$
	$= 9$

Remember

The numerator is the number on top of a fraction. The denominator is the number on the bottom. [Page 287]

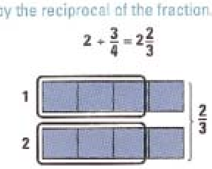
Examples

1 Divide: $2 \div \frac{3}{4}$

$2 \div \frac{3}{4} = \frac{2}{1} \times \frac{4}{3}$ Multiply by the reciprocal of the fraction.

$= \frac{2 \times 4}{1 \times 3}$

$= \frac{8}{3}$ or $2\frac{2}{3}$



2 1 *nail* $= \frac{9}{4}$ in. of cloth. Find the length of 5 in. of cloth in nails.

$5 \div \frac{9}{4} = \frac{5}{1} \times \frac{4}{9}$ Multiply by the reciprocal.

$= \frac{20}{9}$ or $2\frac{2}{9}$ Simplify.

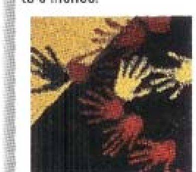
A 5-inch piece of cloth is $2\frac{2}{9}$ nails long.

Try It

Divide. **a.** $4 \div \frac{3}{5}$ **b.** $1 \div \frac{4}{7}$ **c.** $10 \div \frac{17}{4}$ **d.** $3 \div \frac{3}{5}$

DID YOU KNOW?

Three measurements used primarily for cloth include the *nail*, the *finger*, and the *span*. A *finger* is equal to $4\frac{1}{2}$ inches. A *span* is equal to 9 inches.



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Connecting and Exploring: SMPs, Task Demand, and Content Development

Task Number	Level of Task Demand	Standards of Mathematical Practice	Opportunity to Learn Content Through SMPs Practices → Content Understanding and Procedural Skills	Opportunity to develop proficiency with the SMPs Content → Practices

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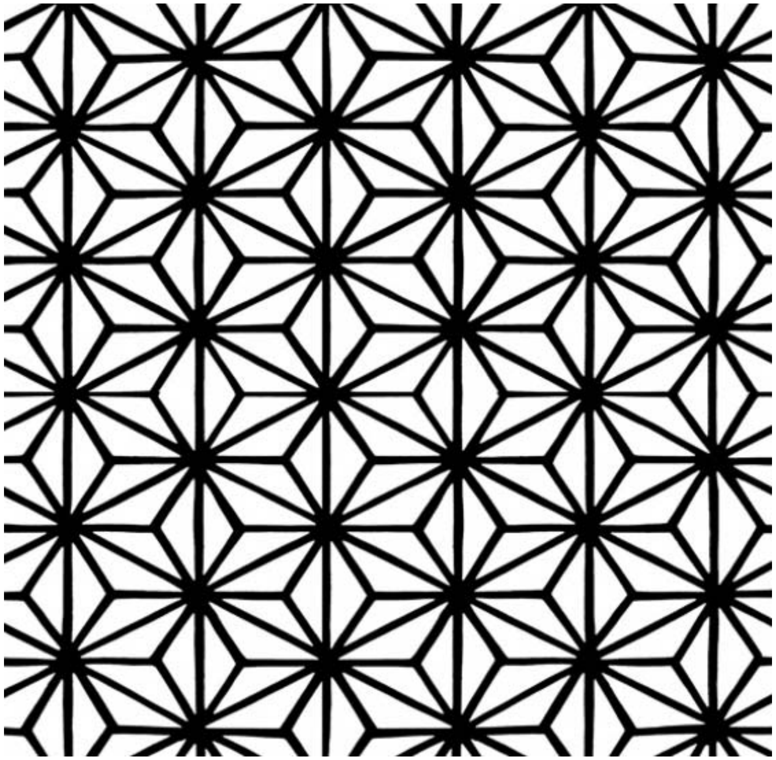
Looking for Lesson Patterns: SMPs, Task Demand, and Content Development

- What patterns do you notice in the use of various task types (levels of demand) in each of the sample lessons?
- What patterns do you notice in the relationship between the levels of task demand, opportunities to use the SMPs, and opportunities to develop mathematical understanding and fluency?



Looking for Lesson Patterns: SMPs, Task Demand, and Content Development

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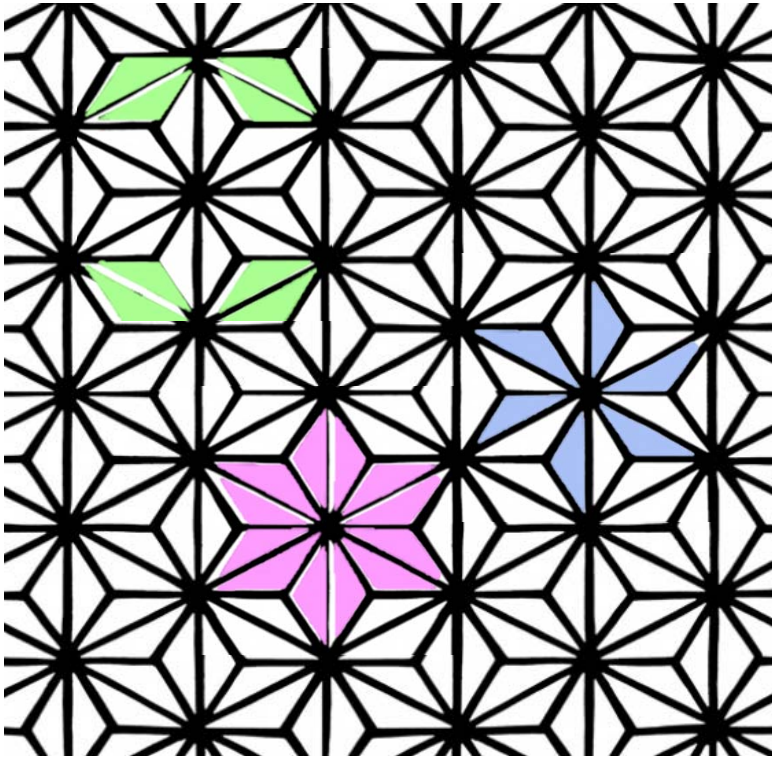
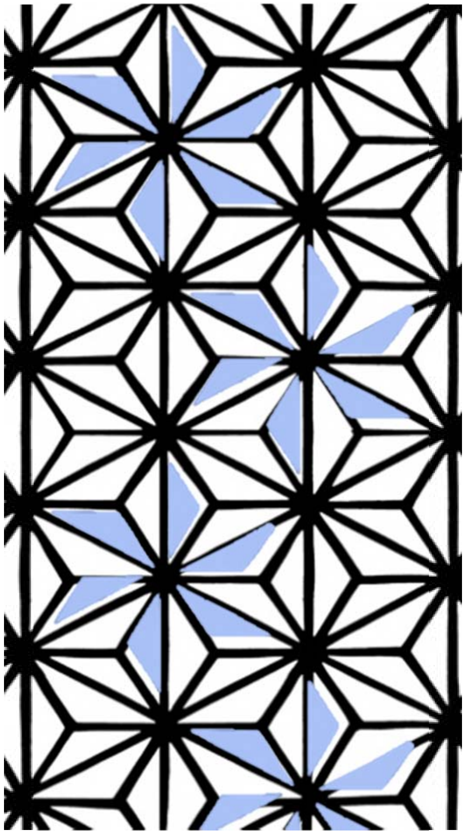


Summarizing

- *Generally, what patterns in the interplay of task demand and the use of the Standards for Mathematical Practice might you expect to find in a collection of tasks that offer students the opportunity to develop mathematical understanding looking over a lesson? a group of lessons?*
- *Describe for a colleague the two-sides of an SMP coin in a mathematics classroom:*
  - *Skills to be developed;*
  - *A vehicle to develop mathematics content.*



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Tool 2 Evidence

Tool 2 Evidence Template	Mathematical Practices Used to Develop Content Practices → Content	Opportunities to Develop SMPs as Habits of Mind Content → Practices	Assessment of SMPs and Teacher Support
Solve Problems & Persevere Attend to Precision Reason & Explain • Reason Abstractly and Quantitatively • Arguments and Reasoning of Others Model & Use Tools • Model with Mathematics • Use Tools Strategically See Structure and Generalize • Look For and Use Structure • Regularity and Repeated Reasoning			

Mills & Briars, April 2017

Textbook analysis requires ....

A knowledgeable eye for critical features of instructional materials in order to assess the degree to which a series will support a faithful and effective implementation of the CCSSM or your state's college and career readiness standards.

Adopting New Math Books?

Start by Selecting an Effective Textbook Analysis Toolkit to Inform Your Work!

NCTM
Annual Meeting
& Exhibition

April 2017
San Antonio, TX

Thank you!

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