NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Adopting New Math Books? Start by Selecting an Effect Textbook Analysis Toolkit to Inform Your Work!

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Curriculum Materials Matter: Evaluating the Evaluation Process

By NCTM President Diane J. Briars, November 2014

Adoption of curriculum materials is one of the most important decisions a teacher, school, or district can make. While state standards describe what students are expected to learn and be able to do, what is taught in classrooms—the implemented curriculum—is heavily influenced by textbooks and other instructional materials. The instructional materials affect lesson content, depth and duration of instruction for particular topics, and topic sequence. So, while we may talk about curriculum materials as just "resources," the fact is that they strongly influence classroom instruction—for better or worse.

Not surprisingly, evaluating curriculum materials has been a hot topic of conversation at recent meetings I've attended. "Which materials are best aligned with 'the Standards'—Common Core or other state standards?" "What criteria, rubrics, or evaluation processes will result in the selection of the 'best' curriculum materials for implementing 'the Standards'?"

During my tenure as mathematics director for the Pittsburgh Public Schools, I led many mathematics materials adoption committees—and I learned a great deal about productive and nonproductive practices. From that work and my experiences with other districts and states, large-scale materials review projects, and national recommendations, I offer my "Top Lessons Learned" about effective curriculum materials evaluation.

Review Criteria and Process: Top Lessons Learned

1. Focus on the central evaluation question: What curriculum materials best support students' learning of the standards? Wording the question in terms of students' learning of content, rather than implementation of standards, puts students' learning front and center. What students learn and how well they learn it depend on both mathematics content and instruction. Framing the review in terms of students' learning makes support for effective teaching and learning a critical feature for review, along with content.
2. Remember that content analysis is much more than alignment. Alignment of content with standards is often represented through "crosswalks" that connect the two, indicating where and when content addresses particular standards. While such an approach can be useful, effective content analysis examines how materials address standards, that is, it looks for the following:
   - The treatment of content is consistent with that described in the standards. For example, the grade 7 Common Core State Standards for Mathematics (CCSSM) emphasize understanding and using unit rates and equivalent ratios to solve problems involving proportional relationships, building a foundation for understanding slope. Thus, a critical content "look for" is whether materials build this understanding and emphasize use of these methods, instead of emphasizing solving proportions by using cross multiplication, with little or no attention to unit rates and equivalent ratios.
   - The development of conceptual understanding, procedural fluency, and applications is balanced, with explicit connections among the three ("rigor" in CCSSM). A critical review criterion is the extent to which procedural fluency builds on conceptual understanding. With respect to applications, important "look fors"

include applications that require problem solving and reasoning, as well as more routine use of concepts and skills; the use of applications to introduce new content, as well as to apply concepts and skills after initial instruction; real-world" applications; and, especially at the high school level, opportunities for using mathematics to model real-world situations.

- The development of content reflects what is known about how students learn that content most effectively. Ideally, this knowledge is incorporated in the standards, so it would be addressed in content treatment review. (CCSSM's attention to learning progressions, especially in grades K–8, is one of its strengths.) If it is not, or standards do not provide sufficient detail to reflect this knowledge, it is an important review criterion. For example, research clearly indicates that students learn their basic facts more efficiently and effectively when instruction focuses on fact families and strategies that relate unknown facts to known facts (doubles plus one, for example), instead of rote memorization of individual facts. Although CCSSM explicitly includes such strategies, other college- and career-ready standards may not. Regardless, the treatment of basic facts is an important consideration in materials review for grades K–4.
- The development of students' problem solving, reasoning, and other mathematical habits of mind—the set of processes identified in the CCSSM Standards for Mathematical Practice—receives explicit and regular attention. These experiences should be embedded in content development, not separate activities or lessons that can easily be skipped. This analysis is also part of the review of support for effective instructional practices described in #3 below.
- The materials are focused. Curriculum materials should give sufficient attention to the critical topics identified in the standards for each grade (in CCSSM, the "major work" of the grade), so that students have the time and support to develop the identified proficiencies. That does not mean simply adding more content to each grade so the books become larger! It means devoting more attention to focus topics and less to secondary topics, while omitting topics that are not in the standards.
- Content treatment is coherent. The content is effectively organized so that students can clearly see how ideas build upon, or connect with, other ideas both within and across grades. This analysis requires looking at the development of content across grades and courses, in addition to looking at the development within a grade or a course.
- The mathematics in the materials is accurate. That the materials should be as close to error-free as possible goes without saying.

Student Achievement Partners' Publishers Criteria provides a more detailed discussion of the preceding criteria.

3. Analyze the nature of the instructional tasks and activities—this is as important as analyzing content. This analysis examines how the materials support students' learning though opportunities to engage in tasks that promote reasoning and problem solving and teachers' implementation of effective teaching practices as described in NCTM's Principles to Actions: Ensuring Mathematical Success for All. Critical questions include the following:

   - To what extent do lessons regularly feature tasks that engage students in problem solving, reasoning, and making sense of mathematics as core instructional activities, rather than special features that can be omitted?
   - What is the quality of these tasks? Do they permit multiple entry points and approaches? To what extent do they address the learning goals of the lesson?
   - Do the tasks constitute a coherent series designed to address specific mathematical goals across lessons? Do the tasks build procedural fluency from conceptual understanding across lessons?
   - What supports do the teachers' editions provide for effective implementation of these lessons? Do they provide, for example, information about likely student solutions, questions to support students as they work on tasks and in subsequent debriefing discussions, and suggestions about ways to structure the summary discussion? Understanding the intended instructional model is essential for this analysis. Be sure to read the teacher's edition or other explanatory materials, view supporting webinars, etc., that describe the instructional model and where particular supports are located. Reviewing only the student materials may

not provide sufficient understanding of how the materials are intended for use in the classroom to support an adequate analysis.

4. Focus initial reviews on student materials and teacher editions of the materials. These have the primary influence on classroom teaching and learning. Analyze ancillary materials and other supports for effective teaching and learning—such as assessments, technology integration, additional practice, and professional learning—after you have narrowed your choices to materials that adequately meet the content and instructional support criteria. All the flashy supplementary materials in the world won't make up for flawed content or lack of high-quality instructional activities.
5. Consider equity, diversity, and access. High-quality content and instructional practices are critical for the success of all students; therefore, reviews of these aspects are essential first steps in addressing equity and access. After narrowing your choices, however, consider specific ways in which materials promote equity and access. To what extent, for example, do they—
   - provide teachers with strategies and materials for meeting the needs of a range of learners, including both struggling and advanced learners?
   - suggest accommodations and modifications for English language learners that will support their regular and active participation in learning mathematics?
   - provide a balanced portrayal of various demographic and personal characteristics?

See the CCSSO-NCSM Common Core State Standards (CCSS) Mathematics Curriculum Materials Analysis Tools for a more complete list of equity, diversity, and access criteria.

6. Recognize that all omissions or gaps are not the same. No materials are perfect. Inevitably, an evaluation process will uncover gaps, omissions, or inadequate treatment of some content. The key question is how easily teachers, the school, or the district can fill the gaps. For example, providing additional practice on a skill may be relatively easy; providing lessons to address a gap in concept development is probably more difficult. Gaps that are most difficult or impossible to fill are consistent lack of instructional tasks that engage students in problem solving, reasoning, and the mathematical practices. Expecting teachers, schools, or districts to create or find high-quality tasks for almost every lesson is unreasonable—and, most likely, will not provide the consistent quality or coherence needed for effective teaching and learning.
7. Recognize that additional content is less problematic than gaps that are difficult to fill. Given the variation in standards across states, materials are likely to contain content beyond that addressed in your standards. The issue is how that extra content affects the treatment of content addressed in the standards. If the extra content can easily be skipped, or if it contributes positively to students' learning the content addressed in the standards, then it doesn't matter. It does matter, however, when it decreases time and attention on content addressed in the standards, disrupts the focus and coherence of the materials, or is so great that the books are huge.
8. Request all series and materials produced by each publisher. When you call for materials to review, remember that some of the large companies publish more than one program, so you may have to ask to see them all. Also, request programs from smaller, alternative publishers and developers as well as the large publishers. You want to review all the options, not just the traditional best sellers.
9. Allocate sufficient time for your review process. Thoughtful analysis of the content, instructional activities, and other features of curriculum materials described above takes time. Materials that are adopted are likely to be used—and to influence instruction—for a number of years. So time spent reviewing materials carefully is time well spent.
10. Use a "narrowing choices" strategy to make the review process as efficient as possible. Clearly, thorough content analyses are time-consuming—and may seem overwhelming. To make the process manageable, first review all materials for their treatment of only one or two key content domains. Retain for further review only those materials that give adequate treatment to those domains. Then make a second cut based on your

evaluation of the nature of the instructional tasks and support for effective teaching practices within those domains. After these cuts, you're likely to have a manageable number of materials for further review. For example, to review middle school materials with respect to CCSSM, you might first review all materials for their treatment of ratios and proportional relationships (grades 6 and 7) and functions and expressions and equations related to proportional relationships (grade 8). Then review materials that treat that content well from the standpoint of the nature of their instructional tasks, and so on, for that content. Submit the materials that adequately address both criteria to additional review, starting with the remaining content domains, instructional tasks, and other review criteria such as equity, diversity, and access, ancillary materials, and so on.

11. Rate and discuss rather than score. Analysis of materials is qualitative, rather than quantitative; that is, reviewers are judging the quality of content treatment, instructional activities, and so forth, in different materials. Consequently, qualitative rubrics with categories such as "Not Found," "Low," "Marginal," "Acceptable," and "High" can be more useful than numeric scales. Qualitative ratings also provide useful guidance for subsequent within- and across-grade discussions of the quality of different materials.
12. Provide adequate professional learning for the members of the review team. It is essential that all reviewers both understand the standards and are knowledgeable about the effective teaching practices for implementing them. To ensure this common base of knowledge and understanding, consider engaging reviewers in collaborative study of the standards. For CCSSM, read and analyze the progression documents in addition to the standards themselves. APrinciples to Actions book study can be a good way to build knowledge of the effective teaching practices.
13. Try out your top choices in the classroom. The real test of the quality of any materials is the learning that they support in the classroom. If at all possible, try out at least a unit or two from the materials under final consideration in several classrooms. Even if the review committee is in unanimous agreement, using the materials in some classrooms is important before finalizing the decision. When you test the materials in this way, recognize that they may use unfamiliar instructional models, so students—and teachers—will need some adjustment time. My experience has been that trying out materials has been invaluable in helping review committees adopt materials that strongly support effective teaching and learning.

A number of rubrics and tools are available to support materials evaluation. I have used the CCSSO-NCSM Common Core State Standards (CCSS) Mathematics Curriculum Materials Analysis Project Tools referred to earlier. The strengths of these tools are that they provide qualitative rubrics for analysis of different review criteria, along with worksheets that are specifically designed to support cross-grade as well as within-grade analysis of treatment of core content domains. As you consider rubrics for your process, be sure that they (1) support cross-grade analysis of content coherence as well as the quality of individual lessons or units and (2) promote discussion of strengths and weaknesses of particular materials rather than only numerical ratings.

Even though this list of review criteria and processes may seem overwhelming, in practice, these "lessons" have worked very well to guide the review process and support adoption of materials that will promote all students' learning of the standards. Selection of curriculum materials is one of the most important responsibilities of teachers, schools, and districts. And careful analysis of how materials address standards and instruction is a necessary foundation for this work and critical to the learning of all students.

Valerie L. Mills

Foundations for Supporting Teachers and the Work of Teaching

In the last four issues of the *NCSM Newsletter*, I have explored leadership issues that surround curricular and instructional coherence, formative assessment, and most recently, the need to reframe the way we describe and utilize mathematical goals for instruction. In this issue, I connect these previous conversations to a related topic—critical features leaders need to consider as they support the work of teachers. I propose two foundational components in an effective support strategy: First, provide teachers with a coherent curriculum and an aligned set of expertly designed coherent instructional materials to enact that curriculum; second, prioritize time for teachers to discuss and plan for the hard work of teaching in collaboration with colleagues.

One other note for readers to keep in mind as they consider the ideas herein–many of us are grappling with how best to support our colleagues in classrooms and so I am asking that you join this conversation by way of Facebook and Twitter. Please consider sharing your thoughts and suggestions for strategies you believe are foundational in supporting teachers so that we can all benefit from our collective wisdom and experiences.

First, provide teachers with a coherent curriculum and an aligned set of expertly designed *coherent* instructional materials to enact that curriculum.

NCTM published *Curriculum and Evaluation Standards for School Mathematics* in 1989, and by the mid-90s it had prompted the publication of "supplemental books" with rich mathematical tasks that could be used to bring problem solving and discrete mathematics topics into classrooms where they were using traditional textbooks and wanted to more closely align their practice with the new standards. Also by the mid-90s, the new, so-called *Standards-based* textbooks were becoming available. These materials were developed in an entirely new way, as research projects, by teams of university faculty working together to design, pilot, revise, and field test carefully sequenced sets of lessons. These textbooks produced lessons that not only stood the "Is it in the book?" test for a list of required content standards, but far more importantly and for the first time, they helped teachers build mathematical understanding and skills with meticulously structured lessons that worked as a coordinated sequence of challenges. These materials were designed to develop mathematical knowledge and reasoning in far more sophisticated and complex ways than a collated collection of stand alone lessons and their use has now been demonstrated to improve mathematics success for all students.

Much like 25 years ago we find ourselves today in an era of new mathematics standards, and like those times, supplemental materials are widely available to help teachers align their practice to these new standards. Now instead of buying them, you can Google them. They are generally free, and certainly plentiful. Many administrators are looking at their shrinking budgets and once again asking teachers to pull together their own instructional materials using these free resources. The question to be considered, both 25 years ago and today, is: What might you get drawing on these now electronically available lessons in comparison with a research-based, standards-based textbook?

To help answer that question, it is worth reminding ourselves what goes into the development of a *coherent* mathematics textbook series using a research–based approach. These author teams structure lessons to develop a mathematically related constellation of ideas rather than a single discrete skill. The lessons are sequenced beginning with concrete contexts and representations and they move gradually toward greater abstraction and mathematical complexity. This is true for the design of a unit of study, the set of units that compose a textbook, and across a series of textbooks.

Supporting this progression toward greater mathematical sophistication, mathematical representations (drawings, words, tables, graphs, symbols) are intentionally selected and sequenced, lesson and student assignments are composed to encourage the construction of mathematical connections among topics and representations, teacher notes suggest ways to improve the nature of the classroom discourse and planning for possible student misconceptions, and mathematical tools are strategically and appropriately introduced. In addition, great care is given to the tasks in lessons, assignments, and assessments. These tasks are designed to be open enough to provide access to a range of students using a variety of approaches, and scaffolded to support the learning trajectory. They utilize engaging contexts, include an appropriate balance and sequence of items that are cognitively more and less sophisticated, and require students to reason mathematically and to synthesize related concepts

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and strategies. All of these decisions are now based on nearly 30 years of experience building, using, and evaluating these materials.

The work of instructional design and evaluation is highly specialized, expensive, and time intensive. It requires focus and dedication, leadership, and vision. It is not random or opportunistic. It demands far more intention from a *team* of education specialists than can be reasonably accomplished by any single person who has been asked to cobble together a set of lessons created originally as stand alone activities and posted on sites across the Internet. Clearly, the development of coherent instructional materials that are aligned to a particular set of standards is not work that we should expect teachers to tack onto their already overloaded plates during planning time or even two weeks set aside in the summer.

Addressing this same concern 25 years ago, at a time when similarly, principals were asking teachers to find or develop their own good lessons Glenda Lappan wrote in an NCTM Presidential Letter,[1] "… think of the complexity of creating coherent, complete mathematics materials that have an internal structure, a spine—materials that guide the development of mathematical understanding and skill." She concluded then, as I do today, that working with teachers to select an excellent mathematics series, aligned to state and national standards, has to be understood to be a more productive approach to the dilemma of optimizing learning for all students.

As leaders, we need to help those in decision-making roles understand the importance of selecting and using well designed instructional materials. The Internet is a powerful resource but it has limitations that we need to understand, recognize, and articulate for others as it concerns instructional materials design. The work of teaching is far too challenging on its own. How can we allow others to distract from that work with the addition of highly specialized design responsibilities? With this reasoning, a first critical step in supporting the work of teachers is to ensure that teachers have access to a coherent curriculum and an aligned set of expertly designed *coherent* instructional materials to enact that curriculum. Equipped with a coherent set of instructional resources, we free teachers to take up the considerable challenges of teaching.

Second, prioritize time for teachers to explore, discuss, and plan for the hard work of teaching in collaboration with colleagues.

This leads me to the second aspect of supporting teachers and the work of teaching—prioritizing time for teachers to consider the hard work of teaching in collaboration with colleagues. This includes time to explore the mathematics they teach, as well as the mathematical progressions that expand above and below theirs, to understand how best to leverage the intentional designs of the textbook authors, to carefully analyze student work to understand students' current thinking, to consider and then provide actionable feedback to students, and to select student work samples as contexts for follow-up lessons to extend student understanding. I could go on, but by now you will see where I am going. Teachers need time and support to continuously reflect on the myriad of instructional decisions they make for particular students. As leaders, it is our responsibility to prioritize and facilitate these discussions in the scarce time available.

In these recommendations I want to make clear that I do not intend to denigrate the knowledge or expertise or capacity of the dedicated women and men charged with educating our children. Neither do I want to suggest that using resources collected from the Internet is always unproductive. I taught high school for 20 years; I understand deeply what it takes to ensure that every child is successful in my classroom. My intent with these recommendations is to make explicit the challenging complex nature of designing/selecting *coherent* instructional materials and to ask that we prioritize time for aspects of teaching that are most closely related to the needs of our particular students.

Ensuring access to great instructional resources and opportunities to develop the expertise needed to optimize their use, this is the work of mathematics education leaders. This is the foundation teachers deserve.

Once again, I invite you to join colleagues in sharing your views about the foundations leaders need to provide for their teachers by joining us online through Facebook [facebook.com/mathedleadership.org] or Twitter [@MathEdLeaders, @VMillsMath, #NCSMHT (Hot Topics)].

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1 *Texts and Teacher: Keys to Improved Mathematics Learning*, Glenda Lappan, NCTM New Bulletin, July/August 1998.

CCSSM Curriculum Analysis Tool 1— Ratios and Proportional Relationships for Grades 6-8
Name of Reviewer ______________________ **School/District** ____ **Date** ________________
Name of Curriculum Materials ____ **Publication Date** _____________________ **Grade Level(s)** _____________

Content Coverage Rubric (Cont):	**Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):**
Not Found (N) -The mathematics content was not found. Low (L) - Major gaps in the mathematics content were found. Marginal (M) - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled. Acceptable (A) - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled. High (H) - The content was fully formed as described in the Standards.	Not Found (N) -The content was not found. Low (L)- The content was not developed or developed superficially. Marginal (M) - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills. Acceptable (A)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed. High (H)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.

CCSSM Grade 6				CCSSM Grade 7				CCSSM Grade 8			
6.RP Ratios and Proportional Relationships	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.RP Ratios and Proportional Relationships**	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.EE Expressions and Equations**	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Understand ratio concepts and use ratio reasoning to solve problems.				**Analyze proportional relationships and use them to solve real-world and mathematical problems.**				**Understand connections between proportional relationships, lines, and linear equations.**			
1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak."				1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.				5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.			

CCSSM Curriculum Analysis Tool 1—Ratios and Proportional Relationships for Grades 6-8											
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
Understand ratio concepts and use ratio reasoning to solve problems.				**Analyze proportional relationships and use them to solve real-world and mathematical problems.**				**Understand connections between proportional relationships, lines, and linear equations.**			
2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”				2. Recognize and represent proportional relationships between quantities. 2a. Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. 2d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation.				6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.			
3. Use ratio and rate reasoning to solve real-world and mathematical problems by reasoning. 3c. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.				2b. Identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. 2c. Represent proportional relationships by equations.							
3a. Make tables of equivalent ratios relating quantities with whole umber measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.				3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease.*							
3b. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.											
3d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.											

CCSSM Curriculum Analysis Tool 1—Ratios and Proportional Relationships for Grades 6-8	
Notes and Examples:	
Overall Impressions: 1. What are your overall impressions of the curriculum materials examined? 2. What are the strengths and weaknesses of the materials you examined? **Standards Alignment:** 3. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation? 4. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard? 5. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?	**Balance between Mathematical Understanding and Procedural Skills** 6. Do the curriculum materials support the development of students' mathematical understanding? 7. Do the curriculum materials support the development of students' proficiency with procedural skills? 8. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills? 9. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills? 10. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

CCSSM Curriculum Analysis Tool 1—Interpreting Functions in Grades 9-12

Name of Reviewer ____ School/District _________________________ Date ______________

Name of Curriculum Materials _______________________________________ Publication Date __________ Course(s) _______________

Content Coverage Rubric (Cont):
Not Found (N) -The mathematics content was not found.
Low (L) - Major gaps in the mathematics content were found.
Marginal (M) -Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled.
Acceptable (A)-Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled.
High (H)-The content was fully formed as described in the standards.

Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):
Not Found (N) -The content was not found.
Low (L)-The content was not developed or developed superficially.
Marginal (M)-The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills.
Acceptable (A)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed.
High (H)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.

CCSSM Standards Grades 9-12	Chapter pages	Cont N-L-M-A-H	Bal N-L-M-A-H	Notes/Explanation
Interpreting Functions (F-IF)				
Understand the concept of a function and use function notation				
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.				
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.				
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.				
Interpret functions that arise in applications in terms of the context				
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*				

CCSSM Curriculum Analysis Tool 1—Interpreting Functions in Grades 9-12				
CCSSM Standards Grades 9-12	**Chapter pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**Notes/Explanation**
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*				
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.				
Analyze functions using different representations				
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.				
8. Graph linear and quadratic functions. Show intercepts, maxima, & minima.				
9. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.				
10. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.				
11. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.				
12. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.				
13. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.				
14. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.				
15. Use the properties of exponents to interpret expressions for exponential functions.				
16. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of a quadratic function and an algebraic expression for another, say which has larger maximum.*				

CCSSM Curriculum Analysis Tool 1—Interpreting Functions in Grades 9-12	
Overall Impressions: 1. What are your overall impressions of the curriculum materials examined? 2. What are the strengths and weaknesses of the materials you examined? **Standards Alignment:** 3. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation? 4. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard? 5. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?	**Balance between Mathematical Understanding and Procedural Skills** 6. Do the curriculum materials support the development of students' mathematical understanding? 7. Do the curriculum materials support the development of students' proficiency with procedural skills? 8. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills? 9. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills? 10. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

NCSM-NCTM-Curriculum Materials Analysis Tools

Tool 1: Content Analysis

Purpose:

- Analyze the extent to which the content (i.e., concepts, skills, applications) is treated in the materials as described in CCSSM.
- Determine the extent to which CCSS are sequenced appropriately in the materials
- Determine the extent to which the materials provide a balanced treatment of the CCSS in terms of conceptual development and procedural fluency.

1A. Content Coverage/Treatment Rubric:	Key Evidence and Where to Find It!	Look Fors:
In the rubric below, "gap" refers to IF, WHERE, and HOW content is treated in the materials. Not Found (N) - The mathematics content was not found. Low (L) - Major gaps in the mathematics content were found. Marginal (M) - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled. Acceptable (A) - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled. High (H) - The content was fully formed as described in the standards	• Base this analysis on lessons as presented in the student and teachers' editions, since these determine students' core instructional experiences. • This analysis addresses IF, WHERE, and HOW content is treated in the materials. Examining whether content is included is insufficient to determine whether students will have the opportunity to learn content as specified in CCSSM. • This analysis must be done not only within grades, but across grades to determine whether the materials adequately address and connect the mathematical ideas as they develop within and across grades, as described in the standards. (The complete the *CCSS Curriculum Materials Analysis Toolkit* contains grade-band analysis sheets for specific CCSS content domains.) • For High School – in addition reviewers will need to explore and understand the author's rationale for distributing content into and cross the three HS courses. Noting particularly *focus* - extensive course level experiences without re-teaching, and *coherence* - building on prior knowledge from within and across courses.	Content development is focused, coherent, and rigorous: 1. CCSS Content: CCSS Content Standards for the grade range are thoroughly developed 2. Focus: Content present respects the foci and learning progressions built into CCSS grade level standards, so that the content present outside this is limited to: connecting to prior knowledge without re-teaching, and previewing future content without expecting proficiency. 3. Mathematical Range: In major topics, lessons pursue conceptual understanding, procedural skill, and fluency, and application 4. Representations: Types and range of representations, sequence of representations, and the use of critical representations as identified in the CCSSM 5. Connections: Degree to which lessons support students in making connections among related mathematical concepts and algorithms as described in CCSSM. (E.g., Content cluster heads that begin with "Extend and apply . . . . ")

Summary Questions—Content Coverage/Treatment

1. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation?
2. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard?
3. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?

Tool 1: Content Analysis

1B. Balance of Mathematical Understanding & Procedural Skills Rubric	Key Evidence and Where to Find It!	Look Fors:
Not Found (N) - The content was not found. Low (L) - The content was not developed or developed superficially. Marginal (M) - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills. Acceptable (A) - The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed. High (H)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.	*Conceptual Understanding* – comprehension of mathematical concepts, operations, and relations. "*Understand"* means that students can explain the concept with mathematical reasoning including concrete illustrations, mathematical representations, and example applications. *Procedural Fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.	1. Procedures from Concepts: Activities designed to develop conceptual understanding are leveraged and explicitly connected to the development of related procedures and algorithms 2. Task Range: Tasks are designed and sequenced so that students are ask to work across the full range of cognitive demand levels Opportunities for students to: 3. Model: Use concepts to make sense of and explain quantitative situations ("Model with mathematics") 4. Reason: Incorporate concepts into their own arguments and use them to evaluate the arguments of others (see "Construct viable arguments and critique the reasoning of others") 5. Problem Solve: Bring them to bear on the solutions to problems (see "Make sense of problems and persevere in solving them") 6. Connect: Make connections between related concepts

Summary Questions: Balance between Mathematical Understanding and Procedural Skills:

1. Do the curriculum materials support the development of students' mathematical understanding?
2. Do the curriculum materials support the development of students' proficiency with procedural skills?
3. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills?
4. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills?
5. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

Overall Impressions:

1. What are your overall impressions of the curriculum materials examined?
2. What are the strengths and weaknesses of the materials you examined?

NCSM-NCTM-Curriculum Materials Analysis Tools

Tool 2: The Mathematical Practices Analysis

Purpose:

- Analyze the extent to which the Standards for Mathematical Practice are treated in the materials as described in CCSSM.
- Determine the extent to which the materials demand that students engage in the Standards for Mathematical Practice as the primary vehicle for learning the Content Standards.
- Determine the extent to which the materials provide opportunities for students to develop the Standards for Mathematical Practice as "habits of mind" throughout the development of the Content Standards.

<table>
<tr><th>2. The Practices</th><th>Key Evidence and Where to Find It!</th><th>Look Fors:</th></tr>
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Low – The Standards for Mathematical Practice are not addressed or are addressed superficially.
Marginal - The Standards for Mathematical Practice are addressed, but not consistently in a way that is embedded in the development of the Content Standards.
Acceptable – Attention to the Standards for Mathematical Practice is embedded throughout the curriculum materials in ways that may help students to develop them as habits of mind.
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Content standards that explicitly refer to "understand" or "understanding" are especially good opportunities to connect the practices to the content. (CCSS, p. 8)
Instructional Tasks:
Examine the extent to which lessons consistently are built around tasks that promote problem solving, reasoning, and engagement in standards for mathematical practice.
SMPs should be treated in two ways:

Students should engage in the SMPs as they work on tasks to learn specific content; and
Developing proficiency in the SMPs should be the explicit goal of some lessons.

Occasional opportunities—once a week; a few times a chapter—for students to engage in the SMPs are not sufficient.
Explicitly labeling lessons or tasks with particular mathematical practices ("call-outs") is irrelevant.
Formative Assessment:
Formal and informal assessments should provide evidence about students' proficiency with the SMPs as well as the content standards.
Resources:

The "Elaborations" on the Standards for Mathematical Practice for Grades K-5 and Grades 6-8 (Illustrative Mathematics) provide additional interpretation of the SMPs for these grade levels.
— Grades K-5: http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards/
— Grades 6-8: ommoncoretools.me/2014/05/04/6-8-elaborations-of-the-practice-standards/
"Model" and "modeling" are used in a variety of ways in mathematics education. See the NCTM-SIAM Committee on Modeling Across the Curriculum's "How to Identify Tasks that Engage Students in Mathematical Modeling" for clarification of SMP 4. Modeling with mathematics

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Opportunities for students to:

Mathematical Practices → Content: To what extent do the materials demand that students engage in the Standards for Mathematical Practice as the primary vehicle for learning the Content Standards?
Content → Mathematical Practices: To what extent do the materials provide opportunities for students to develop the Standards for Mathematical Practice as "habits of mind" (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards?
Opportunities to Elicit Evidence of Student Thinking: To what extent do accompanying assessments of student learning (such as homework, observation checklists, portfolio recommendations, extended tasks, tests, and quizzes) provide evidence regarding students' proficiency with respect to the Standards for Mathematical Practice?
Teacher Support: What is the quality of the instructional support for students' development of the Standards for Mathematical Practice as habits of mind?

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Tool 2: The Mathematical Practices Analysis

Summary Questions: Balance between Mathematical Understanding and Procedural Skills:

1. Do the curriculum materials support the development of students' mathematical understanding?
2. Do the curriculum materials support the development of students' proficiency with procedural skills?
3. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills?
4. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills?
5. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

Overall Impressions:

1. What are your overall impressions of the curriculum materials examined?
2. What are the strengths and weaknesses of the materials you examined?

Mathematical Tasks Analysis Guide

Levels of Demand of Mathematical Tasks

Lower-level demands

Memorization:

a. Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
b. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
c. Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
d. Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.

Procedures without connections:

a. Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
b. Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
c. Have no connection to the concepts or meaning that underlie the procedures being used.
d. Are focused on producing correct answers vs developing mathematical understanding.
e. Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands

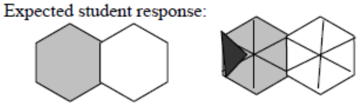
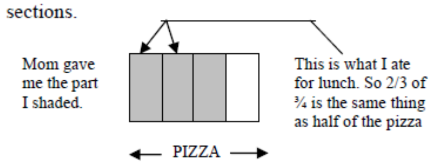
Procedures with connections:

a. Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
b. Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
c. Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols and problem situations. Making connections among multiple representations helps develop meaning.
d. Requires some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Doing mathematics:

a. Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
b. Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
c. Demand self-monitoring or self-regulation of one's own cognitive processes.
d. Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
e. Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
f. Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Examples of Higher and Lower Cognitive Demand Tasks

Lower-Level Demands	Higher-Level Demands
Memorization What is the rule for multiplying fractions? Expected student response: You multiply the numerator times the numerator and the denominator times the denominator. or You multiply the two top numbers and then the two bottom numbers. ***Procedures without Connections*** Multiply: $\frac{2}{3} x \frac{3}{4}$ $\frac{5}{6} x \frac{7}{8}$ $\frac{4}{9} x \frac{3}{5}$ Expected student response: $\frac{2}{3} x \frac{3}{4} = \frac{2x3}{3x4} = \frac{6}{12}$ $\frac{5}{6} x \frac{7}{8} = \frac{5x7}{6x8} = \frac{35}{48}$ $\frac{4}{9} x \frac{3}{5} = \frac{4x3}{9x5} = \frac{12}{45}$ Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase? Expected student response: The formula for area is l x w. 15 x 10 = 150. She will need 150 square feet of carpet.	***Procedures with Connections*** Find $\frac{1}{6}$ of $\frac{1}{2}$. Use pattern blocks. Draw your answer and explain your solution. Expected student response:  First you take half of the whole, which would be one hexagon. Then you take one-sixth of that half. So I divided the hexagon into six pieces, which would be six triangles. I only needed one-sixth, so that would be one triangle. Then I needed to figure out what part of the two hexagons one triangle was, and it was 1 out of 12. So 1/6 of 1/2 is 1/12. ***Doing Mathematics*** Create a real-world situation for the following problem; $\frac{2}{3} x \frac{3}{4}$. Solve the problem you have created without using the rule, and explain your solution. One possible student response: For lunch Mom gave me three-fourths of a pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat? I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part Mom gave me. Since I only ate two-thirds of what she gave me, that would be only two of the shaded sections. 

From: Smith, M.S., & Stein, M.K. (1998). Mathematical tasks as a framework for reflection: From research to practice. Mathematics Teaching in the Middle School,, 3(4), pp. 268-275.

Tool #2
Connecting and Exploring:
SMPs, Task Demand, and Content Development

Task Number	Level of Task Demand	Standard for Mathematical Practice	Opportunity to Develop Proficiency with the SMPs Content ⇨ Practices	Opportunity to Learn Content through SMPs Practices ⇨ Content

From Valerie Mills, Oakland Schools, MI.

Tool #2 Evidence Template

Standards for Mathematical Practice (Grouped)	Opportunities to Develop Proficiency with the SMP as a Habit of Mind Content ➡ Practices	Mathematical Practices Used to Develop Content Practices ➡ Content	Assessment of SMP and Teacher Support
Solve Problems & Persevere			
Attend to Precision			
Reason & Explain • Reason Abstractly and Quantitatively • Arguments and Reasoning of Others			
Model & Use Tools • Modeling with Mathematics • Use Tools Strategically			
See Structure and Generalize • Look For & Use Structure • Regularity & Repeated Reasoning			

Adapted from Valerie Mills, Oakland Schools, MI.