NCTM ANNUAL MEETING
& EXPOSITION 2017
April 5-8 | San Antonio

HOW TO TINKER WITH THE ALGEBRAIC THINKER

Event Title: 0269 - How to Tinker with the Algebraic Thinker

Session Type: Session
Time: 9:30 AM - 10:30 AM
Location: Grand Hyatt San Antonio, Bowie ABC

Dr. David Feikes

- Dr. David Feikes, Professor of Mathematics at Purdue University Northwest

Ken Evans

- **School:** Hobart Middle School
- **School Corp**: School city of Hobart
- **Classes Taught**: Math- Algebra and lowest 25% of the grade in math.

Martin Briggs

- Martin Briggs, Fourth Grade Teacher

  La Porte Community Schools

  La Porte, Indiana

CARE

- Conceptual Algebra Readiness for Everyone, CARE, is a curriculum development project funded through a grant from the Indiana Mathematics and Science Partnership with the Michigan City Area Schools in collaboration with Purdue University Northwest for grades 3 – 8.

- The goal of the project is to help children develop conceptual algebra readiness through weekly classroom activities and by providing extensive professional development for teachers.

Conceptual Algebra Readiness for Everyone

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Conceptual Algebra Readiness for Everyone, CARE, is a supplemental curriculum for students in grades 3-8. The goal of the project is to help children develop conceptual algebra readiness through weekly classroom activities and by providing extensive professional development for teachers.

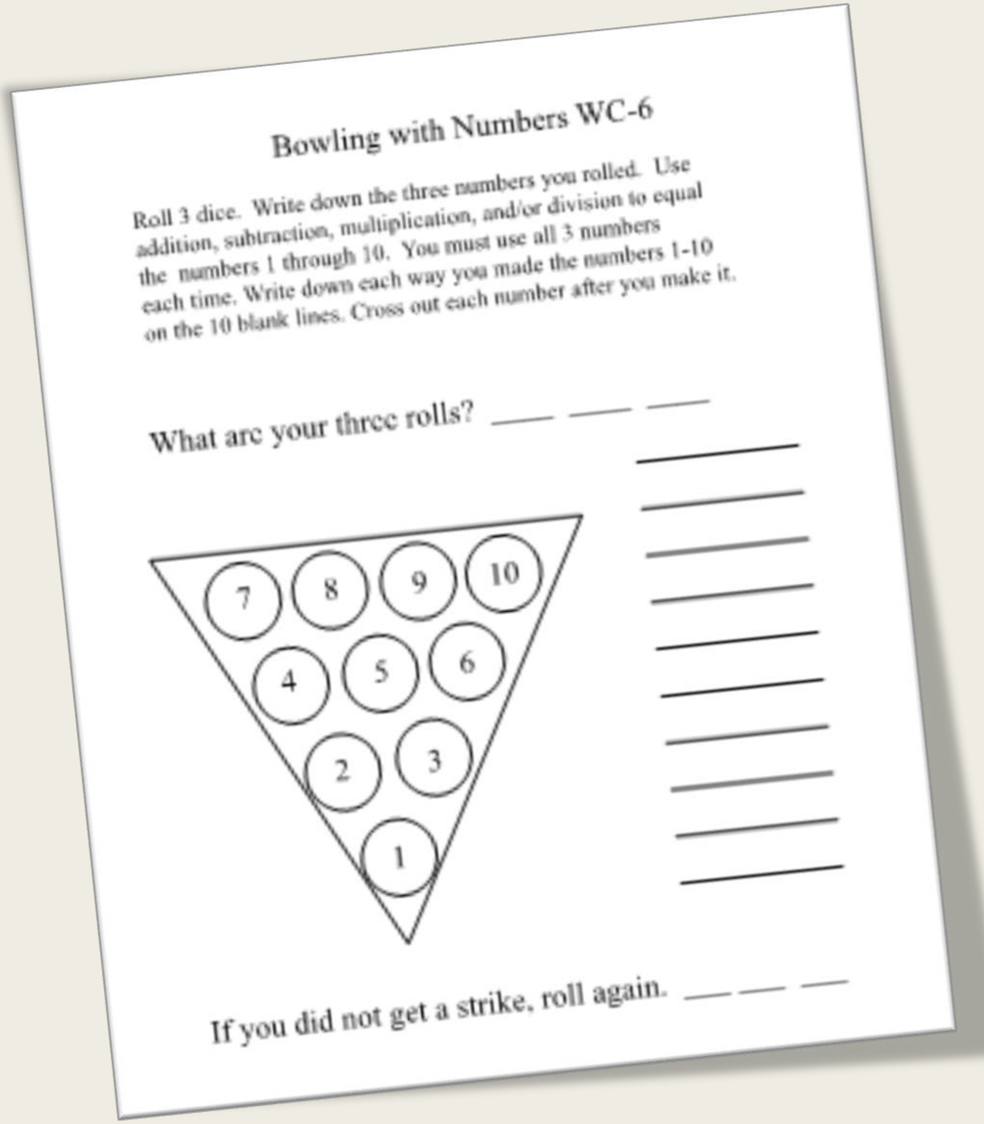
The project attempts to help teachers develop deeper understandings of mathematics and models instructional strategies to enhance mathematics instruction. The student activities vary from rich problem sets to games that engage students in learning mathematics. The CARE Project has been made possible by Indiana Mathematics and Science Partnership Grants to Purdue University Northwest and local schools.

Conceptua Algebra Readiness for Everyone

- Conceptual algebra readiness is sometimes referred to as “early algebra” or “algebraic reasoning.”
- Key Point: not formal algebra early
- Not to teach x’s and y’s-
- Help children understand the **underlying concepts** of algebra so that when they do solve equations in algebra, they will have a **conceptual basis** stemming from their work with whole numbers, fractions, decimals, and percents.

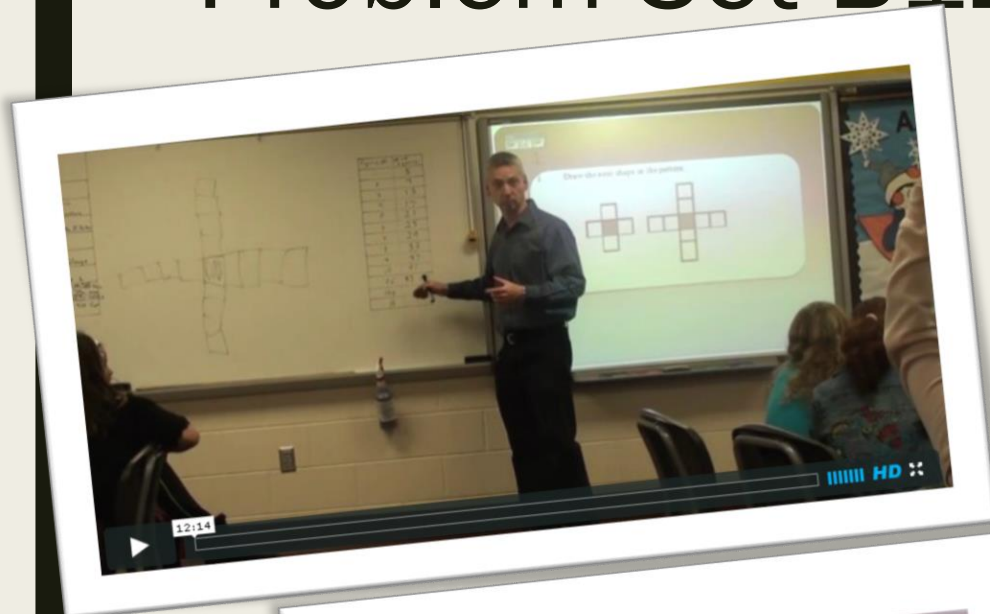
Activities:

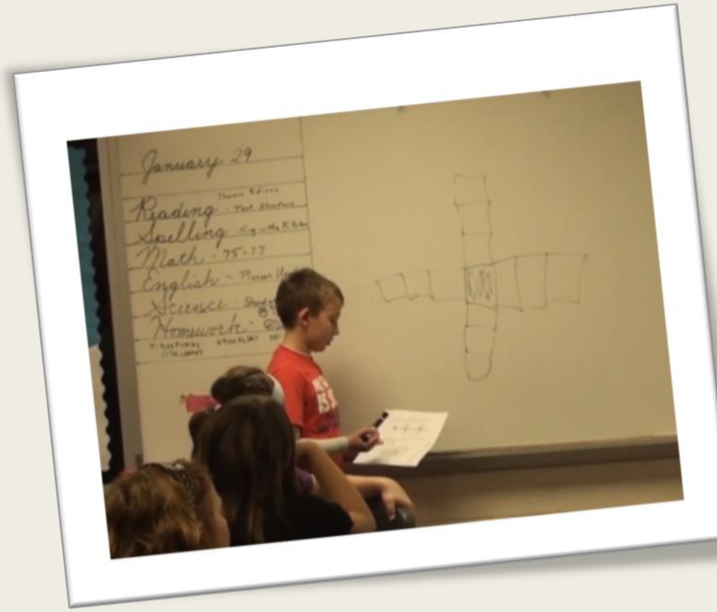
- Whole Class Activities- vary in form from games to a couple of problems to discuss

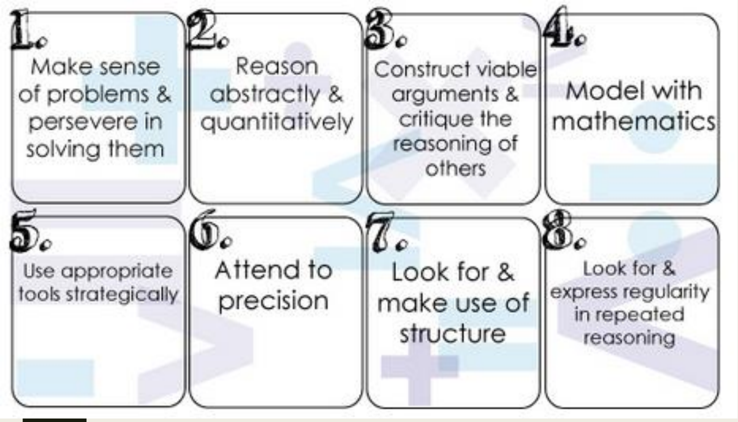


- Problem Sets- typically consists of 10 – 15 problems for students to solve in a mixed review format

Problem Set B12



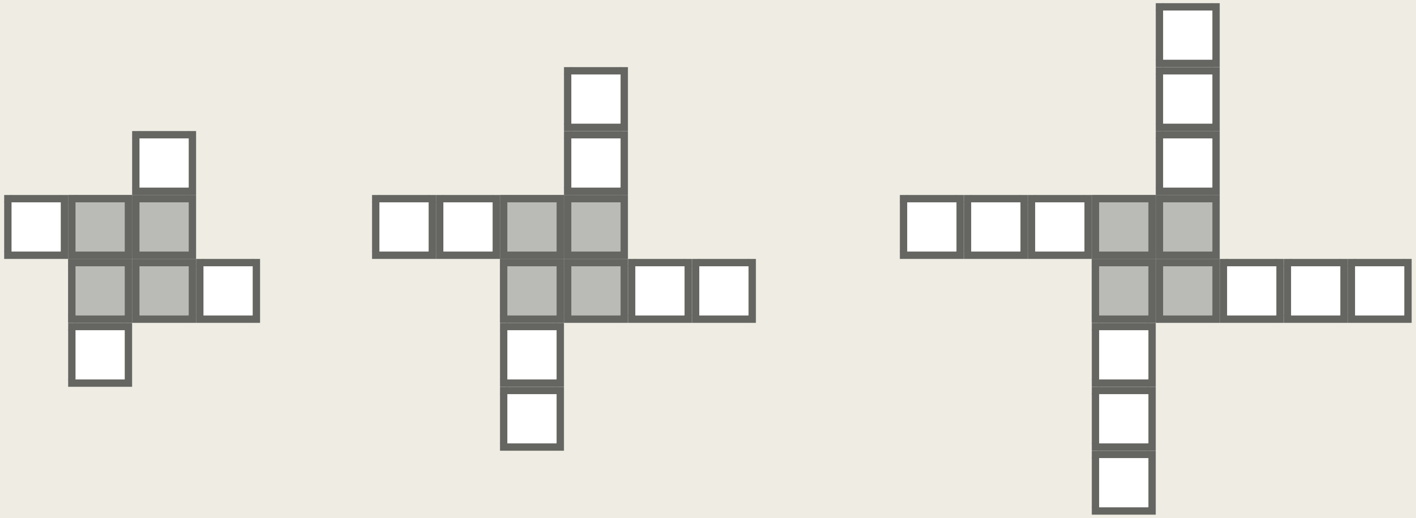




A LOOK AT PROBLEM B12

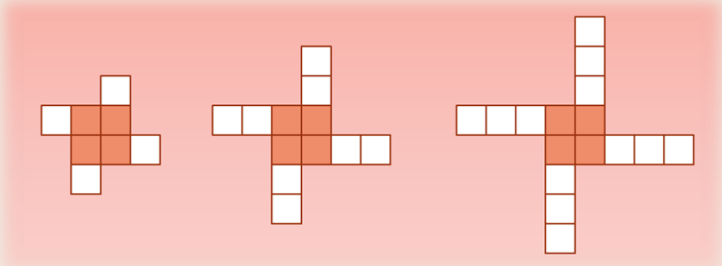
Generalization, symbolic representation, using tables, and geometric patterning

1. Draw the next shape in the pattern.



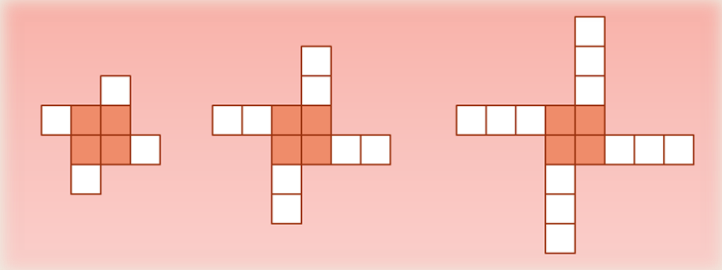
Fill in the table:

Figure #	Number of Squares in the Shape
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
25	
100	
n	

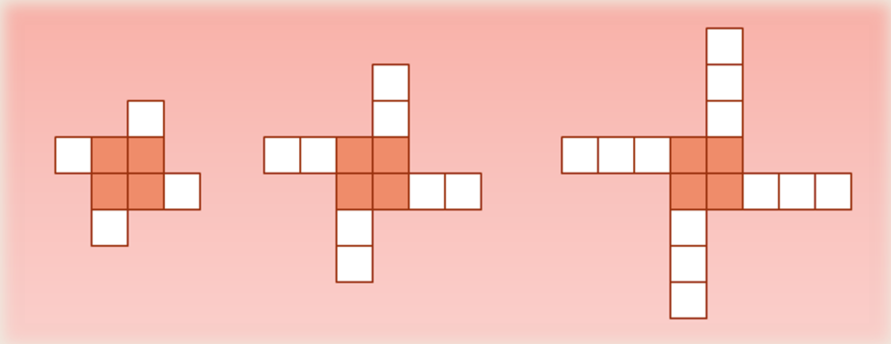


Fill in the table:

Figure #	Number of Squares in the Shape
1	8
2	12
3	16
4	20
5	24
6	28
7	32
8	36
9	40
10	44
25	104
100	404
n	4n + 4

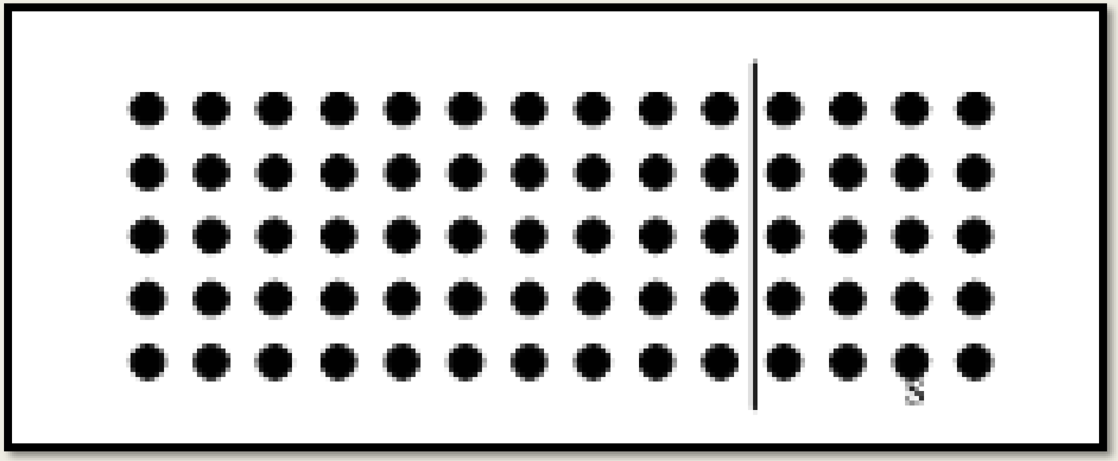


Look at those shades…



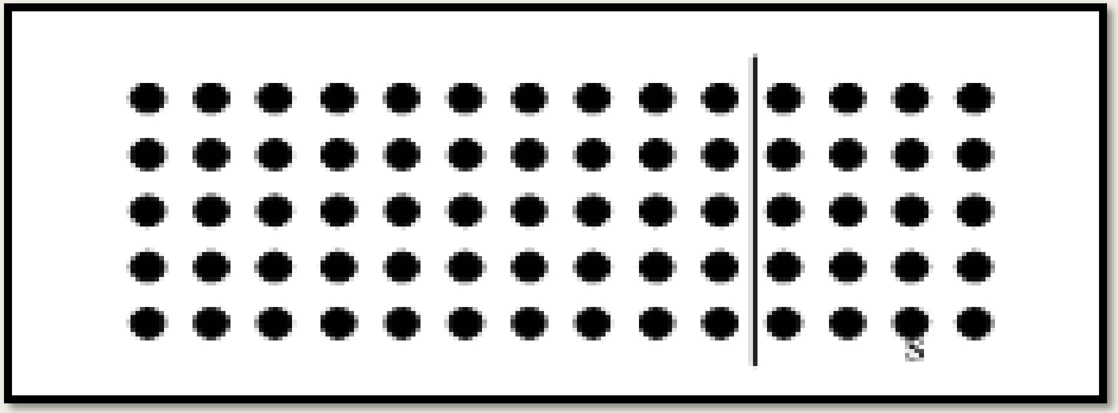
- Using the figure, explain why the rule for the n^{th} shape works. Remember to look at those shades. (Shading is used early in the program, but it is discontinued as students become more adept at seeing patterns in pictures.)

2. How many dots are in this figure?



- Cindy multiplied 5 x 14 to find the number of dots in the picture.
- Charles multiplied 5 x 10 and then added it to 5 x 4 to find the number of dots in the picture.
- Why do both methods give the same answer?

2. How many dots are in this figure?

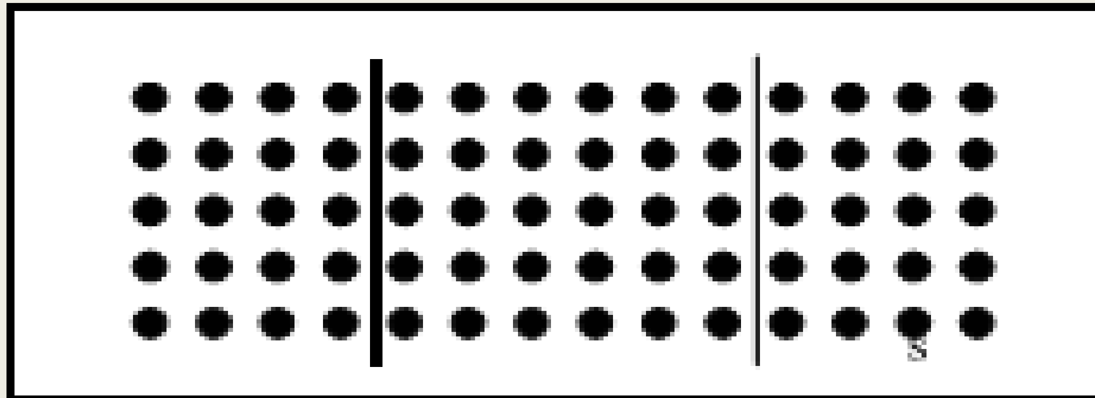


- Cindy multiplied 5 x 14 to find the number of dots in the picture.
- Charles multiplied 5 x 10 and then added it to 5 x 4 to find the number of dots in the picture.

$5 \times 14 = (5 \times 10) + (5 \times 4)$

$70 = 50 + 20$ (Distributive Property)

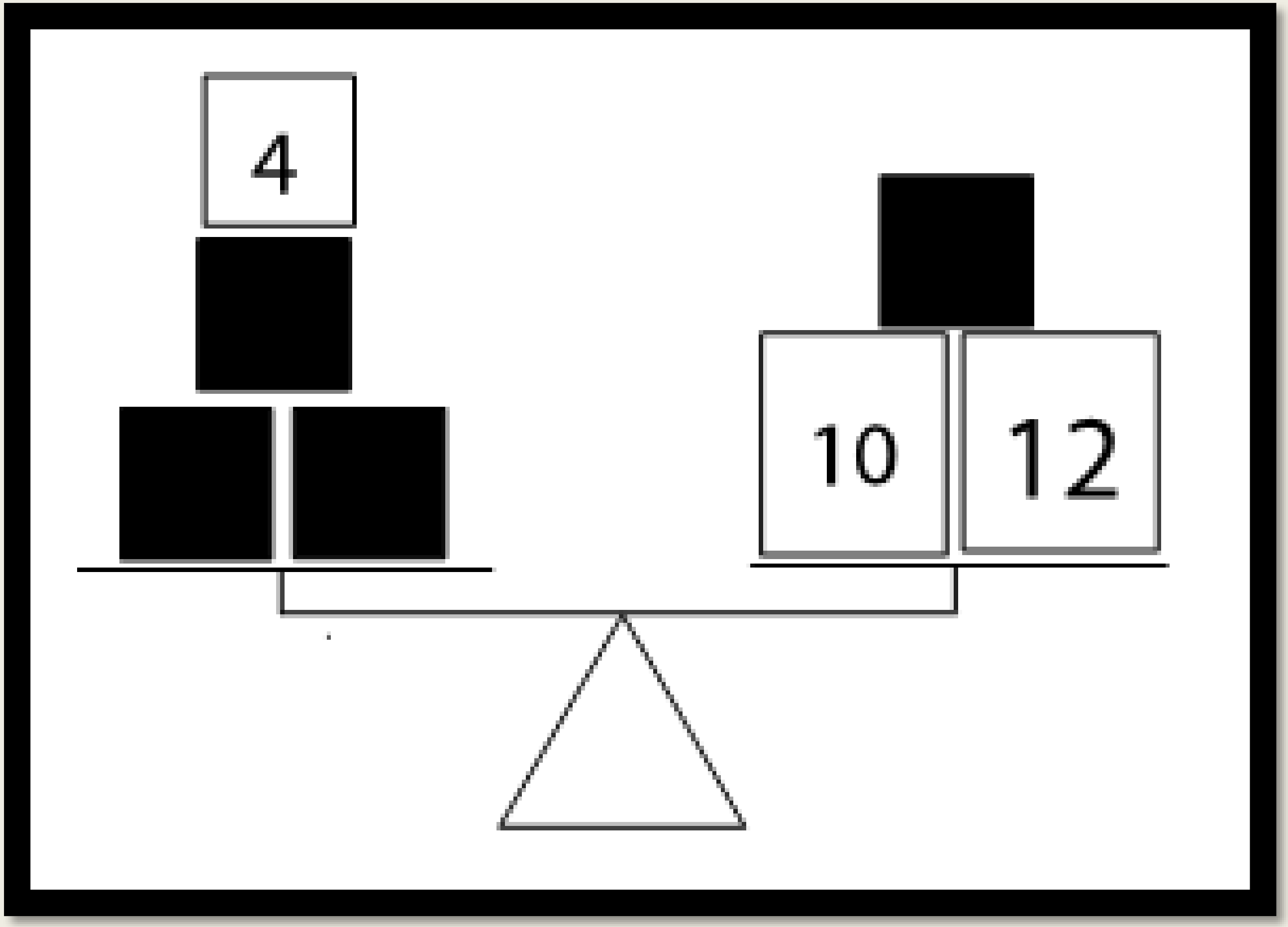
2. How many dots are in this figure?



Another line could be drawn…

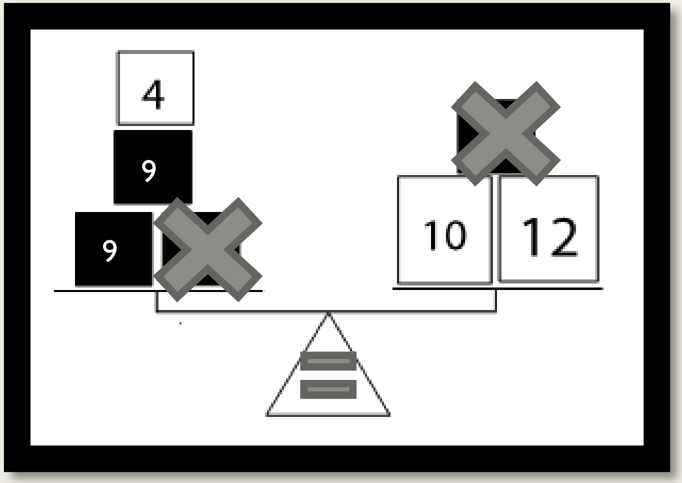
5 x 14 = (5x5) + (5x5) + (5x4)

3. Find the weight of the black square to make the scale balance?

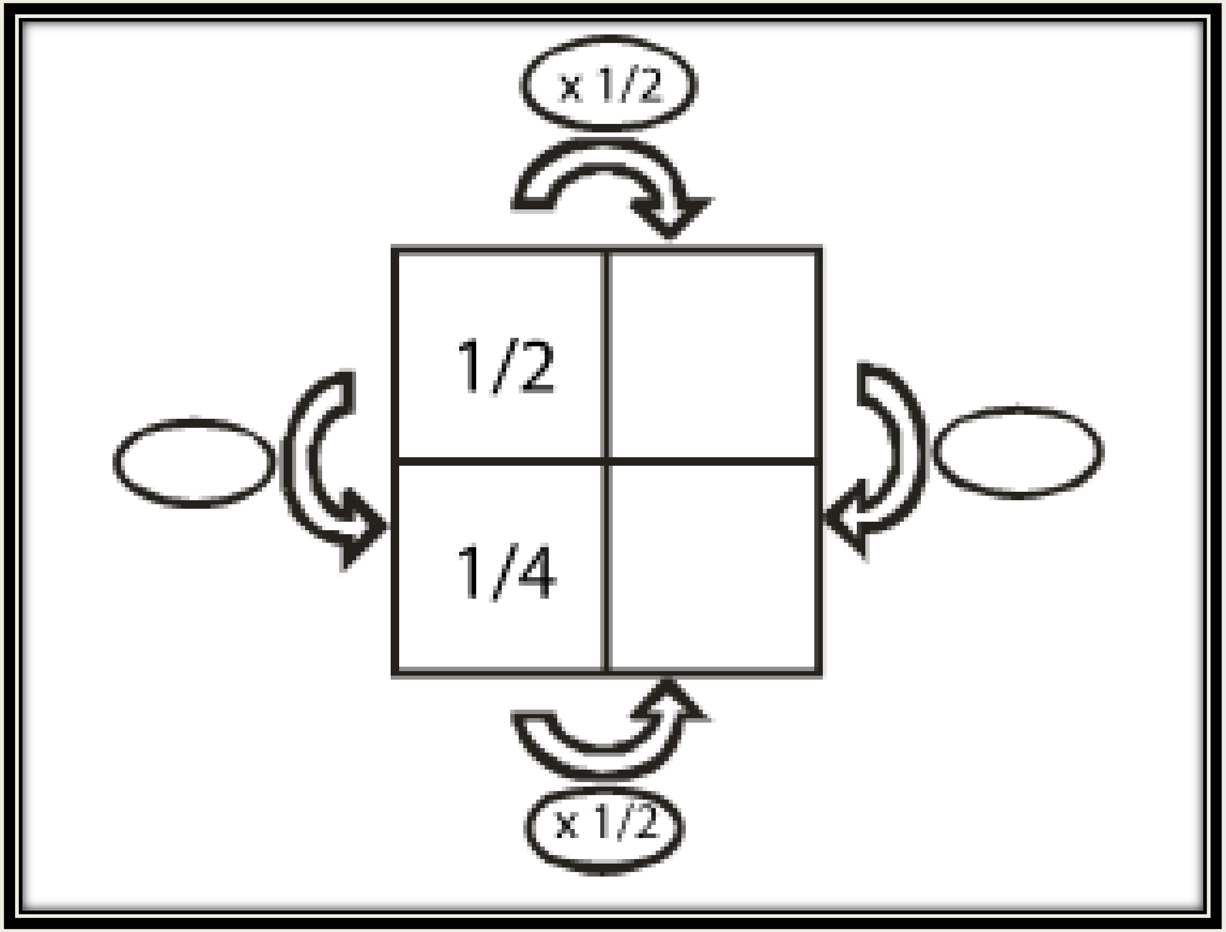


3. Find the weight of the black square to make the scale balance?

- In this problem one black square cancels out similar to an equation like x + x + x + 4 = x + 10 + 12; subtracting x from both sides of the equation eliminates it just like one black square is eliminated without affecting the solution.



4. Find the missing numbers and the operations in the ratio box.



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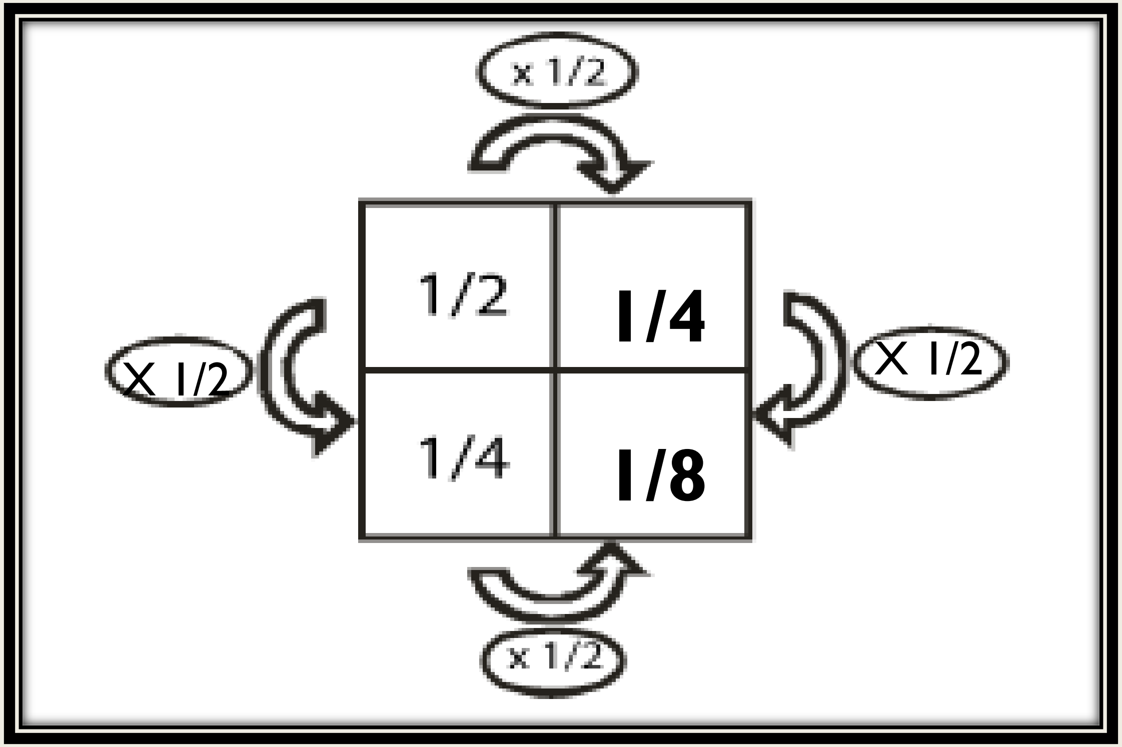
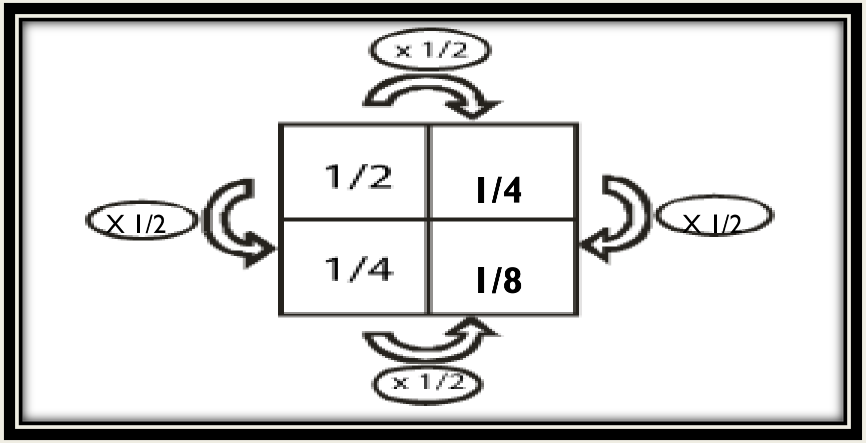


Find the missing numbers and the operations in the ratio box.

- Proportional reasoning and Multiplication of Fractions
- <u>Suggestions:</u> In this problem all the numbers and operations are fractional.



5.

- A loaded trailer truck weighs 26,643 kilograms. When the trailer is empty it weighs 10,547 kilograms. About how much does the load weigh?

- A.	14,000 kilograms
- B.	16,000 kilograms
- C.	18,000 kilograms
- D.	36,000 kilograms

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- A.	14,000 kilograms
- <u>B.	16,000 kilograms</u> (16,096)
- C.	18,000 kilograms
- D.	36,000 kilograms

5.

- Estimation and Number Sense.
- Suggestions: This problem was given on the 2009 NAEP Assessment and 53% of fourth grade students had a correct solution. This problem was also given to eighth graders and 84% had it correct.

A loaded trailer truck weighs 26,643 kilograms. When the trailer is empty it weighs 10,547 kilograms. About how much does the load weigh?

A. 14,000 kilograms

B. 16,000 kilograms (16,096)

C. 18,000 kilograms

D. 36,000 kilograms

6. Fill in the missing numbers in the pattern and give the rule.

___ ___ 8 ___ ___ 26 ___ ___ ___

Rule: ___________

6. Fill in the missing numbers in the pattern and give the rule.

___ ___ 8 ___ ___ 26 ___ ___ ___

-4 2 14 20 32 38 44

Rule: +6_

Number Sense.

Suggestions: The process of taking the difference between two numbers that are three apart and dividing by 3, (26 – 8) ÷ 3 = 6 is a **generalization**. Students have generalized a process, and this is algebraic reasoning.

7. I have exactly $10.00 in nickels, dimes, and quarters. If I have the same number of nickels, dimes, and quarters, how many of each kind of coin do I have?

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- Problem Solving.
- <u>Suggestions:</u> A table or list is a good way to attempt this problem. Is the use of tables in generalization problems like #1 transferring to other problems like this one?

Table: Recursive pattern + $.40

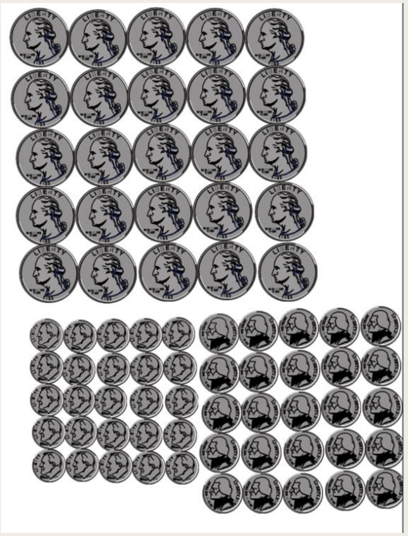
Nickels	Dimes	Quarters	Total
1	1	1	$.40
2	2	2	$.80
3	3	3	$1.20
4	4	4	$1.60
5	5	5	$2.00
6	6	6	$2.40
7	7	7	$2.80
8	8	8	$3.20
9	9	9	$3.60
10	10	10	$4.00
11	11	11	$4.40
12	12	12	$4.80
13	13	13	$5.20
14	14	14	$5.60
15	15	15	$6.00
16	16	16	$6.40
17	17	17	$6.80
18	18	18	$7.20
19	19	19	$7.60
20	20	20	$8.00
21	21	21	$8.40
22	22	22	$8.80
23	23	23	$9.20
24	24	24	$9.60
25	25	25	$10.00

Another strategy…

- One other solution offered was to take the sum total of one quarter, one dime, and one nickel. Since the total is forty cents, this person divided to see the number of “forties” in one thousand, the number of pennies in ten dollars.

7. I have exactly $10.00 in nickels, dimes, and quarters. If I have the same number nickels, dimes, and quarters, how many of each kind of coin do I have?

Another Strategy: One person solved this problem by using the guess and check method. She started with twenty of each coin which left a total of $8.00. She then tried 25 of each, and she ended up with $10.00.

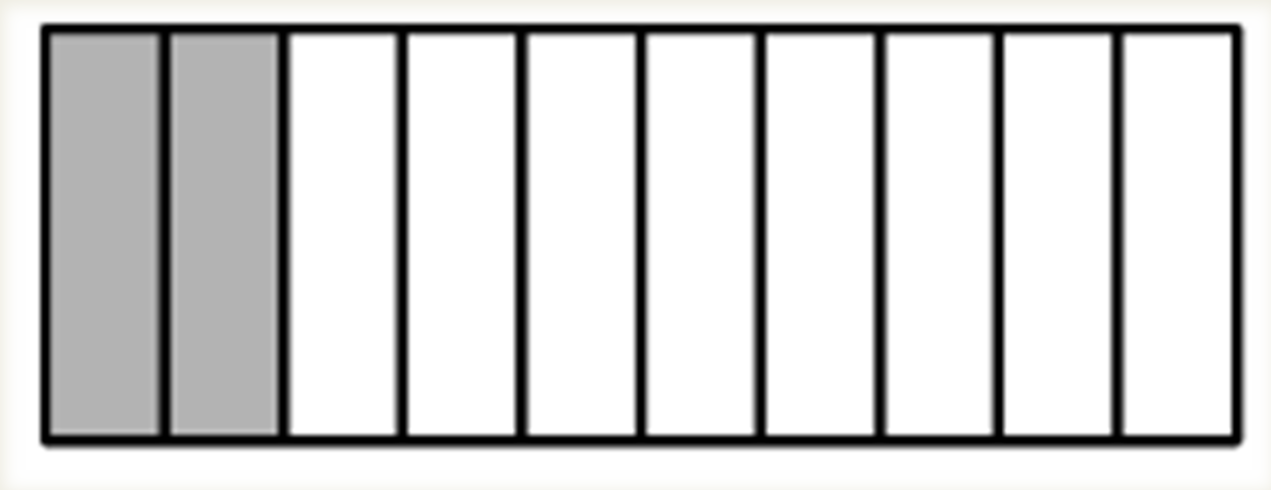


Use Problem Solving Strategies

- Find a pattern.
- Draw a picture.
- Guess and Check.
- Make a Table or List.
- Solve a Similar Problem.
- Use Logical Reasoning.

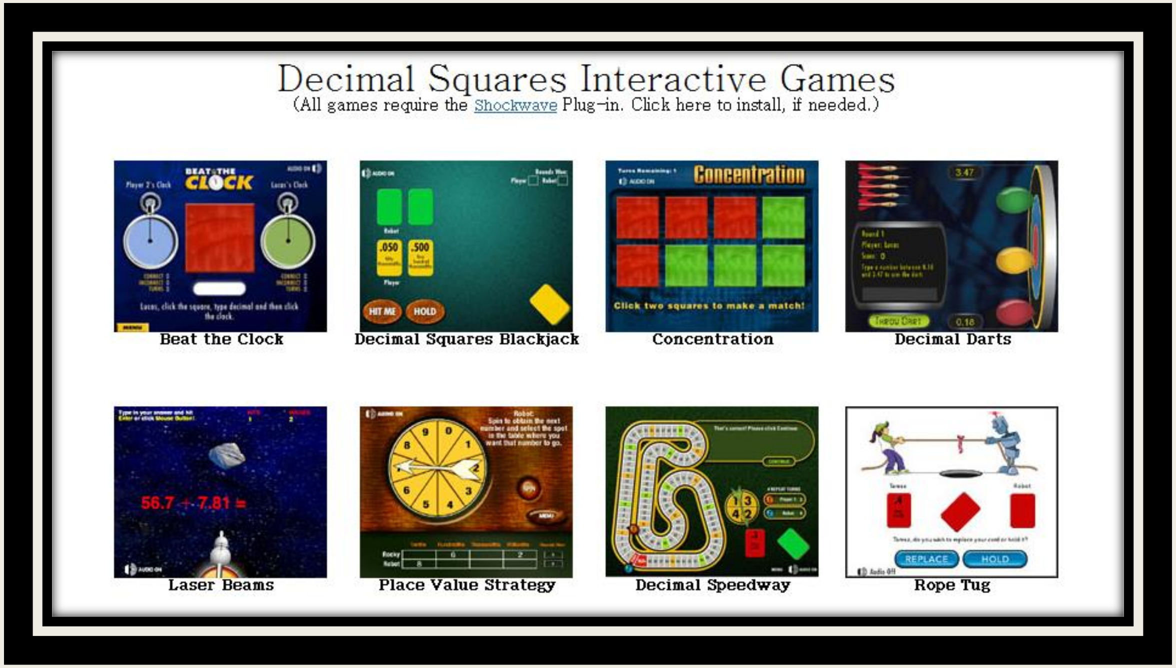
8. Which number represents the shaded part of the figure (TIMSS, 1995)?

- a. 2.8
- b. 0.5
- c. 0.2
- d. 0.02



Purpose:

- Purpose: Decimals.
- Suggestions: Internationally, 33% of third graders and 40% of fourth graders answered correctly on the 1995 TIMSS Assessment.



9. Multiplication

- If you could not multiply the two numbers on each side of the equation to show they are equal, how could you convince someone that the following equation is equal?
- 20 x 16 = 40 x 8

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- If you could not multiply the two numbers on each side of the equation to show they are equal, how could you convince someone that the following equation is equal?
- 20 x 16 = 40 x 8
- Purpose: Relational Understanding.
- Suggestions: When multiplying two numbers, doubling one and halving the other gives the same result. This is a generalization.

The Use of Strings Could Help…

4 X 2 = 8 X 1

6 X 4 = 12 X ___

10 X 6 = 5 X ___

8 X 2 = 4 X ____

10. List all the pairs of whole numbers that you can multiply together to get 72.

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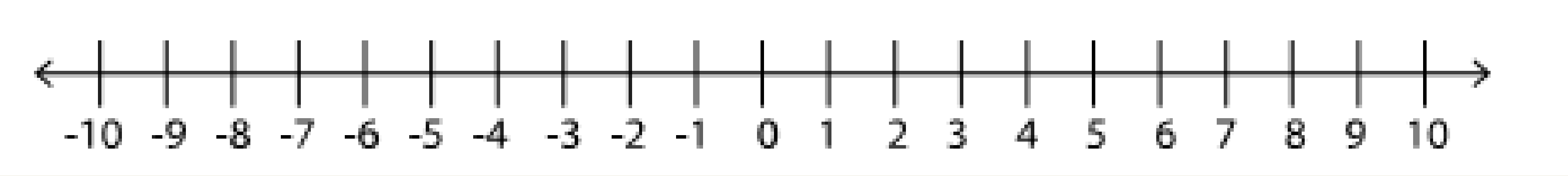
The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

10. List all the pairs of whole numbers that you can multiply together to get 72.

- Purpose: Factors.
- Suggestions: Check to see if students have all the solutions. A good question to ask is: How do you know if you have all the solutions. Students should be confident they have them all, but how do they know?

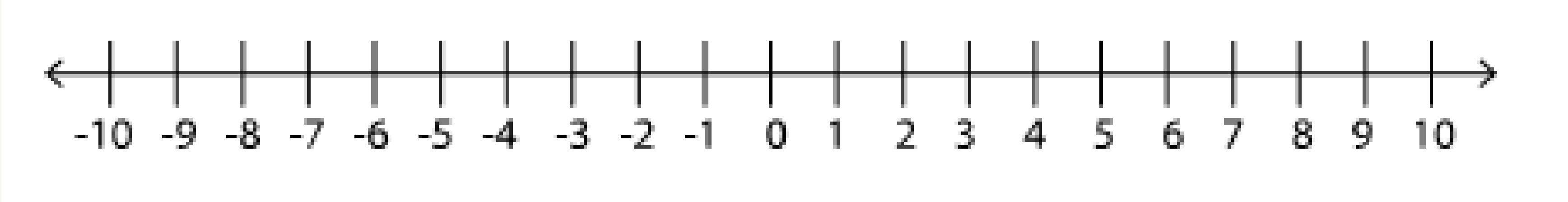
11. Integer Express

- The Integer Express is a train that travels back and forth on the number line railroad. The engineer makes the train move to the right by entering a positive number and to the left by entering a negative number. If he started at 0 and pulled -8 and then + 5, where would he end up?



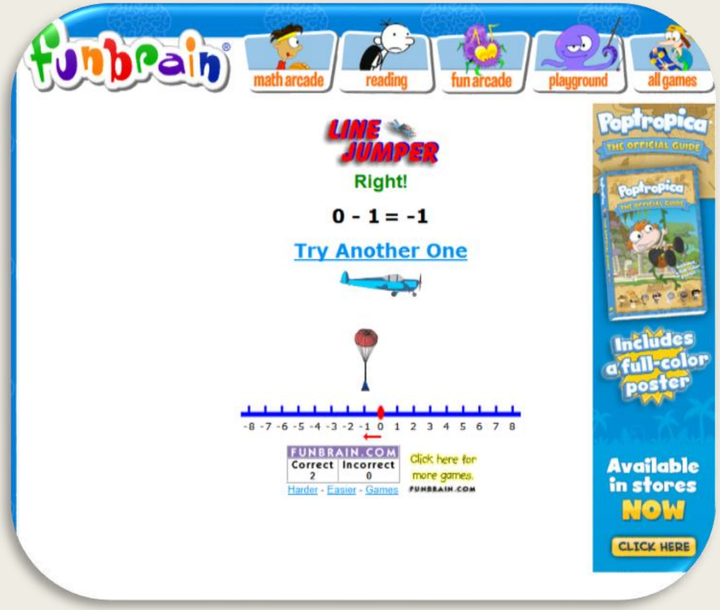
11 b. Integer Express

- The engineer sometimes forgets where he started. He knew he pushed -8 and +5. He ended up at -1. Where did he start?



11. Integer Express

- Purpose: Integers
- Suggestions: We have expanded the number line from -10 to 10 on these problems



Through lots of modeling and practice, some students are able to generalize.

$$4n + 1$$

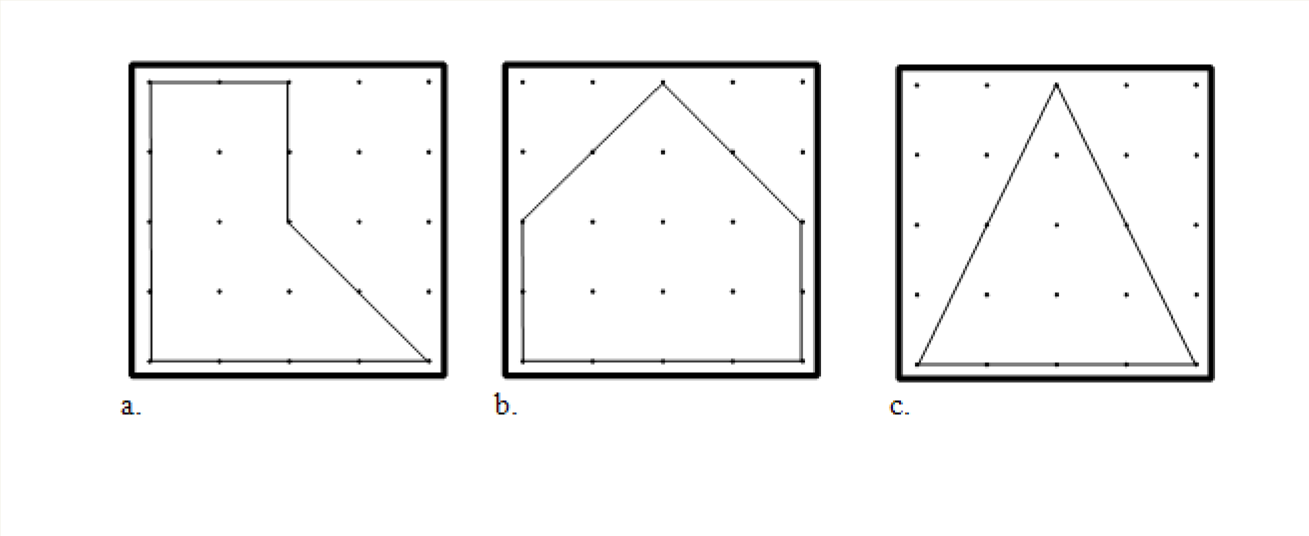
$$(4 \times n) + 1$$

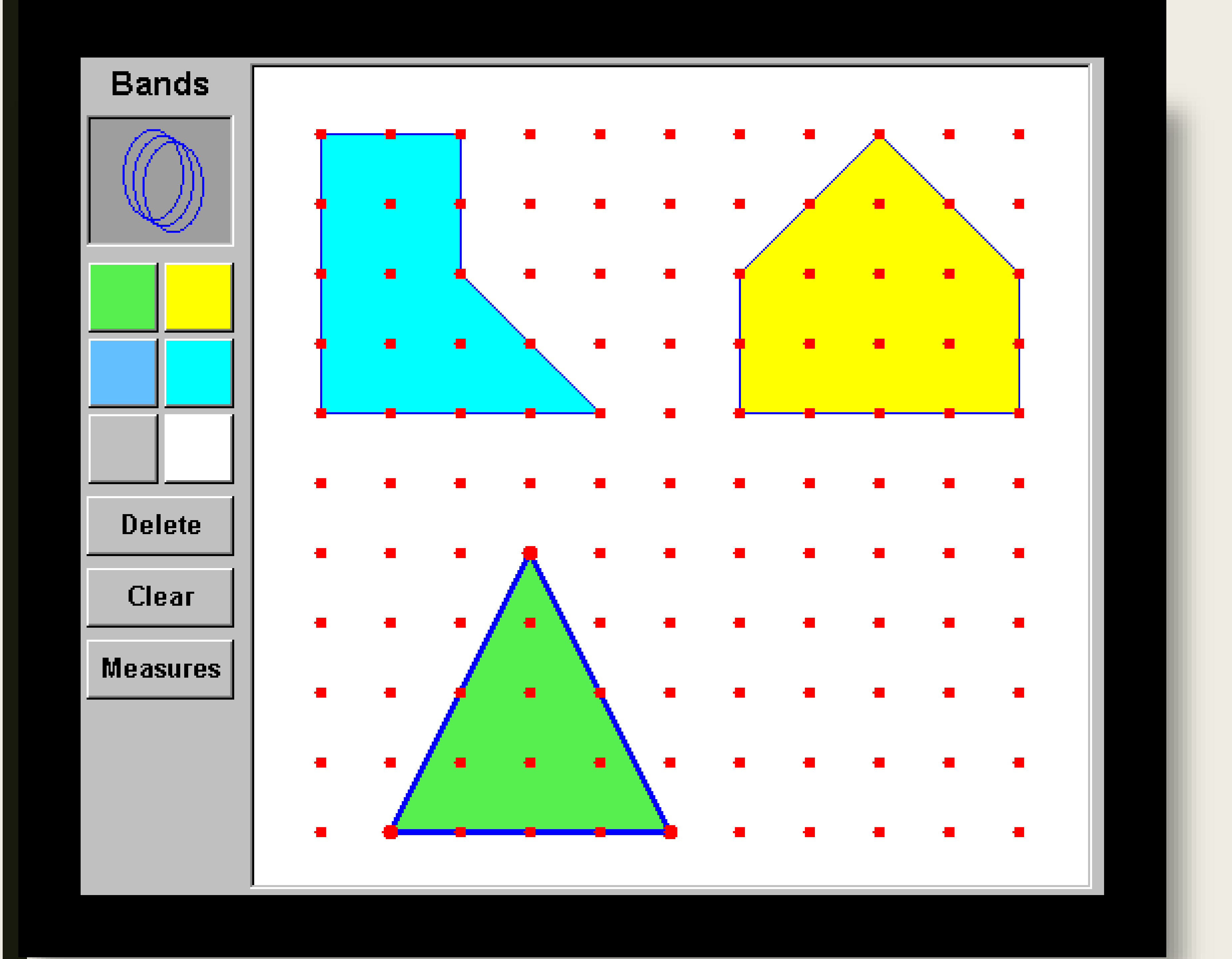
$$(n + n + n + n) + 1$$

Standards for Mathematical Practice

1. Make sense of problems & persevere in solving them
2. Reason abstractly & quantitatively
3. Construct viable arguments & critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for & make use of structure
8. Look for & express regularity in repeated reasoning

From a fourth grade problem set…

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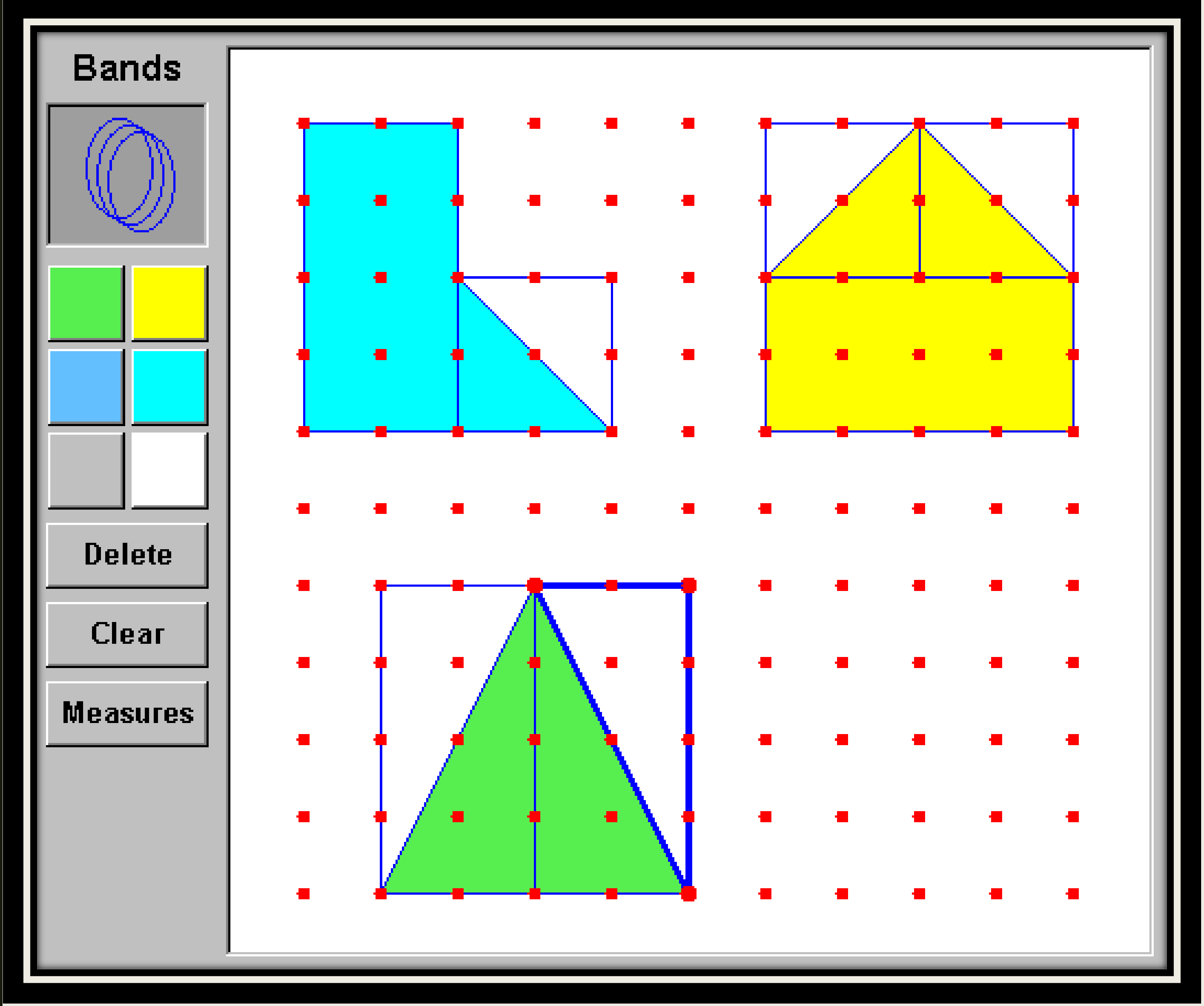
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Common Core Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
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7. Look for and make use of structure.
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